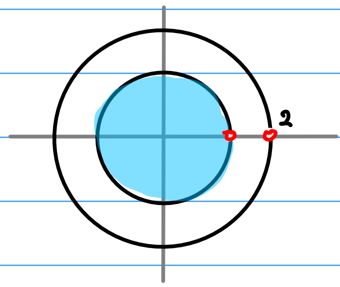
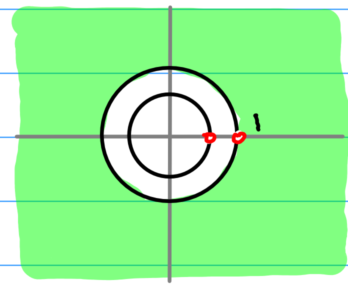
Laurent Series and z-Transform Examples case 0.A

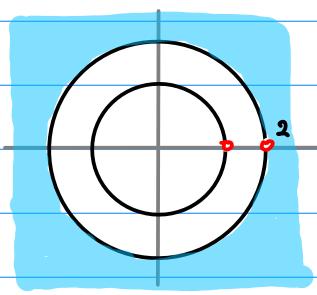
20171014

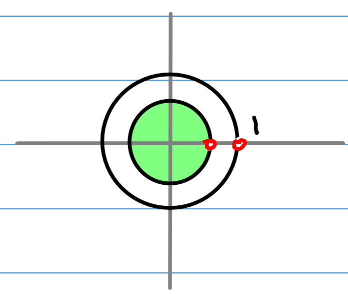
1. A $\quad f(z) = \dfrac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \dfrac{-0.5\, z^2}{(z-1)(z-0.5)}$



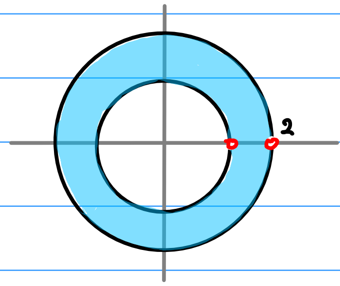


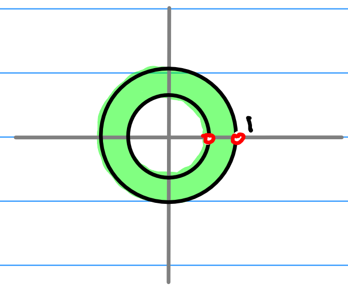
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n = \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n$$





$$\sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n = \sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n$$

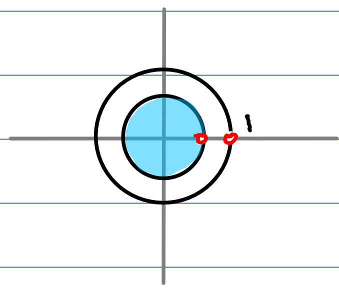


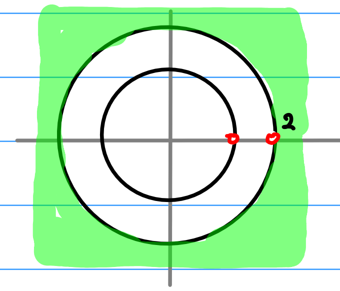


$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n$$

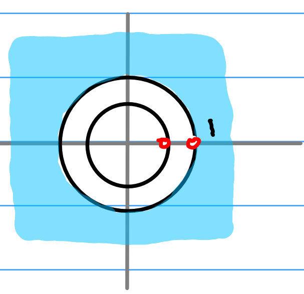
2.A

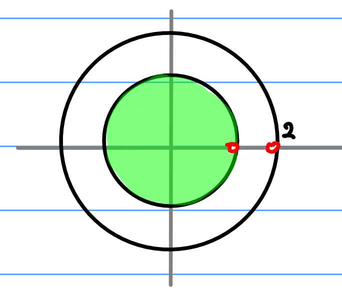
$$f(z) = \frac{-0.5\, z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$



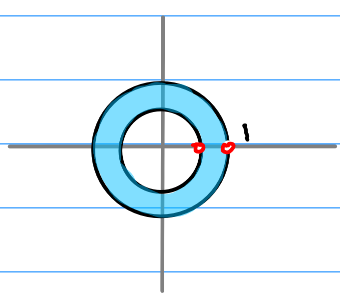


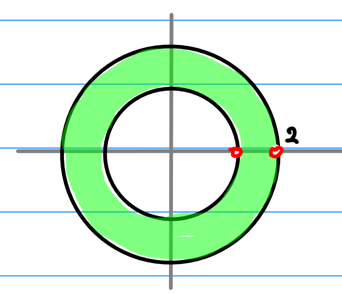
$$\sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n = \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n$$





