Wright State University Lake Campus/2018-9/Phy2410/Equation sheet

lintHint

Also available as File:Wsul file pdf 04.pdf

T1

2.5 Electric charges and fields (https://cnx.org/contents/eg-XcBxE@14.1:Ai7EWAra@5/Introduction)

$\varepsilon_0 = 8.85\times10^{-12}$ F/m = vacuum permittivity.

e = 1.602×10^{-19}C: negative (positive) charge for electrons (protons)

$k_e = \frac{1}{4\pi\varepsilon_0}$ = = 8.99×10^9 m/F

$\vec{F} = Q\vec{E}$ where $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_{Pi}^2}\hat{r}_{Pi}$

Find E

$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{\widehat{\mathcal{R}}_i Q_i}{|\vec{\mathcal{R}}_i|^2} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{\vec{\mathcal{R}}_i Q_i}{|\vec{\mathcal{R}}_i|^3}$ is the electric field at the field point, $\vec{r}$, due to point charges at the source points, $\vec{r}_i$, and $\vec{\mathcal{R}}_i = \vec{r} - \vec{r}_i$, points from source points to the field point.

$\vec{E} = \int \frac{dq}{r^2}\hat{r}$ where $dq = \lambda d\ell = \sigma da = \rho dV$

$E = \frac{\sigma}{2\varepsilon_0}$ = field above an infinite plane of charge.

T2

2.6 Gauss's law (https://cnx.org/contents/eg-XcBxE@14.1:xakXl9gb@5/Introduction):

$\Phi = \vec{E}\cdot\vec{A} \to \int \vec{E}\cdot d\vec{A} = \int \vec{E}\cdot\hat{n}\,dA$ = electric flux

$q_{enclosed} = \varepsilon_0 \oint \vec{E}\cdot d\vec{A}$

$d\,\mathrm{Vol} = dxdydz = r^2\,drdA$ where $dA = r^2\,d\phi d\theta$

$A_{\text{sphere}} = r^2\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\phi = 4\pi r^2$

Surface Integrals

Calculating $\int fdA$ and $\int fdV$ with angular symmetry

Cyndrical: $dA = 2\pi r\,dz$; $dV = dA\,dr$. Spherical: $\int dA = 4\pi r^2,\ dV = 4\pi r^2\,dr$

More Gauss Law

Calculating $\int fdA$ and $\int fdV$ with angular symmetry

Cyndrical: $dA = 2\pi r\,dz$; $dV = dA\,dr$. Spherical: $\int dA = 4\pi r^2,\ dV = 4\pi r^2\,dr$

T3

2.7 Electric potential (https://cnx.org/contents/eg-XcBxE@14.1:bMxnTeM5@6/Introduction) The alpha-particle is made up of two protons and two neutrons.

$\Delta V_{AB} = V_A - V_B = -\int_A^B \vec{E}\cdot d\vec{\ell}$ = electric potential

$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = -\vec{\nabla}V$

$q\Delta V$ = change in potential energy (or simply $U = qV$)

$Power = \frac{\Delta U}{\Delta t} = \frac{\Delta q}{\Delta t}V = IV = e\frac{\Delta N}{\Delta t}$

Electron (proton) mass = 9.11×10^{-31}kg (1.67×10^{-27}kg). Elementary charge = e = 1.602×10^{-19}C.

$K = \frac{1}{2}mv^2$ =kinetic energy. 1 eV = 1.602×10^{-19}J

$V(r) = k\frac{q}{r}$ near isolated point charge

Many charges: $V_P = k\sum_1^N \frac{q_i}{r_i} \to k\int\frac{dq}{r}$.

2.8 Capacitance (https://cnx.org/contents/eg-XcBxE@14.1:FYJxWFC_@6/8-1-Capacitors-and-Capacitance)

$Q = CV$ defines capacitance.

$C = \varepsilon_0\frac{A}{d}$ where A is area and d<<$A^{1/2}$ is gap length of parallel plate capacitor

Series : $\frac{1}{C_S} = \sum\frac{1}{C_i}$. Parallel: $C_P = \sum C_i$.

$u = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2C}Q^2$ = stored energy

$u_E = \frac{1}{2}\varepsilon_0 E^2$ = energy density

T4

2.9 Current and resistors (https://cnx.org/contents/eg-XcBxE@14.10:WS3a1YbB@5/Introduction)

Electric current: 1 Amp (A) = 1 Coulomb (C) per second (s)

Current=$I = dQ/dt = nqv_dA$, where

(n, q, v_d, A) = (density, charge, speed, Area)

$I = \int\vec{J}\cdot d\vec{A}$ where $\vec{J} = nq\vec{v}_d$ =current density.

