# Applicatives Methods (3B)

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# The definition of Applicative

```
class (Functor f) => Applicative f where

pure :: a -> f a

(<*>) :: f (a -> b) -> f a -> f b
```

f (a -> b) :: a function wrapped in f
f a :: a value wrapped in f

The class has a two methods:

pure brings arbitrary values into the functor

(<\*>) takes a function wrapped in a functor f and a value wrapped in a functor f and returns the result of the application which is also wrapped in a functor f

# The Maybe instance of Applicative

```
instance Applicative Maybe where

pure = Just

(Just f) <*> (Just x) = Just (f x)

_ <*> _ = Nothing
```

```
pure wraps the value with Just;
```

```
(<*>) applies
the <u>function</u> wrapped in <u>Just</u>
to the <u>value</u> wrapped in <u>Just</u> if both exist,
and results in <u>Nothing</u> otherwise.
```

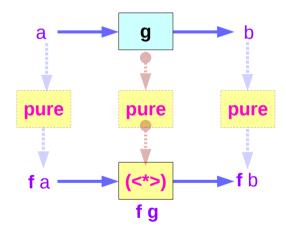
# An Instance of the Applicative Typeclass

f : Functor, Applicative

(Functor f) => Applicative f

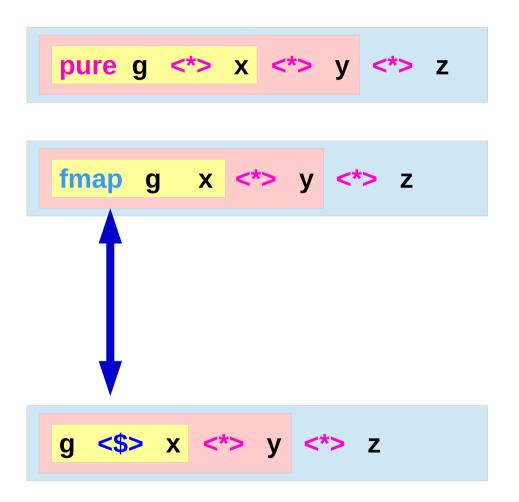
```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just <u>f</u>) <*> something = fmap <u>f</u> something
```

 $\underline{\mathbf{f}}$ : function in a context



(Functor f) => Applicative f

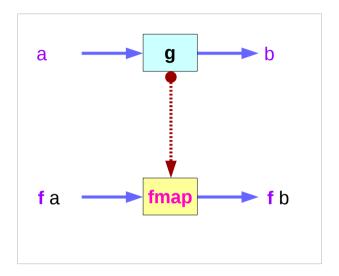
# Left associative <\*>, fmap, and <\$>

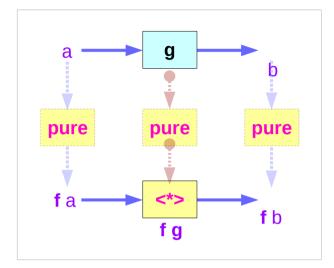


g :: f (a -> b -> c -> d) x :: f a y :: f b z :: f c

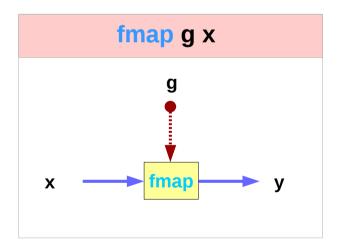
infix operator <\$>

# fmap g x = (pure g) <\*> x

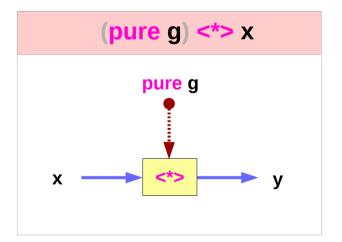




pure = f



x :: f a y :: f b



# Left associative <\*> examples

# Infix Operators <\*> vs <\$> - a type view

# h <\*> x <\*> y

h :: f (a -> b -> c)

x :: f a

y :: **f** b

x :: f a

h <\*> x :: f (b -> c)

x :: f a

h <\*> x :: **f** (b -> c)

y :: f b

h <\*> x <\*> y :: f c

x :: f a

y :: **f** b

x :: f a

g :: (a -> b -> c)

