# OpenStax University Physics Volume 1/Formulas

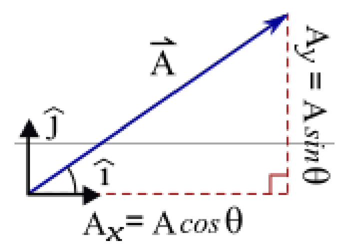
**Introduction**:

**metric prefixes**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| da | h | k | M | G | T | P | E | Z | Y |
| deca | hecto | kilo | mega | giga | tera | peta | exa | zetta | yotta |
| 1E+01 | 1E+02 | 1E+03 | 1E+06 | 1E+09 | 1E+12 | 1E+15 | 1E+18 | 1E+21 | 1E+24 |
| | | | | | | | | | |
| d | c | m | μ | n | p | f | a | z | y |
| deci | centi | milli | micro | nano | pico | femto | atto | zepto | yocto |
| 1E-01 | 1E-02 | 1E-03 | 1E-06 | 1E-09 | 1E-12 | 1E-15 | 1E-18 | 1E-21 | 1E-24 |

**1. Units_and_Measurement**: The base SI units are mass: kg (kilogram); length: m (meter); time: s (second). [1]Percent error] is $(\delta A/A)\times 100\%$

**2. Vectors**: Vector $\vec{A}=A_x\,\hat{i}+A_y\,\hat{j}+A_z\,\hat{k}$ involves components ($A_x$,$A_y$,$A_z$) and [2] unit vectors.[3] □ If $\vec{A}+\vec{B}=\vec{C}$, then $A_x+B_x=C_x$, etc, and vector subtraction is defined by $\vec{B}=\vec{C}-\vec{A}$.



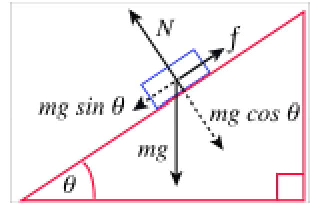

□ The two-dimensional displacement from the origin is $\vec{r}=x\hat{i}+y\hat{j}$. The magnitude is $A\equiv|\vec{A}|=\sqrt{A_x^2+A_y^2}$. The angle (phase) is $\theta=\tan^{-1}(y/x)$. □ Scalar multiplication $\alpha\vec{A}=\alpha A_x\hat{i}+\alpha A_y\hat{j}+\ldots$ □ Any vector divided by its magnitude is a unit vector and has unit magnitude: $|\hat{V}|=1$ where $\hat{V}\equiv\vec{V}/V$ □ Dot product $\vec{A}\cdot\vec{B}=AB\cos\theta=A_xB_x+A_yB_y+\ldots$ and $\vec{A}\cdot\vec{A}=A^2$ □ Cross product $\vec{A}=\vec{B}\times\vec{C}\Rightarrow A_\alpha=B_\beta C_\gamma-C_\gamma A_\beta$ where $(\alpha,\beta,\gamma)$ is any cyclic permutation of $(x,y,z)$, i.e., (α,β,γ) represents either (x,y,z) or (y,z,x) or (z,x,y). □ Cross-product magnitudes obey $A=BC\sin\theta$ where $\theta$ is the angle between $\vec{B}$ and $\vec{C}$, and $\vec{A}\perp\{\vec{B},\vec{C}\}$ by the right hand rule. □ Vector identities $c(\mathbf{A}+\mathbf{B})=c\mathbf{A}+c\mathbf{B}$ □ $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$ □ $\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{B})+\mathbf{C}$ □ $\mathbf{A}\cdot\mathbf{B}=\mathbf{B}\cdot\mathbf{A}$ □ $\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$ □ $(\mathbf{A}+\mathbf{B})\cdot\mathbf{C}=\mathbf{A}\cdot\mathbf{C}+\mathbf{B}\cdot\mathbf{C}$ [4]

**3. Motion_Along_a_Straight_Line**: [5] □ Average velocity $\bar{v}=\Delta x/\Delta t\to v=dx/dt$ (instantaneous velocity) □ Acceleration $\bar{a}=\Delta v/\Delta t\to a=dv/dt$. □ WLOG set $\Delta t=t$ and $\Delta x=x-x_0$ if $t_i=0$. Then $\Delta v=v-v_0$, and $v(t)=\int_0^t a(t')dt'+v_0$, $x(t)=\int_0^t v(t')dt'+x_0=x_0+\bar{v}t$ [6] □ At constant acceleration: $\bar{v}=\frac{v_0+v}{2}$, $v=v_0+at$, $x=x_0+v_0t+\frac{1}{2}at^2$, $v^2=v_0^2+2a\Delta x$. □ For free fall, replace $x\to y$ (positive up) and $a\to -g$, where $g$ = 9.81 m/s$^2$ at Earth's surface).

