

Laurent Series and z-Transform

- Geometric Series

Double Pole Properties B

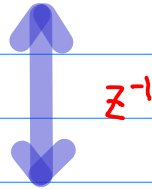
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2 formulas of z

$$\textcircled{1} \quad \frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$



$$\textcircled{2} \quad \frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)} \right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases}$$

$$g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases}$$

$$Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

$$\textcircled{1} \quad \frac{-1}{(z-1)(z-2)}$$

$$\textcircled{2} \quad \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$+ \frac{1}{z-1} - \frac{1}{z-2}$$

$$- \frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$$- \frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

causal $f(z)$

anti-causal $X(z)$

$$- \frac{1}{-z^{-1}} + \frac{0.5}{-0.5z^{-1}} \quad |z| > 1$$

anti-causal $f(z)$

causal $X(z)$

$$+ \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

anti-causal $f(z)$

causal $X(z)$

$$+ \frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

causal $f(z)$

anti-causal $X(z)$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)}$$

$$\textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$+\frac{1}{z-\textcircled{1}} - \frac{1}{z-\textcircled{2}}$$

$$-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$$\cdot 1 \downarrow \quad \cdot \frac{1}{z} \downarrow$$

$$\cdot \frac{1}{z} \downarrow \quad \cdot \frac{1}{z} \downarrow$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$\uparrow \cdot \frac{1}{z} \quad \uparrow \cdot \frac{1}{z}$$

$$\uparrow \cdot \frac{1}{z} \quad \uparrow \cdot 2$$

$$+\frac{1}{(z-1)} - \frac{1}{(z-2)}$$

$$-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)}$$

$$\textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\begin{array}{ll} |z| < 1 & |0.5z| < 1 \\ |z| < 1 & |z| < 2 \end{array}$$

$$\begin{array}{ll} |z^{-1}| < 1 & |0.5z^{-1}| < 1 \\ |z| > 1 & 0.5 < |z| \end{array}$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$\begin{array}{ll} |z^{-1}| < 1 & |2z^{-1}| < 1 \\ |z| > 1 & 2 < |z| \end{array}$$

$$\begin{array}{ll} |z| < 1 & |2z| < 1 \\ |z| < 1 & |z| < 0.5 \end{array}$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$\cdot \frac{1}{z} \downarrow$$

$$\cdot \frac{z}{2} \downarrow$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$\cdot z \downarrow$$

$$\cdot 2z \downarrow$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$\uparrow \cdot z$$

$$\uparrow \cdot \frac{z}{2}$$

$$+\frac{z^{-1}}{1-\cancel{z^{-1}}} - \frac{z^{-1}}{1-\cancel{2z^{-1}}} \quad |z| > 2$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$\uparrow \cdot \frac{1}{z}$$

$$\uparrow \cdot \frac{1}{2z}$$

$$+\frac{z}{1-\cancel{z}} - \frac{z}{1-\cancel{2z}} \quad |z| < 0.5$$

Causal sequence a_n & x_n

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

causal $f_1(z) =$

$$-\left[1 + 1^2 z^1 + 1^3 z^2 + \dots\right] - 1^{n+1}$$

$$+\left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

0 1 2

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

causal $Y_1(z) =$

$$-\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \dots\right] - 1^{n+1}$$

$$+\left[\left(\frac{1}{2}\right)^1 z^0 + \left(\frac{1}{2}\right)^2 z^{-1} + \left(\frac{1}{2}\right)^3 z^{-2} + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

0 1 2

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

causal $X_2(z)$

$$+\left[\left(\frac{1}{1}\right)^0 z^1 + \left(\frac{1}{1}\right)^1 z^2 + \left(\frac{1}{1}\right)^2 z^3 + \dots\right] + \left(\frac{1}{1}\right)^{n+1}$$

$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

1 2 3

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

causal $g_2(z)$

$$+\left[\left(\frac{1}{1}\right)^0 z^1 + \left(\frac{1}{1}\right)^1 z^2 + \left(\frac{1}{1}\right)^2 z^3 + \dots\right] + \left(\frac{1}{1}\right)^{n+1}$$

$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

1 2 3

Anti-causal sequence a_n & x_n

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

anti-causal $x_1(z)$

$$-\left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^{-1} z^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-2} + \dots \right] - \left(\frac{1}{2}\right)^{n-1}$$

$$+ \left[2^0 + 2^{-1} z^{-1} + 2^{-2} z^{-2} + \dots \right] + 2^{n-1}$$

0 -1 -2

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

anti-causal $g_1(z)$

$$-\left[\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^{-1} z^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-2} + \dots \right] - \left(\frac{1}{2}\right)^{n-1}$$

$$+ \left[2^0 z^0 + 2^{-1} z^{-1} + 2^{-2} z^{-2} + \dots \right] + 2^{n-1}$$

0 -1 -2

$$2 = \left(\frac{1}{2}\right)^{-1}$$

$$\left(\frac{1}{2}\right) = 2^{-1}$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

anti-causal $f_2(z)$

$$+ \left[1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \dots \right] + 1^{n+1}$$

$$- \left[\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^{-1} z^{-2} + \left(\frac{1}{2}\right)^{-2} z^{-3} + \dots \right] - \left(\frac{1}{2}\right)^{n+1}$$

-1 -2 -3

$$2 = \left(\frac{1}{2}\right)^{-1}$$

$$\left(\frac{1}{2}\right) = 2^{-1}$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

anti-causal $Y_2(z)$

$$+ \left[1^0 z^1 + 1^{-1} z^2 + 1^{-2} z^3 + \dots \right] + 1^{n+1}$$

$$- \left[\left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^{-1} z^2 + \left(\frac{1}{2}\right)^{-2} z^3 + \dots \right] - \left(\frac{1}{2}\right)^{n+1}$$

-1 -2 -3

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

causal $f_1(z) =$

$$-\left[1 + 1^2 z^1 + 1^3 z^2 + \dots\right] - 1^{n+1}$$

$$+\left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

0 1 2

anti-causal $X_1(z)$

$$-\left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \dots\right] - \left(\frac{1}{2}\right)^{n+1}$$

$$+\left[2^0 + 2^{-1} z^{-1} + 2^{-2} z^{-2} + \dots\right] + 2^{n+1}$$

0 -1 -2

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

anti-causal $g_1(z)$

$$-\left[\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \dots\right] - \left(\frac{1}{2}\right)^{n+1}$$

$$+\left[2^0 z^0 + 2^{-1} z^{-1} + 2^{-2} z^{-2} + \dots\right] + 2^{n+1}$$

0 -1 -2

causal $Y_1(z) =$

$$-\left[1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \dots\right] - 1^{n+1}$$

$$+\left[\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

0 1 2

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

anti-causal $f_2(z)$

$$+\left[1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \dots\right] + 1^{n+1}$$

$$-\left[\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \dots\right] - \left(\frac{1}{2}\right)^{n+1}$$

