RLC Transient Response (H.1)

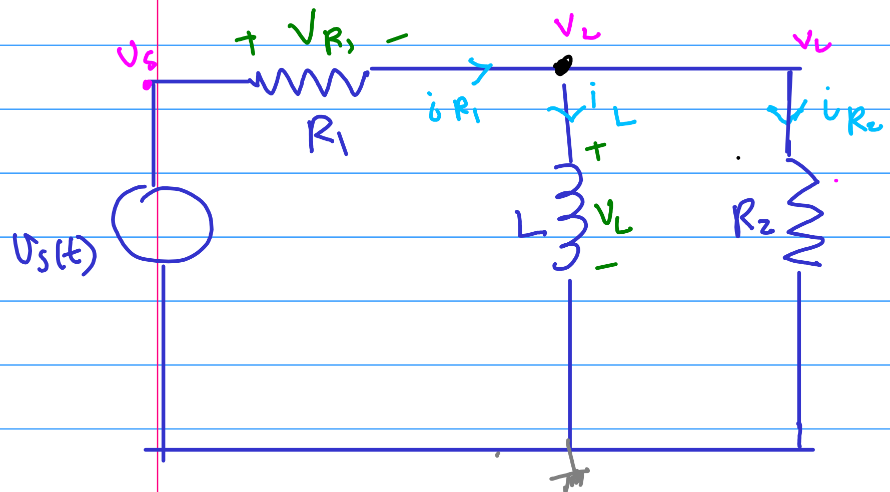
20150612

The necessities in Electric Circuit
wikiversity

$\boxed{DC}$ Steady State Solution

변하지 않는 input에 대한 Steady State 출력값

constant input



$$V_L = L\frac{di_L}{dt}$$

KCL $\quad i_{R_1} - i_L - i_{R_2} = 0$

$$\frac{V_s - V_L}{R_1} - i_L - \frac{V_L}{R_2} = 0$$

$$\frac{v_s}{R_1} - \frac{L}{R_1}\frac{di_L}{dt} - i_L - \frac{L}{R_2}\frac{di_L}{dt} = 0$$

$$-\left(\frac{L}{R_1} + \frac{L}{R_2}\right)\frac{di_L}{dt} - i_L + \frac{v_s}{R_1} = 0$$

$$\boxed{\boxed{\frac{L(R_1+R_2)}{R_1R_2}}\frac{di_L}{dt} + i_L = \boxed{\frac{1}{R_1}}v_s}$$

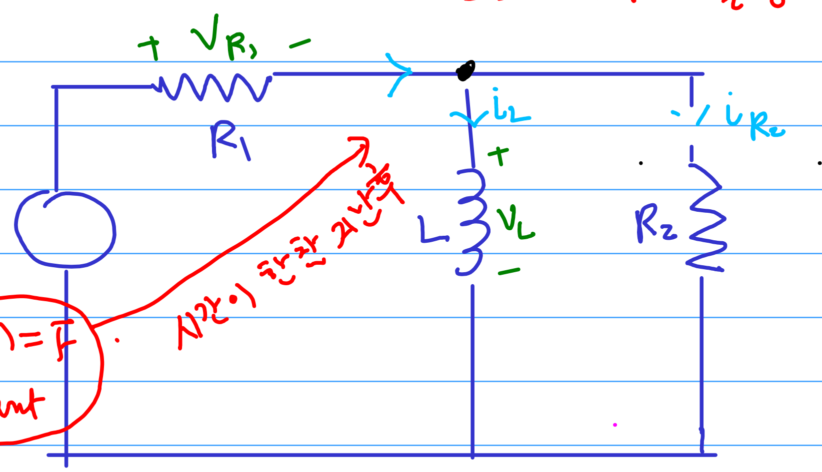
$$\boxed{\boxed{\tau}\frac{dx}{dt} + 1\,x(t) = \boxed{K_s}\,F}$$

$$\boxed{1\cdot\frac{di_L}{dt} + \frac{R_1R_2}{L(R_1+R_2)}i_L = \frac{R_2}{L(R_1+R_2)}v_s}$$

$$\boxed{\frac{L(R_1+R_2)}{R_1R_2}}\frac{di_L}{dt}+i_L=\boxed{\frac{1}{R_1}}v_s$$

$$\boxed{\tau}\frac{dx}{dt}+1\,x(t)=\boxed{K_s}\,F$$

흐르는 전류가 일정 (변함 X) → $\boxed{\frac{di}{dt}=0}$



만약 $v_s(t)=F$ constant

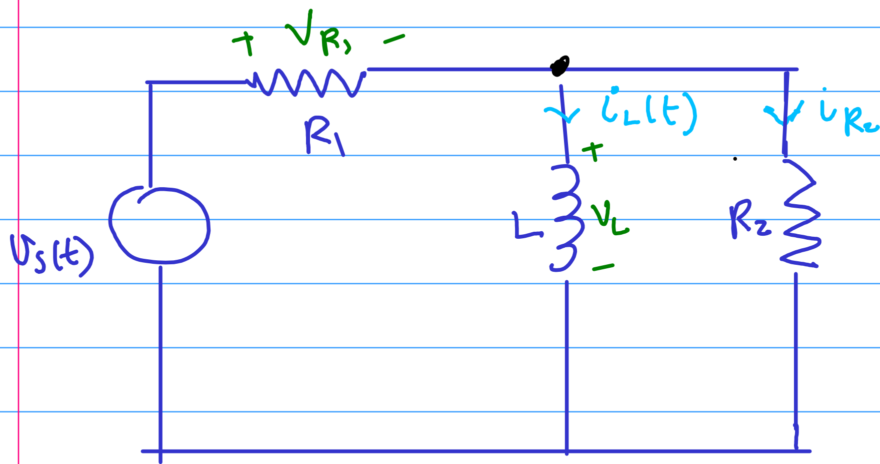
시간이 한참 지난 후

$$\boxed{\frac{L(R_1+R_2)}{R_1R_2}}\cancel{\frac{di_L(\infty)}{dt}}^{\,0}+i_L(\infty)=\boxed{\frac{1}{R_1}}F$$