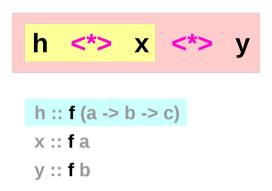
x :: f a

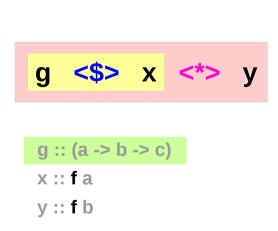
g <\$> x :: **f** (b -> c)

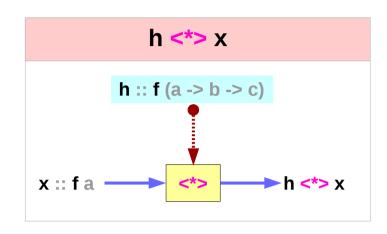
y :: **f** b

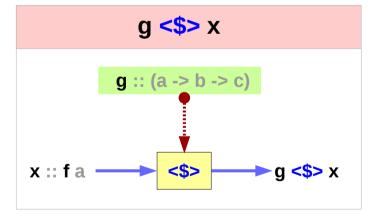
g <\$> x <\*> y :: f c

# Infix Operators <\*> vs <\$> - a curried function view





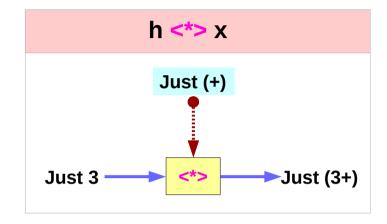




# Infix Operators <\*> vs <\$> examples

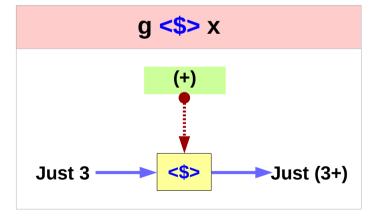


Just (+) <\*> Just 3 <\*> Just 2 Just (+3) <\*> Just 2 Just 5





(+) <\$> Just 3 <\*> Just 2 Just (+3) <\*> Just 2 Just 5



# the minimal complete definition

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

the minimal complete definition

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b
g <$> x = fmap g x
```

Not in the minimal complete definition

```
instance Applicative Maybe where
pure = Just
Nothing <*> _ = Nothing
(Just g) <*> something = fmap g something
```

# The Applicative Typeclass

```
Applicative is a <u>superclass</u> of Monad.
every Monad is also a Functor and an Applicative fmap, pure, (<*>) can all be used with monads.
```

a Monad instance

requires **Functor** and **Applicative** instances. defines the types and roles of **return** and (>>)

fmap: defined in Functors

pure, (<\*>): defined in Applicatives

return, (>>): defined in Monads

# (<\$>) vs (\$)

#### (<\$>) infix operator

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b
g <$> x = fmap g x
```

#### The \$ operator is for avoiding parentheses

```
putStrLn (show (1 + 1))
putStrLn $ show (1 + 1)
putStrLn $ show $ 1 + 1 — right associative
```

(\$) calls the <u>function</u> which is its left-hand argument of \$ on the <u>value</u> which is its right-hand argument of \$

# The Applicative Laws

The identity law: pure id <\*> v = v

u < > v < > w = fa

Homomorphism: pure g < pure x = pure (g x) g :: a -> b x :: a

# The Identity Law

The identity law

**pure** id <\*> v = v

id :: a -> a

v :: f a

pure to inject <u>values</u> into the <u>functor</u> in a *default*, *featureless* way, so that the result is as close as possible to the <u>plain</u> value.

applying the **pure id** morphism does nothing, exactly like with the plain **id** function.

# The Homomorphism Law

The homomorphism law

**pure g <\*> pure x = pure (g x)** 

g :: a -> b

x :: a

applying a "pure" <u>function</u> to a "pure" <u>value</u> is the same as applying the <u>function</u> to the <u>value</u> in the <u>ordinary way</u> and then using **pure** on the result.

means **pure** <u>preserves</u> function application.

applying a non-effectful function g

to a <u>non-effectful</u> argument **x** in an <u>effectful</u> <u>context</u> <u>pure</u> is the same as just **applying** the function **g** to the argument **x** and then injecting the result **(f x)** into the <u>effectual</u> <u>context</u> with <u>pure</u>.