-1 -2 -3

causal $X_2(z)$

$$+\left[\left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

1 2 3

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

causal $g_2(z)$

$$+\left[\left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

1 2 3

anti-causal $Y_2(z)$

$$+\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots\right] + 1^{n+1}$$

$$-\left[\left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \dots\right] - \left(\frac{1}{2}\right)^{n+1}$$

-1 -2 -3

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$f(z) = -[1 + 1^2z + 1^3z^2 + \dots] + \left[\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2z + \left(\frac{1}{2}\right)^3z^2 + \dots\right]$$

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$X(z) = -\left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1z + \left(\frac{1}{2}\right)^2z^2 + \dots\right] + [2^1 + 2^2z + 2^3z^2 + \dots]$$

$$x_n = -\left(\frac{1}{2}\right)^{n+1} + 2^{n+1} \quad (n < 1)$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$f(z) = -\left[\left(\frac{1}{2}\right)^1z^0 + \left(\frac{1}{2}\right)^2z^{-1} + \left(\frac{1}{2}\right)^3z^{-2} + \dots\right] + [2^1z^0 + 2^2z^{-1} + 2^3z^{-2} + \dots]$$

$$a_n = -\left(\frac{1}{2}\right)^{n+1} + 2^{n+1} \quad (n < 1)$$

$$X(z) = -[1z^0 + 1z^{-1} + 1z^{-2} + \dots] + \left[\left(\frac{1}{2}\right)^1z^0 + \left(\frac{1}{2}\right)^2z^{-1} + \left(\frac{1}{2}\right)^3z^{-2} + \dots\right]$$

$$x_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

$$f(z) = +[1^0z^1 + 1^1z^2 + 1^2z^3 + \dots] - \left[\left(\frac{1}{2}\right)^0z^1 + \left(\frac{1}{2}\right)^1z^2 + \left(\frac{1}{2}\right)^2z^3 + \dots\right]$$

$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$X(z) = +\left[\left(\frac{1}{2}\right)^0z^1 + \left(\frac{1}{2}\right)^1z^2 + \left(\frac{1}{2}\right)^2z^3 + \dots\right] - [2^0z^1 + 2^1z^2 + 2^2z^3 + \dots]$$

$$x_n = +\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \quad (n \geq 1)$$

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$f(z) = +\left[\left(\frac{1}{2}\right)^0z^1 + \left(\frac{1}{2}\right)^1z^2 + \left(\frac{1}{2}\right)^2z^3 + \dots\right] - [2^0z^1 + 2^1z^2 + 2^2z^3 + \dots]$$

$$a_n = +\left(\frac{1}{2}\right)^{n+1} - 2^{n+1} \quad (n \geq 1)$$

$$X(z) = +[1^0z^1 + 1^1z^2 + 1^2z^3 + \dots] - \left[\left(\frac{1}{2}\right)^0z^1 + \left(\frac{1}{2}\right)^1z^2 + \left(\frac{1}{2}\right)^2z^3 + \dots\right]$$

$$x_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$f(z) \quad |z| < 0.5 \quad |z| > 2$$

causal anticausal

$$\frac{-1}{(z-1)(z-2)} = +\frac{1}{z-1} - \frac{1}{z-2}$$

① - A

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$|z| < 1$$

$$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$$

$$(n \geq 0)$$

$$-\left(1 + 1^1 z + 1^2 z^2 + \dots\right) + \left(\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z + \left(\frac{1}{2}\right)^2 z^2 + \dots\right)$$

$n=0 \quad n=1 \quad n=2$ $n=0 \quad n=1 \quad n=2$

① - B

$$\frac{z^{-1}}{1-z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$|z| > 2$$

$$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$$

$$(n < 0)$$

$$\left(z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \dots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \dots\right)$$

$$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \dots\right) - \left(\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \dots\right)$$

$n=-1 \quad n=-2 \quad n=-3$ $n=-1 \quad n=-2 \quad n=-3$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = -\frac{z}{z-1} + \frac{0.5z}{z-0.5}$$

② - A

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$|z| > 1$$

$$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$$

$$(n < 1)$$

$$-\left(1 + 1^2 z^{-1} + 1^3 z^{-2} + \dots\right) + \left(\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \dots\right)$$

$$-\left(\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 z^{-1} + \left(\frac{1}{2}\right)^3 z^{-2} + \dots\right) + \left(2^1 + 2^2 z^{-1} + 2^3 z^{-2} + \dots\right)$$

$n=0 \quad n=-1 \quad n=-2$ $n=0 \quad n=-1 \quad n=-2$

② - B

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$|z| < 0.5$$

$$+1^{n-1} - 2^{n-1}$$

$$(n \geq 1)$$

$$+\left(z + 1z^2 + 1^2 z^3 + \dots\right) - \left(z + 2z^2 + 2^2 z^3 + \dots\right)$$

$n=1 \quad n=2 \quad n=3$ $n=1 \quad n=2 \quad n=3$

$$X(z) \quad |z| < 0.5 \quad |z| > 2$$

anticausal causal

$$\frac{-1}{(z-1)(z-2)} = + \frac{1}{z-1} - \frac{1}{z-2}$$

① - (A)

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$|z| < 1$$

$$-\left(\frac{1}{1}\right)^{n-1} + 2^{n-1}$$

$$(n < 1)$$

$$-\left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \dots\right) + \left(\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right)$$

$$-\left(\left(\frac{1}{1}\right)^1 z^0 + \left(\frac{1}{1}\right)^2 z^1 + \left(\frac{1}{1}\right)^3 z^2 + \dots\right) + \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \dots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

① - (B)

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$|z| > 2$$

$$+\left(\frac{1}{1}\right)^{n-1} - 2^{n-1}$$

$$(n \geq 1)$$

$$\left(\left(\frac{1}{1}\right)^0 z^1 + \left(\frac{1}{1}\right)^1 z^2 + \left(\frac{1}{1}\right)^2 z^3 + \dots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = -\frac{z}{z-1} + \frac{0.5z}{z-0.5}$$

② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$|z| > 1$$

$$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$$

$$(n \geq 0)$$

$$-\left(1 + 1^2 z^{-1} + 1^3 z^{-2} + \dots\right) + \left(\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \dots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$|z| < 0.5$$

$$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$$

$$(n < 0)$$

$$+\left(z + \left(\frac{1}{1}\right)z^2 + \left(\frac{1}{1}\right)^2 z^3 + \dots\right) - \left(z + 2z^2 + 2^2 z^3 + \dots\right)$$

$$+\left(1^0 z + 1^1 z^2 + 1^2 z^3 + \dots\right) - \left(\left(\frac{1}{2}\right)^0 z + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \dots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$$f(z) \longleftrightarrow a_n$$

$$X(z) \longleftrightarrow x_n$$

① - (A)