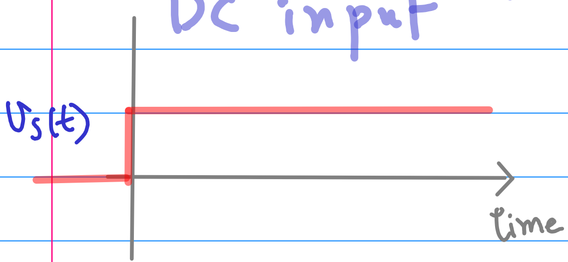
$$\boxed{\tau}\cancel{\frac{dx(\infty)}{dt}}^{\,0}+1\,x(\infty)=\boxed{K_s}\,F$$

$$i_L(\infty)=\boxed{\frac{1}{R_1}}F$$

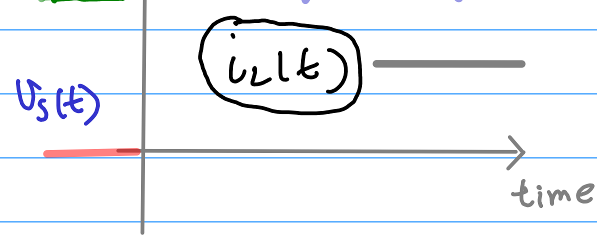
$\boxed{DC}$ steady state 값



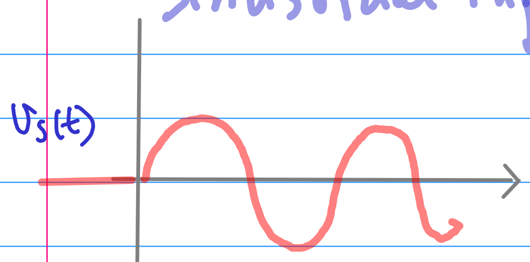
DC input



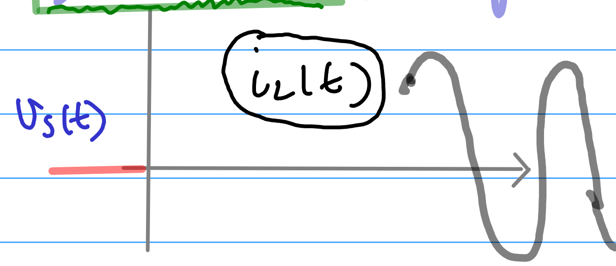
$\boxed{\text{DC}}$ steady state



Sinusoidal input



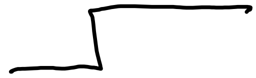
$\boxed{\text{Sinusoidal}}$ steady state



phasor

+

Impedance



DC Steady state

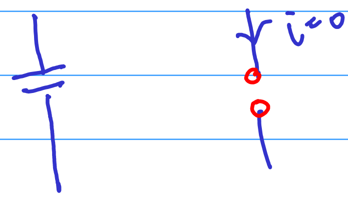
at $t=\infty$ $\quad \frac{dv_c(\infty)}{dt} = 0$

→ $i_c(\infty) = 0$

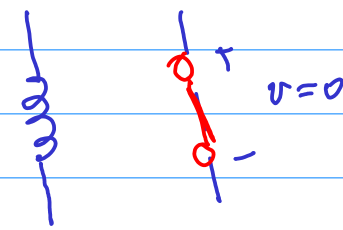
at $t=\infty$ $\quad \frac{di_L(\infty)}{dt} = 0$

→ $v_L(\infty) = 0$

at $t=\infty$



at $t=\infty$



$$\tau \frac{dx}{dt} + 1\,x(t) = \boxed{K_s \; F} \qquad x(\infty) = K_sF$$

$$\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt^2} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = \boxed{Kf(t)}$$

$$x(\infty) = Kf(t)$$

constant input을 갖는
steady state 에서 x(t) 는 변화가 없음
$t=\infty$

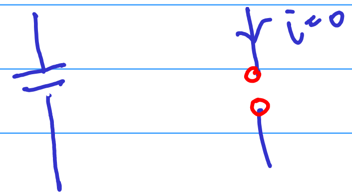
$$\boxed{\frac{d^2x}{dt^2} = \frac{dx}{dt} = 0}$$

Sinusoidal Steady state

$\boxed{\text{at } t=\infty}$ $\quad \cancel{\frac{dv_c(\infty)}{dt}} = 0$

$\rightarrow i_c(\infty) = 0$

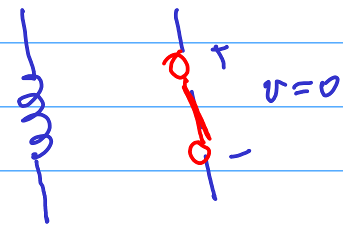
$\boxed{\text{at } t=\infty}$



$\boxed{\text{at } t=\infty}$ $\quad \cancel{\frac{di_L(\infty)}{dt}} = 0$

$\rightarrow v_L(\infty) = 0$

$\boxed{\text{at } t=\infty}$



$$i_c = C\frac{dv_c}{dt}$$

$$v_c(t) = A\cos\omega t$$

$$I = \frac{1}{j\omega C}V$$

Transient Response : 1st Order System

$$\boxed{\tau \frac{dx}{dt} + 1\, x(t) = K_s\, F}$$

$$x(t) \begin{cases} \rightarrow V_c(t) \\ \rightarrow i_L(t) \end{cases}$$

① find $\boxed{x(0^-)}$ & $\boxed{x(\infty)}$ 2 steady state values

② find $\boxed{x(0^+)}$ & $\boxed{\dot{x}(0^+)}$ initial conditions

③ find 1st order Diff Eq.

$$\boxed{\tau \frac{dx}{dt} + 1\, x(t) = K_s\, F}$$

④ find $\boxed{\tau}$

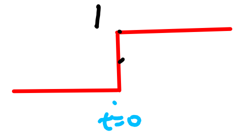
⑤ find $\boxed{x(t) = x_h(t) + \underbrace{x_p(t)}_{x(\infty)}}$

⑥ find $\boxed{\alpha} = [x(0) - x(\infty)]$

$$\boxed{\tau \frac{dx}{dt} + 1\, x(t) = K_s\, F}$$

$$x(t) = \left[x(0) - x(\infty) \right] e^{-t/\tau} + x(\infty) \qquad t \geq 0$$