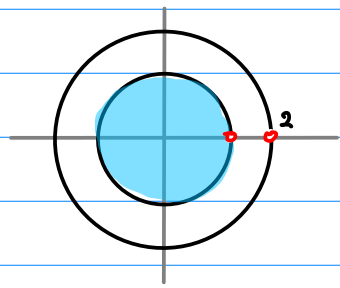
$$\sum_{n=0}^{-\infty} \left[2^{n-1} - 1\right] z^n = \sum_{n=0}^{-\infty} \left[2^{n-1} - 1\right] z^n$$

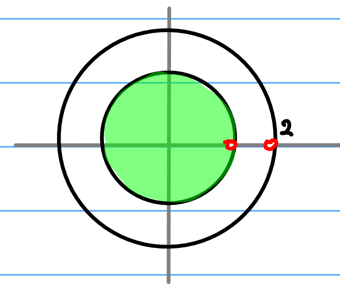




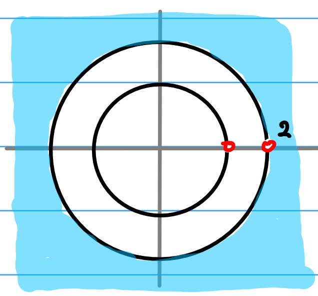
$$\sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n = \sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

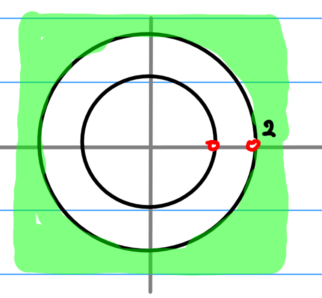
3 . A $\quad f(z) = \frac{-1}{(z-1)(z-2)} = X(z) = \frac{-1}{(z-1)(z-2)}$



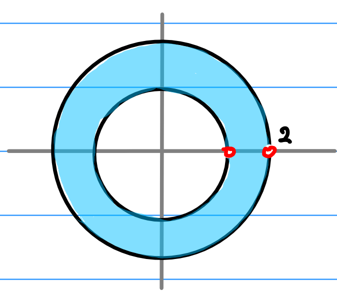


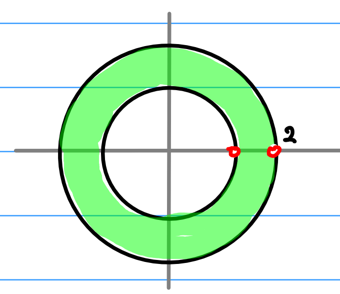
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^n \xrightarrow{-n} \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1} \right] z^n \xrightarrow{-n} \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$

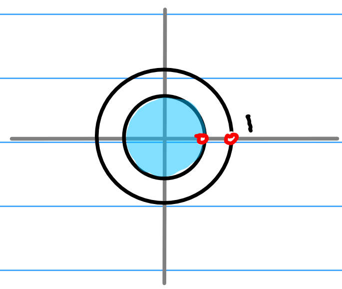


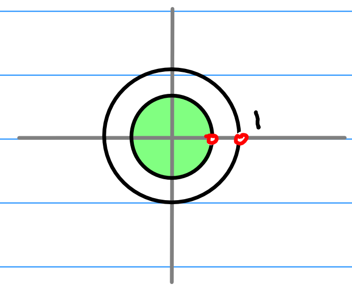


$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n \xrightarrow{-n} + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

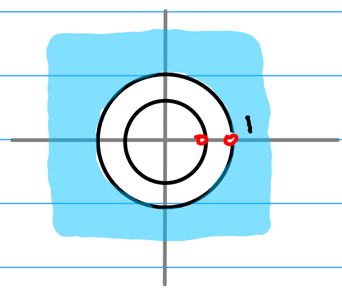
4.A

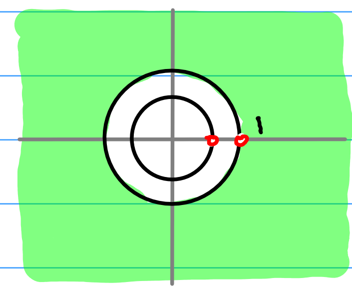
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



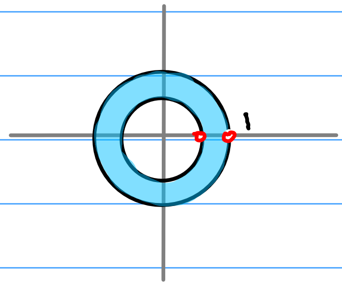


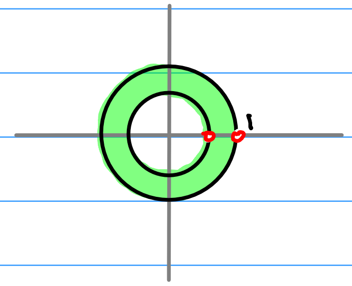
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^{n} \xrightarrow{-n} \sum_{n=-1}^{\infty}\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]z^{-n}$$





$$\sum_{n=-1}^{-\infty}\left[2^{n-1}-1\right]z^{n} \xrightarrow{-n} \sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1}-1\right]z^{-n}$$





$$+\sum_{n=1}^{\infty}z^{n} + \sum_{n=0}^{-\infty}2^{n-1}z^{n} \xrightarrow{-n} +\sum_{n=-1}^{-\infty}z^{-n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n}$$