$\vec{E} = \rho\vec{J}$ = electric field where ρ = resistivity

$\rho = \rho_0\left[1 + \alpha(T - T_0)\right]$, and $R = R_0\left[1 + \alpha\Delta T\right]$,

where $R = \rho\frac{L}{A}$ is resistance

$V = IR$ and Power=$P = IV = I^2R = V^2/R$

2.10 Direct current circuits (https://cnx.org/contents/eg-XcBxE@14.1:BRLHpZ5e@6/Introduction)

$V_{terminal} = \varepsilon - Ir_{eq}$ where r_{eq}=internal resistance and ε=*emf*.

$R_{series} = \sum_{i=1}^N R_i$ and $R_{parallel}^{-1} = \sum_{i=1}^N R_i^{-1}$

Kirchhoff Junction:$\sum I_{in} = \sum I_{out}$ and Loop: $\sum V = 0$

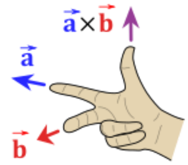
Charging an RC (resistor-capacitor) circuit: $q(t) = Q\left(1 - e^{t/\tau}\right)$ and $I = I_0e^{-t/\tau}$ where $\tau = RC$ is RC time, $Q = \varepsilon C$ and $I_0 = \varepsilon/R$.

Discharging an RC circuit: $q(t) = Qe^{-t/\tau}$ and $I(t) = -\frac{Q}{RC}e^{-t/\tau}$

T5

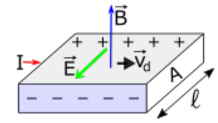
2.11 Magnetic forces and fields (https://cnx.org/contents/eg-XcBxE@14.1:-cf9Ogkt@5/Introduction)

$|\vec{a} \times \vec{b}| = ab \sin\theta \Leftrightarrow$
$(\vec{a} \times \vec{b})_x = (a_y b_z - a_z b_y)$,
$(\vec{a} \times \vec{b})_y = (a_z b_x - a_x b_z)$,
$(\vec{a} \times \vec{b})_z = (a_x b_y - a_y b_x)$
Magnetic force: $\vec{F} = q\vec{v} \times \vec{B}$, $d\vec{F} = I\vec{d\ell} \times \vec{B}$.
$\vec{v}_d = \vec{E} \times \vec{B}/B^2$=EXB drift velocity
Circular motion (uniform B field): $r = \frac{mv}{qB}$. Period=$T = \frac{2\pi m}{qB}$.



cross product

Dipole moment=$\vec{\mu} = NIA\hat{n}$. Torque= $\vec{\tau} = \vec{\mu} \times \vec{B}$. Stored energy=$U = \vec{\mu} \cdot \vec{B}$.
Hall field =$E = V/\ell = Bv_d = \frac{IB}{neA}$
Lorentz force =$q\left(\vec{E} + \vec{v} \times \vec{B}\right)$



Hall effect

2.12 Sources of magnetic fields (https://cnx.org/contents/eg-XcBxE@14.1:BDvPdDpp@5/Introduction)

Free space permeability $\mu_0 = 4\pi \times 10^{-7}$ T·m/A
Force between parallel wires $\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}$
Biot–Savart law $\vec{B} = \frac{\mu_0}{4\pi} \int_{wire} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$

Ampère's Law: $\oint \vec{B} \cdot d\vec{\ell} = 4\pi\mu_0 I_{enc}$
Magnetic field inside solenoid with paramagnetic material = $B = \mu n I$ where $\mu = (1 + \chi)\mu_0$ = permeability

(we skip T6 because it was a review of previous chapters)

T7

2.13 Electromagnetic induction (https://cnx.org/contents/eg-XcBxE@14.1:az4UJL6l@6/Introduction)

Magnetic flux $\Phi_m = \int_S \vec{B} \cdot \hat{n} dA$
Motional $\varepsilon = B\ell v \; if \, \vec{v} \perp \vec{B}$
Electromotive "force" (volts) $\varepsilon = -N\frac{d\Phi_m}{dt} = \oint \vec{E} \cdot d\vec{\ell}$

rotating coil $\varepsilon = NBA\omega \sin \omega t$

2.14 Inductance (https://cnx.org/contents/eg-XcBxE@14.1:gBxAb_6h@6/Introduction)

Unit of inductance = Henry (H)=1V·s/A

Mutual inductance: $M\frac{dI_2}{dt} = N_1\frac{d\Phi_{12}}{dt} = -\varepsilon_1$ where Φ_{12}=flux through 1 due to current in 2. Reciprocity: $M\frac{dI_1}{dt} = -\varepsilon_2$

Self-inductance: $N\Phi_m = LI \rightarrow \varepsilon = -L\frac{dI}{dt}$

$L_{\text{solenoid}} \approx \mu_0 N^2 A\ell$, $L_{\text{toroid}} \approx \frac{\mu_0 N^2 h}{2\pi} \ln \frac{R_2}{R_1}$, Stored energy= $\frac{1}{2}LI^2$

$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$ in LR circuit where $\tau = L/R$.