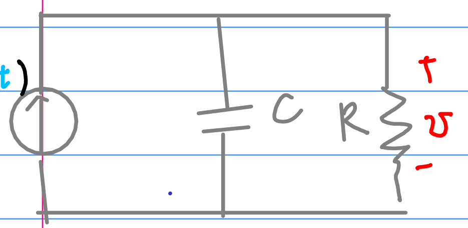
DC input

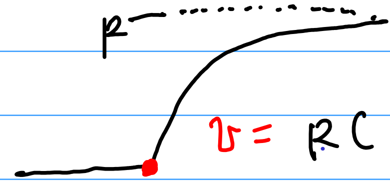


$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

step function

$i_s(t) = u(t)$

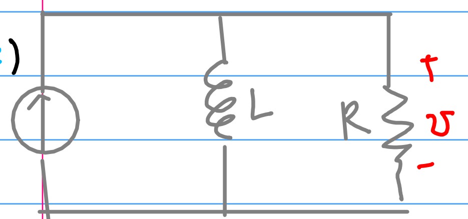




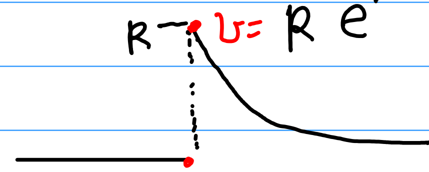
$$v = R(1 - e^{-t/RC})$$

$t \geq 0$

$i_s(t) = u(t)$

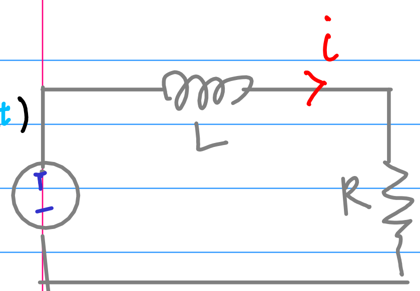


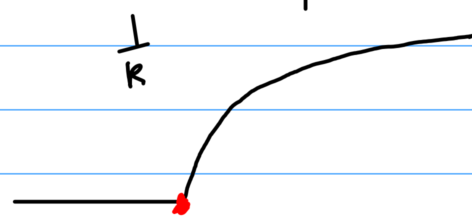
$$v = R e^{-(R/L)t}$$



$$i = \frac{1}{R}(1 - e^{-(R/L)t})$$

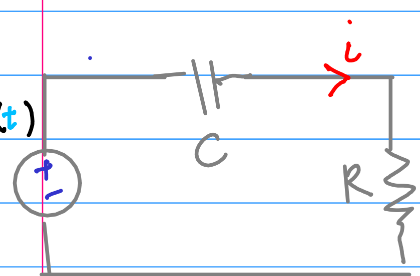
$v_s(t) = u(t)$

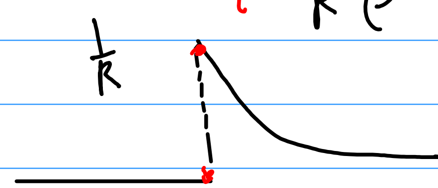




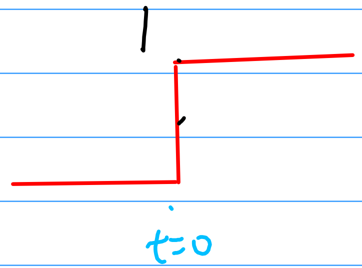
$$i = \frac{1}{R} e^{-t/RC}$$

$v_s(t) = u(t)$

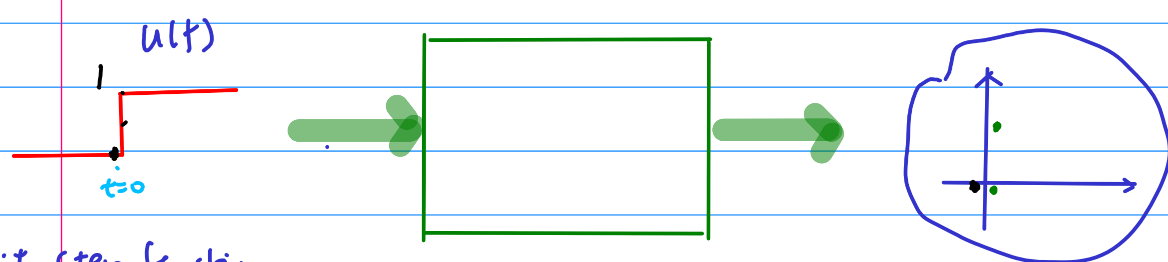




unit step function

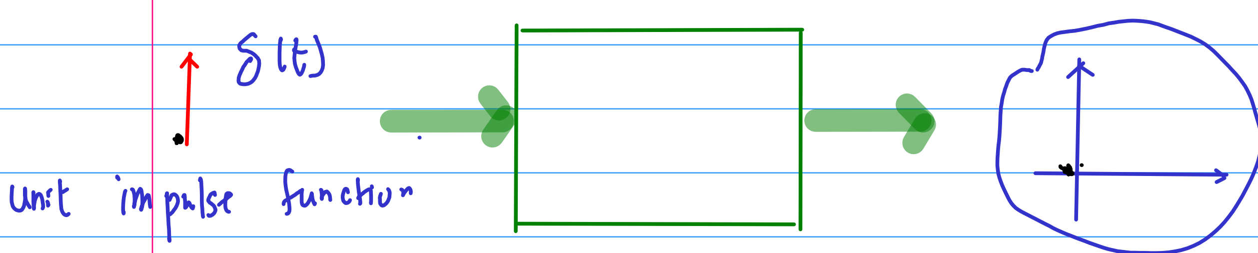


$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$



unit step function

step response



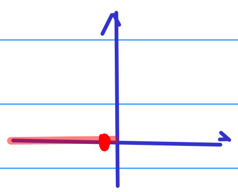
impulse response $h(t)$

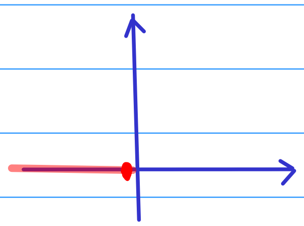
Zero State Response (ZSR)

no initial condition.