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$x_n = -1^{n-1} + 2^{n-1} \quad (n < 1)$$

② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)$$

$$x_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

① - (B)

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$x_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$a_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

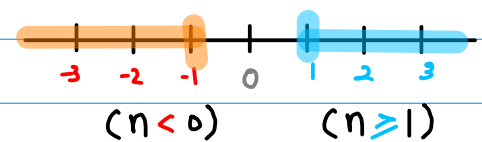
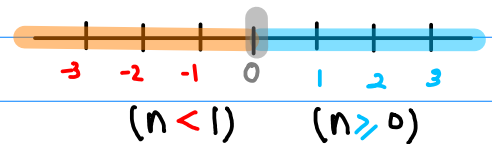
$$x_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$x_n = a_{-n}$$

$$a_n = x_{-n}$$

$$(n \geq 0) \longleftrightarrow (n < 1)$$

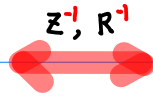
$$(n \geq 1) \longleftrightarrow (n < 0)$$



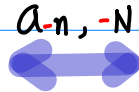
$$\textcircled{I} \quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

①-A

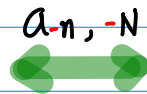
$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$



$$a_n = -|^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$



$$x_n = -|^{n-1} + 2^{n-1} \quad (n < 1)$$



②-A

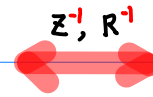
$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$a_n = -|^{n-1} + 2^{n-1} \quad (n < 1)$$

$$x_n = -|^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

①-B

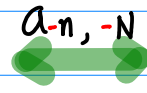
$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$



$$a_n = +|^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$



$$x_n = +|^{n-1} - 2^{n-1} \quad (n \geq 1)$$



②-B

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$a_n = +|^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$x_n = +|^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

II $(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

① - (A)

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

① - (B)

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

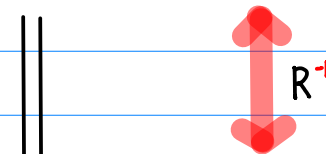


② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

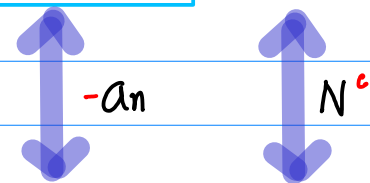


① - (A)

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

① - (B)

$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

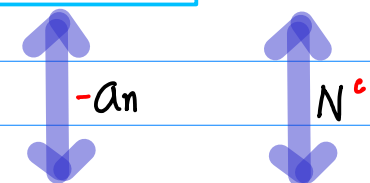


② - (A)

$$a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)$$

② - (B)

$$a_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

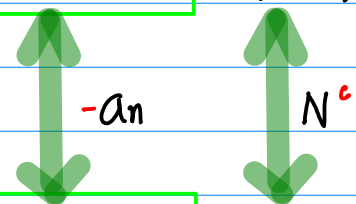


① - (A)

$$x_n = -1^{n-1} + 2^{n-1} \quad (n < 1)$$

① - (B)

$$x_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

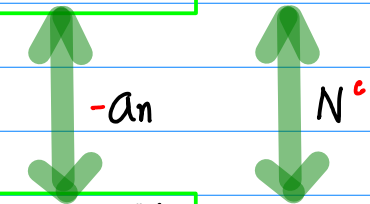


② - (A)

$$x_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

② - (B)

$$x_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$



III

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \quad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

① - (A)

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$a_n = -|^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

① - (B)

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

$$a_n = +|^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$a_n = -|^{n-1} + 2^{n-1} \quad (n < 1)$$

② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$a_n = +|^{n-1} - 2^{n-1} \quad (n \geq 1)$$

① - (A)

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$

$$x_n = +|^{n-1} - 2^{n-1} \quad (n \geq 1)$$

① - (B)

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

$$x_n = -|^{n-1} + 2^{n-1} \quad (n < 1)$$

② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$x_n = +|^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$x_n = -|^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$



②

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \quad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

① - (A)

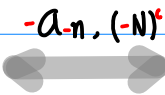
$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$



② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$a_n = -|^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$$



$$a_n = +|^{n-1} - 2^{n-1} \quad (n \geq 1)$$

① - (B)

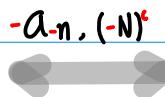
$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$



② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$a_n = +|^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$$



$$a_n = -|^{n-1} + 2^{n-1} \quad (n < 1)$$

① - (A)

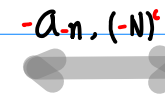
$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$



② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$x_n = +|^{n-1} - 2^{n-1} \quad (n \geq 1)$$



$$x_n = -|^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$$

① - (B)

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$



② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

$$x_n = -|^{n-1} + 2^{n-1} \quad (n < 1)$$



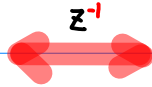
$$x_n = +|^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$$

IV

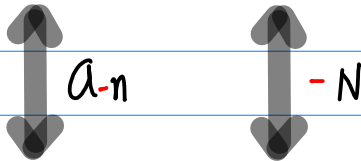
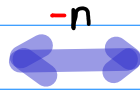
$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

① - (A)

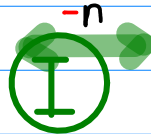
$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad |z| < 1$$



$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$



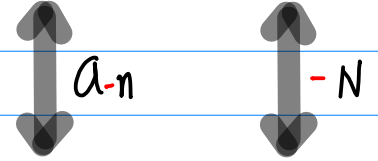
$$x_n = -1^{n+1} + 2^{n+1} \quad (n < 1)$$



② - (A)

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 1$$

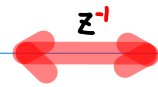
$$a_n = -1^{n+1} + 2^{n+1} \quad (n < 1)$$



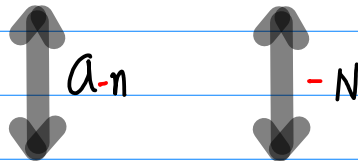
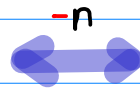
$$x_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

① - (B)

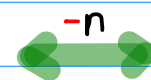
$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$



$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$



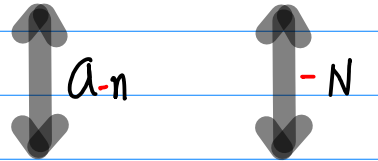
$$x_n = +1^{n+1} - 2^{n+1} \quad (n \geq 1)$$