**4. Motion_in_Two_and_Three_Dimensions**: Instantaneous velocity: $\vec{v}(t)=v_x(t)\hat{i}+v_y(t)\hat{j}+v_z(t)\hat{k}=\dfrac{dx}{dt}\hat{i}+\dfrac{dy}{dt}\hat{j}+\dfrac{dz}{dt}\hat{k}$ □ $\vec{v}(t)=\lim\limits_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t}=\lim\limits_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$, where $\vec{r}(t)=x(t)\hat{i}+y(t)\hat{j}+z(t)\hat{k}$ □ Acceleration $\vec{a}=a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$, where $a_x(t)=dv_x/dt=d^2x/dt^2$. [7] □ Uniform circular motion: position $\vec{r}(t)$, velocity $\vec{v}(t) = d\vec{r}(t)/dt$, and acceleration $\vec{a}(t) = d\vec{v}(t)/dt$: $\vec{r} = A\cos\omega t\hat{\imath} + A\sin\omega t\hat{\jmath}$, $\vec{v} = -A\omega\sin\omega t\hat{\imath} + A\omega\cos\omega t\hat{\jmath}$, $\vec{a} = -A\omega^2\cos\omega t\hat{\imath} - A\omega^2\sin\omega t\hat{\jmath}$. Note that if $A = r$ then $|\vec{a}| = a_C = \omega^2 r = v^2/r$ where $v \equiv |\vec{v}| = \omega r$. [8] ☐ Relative motion: [9] $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$, [10]

**5. Newton's_Laws_of_Motion**: [11] $m\vec{a} = d\vec{p}/dt = \sum \vec{F}_j$, where $\vec{p} = m\vec{v}$ is momentum, [12] $\sum \vec{F}_j$ is the sum of all forces This sum needs only include external forces [13] $\vec{F}_{AB} = -\vec{F}_{BA}$.[14]

☐ Weight= $\vec{w} = m\vec{g}$. ☐ normal force[15] $|\vec{N}| = N = mg\cos\theta$[16] ☐ [17]$F = -kx$ where $k$ is the spring constant.




**6. Applications_of_Newton's_Laws**: $f_s \le \mu_s N$ and $f_k = \mu_k N$: $f =$ friction, $\mu_{s,k} =$ coefficient of (static,kinetic) friction, $N =$ normal force. ☐ Centripetal force $F_c = mv^2/r = mr\omega^2$ for uniform circular motion. Angular velocity $\omega$ is measured in radians per second. [18]☐ Drag equation $F_D = \frac{1}{2}C\rho A v^2$ where $C =$ Drag coefficient, $\rho =$ mass density, $A =$ area, $v =$ speed. Holds approximately for large Reynold's number[19]

**7. Work_and_Kinetic_Energy**: Infinitesimal work[20] $dW = \vec{F}\cdot d\vec{r} = |\vec{F}|\,|d\vec{r}|\cos\theta$ leads to the path integral $W_{AB} = \int_A^B \vec{F}\cdot d\vec{r}$ ☐ Work done from A→B by friction $-f_k|\ell_{AB}|$, gravity $-mg(y_B - y_A)$, and spring $-\frac{1}{2}k\left(x_B^2 - x_A^2\right)$

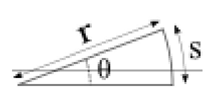
☐ Work-energy theorem: [21] $W_{net} = K_B - K_A$ where kinetic energy $= K = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$. ☐ Power $= P = dW/dt = \vec{F}\cdot\vec{v}$.

**8. Potential_Energy_and_Conservation_of_Energy**: Potential Energy: $\Delta U_{AB} = U_B - U_A = -W_{AB}$; PE at $\vec{r}$ WRT $\vec{r}_0$ is $\Delta U = U(\vec{r}) - U(\vec{r}_0)$ $U = mgy + C$ (gravitational PE Earth's surface. $U = \frac{1}{2}kx^2 + C$ (ideal spring)

☐ Conservative force: $\oint \vec{F}_{\text{cons}}\cdot d\vec{r} = 0$. In 2D, $\vec{F}(x,y)$ is conservative if and only if $\vec{F} = -(\partial U/\partial x)\,\hat{\imath} - (\partial U/\partial y)\,\hat{\jmath} \iff \partial F_x/\partial y = \partial F_y/\partial x$ ☐ Mechanical energy is conserved if no non-conservative forces are present: $0 = W_{nc,AB} = \Delta(K+U)_{AB} = \Delta E_{AB}$

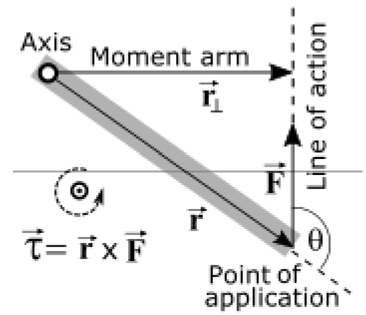
**9. Linear_Momentum_and_Collisions**: $\vec{F}(t) = d\vec{p}/dt$, where $\vec{p} = m\vec{v}$ is momentum. ☐ Impulse-momentum theorem $\vec{J} = F_{ave}\Delta t = \int_{t_i}^{t_f}\vec{F}dt = \Delta\vec{p}$. ☐ For 2 particles in 2D If $\vec{F}_{ext} = 0$ then $\sum_{j=1}^{N}\vec{p}_j = 0 \Rightarrow p_{f,\alpha} = p_{1,i,\alpha} + p_{2,i,\alpha}$ where (α,β)=(x,y) ☐ Center of mass: $\vec{r}_{CM} = \frac{1}{M}\sum_{j=1}^{N} m_j\vec{r}_j \to \frac{1}{M}\int \vec{r}dm$, $\vec{v}_{CM} = \frac{d}{dt}\vec{r}_{CM}$, and $\vec{p}_{CM} = \sum_{j=1}^{N} m_j\vec{v}_j = M\vec{v}_{CM}$. ☐ $\vec{F} = \frac{d}{dt}\vec{p}_{CM} = m\vec{a}_{CM} = \sum_{j=1}^{N} m_j\vec{a}_j$ [22]

**10. Fixed-Axis_Rotation**:

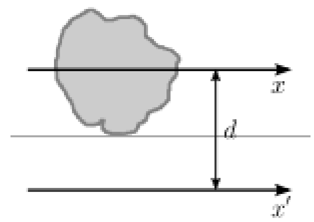



$\theta = s/r$ is angle in radians, $\omega = d\theta/dt$ is angular velocity; ☐ $v_t = \omega r = ds/dt$ is tangential speed. Angular acceleration is $\alpha = d\omega/dt = d^2\theta/dt^2$. $a_t = \alpha r = d^2s/dt^2$ is the tangential acceleration. ☐ Constant angular acceleration $\bar{\omega} = \frac{1}{2}(\omega_0 + \omega_f)$ is average angular velocity. ☐ $\theta_f = \theta_0 + \bar{\omega}t = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$. ☐ $\omega_f = \omega_0 + \alpha t$. $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$. ☐ Total acceleration is centripetal plus tangential: $\vec{a} = \vec{a}_c + \vec{a}_t$. ☐ Rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, where $I = \sum_j m_j r_j^2 \to \int r^2 dm$ is the Moment of inertia. ☐ parallel axis theorem

$I_{parallel-axis} = I_{center\ of\ mass} + md^2$ □ Restricting ourselves to fixed axis rotation, $r$ is the distance from a fixed axis; the sum of torques, $\vec{\tau} = \vec{r} \times \vec{F}$ requires only one component, summed as $\tau_{net} = \sum \tau_j = \sum r_{\perp_j} F_j = I\alpha$. □ Work done by a torque is $dW = \left(\sum \tau_j\right) d\theta$. The Work-energy theorem is $K_B - K_A = W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_j \tau_j\right) d\theta$. □ Rotational power $= P = \tau\omega$.




**11. Angular_Momentum**: Center of mass (rolling without slip) $d_{CM} = r\theta$, $v_{CM} = r\omega$, $a_{MC} = R\alpha = \frac{mg \sin\theta /}{m + (I_{cm}/r^2)}$ □ Total angular momentum and net torque: $d\vec{L}/dt = \sum \vec{\tau} = \vec{l}_1 + \vec{l}_2 + \ldots;\ \vec{l} = \vec{r} \times \vec{p}$ for a single particle. $L_{total} = I\omega$. □ Precession of a top $\omega_P = mrg/(I\omega)$. **12. Static_Equilibrium_and_Elasticity**: Equilibrium $\sum \vec{F}_j = 0 = \sum \vec{\tau}_j$. *Stress = elastic modulus · strain* (analogous to *Force = k · Δ x* ) □ (Young's , Bulk , Shear) modulus: $\left(\frac{F_\perp}{A} = Y \cdot \frac{\Delta L}{L_0},\ \Delta p = B \cdot \frac{-\Delta V}{V_0},\ \frac{F_\parallel}{A} = S \cdot \frac{\Delta x}{L_0}\right)$



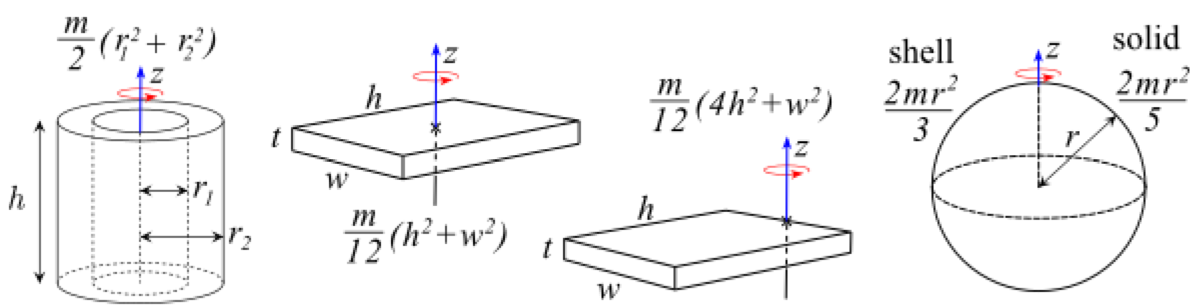

**13. Gravitation**: Newton's law of gravity $\vec{F}_{12} = G\frac{m_1 m_2}{r^2}\hat{r}_{12}$ □ Earth's gravity $g = G\frac{M_E}{r^2}$ □ Gravitational PE beyond Earth $U = -G\frac{M_E m}{r}$ □ Energy conservation $\frac{1}{2}mv_1^2 - G\frac{Mm}{r_1} = \frac{1}{2}mv_2^2 - G\frac{Mm}{r_2}$ □ Escape velocity $v_{esc} = \sqrt{\frac{2GM_E}{r}}$ □ Orbital speed $v_{orbit} = \sqrt{\frac{GM_E}{r}}$ □ Orbital period $T = 2\pi\sqrt{\frac{r^3}{GM_E}}$ □ Energy in circular orbit $E = K + U = -\frac{GmM_E}{2r}$ □ Conic section $\frac{\alpha}{r} = 1 + e\cos\theta$ □ Kepler's third law$T^2 = \frac{4\pi^2}{GM}a^3$ □ Schwarzschild radius $R_S = \frac{2GM}{c^2}$

**14. Fluid_Mechanics**: Mass density $\rho = m/V$ □ Pressure $P = F/A$ □ Pressure vs depth/height (constant density) $p = p_o + \rho g h \Leftarrow dp/dy = -\rho g$ □ Absolute vs gauge pressure $p_{abs} = p_g + p_{atm}$ □ Pascal's principle: $F/A$ depends only on depth, not on orientation of A. □ Volume flow rate $Q = dV/dt$ □ Continuity equation $\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \Rightarrow A_1 v_1 = A_2 v_2$ if $\rho = const.$

**15. Oscillations**: Frequency $f$, period $T$ and angular frequency $\omega$ : $fT = 1$, $\omega T = 2\pi$ □ Simple harmonic motion $x(t) = A\cos(\omega t + \phi)$, $v(t) = -A\omega \sin(\omega t + \phi)$, $a(t) = -A\omega^2 \cos(\omega t + \phi)$ also models the x-component of uniform circular motion. □ For $(A, \omega)$ positive: $x_{max} = A$, $v_{max} = A\omega$, $a_{max} = A\omega^2$ □ Mass-spring $\omega = 2\pi/T = 2\pi f = \sqrt{k/m}$; □ Energy $E_{Tot} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2 \Rightarrow v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$ □ Simple pendulum $\omega \approx \sqrt{g/L}$ □ Physical pendulum $\tau = -MgL\sin\theta \approx -MgL\theta \Rightarrow \omega = \sqrt{mgL/I}$ and $L$ measures from pivot to CM. □ Torsional pendulum $\tau = -\kappa\theta \Rightarrow \omega = \sqrt{I/\kappa}$ □ Damped harmonic oscillator $m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$ $\Rightarrow x = A_0 e^{\frac{b}{2m}t}\cos(\omega t + \phi)$ where $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$ and $\omega_0 = \sqrt{\frac{k}{m}}$. □ [23]Forced harmonic oscillator (**MIT wiki!**)] $m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \sin\omega t \Rightarrow x = Ae^{\frac{b}{2m}t}\cos(\omega t + \phi)$ where $A = \frac{F_0}{\sqrt{m^2(\omega - \omega_0)^2 + b^2\omega^2}}$.

**16. Waves**: [24] Wave speed] (phase velocity) $v = \lambda/T = \lambda f = \omega/k$ where $k = 2\pi/\lambda$ is wavenumber. □ Wave and pulse speed of a stretched string $= \sqrt{F_T/\mu}$ where $F_T$ is tension and $\mu$ is linear mass density. □ Speed of a compression wave in a fluid $v = \sqrt{B/\rho}$. □ Periodic travelling wave $y(x, t) = A\sin(kx \mp \omega t)$ travels in the positive/negative direction. The phase is $kx \mp \omega t$ and the amplitude is $A$. □ The resultant of two waves with identical amplitude and frequency $y_R(x,t) = \left[2A\cos\left(\frac{\phi}{2}\right)\right]\sin\left(kx - \omega t + \frac{\phi}{2}\right)$ where $\phi$ is the phase shift. □ This wave equation $\partial^2 y/\partial t^2 = v_w^2\,\partial^2 y/\partial x^2$ is linear in $y = y(x,t)$ □ Power in a tranverse stretched string wave $P_{ave} = \frac{1}{2}\mu A^2\omega^2 v$. □ Intensity of a plane wave $I = P/A \Rightarrow \frac{P}{4\pi r^2}$ in a spherical wave. □ Standing wave $y(x,t) = A\sin(kx)\cos(\omega t + \phi)$ For symmetric boundary conditions $\lambda_n = 2\pi/k_n = \frac{2}{\pi}L$ $n = 1, 2, 3, \ldots$, or equivalently $f = nf_1$ where $f_1 = \frac{v}{2L}$ is the fundamental frequency.

**17. Sound**: Pressure and displacement fluctuations in a sound wave $P = \Delta P_{max}\sin(kx \mp \omega t + \phi)$ and $s = s_{max}\cos(kx \mp \omega t + \phi)$ □ Speed of sound in a fluid $v = f\lambda = \sqrt{\beta/\rho}$, □ in a solid $\sqrt{Y/\rho}$, □ in an idal gas $\sqrt{\gamma RT/M}$, □ in air $331\frac{m}{s}\sqrt{\frac{T_K}{273\,K}} = 331\frac{m}{s}\sqrt{1 + \frac{T_C}{273^oC}}$ □ Decreasing intensity spherical wave $I_2 = I_1\left(\frac{r_1}{r_2}\right)^2$ □ Sound intensity $I = \frac{\langle P\rangle}{A} = \frac{(\Delta P_{max})^2}{2\rho v}$ □ ...level $10\log_{10} I/I_0$ □ Resonance tube One end closed: $\lambda_n = \frac{4}{n}L$, $f_n = n\frac{v}{4L}$, $n = 1, 3, 5, \ldots$ □ Both ends open: $\lambda_n = \frac{2}{n}L$, $f_n = n\frac{v}{2L}$, $n = 1, 2, 3, \ldots$ □ Beat frequency $f_{beat} = |f_2 - f_1|$ □ (nonrelativistic) Doppler effect $f_O = f_s\frac{v \pm v_o}{v \mp v_s}$ where $v$ is the speed of sound, $v_s$ is the velocity of the source, and $v_o$ is the velocity of the observer. □ Angle of shock wave $\sin\theta = v/v_s = 1/M$ where $v$ is the speed of sound, $v_s$ is the speed of the source, and $M$ is the Mach number.




I=∫r²dm for a hoop, disk, cylinder, box, plate, rod, and spherical shell or solid *can be found* from this figure.

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