$t < 0$ input $x(t) = 0$

output $y(t) = 0$





$t \geq 0$ 일 때

non-zero input $x(t)$에 의한 output response

Zero Input Response (ZIR)

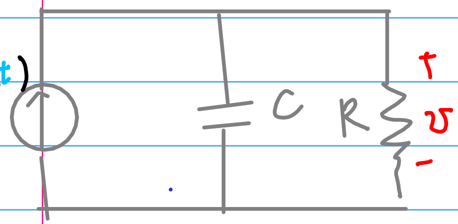
$t > 0$ input $x(t) = 0$ no input

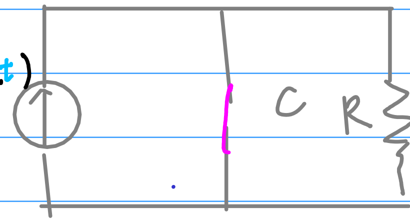
non-zero initial condition 으로 인한

output $t > 0$ 나타남

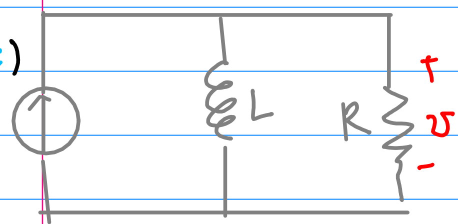
→ transient response 일부분.

