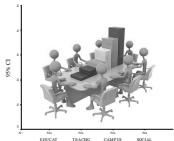


## Analysis of Variance



### Lecture 9

Survey Research & Design in Psychology  
James Neill, 2012

## Readings – Assumed knowledge

Howell (2010):

- Ch3 The Normal Distribution
- Ch4 Sampling Distributions and Hypothesis Testing
- Ch7 Hypothesis Tests Applied to Means
- Ch11 Simple Analysis of Variance
- Ch12 Multiple Comparisons Among Treatment Means
- Ch13 Factorial Analysis of Variance

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## Correlational vs difference statistics

- Correlation and regression techniques reflect the strength of association
- Tests of differences reflect differences in central tendency of variables between groups and measures.

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## Overview



1. Analysing differences
  1. Correlations vs. differences
  2. Which difference test?
  3. Parametric vs. non-parametrics
2. t-tests
  1. One-sample t-test
  2. Independent samples t-test
  3. Paired samples t-test

2

## Readings

Howell (2010):

- Ch14 Repeated-Measures Designs
- Ch16 Analyses of Variance and Covariance as General Linear Models

See also: [Inferential statistics decision-making tree](#)

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## Correlational vs difference statistics

- In MLR we see the world as made of covariation. Everywhere we look, we see relationships.
- In ANOVA we see the world as made of differences. Everywhere we look we see differences.

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## Overview



3. ANOVAs
  1. 1-way ANOVA
  2. 1-way repeated measures ANOVA
  3. Factorial ANOVA
4. Advanced ANOVAs
  1. Mixed design ANOVA (Split-plot ANOVA)
  2. ANCOVA

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## Analysing differences



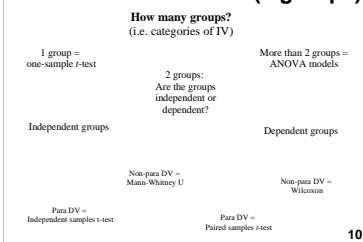
- Correlations vs. differences
- Which difference test?
- Parametric vs. non-parametric

## Correlational vs difference statistics

- LR/MLR e.g.,  
What is the **relationship** between gender and height in humans?
- t-test/ANOVA e.g.,  
What is the **difference** between the heights of human males and females?

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## Which difference test? (2 groups)



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## Parametric vs. non-parametric statistics

- Parametric statistics commonly used for normally distributed interval or ratio dependent variables.
- Non-parametric statistics can be used to analyse DVs that are non-normal or are nominal or ordinal.
- Non-parametric statistics are *less powerful* than parametric tests.

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## *t*-tests



- *t*-tests
- One-sample *t*-tests
- Independent sample *t*-tests
- Paired sample *t*-tests

## Parametric vs. non-parametric statistics

Parametric statistics – *inferential test* that assumes certain characteristics are true of an underlying population, especially the shape of its distribution.

Non-parametric statistics – *inferential test* that makes few or no assumptions about the population from which observations were drawn (distribution-free tests).

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## So, when do I use a non-parametric test?

Consider non-parametric tests when (any of the following):

- Assumptions, like normality, have been violated.
- Small number of observations (*N*).
- DVs have nominal or ordinal levels of measurement.

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## Why a *t*-test or ANOVA?

- A *t*-test or ANOVA is used to determine whether a sample of scores are from the same population as another sample of scores.
- These are inferential tools for examining differences between group means.
- Is the difference between two sample means 'real' or due to chance?

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## Parametric vs. non-parametric statistics

- There is generally at least one non-parametric equivalent test for each type of parametric test.
- Non-parametric tests are generally used when assumptions about the underlying population are questionable (e.g., non-normality).

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Some commonly used parametric & non-parametric tests

Parametric	Non-parametric	Purpose
<i>t</i> test (independent)	Mann-Whitney U; Wilcoxon rank-sum	Compares two independent samples
<i>t</i> test (paired)	Wilcoxon matched pairs signed-rank	Compares two related samples
1-way ANOVA	Kruskal-Wallis	Compares three or more groups
2-way ANOVA	Friedman; $\chi^2$ test of independence	Compares groups classified by two different factors

## *t*-tests

- **One-sample**  
One group of participants, compared with fixed, pre-existing value (e.g., population norms)
- **Independent**  
Compares mean scores on the same variable across different populations (groups)
- **Paired**  
Same participants, with repeated measures

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### Major assumptions

- Normally distributed variables
- Homogeneity of variance

*In general, t-tests and ANOVAs are robust to violation of assumptions, particularly with large cell sizes, but don't be complacent.*

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### One-tail vs. two-tail tests

- Two-tailed test rejects null hypothesis if obtained  $t$ -value is extreme in either direction
- One-tailed test rejects null hypothesis if obtained  $t$ -value is extreme in one direction (you choose – too high or too low)
- One-tailed tests are twice as powerful as two-tailed, but they are only focused on identifying differences in one direction.

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### Independent groups $t$ -test

- Compares mean scores on the same variable across different populations (groups)
- Do Americans vs. Non-Americans differ in their approval of Barack Obama?
- Do males & females differ in the amount of sleep they get?

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### Use of $t$ in $t$ -tests

- $t$  reflects the ratio of between group variance to within group variance
- Is the  $t$  large enough that it is unlikely that the two samples have come from the same population?
- Decision: Is  $t$  larger than the critical value for  $t$ ? (see  $t$  tables – depends on critical  $\alpha$  and  $N$ )



### One sample $t$ -test

- Compare one group (a sample) with a fixed, pre-existing value (e.g., population norms)
- Do uni students sleep less than the recommended amount?  
e.g., Given a sample of  $N = 190$  uni students who sleep  $M = 7.5$  hrs/day ( $SD = 1.5$ ), does this differ significantly from 8 hours hrs/day ( $\alpha = .05$ )?