② - (B)

$$+\frac{z}{1-z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$a_n = +1^{n+1} - 2^{n+1} \quad (n \geq 1)$$



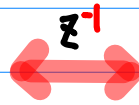
$$x_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

ROC's of interests

$R1(z)$
 $R2(z)$

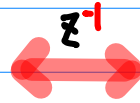
$R1(z^{-1})$
 $R2(z^{-1})$

① $\frac{-1}{(z-1)(z-2)}$



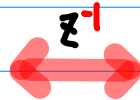
② $\frac{-0.5z^2}{(z-1)(z-0.5)}$

$$+\frac{1}{z-1} - \frac{1}{z-2}$$

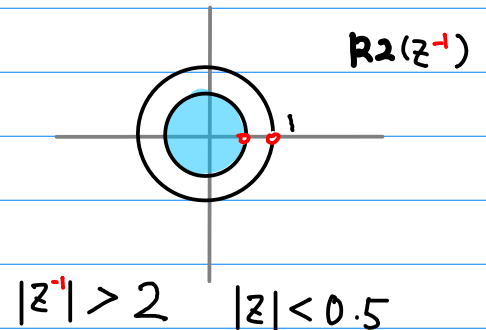
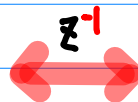
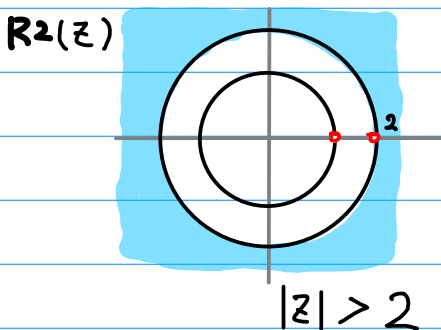
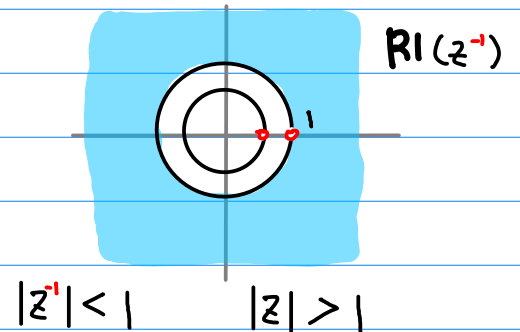
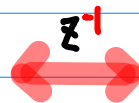
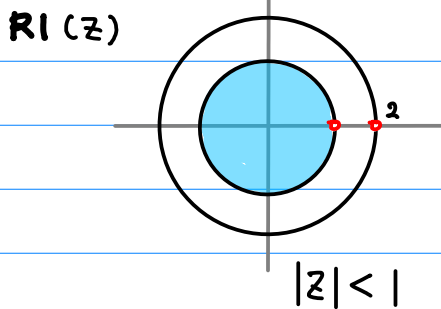


$$-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}$$

$p_1 = 1$
 $p_2 = 2$

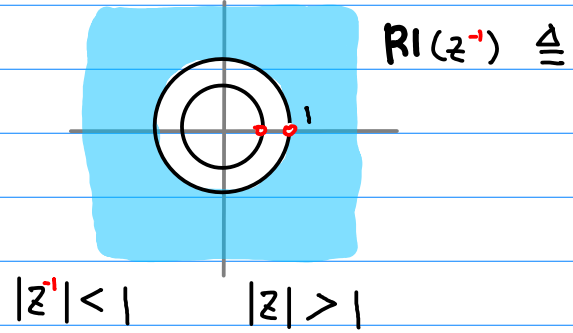
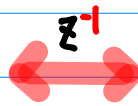
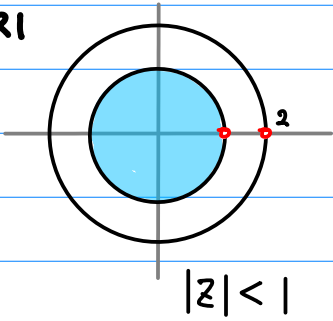


$p_1^{-1} = 1$
 $p_2^{-1} = 0.5$

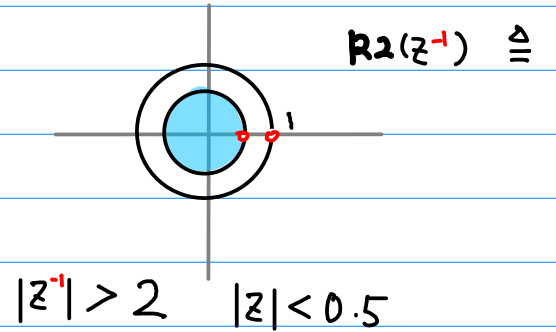
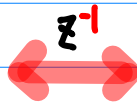
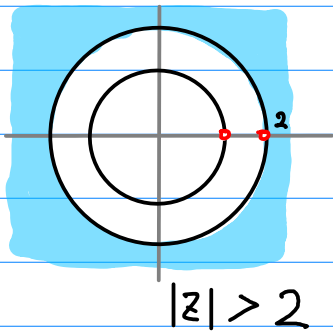


R^{-1}

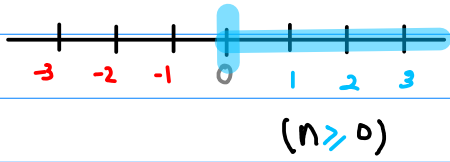
$R_1(z) \triangleq R_1$



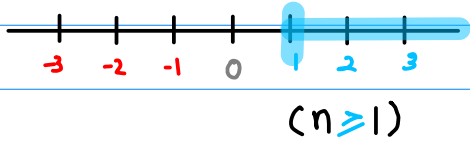
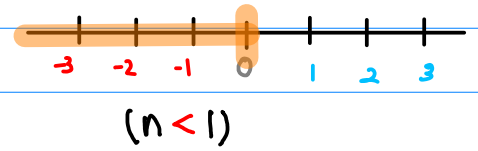
$R_2(z)$



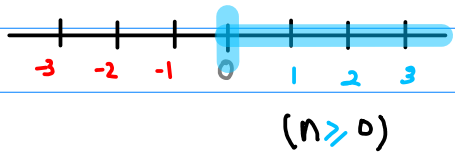
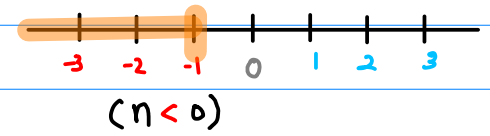
$-N$ N^c