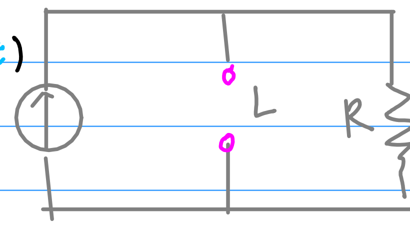
$t=0^+$



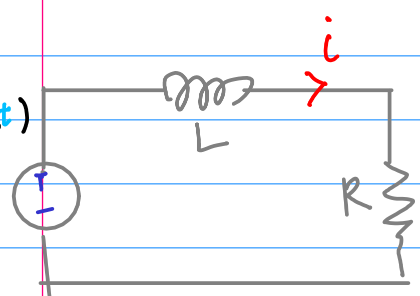


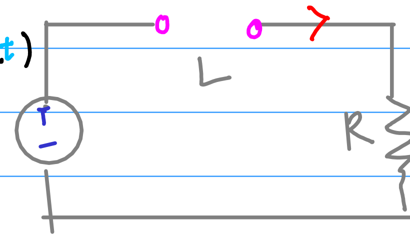
$v(0^+)=0$



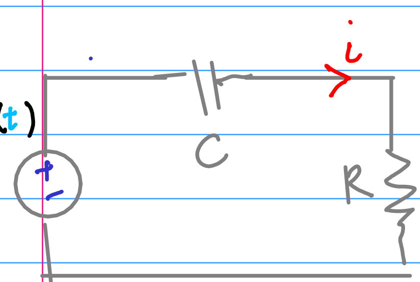


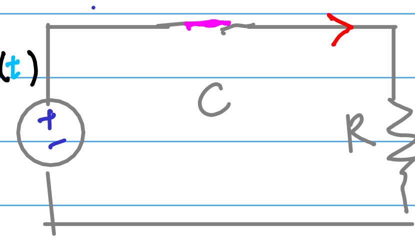
$v(0^+)=R$





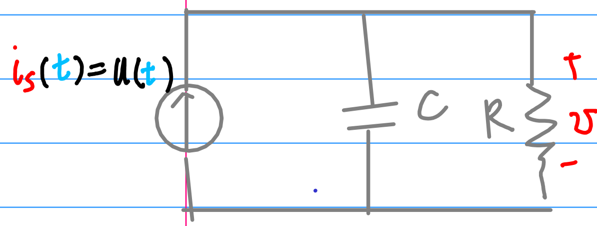
$i(0^+)=0$

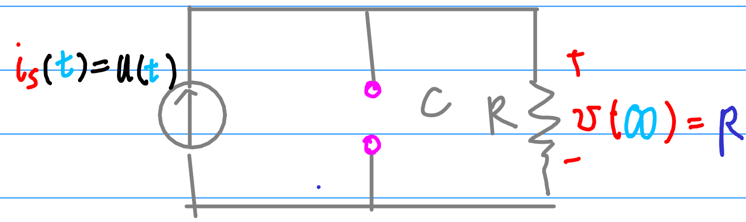


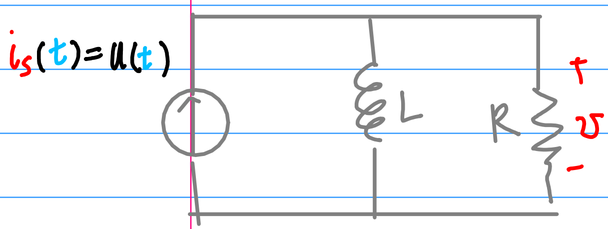


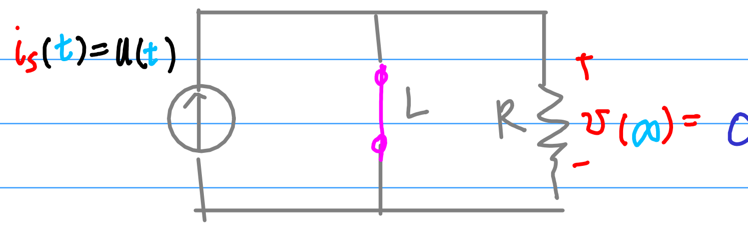
$i(0^+)=\frac{1}{R}$

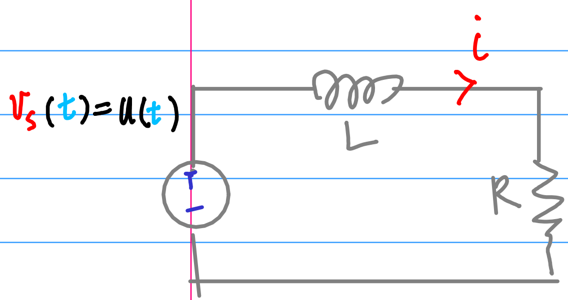
$t=\infty$


$i_s(t)=u(t)$
C
R
+
v
−


$i_s(t)=u(t)$
C
R
+
$v(\infty)=R$
−


$i_s(t)=u(t)$
L
R
+
v
−


$i_s(t)=u(t)$
L
R
+
$v(\infty)=0$
−


$v_s(t)=u(t)$
i
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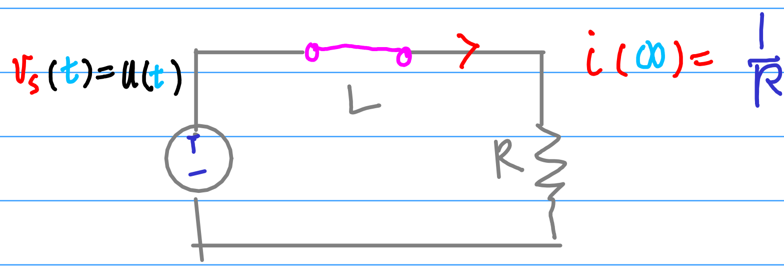
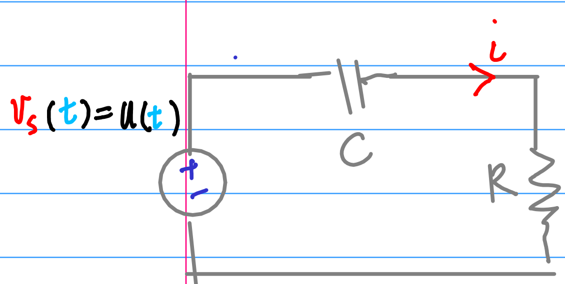
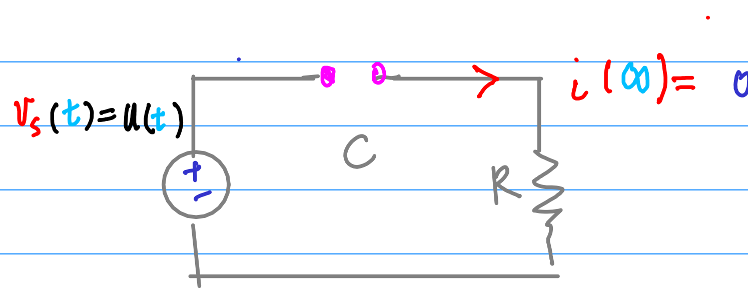
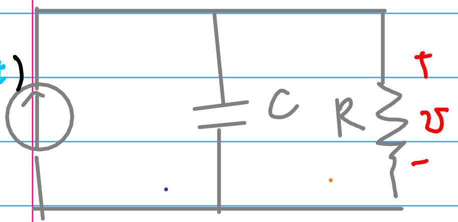

$v_s(t)=u(t)$
L
R
+
−
$i(\infty)=\frac{1}{R}$

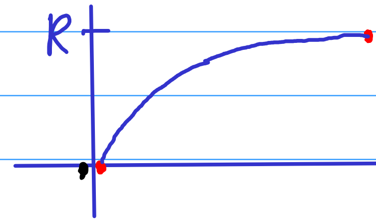

$v_s(t)=u(t)$
i
C
R
+
−


$v_s(t)=u(t)$
C
R
+
−
$i(\infty)=0$

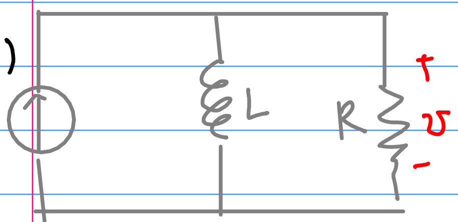
$i_s(t)=u(t)$



$v(0^+)=0 \quad v(\infty)=R$

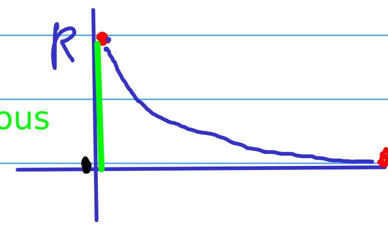


$i_s(t)=u(t)$



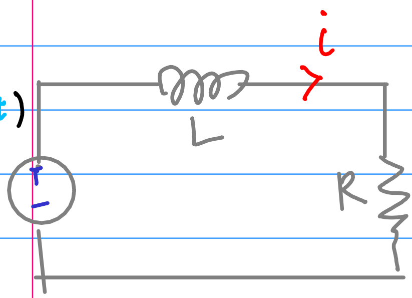
$v(0^+)=R \quad v(\infty)=0$

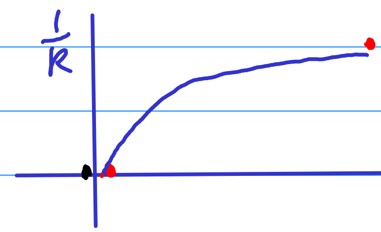
discontinuous



$i(0^+)=0 \quad i(\infty)=\frac{1}{R}$

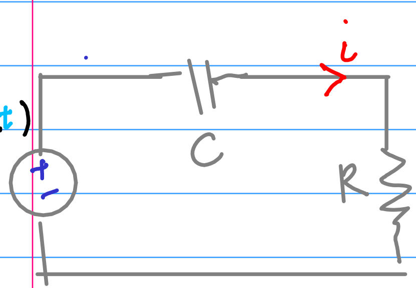
$v_s(t)=u(t)$



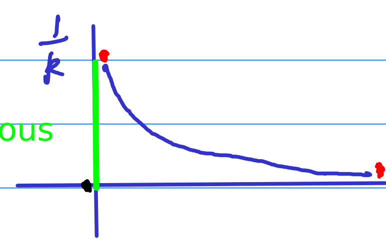


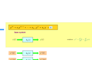
$i(0^+)=\frac{1}{R} \quad i(\infty)=0$

$v_s(t)=u(t)$

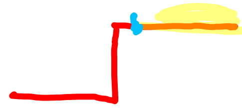


discontinuous

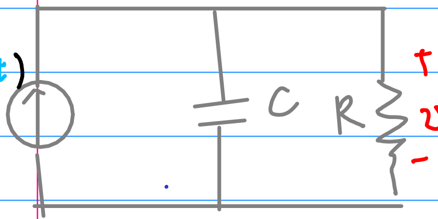




$\boxed{\text{ODE}}$ init cond .... 0^+ $\boxed{t>0}$



$i_s(t) = u(t)$

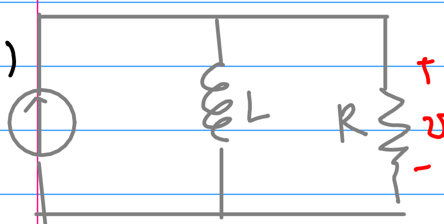


$C\frac{dv}{dt} + \frac{v}{R} = i_s$ $\qquad$ $\frac{dv}{dt} + \frac{v}{RC} = \frac{1}{C} i_s$