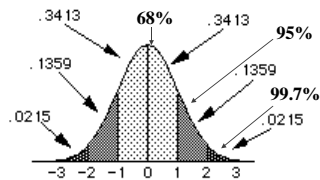
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### Assumptions (Indep. samples $t$ -test)

- **LOM**
  - IV is ordinal / categorical
  - DV is interval / ratio
- **Homogeneity of Variance:** If variances unequal (Levene's test), adjustment made
- **Normality:**  $t$ -tests robust to modest departures from normality, otherwise consider use of Mann-Whitney U test
- **Independence of observations** (one participant's score is not dependent on any other participant's score)

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### Ye good ol' normal distribution



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### One-sample $t$ -test

One-Sample Statistics				
	N	Mean	Std. Deviation	Std. Error Mean
Sleep	190	7.53	1.481	.107

One-Sample Test					
Test Value = 8					
	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Sleep	189	.000	-.468	-.69	-.26

### Do males and females differ in amount of sleep per night?

Group Statistics				
Gender	N	Mean	Std. Deviation	Std. Error Mean
Sleep male	85	7.31	1.640	.178
Sleep female	105	7.71	1.319	.129

Independent Samples Test						
Levene's Test for Equality of Variances						
	F	Sig.	df1	df2	Sig. (2-tailed)	Mean Difference
Sleep	.607	.435	1	189	.435	-.400
Equal variances assumed						
Equal variances not assumed						

### Do males and females differ in memory recall?

**Group Statistics**

gender: R: Gender of respondent	N	Mean	Std. Deviation	Std. Error Mean
1 Male	1169	7.34	2.109	.061
2 Female	1300	8.24	2.255	.062

**Independent Samples Test**

#	Sig.	Levene's Test for Equality of Variances		t-test for Equality of Means		95% Confidence Interval of the Difference
		F	Sig.	t	df	
4.784	.020	1.830	.178	1.900	2287	[-.888, 1.096]

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### Independent samples t-test

- Comparison b/w means of 2 independent sample variables = t-test (e.g., what is the difference in Educational Satisfaction between male and female students?)
- Comparison b/w means of 3+ independent sample variables = 1-way ANOVA (e.g., what is the difference in Educational Satisfaction between students enrolled in four different faculties?)

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### Does an intervention have an effect?

**Paired Samples Statistics**

Pair 1	Pretest	Mean	N	Std. Deviation	Std. Error Mean
1	Posttest	19.80	20	21.867	4.990
		14.40	20	19.198	4.293

**Paired Samples Test**

Pair 1	Pretest - Posttest	Mean	Std. Deviation	t	Sig.	95% Confidence Interval of the Difference	
						Lower	Upper
1		5.40	13.027	1.123	.271	-1.111	11.711

There was no significant difference between pretest and posttest scores ( $t(19) = 1.78, p = .09$ ).

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### Adolescents' Same Sex Relations in Single Sex vs. Co-Ed Schools

**Group Statistics**

Type of School	N	Mean	Std. Deviation	Std. Error Mean
SSR Single Sex	323	4.9955	.7560	4.205E-02
Co-Educational	168	4.9455	.7155	5.523E-02

**Independent Samples Test**

SSR	Equation	Sig.	Levene's Test for Equality of Variances		t-test for Equality of Means		95% Confidence Interval of the Difference
			F	Sig.	t	df	
1	SSR - Co-Educational	.017	.859	.357	.465	1.61E-02	[-.448, .122]

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### Paired samples t-test → 1-way repeated measures ANOVA

- Same participants, with repeated measures
- Data is sampled within subjects. Measures are repeated e.g.,:
  - Time e.g., pre- vs. post-intervention
  - Measures e.g., approval ratings of brand X and brand Y

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### Adolescents' Opposite Sex vs. Same Sex Relations

**Paired Samples Statistics**

Pair 1	SSR	Mean	N	Std. Deviation	Std. Error Mean
1	OSR	4.9787	951	.7560	2.451E-02
		4.2498	951	1.1086	3.595E-02

**Paired Samples Test**

Pair 1	SSR - OSR	Mean	Std. Deviation	t	Sig.	95% Confidence Interval of the Difference	
						Lower	Upper
1		.7289	1.3524	1.778	.078	-.292	1.742

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### Adolescents' Opposite Sex Relations in Single Sex vs. Co-Ed Schools

**Group Statistics**

Type of School	N	Mean	Std. Deviation	Std. Error Mean
OSR Single Sex	327	4.5327	1.0627	5.877E-02
Co-Educational	172	3.9827	1.1543	8.801E-02

**Independent Samples Test**

SSR	Equation	Sig.	Levene's Test for Equality of Variances		t-test for Equality of Means		95% Confidence Interval of the Difference
			F	Sig.	t	df	
1	OSR - Co-Educational	.017	.859	.357	.465	1.61E-02	[-.448, .122]

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### Assumptions (Paired samples t-test)

- LOM:
  - IV: Two measures from same participants (w/in subjects)
    - a variable measured on two occasions or
    - two different variables measured on the same occasion
  - DV: Continuous (Interval or ratio)
- Normal distribution of difference scores (robust to violation with larger samples)
- Independence of observations (one participant's score is not dependent on another's score) 33

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### Paired samples t-test → 1-way repeated measures ANOVA

- Comparison b/w means of 2 within subject variables = t-test
- Comparison b/w means of 3+ within subject variables = 1-way repeated measures ANOVA (e.g., what is the difference in Campus, Social, and Education Satisfaction?)

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### Summary (Analysing Differences)

- Non-parametric and parametric tests can be used for examining differences between the central tendency of two or more variables
- Learn when to use each of the parametric tests of differences, from one-sample *t*-test through to ANCOVA (e.g. use a decision chart).