# The Interchange Law

The interchange law

(\$ y) is the function that supplies yas argument to another functiona higher order function

**Function \$ Argument \$ y**( **y** ) as a single argument

applying a <u>morphism</u> **u** to a <u>"pure" value</u> <u>pure</u> **y** is the same as applying <u>pure</u> (\$ **y**) to the <u>morphism</u> **u** 

Just (+3) <\*> Just 2 Just (\$ 2) <\*> Just (+3)

when evaluating the application of an <u>effectful function</u> (**u**) to a <u>pure argument</u> (<u>pure y</u>), the <u>order doesn't matter</u> – commutative.

# The Composition Law

The composition law pure (.) <\*> u <\*> v <\*> w = u <\*> (v <\*> w) <math>w :: fa v :: f(a -> b) u :: f(b -> c)

pure (.) <u>composes</u> morphisms similarly to how (.) <u>composes</u> functions:

applying the composed mourphism

pure (.) <\*> 
$$\mathbf{u}$$
 <\*>  $\mathbf{v}$  to  $\mathbf{w}$  gives the same result ( $\mathbf{u}$  <\*> ( $\mathbf{v}$  <\*>  $\mathbf{w}$ )) as applying  $\mathbf{u}$  to the result ( $\mathbf{v}$  <\*>  $\mathbf{w}$ ) of applying  $\mathbf{v}$  to  $\mathbf{w}$ 

it is expressing a sort of <u>associativity</u> property of (<\*>).

```
w :: f a -- value
v :: f (a -> b) -- func1
u : f (b -> c) -- func2
```

```
v <*> w :: f b
u <*> (v <*> w) :: f c
```

```
pure (.) <*> u <*> v :: f (a -> c )
pure (.) <*> u <*> v <*> w :: f c
```

# The Composition Law and Left Associativity

```
The composition law
                             pure (.) <*> u <*> v <*> w = u <*> (v <*> w)
                                                                              w :: fa v :: f(a -> b) u :: f(b -> c)
                                                                             f(b \rightarrow c)
                                                                                              f (a -> b)
                                                                                                 pure h
                                                                              pure g
pure (.) <*> pure g <*> pure h <*> pure x
                                                     (g.h) x
                                                                                       pure (.)
((pure (.) <*> pure q) <*> pure h) <*> pure x
                                                                                      f(a \rightarrow c)
= pure q <*> (pure h <*> pure x)
                                                     g (h x)
                                                                              u = pure g :: f (b -> c) g :: (b -> c)
                                                                              v = pure h :: f (a -> b) h :: (a -> b)
                                                                              w = pure x :: f a
                                                                                                          x :: a
Left associative
                       u <*> v <*> w = (u <*> v) <*> w
                                                                             u :: f(c -> b -> a) v :: fc w :: fb
                                                                             u :: f(c -> b -> a)
                                                                             v :: f c
                                                                             u < > v :: f (b -> a)
                                                                             w :: f b
                                                                             u < > v < > w = f a
  https://en.wikibooks.org/wiki/Haskell/Applicative functors
```

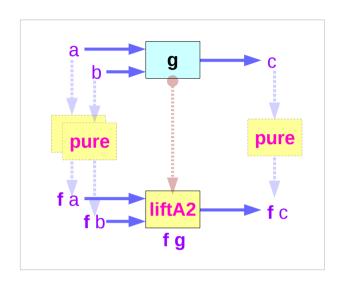
liftA2 :: (a -> b -> c) -> f a -> f b -> f c

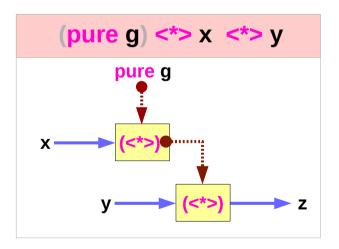
<u>lift</u> a <u>binary</u> <u>function</u> (**a->b->c**) to actions.