1 $\qquad$ $\frac{1}{C}$

$L\frac{di_L}{dt} = v$ $\qquad$ $i_L = \frac{1}{L}\int v(t)\,dt$

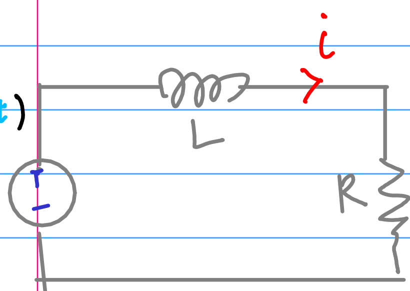
$i_s(t) = u(t)$



$\frac{1}{L}\int_0^t v\,dt + \frac{v}{R} = i_s$ $\qquad$ $\frac{dv}{dt} + \frac{R}{L}v = R\frac{di_s}{dt}$

$\frac{1}{L}v + \frac{1}{R}\frac{dv}{dt} = \frac{di_s}{dt}$

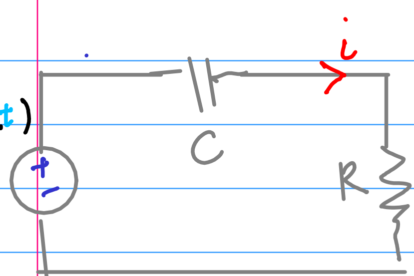
$v_s(t) = u(t)$



$L\frac{di}{dt} + Ri = v_s$ $\qquad$ $\frac{di}{dt} + \frac{R}{L}i = \frac{1}{L}v_s$

$C\frac{dv_c}{dt} = i_c$ $\qquad$ $v_c = \frac{1}{C}\int_0^t i_c\,dt$

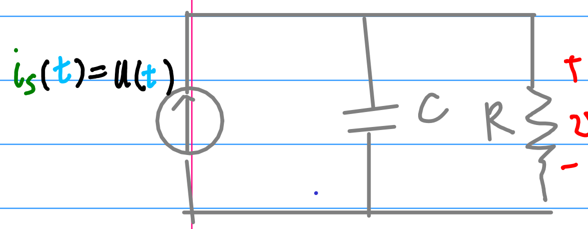
$v_s(t) = u(t)$



$\frac{1}{C}\int_0^t i\,dt + Ri = v_s$ $\qquad$ $\frac{di}{dt} + \frac{1}{RC}i = \frac{1}{R}\frac{dv_s}{dt}$

$\frac{1}{C}i + R\frac{di}{dt} = \frac{dv_s}{dt}$

(I)



$$C\frac{dv}{dt}+\frac{v}{R}=i_s$$

$$\boxed{\frac{dv}{dt}+\frac{v}{RC}=\frac{1}{C}i_s}$$

$$v_h = C_1 e^{-\frac{t}{RC}}$$

$$v_p = A$$

$$0+\frac{A}{RC}=\frac{1}{C}$$

$$A = R$$

initial condition $v(0^+)=0$

$$v(t) = C_1 e^{-\frac{t}{RC}} + R$$

$$v(0^+) = C_1 e^{-\frac{0^+}{RC}} + R = 0$$

$$C_1 = -R$$

$$v(t) = -R e^{-\frac{t}{RC}} + R = R\left(1-e^{-\frac{t}{RC}}\right)$$

$$\boxed{\frac{dv}{dt}+\frac{v}{RC}=\frac{1}{C}i_s}$$

Laplace Transform

$$sV(s) - v(0^-) + \frac{1}{RC}V(s) = \frac{1}{sC}$$

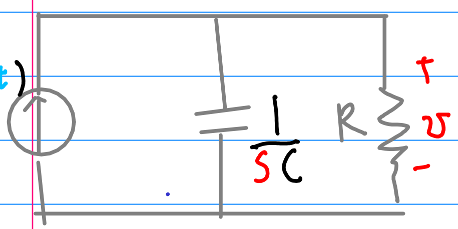
$$\left(s+\frac{1}{RC}\right)V(s) = v(0^-) + \frac{1}{sC}$$

$$V(s) = \underbrace{\frac{v(0^-)}{\left(s+\frac{1}{RC}\right)}}_{\text{initial condition 의한 response}} + \underbrace{\frac{1/C}{s\left(s+\frac{1}{RC}\right)}}_{\text{input } u(t)\text{에 의한 response}}$$

$$V(s) = \frac{1/C}{s\left(s+\frac{1}{RC}\right)} = \frac{R\cancel{C}}{\cancel{C}}\left(\frac{1}{s} - \frac{1}{\left(s+\frac{1}{RC}\right)}\right)$$

$$v(t) = R\left(1 - e^{-\frac{t}{RC}}\right)$$

$i_s(t)=u(t)$



$$\frac{1}{sC} \parallel R \qquad \frac{1}{sC+\frac{1}{R}} = \frac{R}{sRC+1}$$

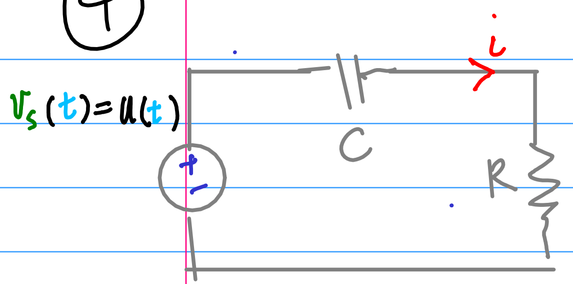
$I_s(s)$

$\frac{1}{s}$

$$\frac{R}{sRC+1} = V(s)$$

$$V(s) = \frac{R}{s(sRC+1)} = \frac{1/C}{s\left(s+\frac{1}{RC}\right)}$$

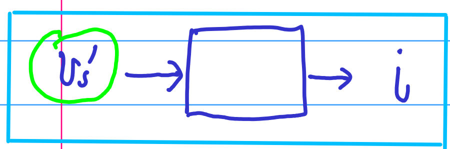
(4)