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 - 2^{n-1} & (\\ 0 & (\end{cases}$$

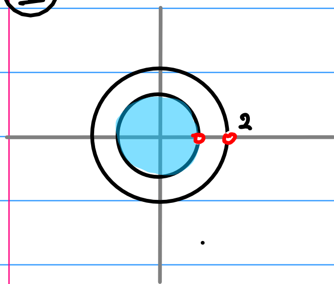
$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

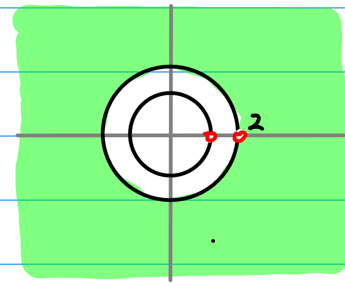
$$x_n = \begin{cases} 0 & (\\ 2^{n-1} - 1 & (\end{cases}$$

Z.T. first

$$f(z) = \frac{-1}{(z-1)(z-2)} = X(z)$$

I

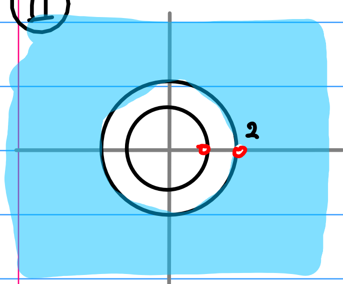


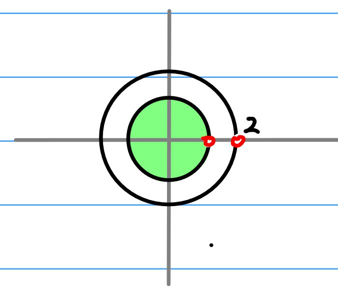


$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

II

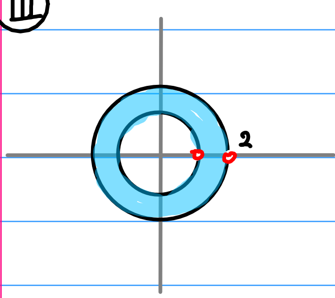


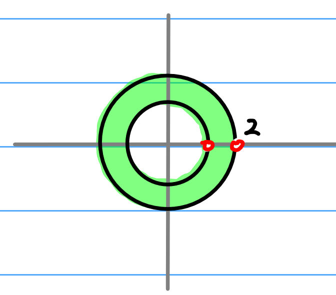


$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

III





$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

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$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{-n+1} - 1 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

$$X(z) = \sum_{n=0}^{-\infty} \left(\frac{1}{2}\right)^{-n+1} z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

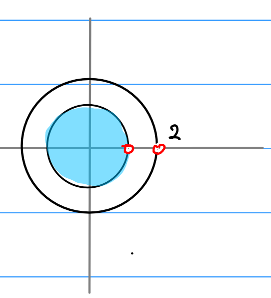
$$= \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{z}{2}\right)} - \frac{1}{1 - z}$$

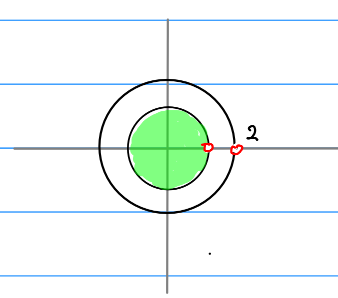
$$= \frac{-1}{z - 2} + \frac{1}{z - 1}$$

$$= \frac{-z + 1 + z - 2}{(z-1)(z-2)}$$

$$= \frac{-1}{(z-1)(z-2)}$$

$$\left|\frac{z}{2}\right| < 1 \qquad \left|\frac{z}{1}\right| < 1$$





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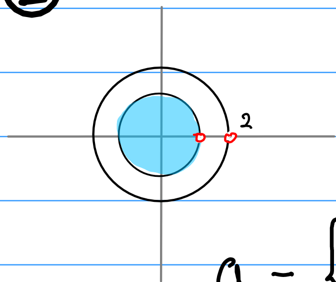
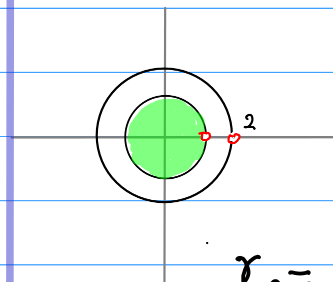