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### Introduction to ANOVA (Analysis of Variance)

- Extension of a *t*-test to assess differences in the central tendency (*M*) of several groups or variables.
- DV variance is partitioned into between-group and within-group variance
- Levels of measurement:
  - Single DV: metric,
  - 1 or more IVs: categorical

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### Example ANOVA research questions

Are there differences in the degree of religious commitment between countries (UK, USA, and Australia) and gender (male and female)?

1. 1-way ANOVA
2. 1-way repeated measures ANOVA
3. Factorial ANOVA
4. Mixed ANOVA
5. ANCOVA

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### *t*-tests

- Difference between a set value and a variable → one-sample *t*-test
- Difference between two independent groups → independent samples *t*-test = BETWEEN-SUBJECTS
- Difference between two related measures (e.g., repeated over time or two related measures at one time) → paired samples *t*-test = WITHIN-SUBJECTS

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### Example ANOVA research question

Are there differences in the degree of religious commitment between countries (UK, USA, and Australia)?

1. 1-way ANOVA
2. 1-way repeated measures ANOVA
3. Factorial ANOVA
4. Mixed ANOVA
5. ANCOVA

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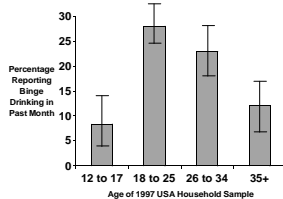
### Example ANOVA research questions

Does couples' relationship satisfaction differ between males and females and before and after having children?

1. 1-way ANOVA
2. 1-way repeated measures ANOVA
3. Factorial ANOVA
4. Mixed ANOVA
5. ANCOVA

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### Are the differences in a sample generalisable to a population?



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### Example ANOVA research question

Do university students have different levels of satisfaction for educational, social, and campus-related domains?

1. 1-way ANOVA
2. 1-way repeated measures ANOVA
3. Factorial ANOVA
4. Mixed ANOVA
5. ANCOVA

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### Example ANOVA research questions

Are there differences in university student satisfaction between males and females (gender) after controlling for level of academic performance?

1. 1-way ANOVA
2. 1-way repeated measures ANOVA
3. Factorial ANOVA
4. Mixed ANOVA
5. ANCOVA

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### Introduction to ANOVA

- Inferential: What is the likelihood that the observed differences could have been due to chance?
- Follow-up tests: Which of the *M*s differ?
- Effect size: How large are the observed differences?

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### Follow-up tests

- ANOVA *F*-tests are a "gateway". If *F* is significant, then...



- interpret (main and interaction) effects and
- consider whether to conduct follow-up tests
  - planned comparisons
  - post-hoc contrasts.

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### One-way ANOVA: Are there differences in satisfaction levels between students who get different grades?

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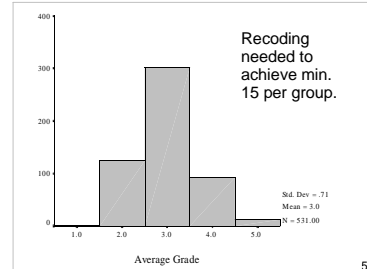
### F test

- ANOVA partitions the sums of squares (variance from the mean) into:
  - Explained variance (between groups)
  - Unexplained variance (within groups) – or error variance
- *F* = ratio between explained & unexplained variance
- *p* = probability that the observed mean differences between groups could be attributable to chance

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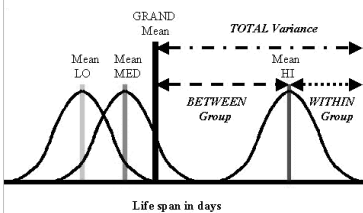
### One-way ANOVA

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*F* is the ratio of between-group : within-group variance



### Assumptions – One-way ANOVA

Dependent variable (DV) must be:

- LOM: Interval or ratio
- Normality: Normally distributed for all IV groups (robust to violations of this assumption if *N*s are large and approximately equal e.g., >15 cases per group)
- Variance: Equal variance across for all IV groups (homogeneity of variance)
- Independence: Participants' data should be independent of others' data

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These groups could be combined.

		AVGRADE Average Grade			
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1 Fail	1	.2	.2	.2
	2 Pass	125	23.5	23.5	23.7
	3 Credit	2	.3	.4	24.1
	4 Distinction	289	48.9	56.3	80.4
	5 High Distinction	4	.7	.8	81.2
	Total	88	14.4	16.6	97.7
Missing	System	12	2.0	2.3	100.0
	Total	531	86.9	100.0	
	Total	80	13.1		
	Total	611	100.0		

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The recoded data has more similar group sizes and is appropriate for ANOVA.

AVGRADX Average Grade (R)

	Frequency	Percent	Valid Percent	Cumulative Percent
Valid	128	20.3	24.1	24.1
2.00 Fall/Pass	299	48.9	56.3	80.4
3.00 Credit	104	17.0	19.6	100.0
4.00 D/HD	531	86.9	100.0	
Total	80	13.1		
Missing System	611	100.0		
Total				

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4.306 <sup>a</sup>	2	2.153	7.854	.000
Intercept	5981.431	1	5981.431	21820.681	.000
AVGRADX	4.306	2	2.153	7.854	.000
Error	144.734	528	.274		
Total	7485.554	531			
Corrected Total	148.040	530			

a. R Squared = .029 (Adjusted R Squared = .028)

Follow-up tests should then be conducted because the effect of Grade is statistically significant ( $p < .05$ ).

The SDs vary between groups (the third group has almost double the SD of the younger group). Levene's test is significant (variances are not homogenous).