Some functors support an implementation of **liftA2** that is more efficient than the default one.

**liftA2** may have an <u>efficient</u> implementation whereas **fmap** is an <u>expensive</u> operation,

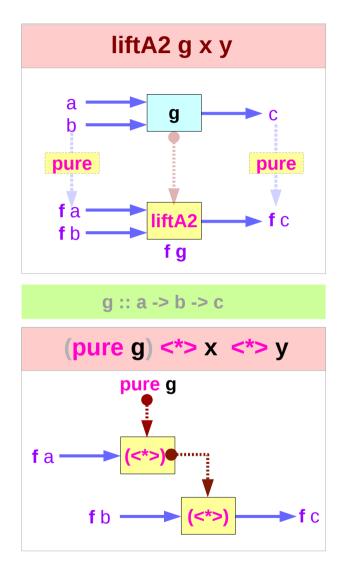
sometimes better to use **liftA2** than to use **fmap** over the structure and then use <\*>.

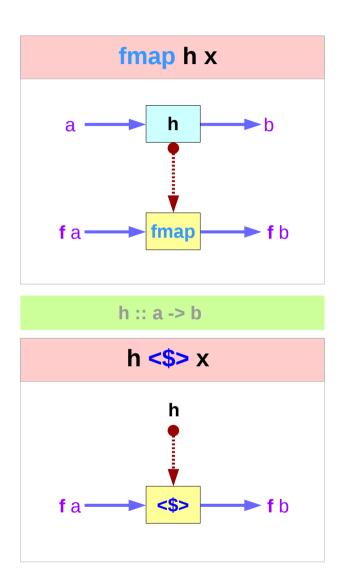




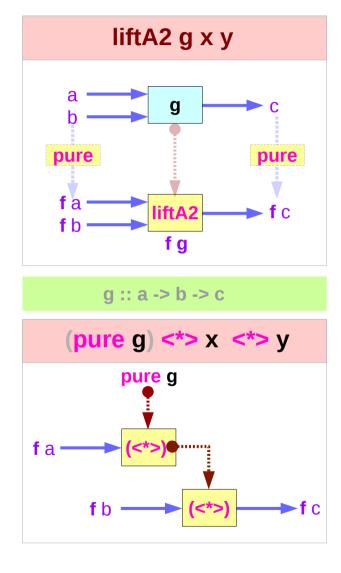
http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

# liftA2, <\*>, fmap, <\$>

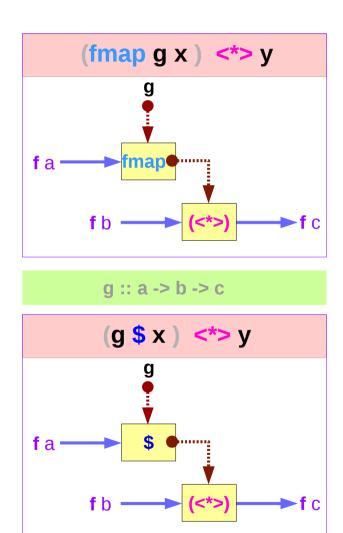




# pure g <\*> x <\*> y = (fmap g x) <\*> y







```
liftA2 g x y
liftA2 :: (a -> b -> c) -> f a -> f b -> f c
g:: a -> b -> c
x :: f a
y :: f b
liftA2 g x y :: f c
pure g <*> x <*> y
g:: a -> b -> c
x :: f a
y :: f b
z :: f c
pure g <*> x <*> y :: f c
```

liftA2 g x y g pure pure liftA2 f b f g g :: a -> b -> c (pure g) <\*> x <\*> y pure g

https://wiki.haskell.org/Applicative\_functor

```
(a -> b -> c) -> (f a -> f b -> f c)
fmap :: (a -> b) -> (f a -> f b)
fmap2 :: Functor f => (a -> b -> c) -> (f a -> f b -> f c)
fmap2 h fa fb = undefined
h :: a -> b -> c
fa :: f a
fb :: f b
h
       :: a -> (b -> c)
fmap h :: f a -> f (b -> c)
fmap h fa :: f (b -> c)
```

