Mtg 13: Tue, 26 Jan 10

Error in Newton-Cotes formula:

Thm: $E_n := I - I_n$ p. 7-2

$$= \int_a^b \underbrace{[f(x) - f_n(x)]}_{\substack{\parallel \\ e_n(x)}} dx$$

$e_n(x)$ (p. 10-2 E)

$$|E_n| \leq \int_a^b |e_n(x)| \, dx$$

$$e_n(x) = \frac{q_{n+1}(x)}{(n+1)!} f^{(n+1)}(\xi) \qquad \xi \in [a,b]$$

Let $M_{n+1} := \max_{\xi \in [a,b]} f^{(n+1)}(\xi)$

$$|E_n| \leq \frac{M_{n+1}}{(n+1)!} \int_a^b |q_{n+1}(x)| \, dx$$

Appl: Simple Trap. rule

$n = 1$, $q_2(x) = (\underbrace{x - a}_{\geq 0})(\underbrace{x - b}_{\leq 0})$, where $a = x_0$, $b = x_1$

$$|E_1| \leq \frac{M_2}{2!} \int_a^b \underbrace{|q_2(x)|}_{\geq 0} dx$$

$$= \frac{M_2}{2!} \int_a^b \underbrace{(x-a)}_{\geq 0}\underbrace{(b-x)}_{\geq 0} dx$$

$$= \frac{(b-a)^3}{12} M_2 = \frac{h^3}{12} M_2$$ A. p.253

↑ HW

$$h := b-a$$

Appl: Simple Simpson's rule

$n = \cancel{3}\, 2$, $q_3(x) = (x-x_0)(x-x_1)(x-x_2)$

$x_0 = a$, $x_2 = b$, $x_1 = \frac{a+b}{2}$

$$|E_2| \leq \frac{M_3}{3!} \int_a^b \left|(x-a)\left(x-\frac{a+b}{2}\right)(x-b)\right| dx$$

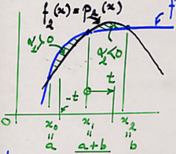
$$= \frac{(b-a)^4}{196} M_3 = \frac{2^4 h^4}{196} M_3$$ ← Too pessimistic. Can do better.

↑ HW

$$h := \frac{b-a}{2} \Rightarrow b-a = 2h$$

Simpson's rule int. exactly poly. deg ≤ 2, but also poly. of deg ≤ 3 (!)

cubic



A.p. 256

$$E_2 = \int_a^b (f - f_2)\,dx$$

$$= \alpha_1 + \alpha_2$$

>0 $\quad$ <0

Cancellation

$\Rightarrow$ int. exactly poly. in $\mathcal{P}_3$

HW: $[a, b] = [0, 1]$

$$f_3(x) = p_3(x) = c_0 + c_1 x^1 + c_2 x^2 + c_3 x^3$$

$x^0 = 1$

f''

$c_0 = 3, \quad c_1 = 8$

$c_2 = -2, \quad c_3 = 6$

Find I, I_2

exact $\quad$ Simpson