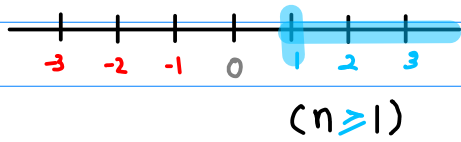
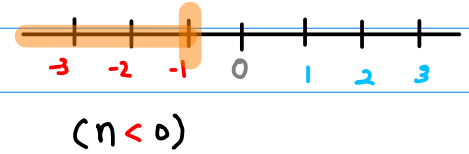
$-N$



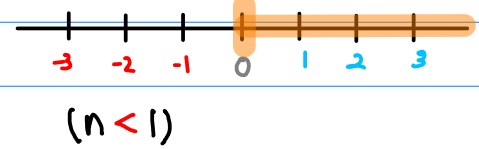
$-N$



N^c



N^c



$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \quad \longleftrightarrow \quad \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$\left(-\frac{z}{z-1} + \frac{0.5z}{z-0.5} \right)$$

$$\boxed{z} \quad - \frac{1}{| -z } + \frac{0.5}{| -0.5z }$$

$$\boxed{z} \quad + \frac{z}{| -z } - \frac{z}{| -2z }$$

$$\boxed{|z| < 1} \quad |z| < 1 \quad |0.5z| < 1$$

$$\boxed{|z| < 0.5} \quad |z| < 1 \quad |2z| < 1$$

$$\boxed{z^{-1}} \quad - \frac{z^{-1}}{| -z^{-1} } - \frac{z^{-1}}{| -2z^{-1} }$$

$$\boxed{z^{-1}} \quad - \frac{1}{| -z^{-1} } + \frac{0.5}{| -0.5z^{-1} }$$

$$\boxed{|z| > 2} \quad |z^{-1}| < 1 \quad |2z^{-1}| < 1$$

$$\boxed{|z| > 1} \quad |z^{-1}| < 1 \quad |0.5z^{-1}| < 1$$

$$- \frac{1}{| -z } + \frac{0.5}{| -0.5z }$$

$$\cdot \frac{1}{z} \quad \cdot z \quad \cdot \frac{z}{z} \quad \cdot \frac{z}{z}$$

$$\frac{z^{-1}}{| -z^{-1} } - \frac{z^{-1}}{| -2z^{-1} }$$

$$+ \frac{z}{| -z } - \frac{z}{| -2z }$$

$$\cdot \frac{1}{z} \quad \cdot z \quad \cdot \frac{1}{2z} \quad \cdot 2z$$

$$- \frac{1}{| -z^{-1} } + \frac{0.5}{| -0.5z^{-1} }$$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$\left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)} \right)$$

$$\boxed{z} \quad -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$\boxed{z} \quad +\frac{z}{1-z} - \frac{z}{1-2z}$$

$$\boxed{|z| < 1} \quad f(z) \text{ causal} \quad (n \geq 0)$$

$$\boxed{|z| < 0.5} \quad f(z) \text{ causal} \quad (n \geq 1)$$

$$\boxed{z^{-1}} \quad \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$\boxed{z^{-1}} \quad -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$\boxed{|z| > 2}$$

$$X(z) \text{ causal} \quad (n \geq 1)$$

$$\boxed{|z| > 1}$$

$$X(z) \text{ causal} \quad (n \geq 0)$$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$\left(-\frac{z}{z-1} + \frac{0.5z}{z-0.5} \right)$$

$$\boxed{z} - \frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$\boxed{z} + \frac{z}{1-z} - \frac{z}{1-2z}$$

$|z| < 1$ $f(z)$ causal ($n \geq 0$)
 $X(z)$ anticausal ($n \leq 0$)

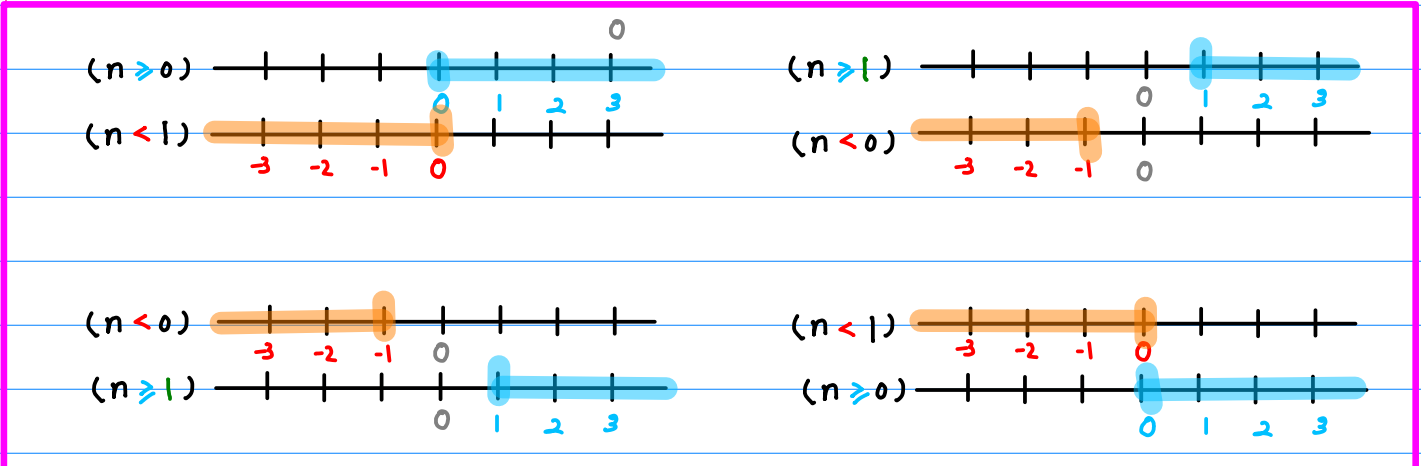
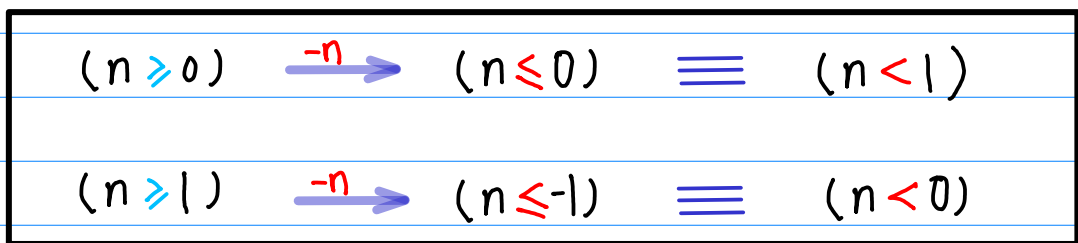
$|z| < 0.5$ $f(z)$ causal ($n \geq 1$)
 $X(z)$ anticausal ($n \leq -1$)

$$\boxed{z^{-1}} - \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$\boxed{z^{-1}} - \frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$|z| > 2$ $f(z)$ anticausal ($n \leq -1$)
 $X(z)$ causal ($n \geq 1$)