	N	Mean	Std. Deviation
2.00 20-25	20	38.1000	5.25258
1.00 40-45	20	38.5500	5.29623
2.00 60-65	20	33.4000	9.29289
Total	60	37.0167	7.24050

Test of Homogeneity of Variances

control	Levene Statistic	df1	df2	Sig.
	13.186	2	57	.000

SDs are similar (homogeneity of variance). Ms suggest that higher grade groups are more satisfied.

Descriptive Statistics

AVGRADX	Mean	Std. Deviation	N
2.00 Fall/Pass	3.57	.53	128
3.00 Credit	3.74	.51	299
4.00 D/HD	3.84	.55	104
Total	3.72	.53	531

### One-way ANOVA: Does locus of control differ between three age groups?

- |                   |                     |
|-------------------|---------------------|
| Age               | Locus of Control    |
| • 20-25 year-olds | • Lower = internal  |
| • 40-45 year olds | • Higher = external |
| • 60-65 year-olds |                     |

ANOVA

control	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	395.433	2	197.717	4.178	.020
Within Groups	2897.550	57	47.325		
Total	3292.983	59			

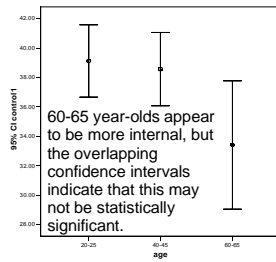
There is a significant effect for Age ( $F(2, 59) = 4.18, p = .02$ ). In other words, the three age groups are unlikely to be drawn from a population with the same central tendency for LOC.

Levene's test indicates homogeneity of variance.

Levene's Test of Equality of Error Variances

Dependent Variable: EDUCAT	F	df1	df2	Sig.
	.748	2	528	.474

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.  
a. Design: Intercept+AVGRADX



Which age groups differ in their mean locus of control scores? (Post hoc tests).

Multiple Comparisons

Dependent Variable: control	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
(I) age	(J) age			Lower Bound	Upper Bound
00 20-25	1.00 40-45	.65200	.217544	-.48950	2.13550
2.00 60-65	5.70000*	2.17544	.039	1.09550	10.30450
1.00 40-45	00 20-25	-.55000	2.17544	-.36800	-1.13200
2.00 60-65	5.15000	2.17544	.055	-.09500	10.39500
2.00 60-65	00 20-25	-5.70000*	2.17544	-.03900	-10.35900
1.00 40-45	-5.15000	2.17544	.055	-10.35900	-.03900

\*. The mean difference is significant at the .05 level.

Conclude: Gps 0 differs from 2; 1 differs from 2

### Follow-up (pairwise) tests

- Post hoc: Compares every possible combination
- Planned: Compares specific combinations  
(Do one or the other; not both)

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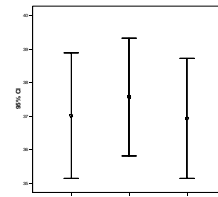
### Assumptions - Repeated measures ANOVA

Repeated measures designs have the additional assumption of Sphericity:

- Variance of the population difference scores for any two conditions should be the same as the variance of the population difference scores for any other two conditions
- Test using Mauchly's test of sphericity (If Mauchly's W Statistic is  $p < .05$  then assumption of sphericity is violated.)

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### Mean LOC scores (with 95% C.I.s) across 3 measurement occasions



Not much variation between means.

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### Post hoc

- Control for Type I error rate
- Scheffe, Bonferroni, Tukey's HSD, or Student-Newman-Keuls
- Keeps experiment-wise error rate to a fixed limit

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### Assumptions - Repeated measures ANOVA

- Sphericity is commonly violated, however the multivariate test (provided by default in PASW output) does not require the assumption of sphericity and may be used as an alternative.
- The obtained F ratio must then be evaluated against new degrees of freedom calculated from the Greenhouse-Geisser, or Huynh-Feld, Epsilon values.

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### Descriptive statistics

Descriptive Statistics

	Mean	Std. Deviation	N
control1	37.0167	7.24040	60
control2	37.5667	6.80071	60
control3	36.9333	6.92788	60

Not much variation between means.

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### Planned

- Need hypothesis before you start
- Specify contrast coefficients to weight the comparisons (e.g., 1<sup>st</sup> two vs. last one)
- Tests each contrast at critical  $\alpha$

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### Example: Repeated measures ANOVA

Does LOC vary over time?

- Baseline
- 6 months
- 12 months

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### Mauchly's test of sphericity

Measure: MEASURE_1		Mauchly's Test of Sphericity <sup>a</sup>					
Within-Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Greenhouse-Geisser Epsilon <sup>b</sup>	Huynh-Feldt Epsilon <sup>b</sup>	Lower-bound Epsilon <sup>b</sup>
Intercept	.000	3.927	3	.045	.750	.911	.500

a. Tests the null hypothesis that the error covariance matrix of the model is proportional to an identity matrix.  
b. May be used to adjust the degrees of freedom for the unadjusted tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

Design: Intercept  
Within-Subjects Design Factor1

Mauchly's test is not significant, therefore sphericity can be assumed.

