Power Density Spectrum - Continuous Time

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Definition

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) e^{j\omega t} d\omega$$

Definition

$$x_T(t) = \begin{cases} x(t) & -T < t < T \\ 0 & otherwise \end{cases}$$

$$E(T) = \int_{-T}^{+T} x^2(t)dt$$

$$P(T) = \frac{1}{2T} \int_{-T}^{+T} x^2(t)dt$$

Definition

$$P(T) = \frac{1}{2T} \int_{-T}^{+T} x^2(t) dt$$

$$P_{XX} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} E\left[x^2(t)\right] dt$$

$$= A\left[E\left[x^2(t)\right]\right]$$

Definition

$$\int_{-T}^{+T} |x^2(t)| dt < \infty$$

$$X_T(\omega) = \int_{-\infty}^{+\infty} x_T(t) e^{-j\omega t} dt$$

$$= \int_{-T}^{+T} x(t) e^{-j\omega t} dt$$

Definition

$$E(T) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X_T(\omega)| d\omega$$

$$P(T) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{|X_T(\omega)|}{2T} d\omega$$

Definition

$$P_{XX} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \lim_{T \to \infty} \frac{|X_T(\omega)|}{2T} d\omega$$

$$S_{XX} = \lim_{T \to \infty} \frac{|X_T(\omega)|}{2T}$$

$$P_{XX} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{XX}(\omega) d\omega$$

- $S_{XX}(\omega) \geq 0$
- $S_{XX}(-\omega) = S_{XX}(\omega)$ $\qquad$ $X(t)$ real
- $S_{XX}(\omega)$ real
- $\frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{XX}(\omega)d\omega = A\left[E\left[X^2(t)\right]\right]$
- $S_{XX}(\omega) = \omega^2 S_{XX}(\omega)$
- $\frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{XX}(\omega)e^{j\omega t}d\omega = A[R_{XX}(t,t+\tau)]$
- $S_{XX}(\omega) = \int_{-\infty}^{+\infty} A[R_{XX}(t,t+\tau)]\, e^{-j\omega\tau}d\tau$

Definition

$$S_{XX} = \int_{-\infty}^{+\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$$

$$R_{XX} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{XX}(\omega) e^{+j\omega\tau} d\omega$$