$|z| > 1$ $f(z)$ anticausal ($n \leq 0$)
 $X(z)$ causal ($n \geq 0$)



$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$|z| < 1$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^0 + 1^1 z^1 + 1^2 z^2 + \dots\right] - 1^{n+1} + \left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

$$|z| < 0.5$$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots\right] + 1^{n+1} - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

$$|z| > 2$$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) = +\left[1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \dots\right] + 1^{n+1} - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

$$* n = \quad 1 \quad 2 \quad 3$$

$$|z| > 1$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^1 + 1^3 z^2 + \dots\right] - 1^{n+1} + \left[\left(\frac{1}{2}\right)^1 z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

$$* n = \quad 0 \quad 1 \quad 2$$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -[1^0 + 1^1 z^1 + 1^2 z^2 + \dots] - 1^{n+1} + [(\frac{1}{2})^0 + (\frac{1}{2})^1 z^1 + (\frac{1}{2})^2 z^2 + \dots] + (\frac{1}{2})^{n+1}$$

$$\begin{aligned} 2 &= (\frac{1}{2})^{-1} \\ (\frac{1}{2}) &= 2^{-1} \end{aligned}$$

$$X(z) = -\left[\binom{-1}{n} (\frac{1}{2})^n + \binom{-1}{n-1} (\frac{1}{2})^{n-1} z^{-1} + \binom{-1}{n-2} (\frac{1}{2})^{n-2} z^{-2} + \dots \right] - 1^{n+1} + \left[\binom{n}{n} 2^n + \binom{n}{n-1} 2^{n-1} z^{-1} + \binom{n}{n-2} 2^{n-2} z^{-2} + \dots \right] + 2^{n+1}$$

$n = \quad 0 \quad -1 \quad -2$

$|z| < 0.5$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots] + 1^{n+1} - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots] - 2^{n+1}$$

$$X(z) = +\left[\binom{n}{n} (\frac{1}{2})^n z^1 + \binom{n}{n-1} (\frac{1}{2})^{n-1} z^2 + \binom{n}{n-2} (\frac{1}{2})^{n-2} z^3 + \dots \right] + 1^{n+1} - \left[\binom{n+1}{n} (\frac{1}{2})^n z^1 + \binom{n+1}{n-1} (\frac{1}{2})^{n-1} z^2 + \binom{n+1}{n-2} (\frac{1}{2})^{n-2} z^3 + \dots \right] - (\frac{1}{2})^{n+1}$$

$n = \quad -1 \quad -2 \quad -3$

$|z| > 2$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$\begin{aligned} 2 &= (\frac{1}{2})^{-1} \\ (\frac{1}{2}) &= 2^{-1} \end{aligned}$$

$$f(z) = +\left[\binom{0}{n} (\frac{1}{2})^n z^{-1} + \binom{0}{n-1} (\frac{1}{2})^{n-1} z^{-2} + \binom{0}{n-2} (\frac{1}{2})^{n-2} z^{-3} + \dots \right] + 1^{n+1} - \left[\binom{1}{n} (\frac{1}{2})^n z^{-1} + \binom{1}{n-1} (\frac{1}{2})^{n-1} z^{-2} + \binom{1}{n-2} (\frac{1}{2})^{n-2} z^{-3} + \dots \right] - (\frac{1}{2})^{n+1}$$

$$X(z) = +\left[\binom{n}{n} 1^n z^{-1} + \binom{n}{n-1} 1^{n-1} z^{-2} + \binom{n}{n-2} 1^{n-2} z^{-3} + \dots \right] + 1^{n+1} - \left[\binom{n+1}{n} 2^n z^{-1} + \binom{n+1}{n-1} 2^{n-1} z^{-2} + \binom{n+1}{n-2} 2^{n-2} z^{-3} + \dots \right] - 2^{n+1}$$

$n = \quad 1 \quad 2 \quad 3$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\binom{-1}{n} (\frac{1}{2})^n z^{-1} + \binom{-1}{n-1} (\frac{1}{2})^{n-1} z^{-2} + \binom{-1}{n-2} (\frac{1}{2})^{n-2} z^{-3} + \dots \right] - 1^{n+1} + \left[\binom{0}{n} 2^n z^{-1} + \binom{0}{n-1} 2^{n-1} z^{-2} + \binom{0}{n-2} 2^{n-2} z^{-3} + \dots \right] + 2^{n+1}$$

$$X(z) = -\left[\binom{-1}{n} 1^n z^{-1} + \binom{-1}{n-1} 1^{n-1} z^{-2} + \binom{-1}{n-2} 1^{n-2} z^{-3} + \dots \right] - 1^{n+1} + \left[\binom{n}{n} (\frac{1}{2})^n z^{-1} + \binom{n}{n-1} (\frac{1}{2})^{n-1} z^{-2} + \binom{n}{n-2} (\frac{1}{2})^{n-2} z^{-3} + \dots \right] + (\frac{1}{2})^{n+1}$$

$n = \quad 0 \quad 1 \quad 2$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$|z| < 1$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -[1^0 + 1^1 z^1 + 1^2 z^2 + \dots] + \left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right]$$

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$X(z) = -\left[\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-1} + \left(\frac{1}{2}\right)^{-3} z^{-2} + \dots\right] + [2^0 + 2^1 z^1 + 2^2 z^2 + \dots]$$

$$a_n = -1^{n+1} + 2^{n+1} \quad (n < 1)$$

$$|z| < 0.5$$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots] - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots]$$

$$a_n = +1^{n+1} - 2^{n+1} \quad (n \geq 1)$$

$$X(z) = +\left[\left(\frac{1}{2}\right)^{-1} z^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-2} + \left(\frac{1}{2}\right)^{-3} z^{-3} + \dots\right] - \left[\left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \dots\right]$$

$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$|z| > 2$$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \dots\right] - \left[\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \dots\right]$$