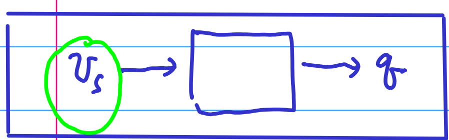
$C\frac{dv_c}{dt} = i_c$ $\qquad v_c = \frac{1}{C}\int_0^t i_c\,dt$

$$\frac{1}{C}\int_0^t i\,dt + Ri = v_s$$

$$\frac{1}{C} i + R\frac{di}{dt} = \frac{dv_s}{dt}$$



$$\boxed{\frac{di}{dt} + \frac{1}{RC} i = \frac{1}{R}\frac{dv_s}{dt}}$$



$$\boxed{\frac{dq}{dt} + \frac{1}{RC} q = \frac{1}{R} v_s}$$

$$q_h(t) = c_1 e^{-\frac{t}{RC}}$$

$$q_p(t) = A \qquad 0 + \frac{1}{RC}A = \frac{1}{R}$$

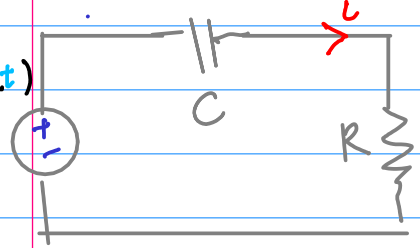
$$= C$$

$$q(t) = c_1 e^{-\frac{t}{RC}} + C$$

$$q(0^+) = c_1 e^{-\frac{0^+}{RC}} + C = 0 \qquad c_1 = -C$$

$$q(t) = C\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i(t) = q'(t) = \frac{1}{R} e^{-\frac{t}{RC}}$$



$C\frac{dv_c}{dt}=i_c \qquad v_c=\frac{1}{C}\int_0^t i_c\,dt$

$$\frac{1}{C}\int_0^t i\,dt + Ri = v_s$$

$$\frac{1}{C}i + R\frac{di}{dt} = \frac{dv_s}{dt}$$

$$\frac{di}{dt} + \frac{1}{RC}i = \frac{1}{R}\frac{dv_s}{dt}$$

$$sI(s) - i(0^-) + \frac{1}{RC}I(s) = \frac{1}{R}\left(sV(s) - v(0^-)\right)$$

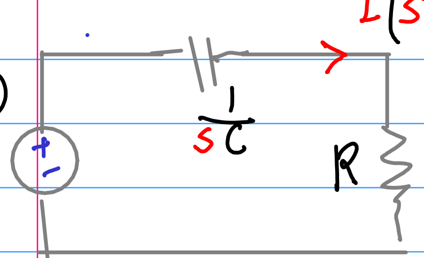
$i(0^-) \to 0$, $v(0^-) \to 0$

$$\left(s+\frac{1}{RC}\right)I(s) = \frac{1}{R}s\frac{1}{s}$$

$$I(s) = \frac{1}{R}\frac{1}{\left(s+\frac{1}{RC}\right)}$$

$$i(t) = \frac{1}{R}e^{-\frac{t}{RC}}$$

$\frac{1}{sC} \qquad sL$



$$I(s) = \frac{V(s)}{\left(\frac{1}{sC}+R\right)} = \frac{\frac{1}{s}}{\left(\frac{1}{sC}+R\right)}$$

$$\frac{1/R}{\frac{1}{RC}+s}$$

$$\frac{1}{R}e^{-\frac{t}{RC}}$$

output

input

$$y^{(N)} + a_1 y^{(N-1)} + \cdots + a_{N-1} y^{(1)} + a_N y = x$$

base system

$$x(t) \rightarrow \boxed{h_0(t)} \rightarrow y(t)$$

notation: $y^{(N)} = \dfrac{d^N y}{dt^N} = \dfrac{d^N}{dt^N} y(t)$

$$x'(t) \rightarrow \boxed{h_0(t)} \rightarrow y'(t)$$

$$x''(t) \rightarrow \boxed{h_0(t)} \rightarrow y''(t)$$

$$x^{(3)}(t) \rightarrow \boxed{h_0(t)} \rightarrow y^{(3)}(t)$$

$$\Uparrow \qquad \Downarrow$$

$$y'' + a y' + b y(t) = x(t)$$

$$\Uparrow \qquad \Downarrow$$

$$y''' + a y'' + b y'(t) = x'(t)$$

$$\frac{d^2}{dt^2}\left(\frac{dy}{dt}\right) + a \frac{d}{dt}\left(\frac{dy}{dt}\right) + b\left(\frac{dy}{dt}\right) = \left(\frac{dx}{dt}\right)$$

$$\Uparrow \qquad \Downarrow$$

$$y'' + a y' + b y(t) = x(t)$$

Transient Response : 2nd Order System

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = Kf(t)}$$

$x(t) \to V_c(t)$

$x(t) \to i_L(t)$

1. find $\boxed{x(0^-)}$ & $\boxed{x(\infty)}$ — 2 steady state values
2. find $\boxed{x(0^+)}$ & $\boxed{\dot{x}(0^+)}$ — initial conditions
3. find 2nd order Diff Eq.

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = Kf(t)}$$

4. find $\boxed{\omega_n}$ & $\boxed{\zeta}$
5. find $\boxed{x(t) = x_h(t) + x_p(t)}$ — $x(\infty)$
6. find $\boxed{\alpha_1}$ & $\boxed{\alpha_2}$

Constant input : DC

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt^2} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = K f(t)}$$

ⓐ $\zeta > 1$	ⓑ $\zeta = 1$	ⓒ $\zeta < 1$
$\alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t}$ $+ \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t}$ $+ x(\infty)$	$\alpha_1 e^{(-\zeta\omega_n)t}$ $+ \alpha_2 t e^{(-\zeta\omega_n)t}$ $+ x(\infty)$	$\alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t}$ $+ \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t}$ $+ x(\infty)$

$x(\infty)$ .... particular
forced

Homogeneous sol → natural response → 0 vanishes

$$\alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t} + \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t}$$

$$\alpha_1 e^{(-\zeta\omega_n)t} + \alpha_2 t e^{(-\zeta\omega_n)t}$$

$$\alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t} + \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t}$$

$$e^{-\zeta\omega_n t}$$

$$e^{-\zeta\omega_n \infty} \to 0$$

$$\boxed{\zeta > 0}$$

$\omega_n > 0$

$t > 0$

$$c_1 e^{-\zeta\omega_n t} \cos\left(\sqrt{1-\zeta^2}\,\omega_n t\right) + c_2 e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right)$$