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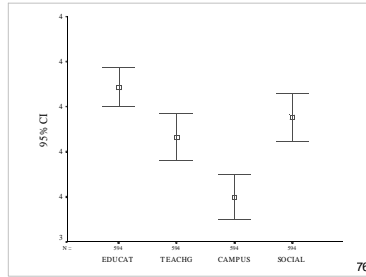


### Tests of within-subject effects

Tests of Within-Subjects Effects						
Measure: MEASURE_1						
Source	Sphericity Assumed	Type III Sum of Squares	df	Mean Square	F	Sig.
Error	Sphericity Assumed	14,211	4	3,553	2,719	.067
	Greenhouse-Geisser	14,211	1,883	7,548	2,426	.067
	Huynh-Feldt	14,211	1,943	7,315	2,739	.067
Error(Interact)	Sphericity Assumed	300,456	118	2,546		
	Greenhouse-Geisser	300,456	111,067	2,705		
	Huynh-Feldt	300,456	114,628	2,621		
Error	Sphericity Assumed	300,456	59,000	5,092		
	Greenhouse-Geisser	300,456	59,000	5,092		
	Huynh-Feldt	300,456	59,000	5,092		

Conclude: Observed differences in means could have occurred by chance ( $F(2, 118) = 2.79, p = .06$ ) if critical alpha = .05

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### Factorial ANOVA

- Levels of measurement
  - 2 or more between-subjects categorical/ordinal IVs
  - 1 interval/ratio DV
- e.g., Does Educational Satisfaction vary according to Age (2) and Gender (2)? 2 x 2 Factorial ANOVA

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### 1-way repeated measures ANOVA

Do satisfaction levels vary between Education, Teaching, Social and Campus aspects of university life?

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### Tests of within-subject effects

Tests of Within-Subjects Effects						
Measure: MEASURE_1						
Source	Sphericity Assumed	Type III Sum of Squares	df	Mean Square	F	Sig.
SATSFP	Sphericity Assumed	18,920	3	6,307	28,389	.000
	Greenhouse-Geisser	18,920	2,520	7,507	28,389	.000
	Huynh-Feldt	18,920	2,532	7,472	28,389	.000
Error(SATSFP)	Sphericity Assumed	18,920	1,000	18,920	28,389	.000
	Greenhouse-Geisser	365,252	1,779	205,222		
	Huynh-Feldt	365,252	1,844,972	264		
Error	Sphericity Assumed	365,252	1501,474	263		
	Greenhouse-Geisser	365,252	1501,474	263		
	Huynh-Feldt	365,252	1501,474	263		

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### Factorial ANOVA

- Factorial designs test Main Effects and Interactions. For a 2-way design:
  - Main effect of IV1
  - Main effect of IV2
  - Interaction between IV1 and IV2
- If
  - significant effects are found and
  - there are more than 2 levels of an IV are involved
 then follow-up tests are required.

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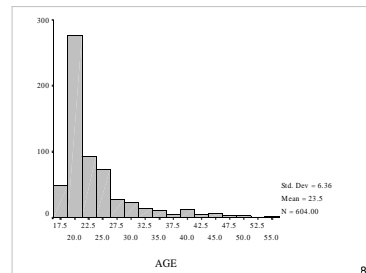
### Descriptive Statistics

	Mean	Std. Deviation
EDUCAT	3.74	.54
TEACHG	3.63	.65
CAMPUS	3.50	.61
SOCIAL	3.67	.65

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Factorial ANOVA (2-way): Are there differences in satisfaction levels between gender and age?

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AGE				
Valid	Frequency	Percent	Valid Percent	Cumulative Percent
17	3	.5	.5	.5
18	46	7.5	7.6	8.1
19	89	11.3	11.4	19.5
20	114	18.7	18.9	38.4
21	94	15.4	15.6	54.0
22	64	10.5	10.6	64.6
23	29	4.7	4.8	69.4
24	29	4.7	4.8	74.2
25	30	4.9	5.0	79.1
26	15	2.5	2.5	81.6
27	16	2.6	2.6	84.3
28	12	2.0	2.0	86.3
29	7	1.1	1.2	87.4
30	7	1.1	1.2	88.6
31	8	1.3	1.3	89.9
32	7	1.1	1.2	91.1
33	7	1.1	1.2	92.2
34	3	.5	.5	92.7

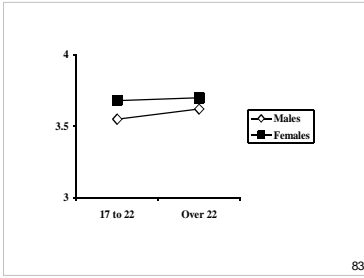
**Descriptive Statistics**

Dependent Variable: TEACHG

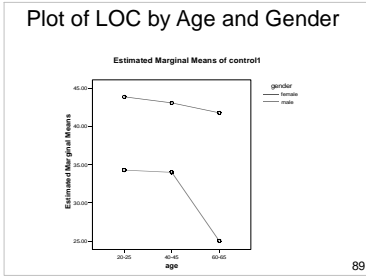
AGE X Age	GENDER	Mean	Std. Deviation	N
1.00 17 to 22	0 Male	3.5494	.6722	156
	1 Female	3.6795	.5895	233
	Total	3.6273	.6264	389
2.00 over 22	0 Male	3.6173	.7389	107
	1 Female	3.7038	.6367	104
	Total	3.6600	.6901	211
Total	0 Male	3.5770	.6995	263
	1 Female	3.6870	.6036	337
	Total	3.6388	.6491	600

**Example: Factorial ANOVA**

- In this example, there are:
  - Two main effects (Age and Gender)
  - One interaction effect (Age x Gender)
- IVs
  - Age recoded into 2 groups (2)
  - Gender dichotomous (2)
- DV
  - Locus of Control (LOC)



**Factorial ANOVA (2-way):**  
Are there differences in LOC between gender and age?



**Tests of Between-Subjects Effects**

Dependent Variable: TEACHG

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4.121 <sup>a</sup>	3	.708	1.886	.163
Intercept	7136.890	1	7136.890	16996.047	.000
AGE X	.287	1	.287	.883	.409
GENDER	1.584	1	1.584	3.771	.053
AGE X / GENDER	6.416E-02	1	6.416E-02	.153	.696
Error	250.269	596			
Total	8196.937	600			
Corrected Total	252.393	599			