$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$X(z) = +[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots] - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots]$$

$$a_n = +1^{n+1} - 2^{n+1} \quad (n \geq 1)$$

$$|z| > 1$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right] + [2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \dots]$$

$$a_n = -1^{n+1} + 2^{n+1} \quad (n < 1)$$

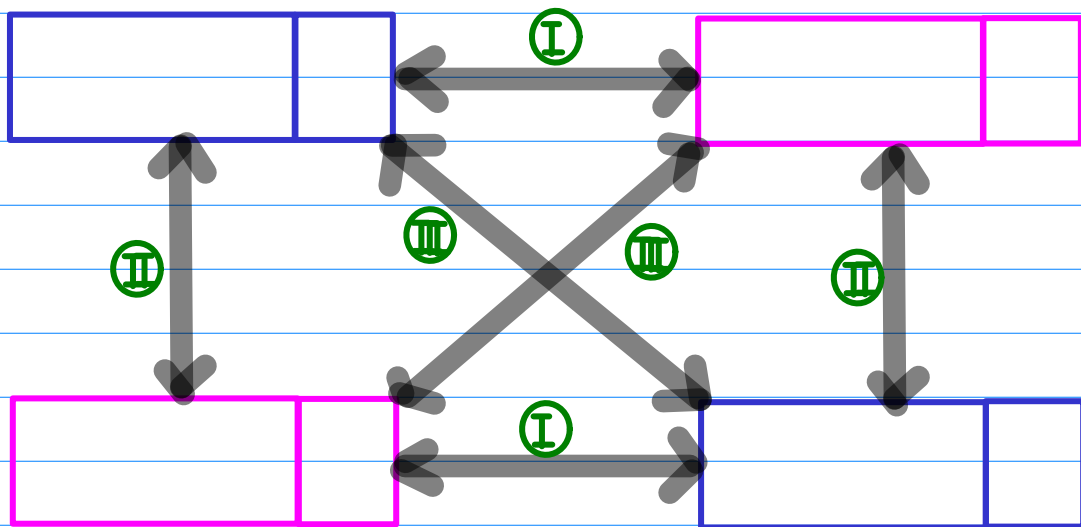
$$X(z) = -[1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \dots] + \left[\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right]$$

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$\textcircled{\text{I}} \quad (z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$\textcircled{\text{II}} \quad (z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$\textcircled{\text{III}} \quad (z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$



$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$f(z)$	$ROC(z)$ $ z < p$	\longleftrightarrow	a_n	$RNG(n)$ $n \geq 0$
--------	-----------------------	-----------------------	-------	------------------------

$$(z, R) \Leftrightarrow (a_n, N)$$

Ⓘ	$f(z^{-1})$	$ROC(z^{-1})$ $ z > \frac{1}{p}$	\longleftrightarrow	a_{-n}	$RNG(-n)$ $n < 1$
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$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

Ⓙ	$f(z)$	$ROC(z^{-1})$ $ z > \frac{1}{p}$	\longleftrightarrow	$-a_n$	$\overline{RNG}(n)$ $n < 0$
---	--------	--------------------------------------	-----------------------	--------	--------------------------------

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

Ⓚ	$f(z^{-1})$	$ROC(z)$ $ z < p$	\longleftrightarrow	$-a_{-n}$	$\ll RNG(n) \gg$ $n \geq 1$
---	-------------	-----------------------	-----------------------	-----------	--------------------------------

$$\textcircled{\text{I}} + \textcircled{\text{II}}$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Ⓛ	$X(z)$	$ROC(z)$ $ z < p$	\longleftrightarrow	a_{-n}	$RNG(-n)$ $n < 1$
---	--------	-----------------------	-----------------------	----------	----------------------

$$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$$\textcircled{\text{III}} = \textcircled{\text{I}} + \textcircled{\text{II}}$$

$\textcircled{\text{III}}$	$f(z^{-1})$	ROC(z) $ z < p$	\longleftrightarrow	$-a_{-n}$	$\ll \text{RNG}(n) \gg$ $n \geq 1$	$\textcircled{\text{I}} + \textcircled{\text{II}}$
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	$f(z)$	ROC(z) $ z < p$	\longleftrightarrow	a_n	RNG(n) $n \geq 0$	
--	--------	---------------------	-----------------------	-------	----------------------	--

$\textcircled{\text{I}}$	$f(z^{-1})$	ROC(z^{-1}) $ z > \frac{1}{p}$	\longleftrightarrow	a_{-n}	RNG(-n) $n < 1$	
--------------------------	-------------	------------------------------------	-----------------------	----------	--------------------	--

$\textcircled{\text{II}}$	$f(z)$	ROC(z^{-1}) $ z > \frac{1}{p}$	\longleftrightarrow	$-a_n$	$\overline{\text{RNG}(n)}$ $n < 0$	
---------------------------	--------	------------------------------------	-----------------------	--------	---------------------------------------	--

	$f(z^{-1})$	ROC(z) $ z < p$	\longleftrightarrow	$-a_{-n}$	$\overline{\text{RNG}(-n)}$ $n \geq 1$	
--	-------------	---------------------	-----------------------	-----------	---	--

$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Compare ① with ④

$$\begin{array}{ccc} \text{ROC}(z) & f(z) & \longleftrightarrow & a_n & \text{RNG}(n) \\ |z| < p & & & & n \geq 0 \end{array}$$

①	$\text{ROC}(z^{-1})$ $ z > \frac{1}{p}$	$f(z)$	\longleftrightarrow	$-a_n$	$\overline{\text{RNG}(n)}$ $n < 0$
---	---	--------	-----------------------	--------	---------------------------------------

- | - |
- | complement

④	$\text{ROC}(z)$	$X(z)$	\longleftrightarrow	a_{-n}	$\text{RNG}(-n)$ $n < $
---	-----------------	--------	-----------------------	----------	-----------------------------

- n - n
Symmetrical

$$X(z) \quad |z| < 0.5 \quad |z| > 2$$

anticausal causal

$$\textcircled{1} - \textcircled{B} \quad \frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$|z| < 1 \quad X(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$-\left(|^1 z^0 + |^2 z^1 + |^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots \right)$$

$$-\left(|^1 z^0 + |^2 z^1 + |^3 z^2 + \dots \right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \dots \right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$$|z| > 2 \quad X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$+ \left(|^0 z^1 + |^1 z^2 + |^2 z^3 + \dots \right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots \right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\textcircled{2} - \textcircled{B} \quad \frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{z-1} + \frac{0.5z}{z-0.5} \right)$$

$$|z| < 0.5 \quad X(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \quad \boxed{+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$+ \left(|^0 z^1 + |^1 z^2 + |^2 z^3 + \dots \right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots \right)$$

$$+ \left(|^0 z + |^1 z^2 + |^2 z^3 + \dots \right) - \left(2^0 z + 2^1 z^2 + 2^2 z^3 + \dots \right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$$|z| > 1 \quad X(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad \boxed{-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$-\left(|^1 z^0 + |^2 z^1 + |^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots \right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

Ⓘ

$ROC(z^{-1})$	$f(z)$	\longleftrightarrow	$-a_n$	$\overline{RNG}(n)$
$ z > \frac{1}{p}$				$n < 0$

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$|z| < 0.5$ $X(z)$

$a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)$

$|z| > 2$ $X(z)$

$b_n = \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)$

$\{ |z| < 0.5 \} \cap \{ |z| > 2 \} = \emptyset \quad \longrightarrow \quad a_n + b_n = 0$

$a_n = -b_n$

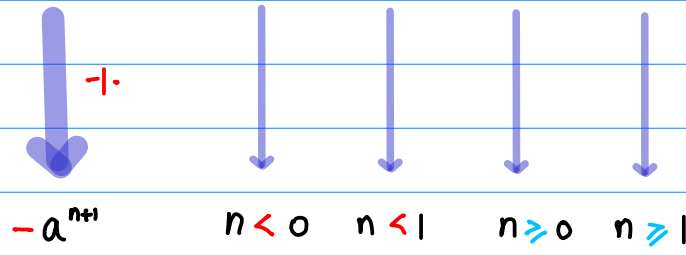
ROC
 $|z| < a$

$X(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$

a^{n+1} $n \geq 0$ $n \geq 1$ $n < 0$ $n < 1$

ROC'
 $|z| > a^{-1}$

$X(z) = -\frac{z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$
 $= -\sum_{k=-1}^{\infty} a^{k+1} z^k$



$\frac{a}{1-az}$	=	$\sum_{n=0}^{\infty} a^{n+1} z^n$	$\frac{z}{1-az}$	=	$\sum_{n=1}^{\infty} a^{n-1} z^n$
$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$	=	$-\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	=	$-\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$
		$= -\sum_{k=-1}^{\infty} a^{k+1} z^k$			$= -\sum_{k=0}^{\infty} a^{k-1} z^k$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$-\frac{z^{-1}}{1-a^1 z^1} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$= -\sum_{k=-1}^{\infty} a^{k+1} z^k$$

$$\frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$-\frac{a^{-1}}{1-a^1 z^1} \Rightarrow -\sum_{n=0}^{\infty} a^{-n+1} z^{-n}$$

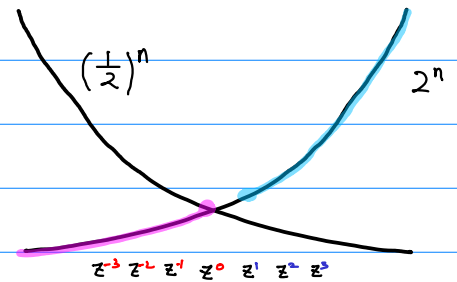
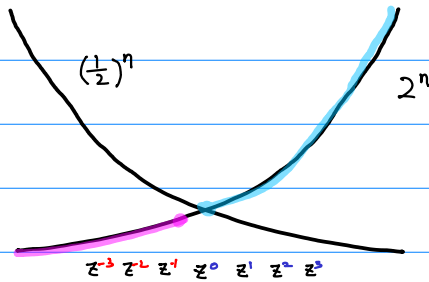
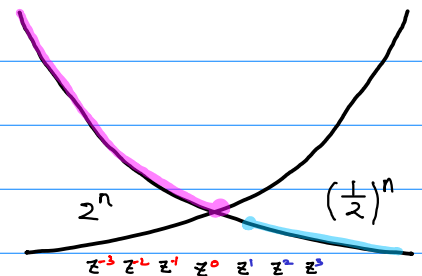
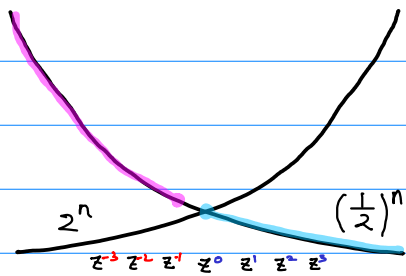
$$= -\sum_{k=0}^{\infty} a^{k-1} z^k$$

$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \dots$$

$$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \dots$$

$$z + a z^2 + a^2 z^3 + a^3 z^4 + \dots$$

$$a^{-1} + a^{-2} z^1 + a^{-3} z^2 + a^{-4} z^3 + \dots$$



IV

$\text{ROC}(z)$ $ z < p$	$X(z)$	\longleftrightarrow	a_{-n}	$\text{RNG}(-n)$ $n \leq 0$
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$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$$|z| < 0.5 \quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots \right)$$

$n=0 \quad n=1 \quad n=2$
 $n=0 \quad n=1 \quad n=2$

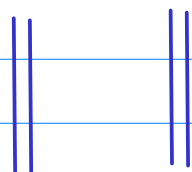
$$|z| < 0.5 \quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots \right)$$

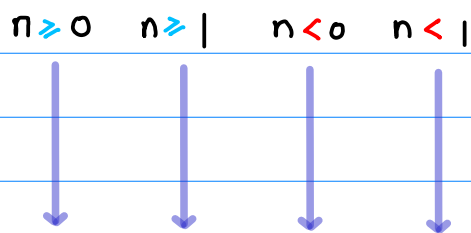
$$-\left(\left(\frac{1}{2}\right)^1 z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots \right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \dots \right)$$

$n=0 \quad n=1 \quad n=2$
 $n=0 \quad n=1 \quad n=2$

$$\text{ROC} \quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \quad a^{n+1} \quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$



$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$$



$$\text{ROC} \quad X(z) = \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$$

$$a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$$

$$n \leq 0 \quad n \leq 1 \quad n > 0 \quad n > 1$$

II

ROC(z^{-1}) $ z > \frac{1}{p}$	$f(z^{-1})$	a_{-n}	RNG($-n$) $n < 1$
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$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -[1 + 1^2 z^1 + 1^3 z^2 + \dots] + [(\frac{1}{2}) + (\frac{1}{2})^2 z^1 + (\frac{1}{2})^3 z^2 + \dots]$$

$$a_n = -|^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

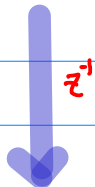
$$f(z) = -[(\frac{1}{z})^1 z^0 + (\frac{1}{z})^2 z^1 + (\frac{1}{z})^3 z^2 + \dots] + [2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \dots]$$

$$a_n = -|^{n+1} + 2^{n+1} \quad (n < 1)$$

ROC

$|z| < a$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$



ROC'

$|z| > a^{-1}$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n} = \sum_{k=0}^{-\infty} a^{-k+1} z^k$$

a^{n+1}



$$a^{-n+1} = (\frac{1}{a})^{n-1}$$

$n \geq 0$

$n \geq 1$

$n < 0$

$n < 1$



$n < 1$

$n < 0$

$n \geq 0$

$n \geq 0$