http://www.openhaskell.com/lectures/applicative.html

```
class Functor f => Applicative f where
 pure :: a -> f a
 (<*>) :: f (a -> b) -> f a -> f b
pure
       :: a -> f a
fmap :: (a -> b) -> f a -> f b
fmap2 :: (a -> b -> c) -> fa -> fb -> fc
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
liftA2 h fa fb = (h `fmap` fa) <*> fb
liftA2 h fa fb = h < $> fa <*> fb
(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
liftA3 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d
liftA3 h fa fb fc = ((h < \$ > fa) < * > fb) < * > fc
```

http://www.openhaskell.com/lectures/applicative.html

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c

liftA2 (+) (Just 5) (Just 6) = Just 11

<*> :: Applicative f => f (a -> b) -> f a -> f b

(Just (+5)) <*> (Just 6) = Just 11

let v1 = IO (Just (+5))

let v2 = IO (Just 6)
```

liftA2 (<\*>) v1 v2 = IO (Just 11)

https://blog.ssanj.net/posts/2014-08-10-boosting-liftA2.html

# <\*> or liftA2 implementations

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

A minimal complete definition:

either one of the two

- **1) pure** and **<\*>**
- 2) pure and liftA2

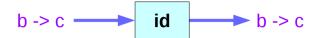
If it defines <u>both</u>, then they must behave the same as their default definitions:

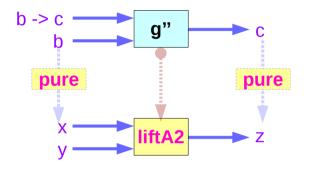
http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

# (<\*>) = liftA2 id

liftA2 id x y = id 
$$<$$
 x  $<$  y = x  $<$  y

liftA2 id 
$$x y = x <*> y$$



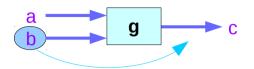


liftA2 g x y = g < $>>$ x < $>>$ y	g :: a -> b -> c	x :: f a y :: f b	
liftA2 g" x y = g" <\$> x <*> y	g" :: (b -> c) -> b -> c	x :: f (b -> c) y :: f b	
liftA2 id x y = id $<$ x $<$ y = x $<$ y	id :: (b -> c) -> (b -> c)	x :: f (b -> c) y :: f b	

(<\*>) = liftA2 id

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

# g":: (b->c) -> b->c



view the function as having one input only

consider the case when  $\mathbf{a}$  is  $(\mathbf{b} \rightarrow \mathbf{c})$ 

id :: 
$$(b \rightarrow c) \rightarrow (b \rightarrow c)$$

Then  $\mathbf{g}$ " is the same as  $\mathbf{id}$ 

$$g" :: (b \rightarrow c) \rightarrow (b \rightarrow c)$$

Actually, using the **liftA** commands

we can pull <u>results</u> of applicative functors

into a scope where we can talk

exclusively about <u>functor results</u> **c** 

and not about effects. **f c** 

Note that functor results can also be functions. c

This scope is simply a function,

which contains the code that we used in the non-functorial setting.

#### liftA3

```
(x g h \rightarrow let y = g x in h y y)
fx fg fh
```

The order of effects is entirely determined by the order of arguments to liftA3

.

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

Consider the non-functorial expression:

x :: x g :: x -> y h :: y -> y -> z let y = g x in h y y

Very simple. Now we like to generalize this to

fx :: f x fg :: f (x -> y) fh :: f (y -> y -> z)

https://wiki.haskell.org/Applicative\_functor

However, we note that

```
let fy = fg <*> fx
in fh <*> fy <*> fy
```

runs the effect of fy

twice. E.g. if fy

writes something to the terminal then fh <\*> fy <\*> fy

writes twice. This could be intended, but how can we achieve, that the effect is run only once and the result is used twice? Actually, using the liftA

commands we can pull results of applicative functors into a scope where we can talk exclusively about functor results and not about effects. Note that functor results can also be functions. This scope is simply a function, which contains the code that we used in the non-functorial setting.

#### liftA3