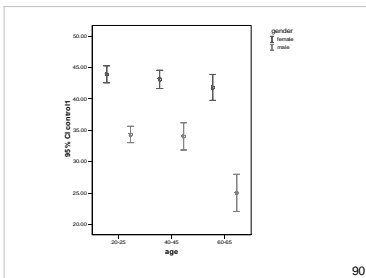
<sup>a</sup>. R Squared = .006 (Adjusted R Squared = .003)

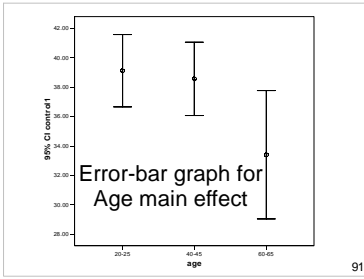
**Example: Factorial ANOVA**

**Main effect 1:**  
- Do LOC scores differ by Age?

**Main effect 2:**  
- Do LOC scores differ by Gender?

**Interaction:**  
- Is the relationship between Age and LOC moderated by Gender? (Does any relationship between Age and LOC vary as a function of Gender?)





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### Descriptives for Gender main effect

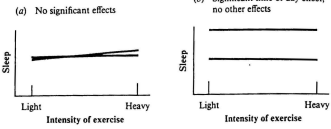
Descriptives

control1		N	Mean	Std. Deviation
.00	female	30	42.9333	2.40593
1.00	male	30	31.1000	5.33272
Total		60	37.0167	7.24040

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### Interactions

- IV1 = Separate lines for morning and evening exercise.
- IV2 = Light and heavy exercise
- DV = Av. hours of sleep per night



### Descriptives for Age main effect

Descriptives

control1		N	Mean	Std. Deviation
.00	20-25	20	39.1000	5.25056
1.00	40-45	20	38.5500	5.29623
2.00	60-65	20	33.4000	9.29289
Total		60	37.0167	7.24040

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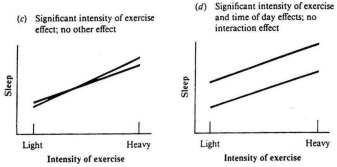
### Descriptives for LOC by Age and Gender

Dependent Variable: control1

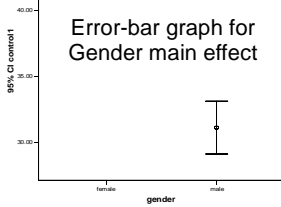
age	gender	Mean	Std. Deviation	N
.00 20-25	.00 female	43.9000	1.91195	10
	1.00 male	34.3000	1.82978	10
	Total	39.1000	5.25056	20
1.00 40-45	.00 female	43.1000	2.02485	10
	1.00 male	34.0000	3.01846	10
	Total	38.5500	5.29623	20
2.00 60-65	.00 female	41.8000	2.89828	10
	1.00 male	25.0000	4.13656	10
	Total	33.4000	9.29289	20
Total	.00 female	42.9333	2.40593	30
	1.00 male	31.1000	5.33272	30
	Total	37.0167	7.24040	60

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### Interactions



### Error-bar graph for Gender main effect



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### Tests of between-subjects effects

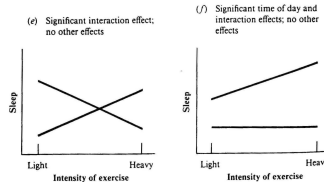
Dependent Variable: control1

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2681.463 <sup>a</sup>	5	536.297	70.377	.000
Intercept	82214.017	1	82214.017	10783.717	.000
age	385.433	2	192.717	25.946	.000
gender	2100.417	1	2100.417	275.632	.000
age * gender	185.633	2	92.817	12.180	.000
Error	411.500	54	7.620		
Total	85337.000	60			
Corrected Total	3092.963	59			

a. R Squared = .867 (Adjusted R Squared = .856)

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### Interactions



### Mixed design ANOVA (SPANOVA)

- Independent groups (e.g., males and females) with **repeated measures** on each group (e.g., word recall under three different character spacing conditions (Narrow, Medium, Wide)).
- Since such experiments have mixtures of between-subject and within-subject factors they are said to be of **mixed design**
- Since output is split into two tables of effects, this is also said to be **split-plot ANOVA (SPANOVA)**

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### Mixed design ANOVA: Design

- If A is Gender and B is Spacing the Reading experiment is of the type A X (B) or 2 x (3)
- Brackets signify a mixed design with repeated measures on Factor B

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### Mixed design ANOVA: Example

Do satisfaction levels vary between gender for education and teaching?

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### Mixed design ANOVA (SPANOVA)

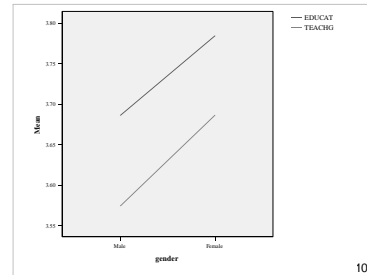
- IV1 is between-subjects (e.g., Gender)
- IV2 is within-subjects (e.g., Social Satisfaction and Campus Satisfaction)
- Of interest are:
  - **Main effect** of IV1
  - **Main effect** of IV2
  - **Interaction** b/w IV1 and IV2
- If significant effects are found and more than 2 levels of an IV are involved, then specific contrasts are required, either:
  - A priori (planned) contrasts
  - Post-hoc contrasts

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### Mixed design ANOVA: Assumptions

- Normality
- Homogeneity of variance
- Sphericity
- Homogeneity of inter-correlations

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### Mixed design ANOVA (SPANOVA)

- An experiment has two IVs:
- Between-subjects = Gender (Male or Female) - varies between subjects
  - Within-subjects = Spacing (Narrow, Medium, Wide)
  - Gender - varies within subjects

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### Homogeneity of intercorrelations

- The pattern of inter-correlations among the various levels of repeated measure factor(s) should be consistent from level to level of the Between-subject Factor(s)
- The assumption is tested using Box's *M* statistic
- Homogeneity is present when the *M* statistic is NOT significant at  $p > .001$ .

