## Signal Processing

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## Based on

Signal Processing with Free Software : Practical Experiments
F. Auger
: y = filter (b, a, x)
: $[\mathrm{y}, \mathrm{sf}]=$ filter (b, a, $\mathrm{x}, \mathrm{si})$
: [y, sf] = filter (b, a, x, [], dim)
: $[y$, sf] $=$ filter ( $b, a, x$, si, dim $)$

## filter (2)

Apply a 1-D digital filter to the data x .
filter returns the solution to the following
linear, time-invariant difference equation:

$$
\sum_{k=0}^{N} a(k+1) y(n-k)=\sum_{k=0}^{M} b(k+1) x(n-k) \quad \text { for } 1 \leq n \leq \operatorname{length}(x)
$$

where $N=$ length( $a$ )- 1 and $M=$ length(b)-1.

$$
\begin{array}{ll}
\boldsymbol{a}=[a(1), a(2), \cdots, a(N+1)] & \boldsymbol{x}=[x(1), x(2), \cdots, x(L+1)] \\
\boldsymbol{b}=[b(1), b(2), \cdots, b(M+1)] & \text { length }(\boldsymbol{x})=L+1 \\
\text { length }(\boldsymbol{a})=N+1 & \\
\text { length }(\boldsymbol{b})=M+1 & 1 \leq n \leq L+1
\end{array}
$$

## filter (3)

The result is calculated over the first non-singleton dimension of $x$ or over dim if supplied.
$x(1), \quad x(2), \quad x(3), \quad \cdots \quad, \quad x(K)$

$x(:, 1), \quad x(:, 2), \quad x(:, 3), \cdots, x(:, K)$

$$
x(1), \quad x(2), \quad x(3), \quad \cdots \quad, \quad x(K)
$$

$$
\begin{array}{lllll}
y(1,1), & y(1,2), & y(1,3), & \cdots & , \\
y(2,1), & y(2,2), & y(2,3), & \cdots & , \\
y(2, K) \\
y(3,1), & y(3,2), & y(3,3), & \cdots & , \\
& & & \\
& & & \\
y(3, K) \\
y(L, 1), & y(L, 2), & y(L, 3), & \cdots & , \\
& y(L, K)
\end{array}
$$

$$
y(:, 1), \quad y(:, 2), \quad y(:, 3), \cdots \quad, \quad y(:, K)
$$

## filter (4)

$$
\begin{aligned}
& \sum_{k=0}^{N} a(k+1) y(n-k)=\sum_{k=0}^{M} b(k+1) x(n-k) \quad \text { for } 1 \leq n \leq \text { length }(x) \\
& a(1) y(n)+\sum_{k=1}^{N} a(k+1) y(n-k)=\sum_{k=0}^{M} b(k+1) x(n-k) \\
& a(1) y(n)=-\sum_{k=1}^{N} a(k+1) y(n-k)+\sum_{k=0}^{M} b(k+1) x(n-k) \\
& y(n)=-\sum_{k=1}^{N} \frac{a(k+1)}{a(1)} y(n-k)+\sum_{k=0}^{M} \frac{b(k+1)}{a(1)} x(n-k) \\
& y(n)=-\sum_{k=1}^{N} c(k+1) y(n-k)+\sum_{k=0}^{M} d(k+1) x(n-k) \quad \text { for } 1 \leq n \leq \operatorname{length}(x)
\end{aligned}
$$

where $c=a / a(1)$ and $d=b / a(1)$.

## filter (5)

$\mathbf{s i}$ : the initial state of the system
sf : the final state
the state vector is a column vector whose length is equal to the length of the longest coefficient vector - 1

No si is presented, the zero initial state.
in terms of the $z$ transform,
$\mathbf{y}$ is the result of passing the discrete-time signal $\mathbf{x}$
through a system characterized
by the following rational system function:

$$
H(z)=\frac{\sum_{k=0}^{M} d(k+1) z^{-k}}{1+\sum_{k=1}^{N} c(k+1) z^{-k}}
$$

## freqz (1)

: $[\mathbf{h}, \mathbf{w}]=$ freqz (b, a, n, "whole")
: $[\mathrm{h}, \mathbf{w}]=$ freqz (b)
: $[h, w]=$ freqz (b, a)
: $[h, w]=$ freqz (b, a, n)
: h = freqz (b, a, w)
: [h, w] = freqz (..., Fs)
: freqz (...)

## freqz (2)

Return the complex frequency response $\mathbf{h}$
of the rational IIR filter
with the numerator coefficients $\mathbf{b}$ and the denominator coefficients a

The response is evaluated
at $\mathbf{n}$ angular frequencies between $\mathbf{0}$ and $\mathbf{2 *}$ pi.
The output value $\mathbf{w}$ is a vector of the frequencies.
$\mathbf{h}$ : the frequency response vector
$\mathbf{w}$ : the frequency vector

## freqz (3)

If $\mathbf{a}$ is omitted, the denominator is assumed to be $\mathbf{1}$
(this corresponds to a simple FIR filter).
If $n$ is omitted, a value of 512 is assumed.
For fastest computation, n should factor
into a small number of small primes.
If the fourth argument, "whole", is omitted the response is evaluated at frequencies between $\mathbf{0}$ and $\mathbf{p i}$.

## freqz (4)

```
freqz (b, a, w)
```

Evaluate the response at the specific frequencies in the vector $\mathbf{w}$. The values for $\mathbf{w}$ are measured in radians.

```
freqz (...)
```

Plot the magnitude and phase response of $\mathbf{h}$ rather than returning them.

## freqz (5)

$$
[\ldots]=\text { freqz }(\ldots, \text { Fs })
$$

Return frequencies in Hz instead of radians assuming a sampling rate Fs.
If you are evaluating the response at specific frequencies $\mathbf{w}$, those frequencies should be requested in $\underline{H z}$ rather than radians.

```
[h, w] = freqz (b, a, n, "whole", Fs)
[h, w] = freqz (b, Fs)
[h,w] = freqz (b, a, Fs)
[h,w] = freqz (b, a, n, Fs)
h = freqz (b, a, w, Fs)
```


## freqz_plot

: freqz_plot (w, h)
: freqz_plot (w, h, freq_norm)

Plot the magnitude and phase response of $\mathbf{h}$.
If the optional freq_norm argument is true, the frequency vector $\mathbf{w}$ is in units of normalized radians.
If freq_norm is false, or not given, then $\mathbf{w}$ is measured in Hertz.

## conv

: conv (a, b)
: conv (a, b, shape)

Convolve two vectors $\mathbf{a}$ and $\mathbf{b}$.
The output convolution is a vector with length equal to length (a) + length (b) - 1 . When $\mathbf{a}$ and $\mathbf{b}$ are the coefficient vectors of two polynomials, the convolution represents the coefficient vector of the product polynomial.

The optional shape argument may be
shape = "full"
Return the full convolution. (default)
shape = "same"
Return the central part of the convolution with the length(a).

## fftconv

: fftconv ( $\mathbf{x}, \mathbf{y}$ )
: fftconv ( $\mathbf{x}, \mathbf{y}, \mathrm{n}$ )

Convolve two vectors using the FFT for computation.
$\mathbf{c}=\mathbf{f f t c o n v}(\mathbf{x}, \mathbf{y})$ returns
a vector of length equal to length $(\mathbf{x})+$ length $(\mathbf{y})-1$
If $\mathbf{x}$ and $\mathbf{y}$ are the coefficient vectors of two polynomials, the returned value is the coefficient vector of the product polynomial.

The computation uses the FFT
by calling the function fftfilt.
If the optional argument n is specified, an n-point FFT is used.

## deconv

: deconv ( $\mathbf{y}, \mathbf{a}$ )

Deconvolve two vectors.
$[\mathbf{b}, \mathbf{r}]=\operatorname{deconv}(\mathbf{y}, \mathbf{a})$ solves for $\mathbf{b}$ and $\mathbf{r}$ such that $\mathbf{y}=\mathbf{c o n v}(\mathbf{a}, \mathbf{b})+\mathbf{r}$.
If $\mathbf{y}$ and $\mathbf{a}$ are polynomial coefficient vectors,
b will contain the coefficients of the polynomial quotient and $r$ will be a remainder polynomial of lowest order.

## --plot gnuplot | octave

sox --plot gnuplot s6s.wav -n fir 0.10 .20 .40 .3 sox --plot gnuplot s6s.wav -n fir coeff.txt sox --plot gnuplot s6s.wav -n biquad .6 .2 . 4 1-1.5 . $6>$ fir3.plt sox --plot gnuplot s6s.wav -n fir 0.2 0.2 0.2 0.2 0.2 >fir4.plt

## --plot gnuplot | octave



## References

[1] F. Auger, Signal Processing with Free Software : Practical Experiments