```
(x g h \rightarrow let y = g x in h y y)
fx fg fh
```

The order of effects is entirely determined by the order of arguments to liftA3

•

https://wiki.haskell.org/Applicative\_functor

# liftA2(<\*>)

10 down vote accepted

The wiki article says that **liftA2** (<\*>) can be used to <u>compose applicative functors</u>. It's easy to see how to use it from its type:

```
o :: (Applicative f, Applicative f1) =>
f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)
o = liftA2 (<*>)
```

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

# liftA2(<\*>) Examples

So to if **f** is **Maybe** and **f1** is **[]** we get:

The other way around is:

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

# **LiftA2 (:)**

your ex function is equivalent to **liftA2 (:)**:

```
test1 = liftA2 (:) "abc" ["pqr", "xyz"]
```

To use (:) with deeper applicative stack you need multiple applications of liftA2:

```
*Main> (liftA2 . liftA2) (:) (Just "abc") (Just ["pqr", "xyz"])

Just ["apqr", "axyz", "bpqr", "bxyz", "cpqr", "cxyz"]
```

However it only works when both operands are equally deep. So besides double liftA2 you should use pure to fix the level:

```
*Main> (liftA2 . liftA2) (:) (pure "abc") (Just ["pqr", "xyz"])

Just ["apqr", "axyz", "bpqr", "bxyz", "cpqr", "cxyz"]
```

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

# related operators

Functor map <\$>

(<\$>) :: Functor f => (a -> b) -> f a -> f b

(<\$) :: Functor f => a -> f b -> f a

(\$>) :: Functor f => f a -> b -> f b

The **<\$>** operator is just a synonym for the **fmap** function from the Functor typeclass.

This function generalizes the **map** function for lists to many other data types, such as **Maybe**, **IO**, and **Map**.

# <\$> examples

```
#!/usr/bin/env stack
-- stack --resolver ghc-7.10.3 runghc
import Data.Monoid ((<>))

main :: IO ()
main = do
   putStrLn "Enter your year of birth"
   year <- read <$> getLine
   let age :: Int
       age = 2020 - year
   putStrLn $ "Age in 2020: " <> show age
```

# <\$, \$> operators

In addition, there are two additional operators provided which replace a value inside a Functor instead of applying a function.

This can be both more convenient in some cases, as well as for some Functors be more efficient.

$$x < y = y > x$$

$$x > y = y < x$$

## <\*> related operators

Applicative function application <\*>

(<\*>) :: Applicative f => f (a -> b) -> f a -> f b

(\*>) :: Applicative f => f a -> f b -> f b

(<\*) :: Applicative f => f a -> f b -> f a

Commonly seen with <\$>, <\*> is an operator that applies a wrapped function to a wrapped value. It is part of the Applicative typeclass, and is very often seen in code like the following:

foo <\$> bar <\*> baz



For cases when you're dealing with a Monad, this is equivalent to:

```
do x <- bar
y <- baz
return (foo x y)</pre>
```

Other common examples including parsers and serialization libraries.

Here's an example you might see using the aeson package:

```
data Person = Person { name :: Text, age :: Int } deriving Show
```

-- We expect a JSON object, so we fail at any non-Object value.

instance FromJSON Person where

```
parseJSON (Object v) = Person <$> v .: "name" <*> v .: "age"
parseJSON _ = empty
```

# \*> operator

To go along with this, we have two helper operators that are less frequently used:

\*> ignores the value from the first argument. It can be defined as:

Or in do-notation:

For Monads, this is completely equivalent to >>.

# <\* operator

<\* is the same thing in reverse: perform the first action then the second,

but only take the value from the first action.

Again, definitions in terms of <\*> and do-notation:

res <- a1

\_ <- a2

return res

# (\*> v.s. >>) and (pure v.s. return)

```
(*>) :: Applicative f \Rightarrow fa \Rightarrow fb \Rightarrow fb
```

$$(>>) :: Monad m => m a -> m b -> m b$$

the constraint changes from **Applicative** to **Monad**.

#### References

- [1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
- [2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf