# Signal Processing

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### Based on

Signal Processing with Free Software : Practical Experiments F. Auger

### filter (1)

```
    : y = filter (b, a, x)
    : [y, sf] = filter (b, a, x, si)
    : [y, sf] = filter (b, a, x, [], dim)
    : [y, sf] = filter (b, a, x, si, dim)
```

## filter (2)

Apply a 1-D digital filter to the data x.

filter returns the solution to the following linear, time-invariant difference equation:

$$\sum_{k=0}^{N} a(k+1)y(n-k) = \sum_{k=0}^{M} b(k+1)x(n-k) \quad \text{for } 1 \le n \le length(x)$$

where N=length(a)-1 and M=length(b)-1.

$$a = [a(1), a(2), \dots, a(N+1)]$$
  $x = [x(1), x(2), \dots, x(L+1)]$   
 $b = [b(1), b(2), \dots, b(M+1)]$   $length(a) = N+1$   
 $length(b) = M+1$   $1 \le n \le L+1$ 

# filter (3)

The result is calculated over the **first** non-singleton dimension of x or over **dim** if supplied.

$$x(1), \quad x(2), \quad x(3), \quad \cdots \quad , \quad x(K)$$
 $x(1,1), \quad x(1,2), \quad x(1,3), \quad \cdots \quad , \quad x(1,K)$ 
 $x(2,1), \quad x(2,2), \quad x(2,3), \quad \cdots \quad , \quad x(2,K)$ 
 $x(3,1), \quad x(3,2), \quad x(3,3), \quad \cdots \quad , \quad x(3,K)$ 
 $x(L,1), \quad x(L,2), \quad x(L,3), \quad \cdots \quad , \quad x(L,K)$ 

$$x(:,1), x(:,2), x(:,3), \dots, x(:,K)$$

$$x(1), \quad x(2), \quad x(3), \quad \cdots \quad , \quad x(K)$$

$$y(1, 1), \quad y(1, 2), \quad y(1, 3), \quad \cdots \quad , \quad y(1, K)$$
 $y(2, 1), \quad y(2, 2), \quad y(2, 3), \quad \cdots \quad , \quad y(2, K)$ 
 $y(3, 1), \quad y(3, 2), \quad y(3, 3), \quad \cdots \quad , \quad y(3, K)$ 
 $y(L, 1), \quad y(L, 2), \quad y(L, 3), \quad \cdots \quad , \quad y(L, K)$ 

$$y(:,1), y(:,2), y(:,3), \dots, y(:,K)$$

### filter (4)

$$\sum_{k=0}^{N} a(k+1)y(n-k) = \sum_{k=0}^{M} b(k+1)x(n-k) \quad \text{for } 1 \le n \le length(x)$$

$$a(1)y(n) + \sum_{k=1}^{N} a(k+1)y(n-k) = \sum_{k=0}^{M} b(k+1)x(n-k)$$

$$a(1)y(n) = -\sum_{k=1}^{N} a(k+1)y(n-k) + \sum_{k=0}^{M} b(k+1)x(n-k)$$

$$y(n) = -\sum_{k=1}^{N} \frac{a(k+1)}{a(1)} y(n-k) + \sum_{k=0}^{M} \frac{b(k+1)}{a(1)} x(n-k)$$

$$y(n) = -\sum_{k=1}^{N} c(k+1) y(n-k) + \sum_{k=0}^{M} d(k+1) x(n-k)$$
 for  $1 \le n \le length(x)$ 

where c = a/a(1) and d = b/a(1).

# filter (5)

**si**: the initial state of the system

**sf**: the final state

the state vector is a <u>column vector</u> whose length is equal to the length of the <u>longest coefficient</u> vector – 1

No **si** is presented, the <u>zero</u> <u>initial</u> <u>state</u>.

in terms of the z transform,
 y is the result of passing the discrete-time signal x
 through a system characterized
 by the following rational system function:

$$H(z) = \frac{\sum_{k=0}^{M} d(k+1)z^{-k}}{1 + \sum_{k=1}^{N} c(k+1)z^{-k}}$$

### freqz (1)

```
: [h, w] = freqz (b, a, n, "whole")
: [h, w] = freqz (b)
: [h, w] = freqz (b, a)
: [h, w] = freqz (b, a, n)
: h = freqz (b, a, w)
: [h, w] = freqz (..., Fs)
: freqz (...)
```

### freqz (2)

Return the <u>complex frequency response</u> **h** of the rational **IIR** filter with the <u>numerator</u> coefficients **b** and the <u>denominator</u> coefficients **a** 

The response is evaluated at **n** angular frequencies between **0** and **2\*pi**.

The output value **w** is a <u>vector</u> of the frequencies.

**h**: the frequency response vector

w : the frequency vector

## freqz (3)

If **a** is omitted, the denominator is assumed to be **1** (this corresponds to a simple **FIR** filter).

If n is omitted, a value of 512 is assumed. For fastest computation, n should factor into a small number of small primes.

If the fourth argument, "whole", is omitted the response is evaluated at frequencies between **0** and **pi**.

### freqz (4)

#### freqz (b, a, w)

Evaluate the response at the <u>specific frequencies</u> in the vector **w**. The values for **w** are measured in <u>radians</u>.

#### freqz (...)

<u>Plot</u> the magnitude and phase response of **h** rather than returning them.

### freqz (5)

```
[...] = freqz(..., Fs)
```

Return frequencies in <u>Hz</u> instead of <u>radians</u> assuming a <u>sampling</u> <u>rate</u> Fs.

If you are evaluating the response at specific frequencies **w**, those frequencies should be requested in <u>Hz</u> rather than <u>radians</u>.

```
[h, w] = freqz (b, a, n, "whole", Fs)
[h, w] = freqz (b, Fs)
[h, w] = freqz (b, a, Fs)
[h, w] = freqz (b, a, n, Fs)
h = freqz (b, a, w, Fs)
```

### freqz\_plot

```
: freqz_plot (w, h)
: freqz_plot (w, h, freq_norm)
```

Plot the magnitude and phase response of **h**.

If the optional freq\_norm argument is true, the frequency vector **w** is in units of <u>normalized radians</u>. If freq\_norm is false, or not given, then **w** is measured in <u>Hertz</u>.

#### conv

```
: conv (a, b)
: conv (a, b, shape)
```

Convolve two vectors **a** and **b**.

```
The output convolution is a vector with length equal to length (a) + length (b) - 1. When a and b are the coefficient vectors of two polynomials, the convolution represents the coefficient vector of the <u>product polynomial</u>.
```

```
The optional shape argument may be shape = "full"

Return the <u>full convolution</u>. (default)

shape = "same"

Return the <u>central part</u> of the convolution with the length(a).
```

### fftconv

```
: fftconv (x, y)
: fftconv (x, y, n)
```

Convolve two vectors <u>using</u> the <u>FFT</u> for computation.

```
c = fftconv(x, y) returns
a vector of length equal to length(x) + length(y) - 1
```

If **x** and **y** are the coefficient vectors of two polynomials, the returned value is the coefficient vector of the <u>product</u> <u>polynomial</u>.

The computation uses the FFT by calling the function **fftfilt**.

If the optional argument n is specified, an n-point FFT is used.

### deconv

```
: deconv (y, a)
```

Deconvolve two vectors.

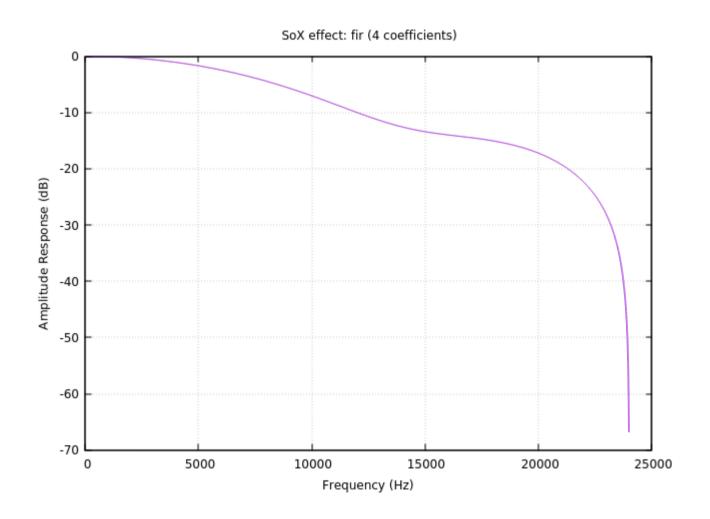
 $[\mathbf{b}, \mathbf{r}] = \mathbf{deconv}(\mathbf{y}, \mathbf{a})$  solves for  $\mathbf{b}$  and  $\mathbf{r}$  such that  $\mathbf{y} = \mathbf{conv}(\mathbf{a}, \mathbf{b}) + \mathbf{r}$ .

If **y** and **a** are polynomial coefficient vectors, **b** will contain the coefficients of the polynomial <u>quotient</u> and **r** will be a <u>remainder</u> polynomial of lowest order.

### --plot gnuplot | octave

```
sox --plot gnuplot s6s.wav -n fir 0.1 0.2 0.4 0.3 >fir1.plt
sox --plot gnuplot s6s.wav -n fir coeff.txt >fir2.plt
sox --plot gnuplot s6s.wav -n biquad .6 .2 .4 1 -1.5 .6 >fir3.plt
sox --plot gnuplot s6s.wav -n fir 0.2 0.2 0.2 0.2 0.2 >fir4.plt
```

# --plot gnuplot | octave



#### References

[1] F. Auger, Signal Processing with Free Software: Practical Experiments