$q(t) = q_0 \cos(\omega t + \phi)$ in LC circuit where $\omega = \sqrt{\frac{1}{LC}}$

2.15 Alternating current circuits (https://cnx.org/contents/eg-XcBxE@14.1:6uMHjFiO@5/Introduction)

AC voltage and current $v = V_0 \sin(\omega t - \phi)$ if $i = I_0 \sin \omega t$.
RMS values $I_{rms} = \frac{I_0}{\sqrt{2}}$ and $V_{rms} = \frac{V_0}{\sqrt{2}}$
Impedance $V_0 = I_0 X$
Resistor $V_0 = I_0 X_R$, $\phi = 0$, where $X_R = R$
Capacitor $V_0 = I_0 X_C$, $\phi = -\frac{\pi}{2}$, where $X_C = \frac{1}{\omega C}$
Inductor $V_0 = I_0 X_L$, $\phi = +\frac{\pi}{2}$, where $X_L = \omega L$

RLC series circuit $V_0 = I_0 Z$ where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\phi = \tan^{-1}\frac{X_L - X_C}{R}$
Resonant angular frequency $\omega_0 = \sqrt{\frac{1}{LC}}$

Quality factor $Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0 L}{R}$

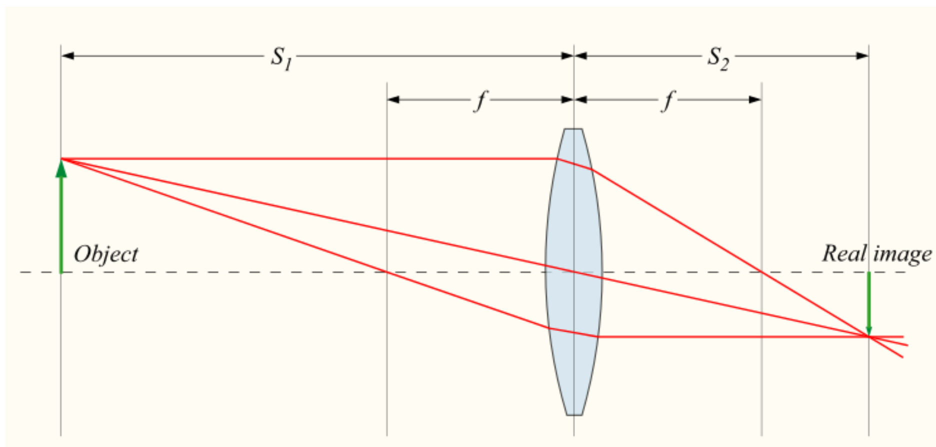
Average power $P_{ave} = \frac{1}{2} I_0 V_0 \cos\phi = I_{rms} V_{rms} \cos\phi$

Transformer voltages and currents $\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$

T8

3.2 Geometric optics and image formation (https://cnx.org/contents/rydUIGBQ@10.14:-YlrAoNe@5/Introduction)

$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$ relates the focal length f of the lens, the image distance S_1, and the object distance S_2. The figure depicts the situation for which (S_1, S_2, f) are all positive: (1)The lens is converging (convex); (2) The real image is to the right of the lens; and (3) the object is to the left of the lens. If the lens is diverging (concave), then f < 0. If the image is to the left of the lens (virtual image), then $S_2 < 0$.



T9

2.16 Electromagnetic waves (https://cnx.org/contents/eg-XcBxE@14.10:-LQJwSUO@6/16-1-Maxwell-s-Equations-and-Electromagnetic-Waves)

Displacement current $I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$ where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ is the electric flux.

Maxwell's equations: $\epsilon_0 \mu_0 = 1/c^2$

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{A} = \frac{1}{\epsilon_0} Q_{in}$$

$$\oint_S \vec{B} \cdot \mathrm{d}\vec{A} = 0$$

$$\oint_C \vec{E} \cdot \mathrm{d}\vec{\ell} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \mathrm{d}\vec{A}$$

$$\oint_C \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I + \epsilon_0 \mu_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2}$ and $\frac{E_0}{B_0} = c$

Poynting vector $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$=energy flux

Average intensity $I = S_{ave} = \frac{c\varepsilon_0}{2} E_0^2 = \frac{c}{2\mu_0} B_0^2 = \frac{1}{2\mu_0} E_0 B_0$

Radiation pressure $p = I/c$ (perfect absorber) and $p = 2I/c$ (perfect reflector).