$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

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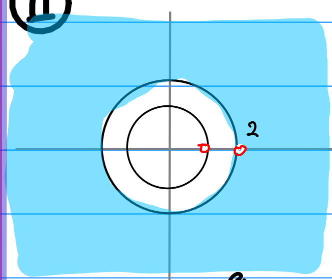
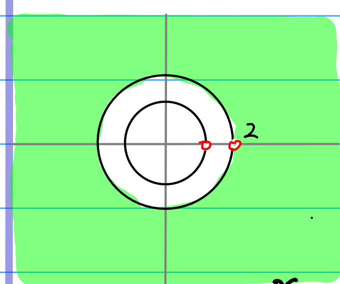


$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} \cdot z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} \cdot z^{-n}$$

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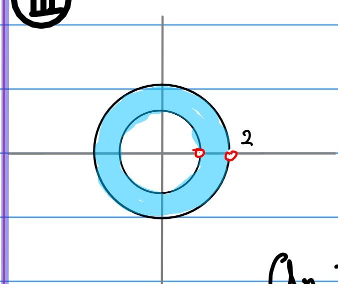

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

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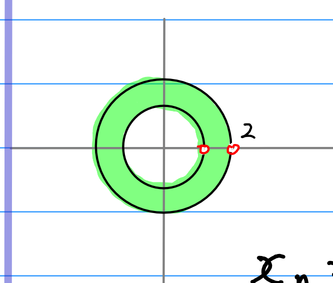


$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n}$$

$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$

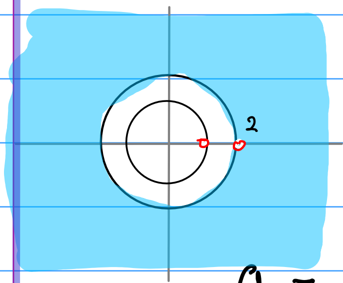
$$= \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{z}{2}\right)} - \frac{\left(\frac{1}{1}\right)}{1 - \left(\frac{z}{1}\right)}$$

$$=$$

$$= \frac{-1}{z-2} + \frac{1}{z-1}$$

$$= \frac{-z-1+z-2}{(z-2)(z-1)}$$

$$= \frac{-1}{(z-2)(z-1)}$$



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1-\left(\frac{1}{2}\right)^{n+1} & (n<0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} z^n$$

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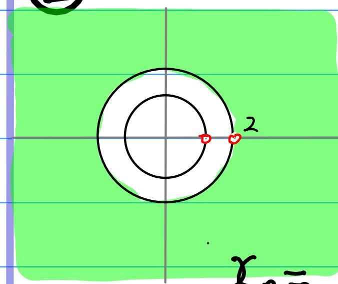
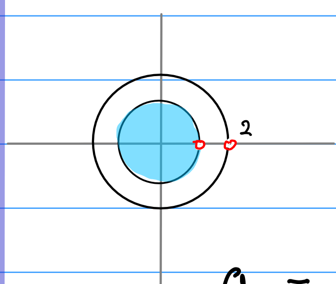


$$x_n = \begin{cases} 1-2^{n-1} & (n>0) \\ 0 & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1}-1 & (n \geq 0) \\ 0 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

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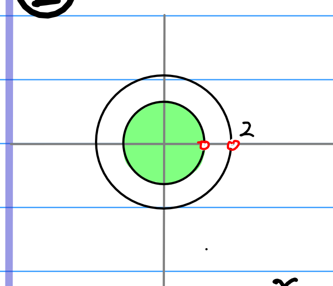
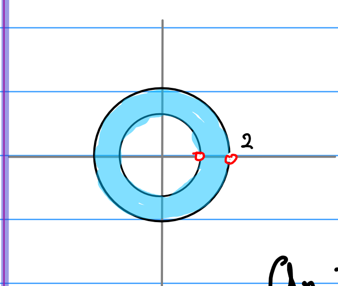


$$x_n = \begin{cases} 0 & (n>0) \\ 2^{n-1}-1 & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$

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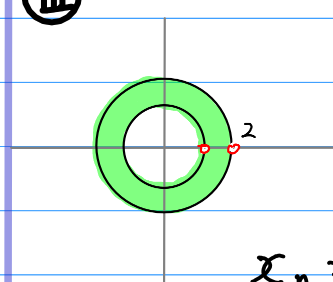

$$x_n = \begin{cases} 1 & (n>0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} + \sum_{n=1}^{\infty} 1 \cdot z^{-n}$$