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### Tests of within-subjects contrasts

Measure: MEASURE_1						
Source	SATISF	Type III Sum of Squares	df	Mean Square	F	Sig.
SATISF	Linear	3.262	1	3.262	22.919	.000
SATISF * GENDER	Linear	1.426(SD)	1	1.426(SD)	.101	.731
Error(SATISF)	Linear	68.901	600	114.835		

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## Tests of between-subjects effects

Tests of Between-Subjects Effects

Measure: MEASURE\_1  
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	16083.714	1	16083.714	29046.815	.000
GENDER	3.288	1	3.288	5.894	.015
Error	332.436	600	.554		

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## ANCOVA (Analysis of Covariance)

- A covariate IV is added to an ANOVA (can be dichotomous or metric)
- Effect of the covariate on the DV is removed (or partialled out) (akin to Hierarchical MLR)
- Of interest are:
  - Main effects of IVs and interaction terms
  - Contribution of CV (akin to Step 1 in HMLR)
- e.g., GPA is used as a CV, when analysing whether there is a difference in Educational Satisfaction between Males and Females.

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## Assumptions of ANCOVA

- As per ANOVA
- Normality
- Homogeneity of Variance (use Levene's test)

Levene's Test of Equality of Error Variances<sup>a</sup>

Dependent Variable: achievement	F	df1	df2	Sig.
	.020	1	78	.882

Tests the null hypothesis that the error-variance of the dependent variable is equal across groups.  
a. Design: Intercept+MOTIV+TEACH

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### 1. gender

Measure: MEASURE\_1

gender	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
0 Male	3.630	.032	3.566	3.693
1 Female	3.735	.029	3.679	3.791

### 2. satisf

Measure: MEASURE\_1

satisf	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	3.735	.022	3.692	3.778
2	3.630	.027	3.578	3.682

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## Why use ANCOVA?

- **Reduces** variance associated with covariate (CV) from the **DV error** (unexplained variance) term
- Increases power of *F*-test
- May not be able to achieve experimental control over a variable (e.g., randomisation), but can measure it and statistically control for its effect.

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## Assumptions of ANCOVA

- Independence of observations
- Independence of IV and CV
- Multicollinearity - if more than one CV, they should not be highly correlated - eliminate highly correlated CVs
- Reliability of CVs - not measured with error - only use reliable CVs

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## What is ANCOVA?

- Analysis of Covariance
- Extension of ANOVA, using 'regression' principles
- Assesses effect of
  - one variable (IV) on
  - another variable (DV)
  - after controlling for a third variable (CV)

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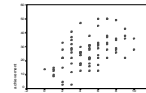
## Why use ANCOVA?

- Adjusts group means to what they would have been if all *F*s had scored identically on the CV.
- The differences between *F*s on the CV are removed, allowing focus on remaining variation in the DV due to the IV.
- Make sure hypothesis (hypotheses) is/are clear.

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## Assumptions of ANCOVA

- Check for linearity between CV & DV - check via scatterplot and correlation.
- If the CV is not correlated with the DV there is no point in using it.

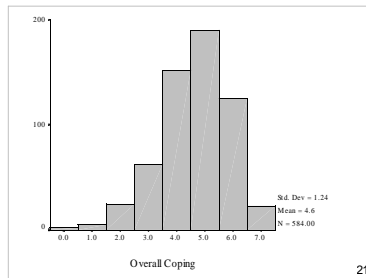


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### Assumptions of ANCOVA

- Homogeneity of regression
- Assumes slopes of regression lines between CV & DV are equal for each level of IV, if not, don't proceed with ANCOVA
- Check via scatterplot with lines of best fit

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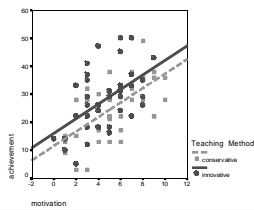
Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	11.884 <sup>a</sup>	3	3.961	15.305	.000
Intercept	302.970	1	302.970	1169.568	.000
AVGRADE	2.860	1	2.860	11.042	.001
COPEX	7.400	2	3.700	14.283	.000
Error	131.595	508	.259		
Total	7206.028	512			
Corrected Total	143.489	511			

<sup>a</sup>. R Squared = .083 (Adjusted R Squared = .077)

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### Assumptions of ANCOVA



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COPEX Coping

	Frequency	Percent	Valid Percent	Cumulative Percent
Valid 1.00 Not Coping	94	15.4	16.1	16.1
2.00 Coping	151	24.7	25.9	42.0
3.00 Coping Well	338	55.3	58.0	100.0
Total	583	95.4	100.0	
Missing System	28	4.6		
Total	611	100.0		

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### ANCOVA Example 2: Does teaching method affect academic achievement after controlling for motivation?

- IV = teaching method
- DV = academic achievement
- CV = motivation
- Experimental design - assume students randomly allocated to different teaching methods.

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### ANCOVA example 1: Does education satisfaction differ between people with different levels of coping ('Not coping', 'Just coping' and 'Coping well') with average grade as a covariate?

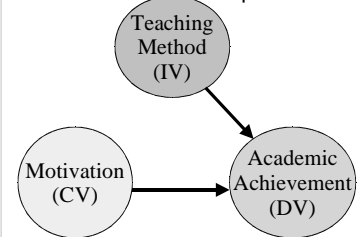
120

Descriptive Statistics

COPEX Coping	Mean	Std. Deviation	N
1.00 Not Coping	3.4586	.6602	83
2.00 Just Coping	3.6453	.5031	129
3.00 Coping Well	3.8142	.4710	300
Total	3.7140	.5299	512

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### ANCOVA example



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### ANCOVA example



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### ANCOVA & hierarchical MLR

- ANCOVA is similar to hierarchical regression – assesses impact of IV on DV while controlling for 3<sup>rd</sup> variable.
- ANCOVA more commonly used if IV is categorical.

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### Effect sizes

Three effect sizes are relevant to ANOVA:

- **Eta-square ( $\eta^2$ )** provides an overall test of size of effect
- **Partial eta-square ( $\eta_p^2$ )** provides an estimate of the effects for each IV.
- **Cohen's  $d$** : Standardised differences between two means.

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### ANCOVA example 2

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Eta Squared
Corrected Model	188.113 <sup>a</sup>	1	188.113	1.622	.207	.020
Intercept	56021.113	1	56021.113	480.457	.000	.890
TEACH	188.113	1	188.113	1.622	.207	.020
Error	9304.775	78	118.600			
Total	65326.000	80				
Corrected Total	9393.888	79				

a. R Squared = .020 (Adjusted R Squared = .008)

- A one-way ANOVA shows a non-significant effect for teaching method (IV) on academic achievement (DV)

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### Summary of ANCOVA

- Use ANCOVA in survey research when you can't randomly allocate participants to conditions e.g., quasi-experiment, or control for extraneous variables.
- ANCOVA allows us to statistically control for *one or more* covariates.

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### Effect Size: Eta-squared ( $\eta^2$ )

- Analogous to  $R^2$  from regression
- =  $SS_{\text{between}} / SS_{\text{total}} = SS_B / SS_T$
- = prop. of variance in Y explained by X
- = Non-linear correlation coefficient
- = prop. of variance in Y explained by X
- Ranges between 0 and 1

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### ANCOVA example 2

Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Eta Squared
Corrected Model	3650.749 <sup>a</sup>	2	1825.372	18.843	.000	.329
Intercept	2754.775	1	2754.775	34.526	.000	.310
MOTIV	2861.632	1	2861.632	35.361	.000	.315
TEACH	421.769	1	421.769	4.210	.025	.063
Error	6223.143	77	80.950			
Total	65326.000	80				
Corrected Total	9283.888	79				

a. R Squared = .329 (Adjusted R Squared = .311)

- An ANCOVA is used to adjust for differences in motivation
- $F$  has gone from 1 to 5 and is significant because the error term (unexplained variance) was reduced by including motivation as a CV.

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### Summary of ANCOVA

- Decide which variable(s) are IV, DV & CV.
- Check assumptions:
  - normality
  - homogeneity of variance (Levene's test)
  - Linearity between CV & DV (scatterplot)
  - homogeneity of regression (scatterplot – compares slopes of regression lines)
- Results – does IV effect DV after controlling for the effect of the CV?

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### Effect Size: Eta-squared ( $\eta^2$ )

- Interpret as for  $r^2$  or  $R^2$
- Cohen's rule of thumb for interpreting  $\eta^2$ :
  - .01 is small
  - .06 medium
  - .14 large

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ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	395.433	2	197.717	4.178	.024
Within Groups	2697.550	57	47.325		
Total	3092.983	59			

$\eta^2 = SS_{\text{between}} / SS_{\text{total}}$   
 $= 395.433 / 3092.983$   
 $= 0.128$

Eta-squared is expressed as a percentage:  
 12.8% of the total variance in control is explained by differences in Age

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- ### Results - Writing up ANOVA
- Report on test results – Size, direction and significance ( $F$ ,  $p$ , partial eta-squared)
  - Conduct planned or post-hoc testing as appropriate, with pairwise effect sizes (Cohen's  $d$ )
  - Indicate whether or not results support hypothesis (hypotheses)
- 139

- ### Summary
- Report on the size of effects potentially using:
    - Eta-square ( $\eta^2$ ) as the omnibus ES
    - Partial eta-square ( $\eta_p^2$ ) for each IV
    - Standardised mean differences for the differences between each pair of means (e.g., Cohen's  $d$ )
- 142

- ### Effect Size: Eta-squared ( $\eta^2$ )
- The eta-squared column in SPSS  $F$ -table output is actually partial eta-squared ( $\eta_p^2$ ). Partial eta-squared indicates the size of effect for each IV (also useful).
  - $\eta^2$  is not provided by SPSS – calculate separately:
    - $= SS_{\text{between}} / SS_{\text{total}}$
    - $= \text{prop. of variance in } Y \text{ explained by } X$
  - $R^2$  at the bottom of SPSS  $F$ -tables is the linear effect as per MLR – if an IV has 3 or more non-interval levels, this won't equate with  $\eta^2$ .
- 137

- ### Summary
- Hypothesise each main effect and interaction effect.
  - $F$  is an omnibus “gateway” test; may require follow-up tests.
  - Conduct follow-up tests where sig. main effects have three or more levels.
- 140

### Open Office Impress

- This presentation was made using Open Office Impress.
- Free and open source software.
- <http://www.openoffice.org/product/impress.html>



143

- ### Results - Writing up ANOVA
- Establish clear hypotheses – one for each main or interaction or covariate effect
  - Test the assumptions, esp. LOM, normality and  $n$  for each cell, homogeneity of variance, Box's  $M$ , Sphericity
  - Present the descriptive statistics ( $M$ ,  $SD$ , skewness, and kurtosis in a table, with marginal totals)
  - Present a figure to illustrate the data (bar, error-bar, or line graph)
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- ### Summary
- Choose from mixed-design ANOVA or ANCOVA for lab report
  - Repeated measure designs include the assumption of sphericity
- 141