① Find $x(0^-)$ & $x(\infty)$

ON

$t = 0$

0^- 0^+

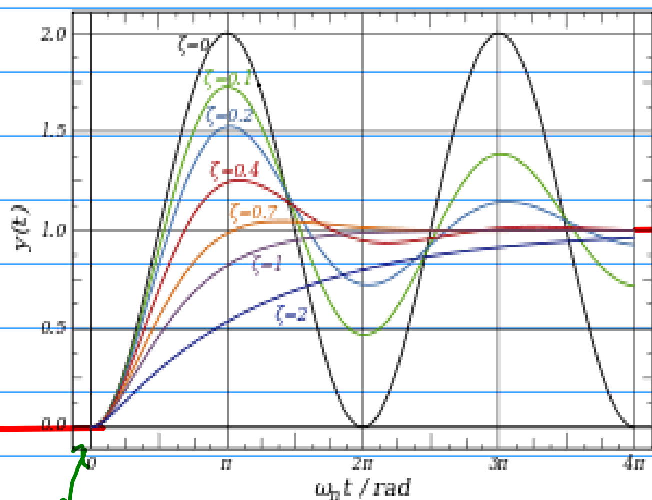
$t = \infty$

$x(0^-)$

steady state Rsp

$x(\infty)$

steady state Rsp



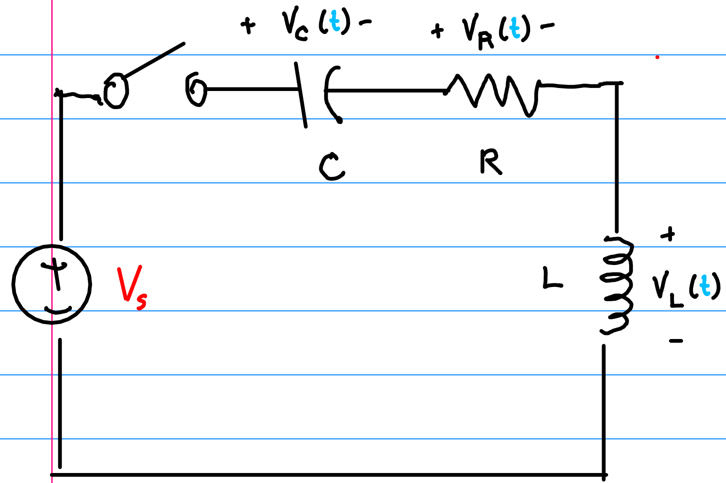
$t = \infty$

$t = 0^-$

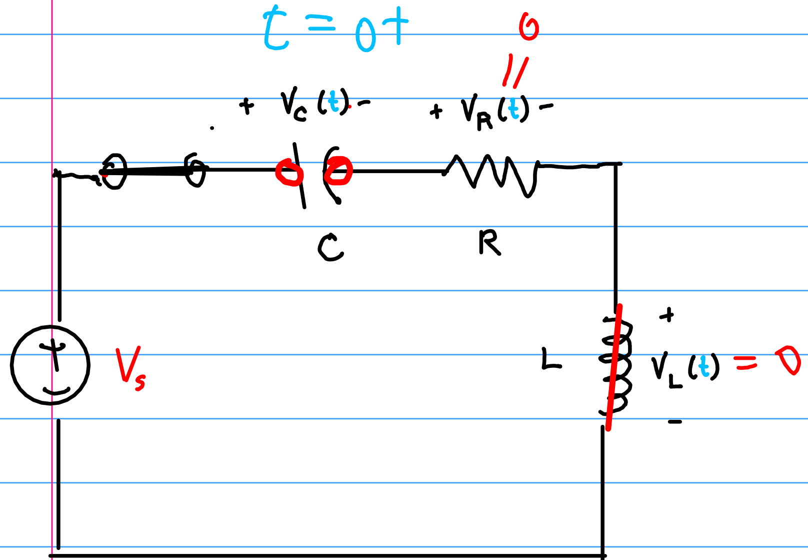
Fig 5.80 $t = 0^-$

문제에서

$$\begin{cases} v_C(0^-) = 5 \\ i_L(0^-) = 0 \end{cases}$$

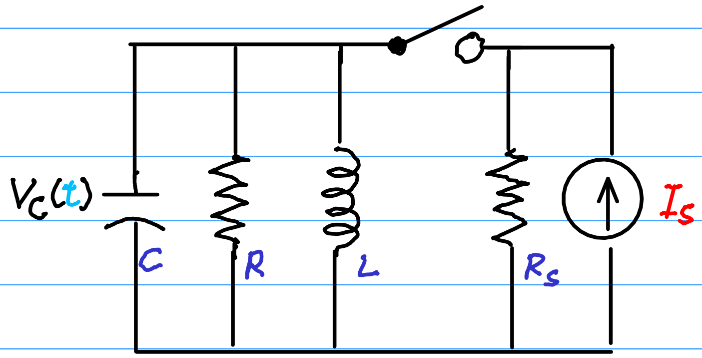


$t = 0^+$



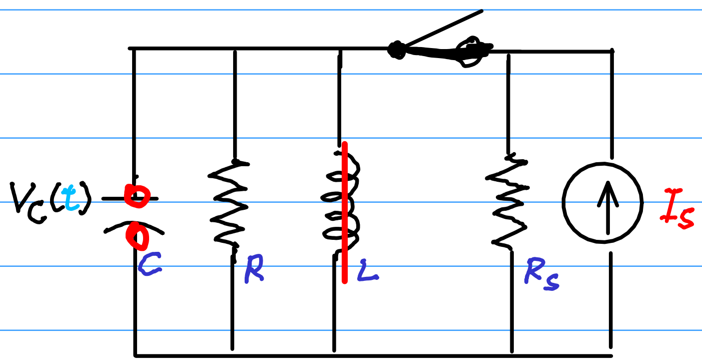
$$\begin{cases} v_c(\infty) = 25 \\ i_L(\infty) = 0 \end{cases}$$

$t = 0^-$



$i_L(0^-) = 0$
$v_C(0^-) = 0$

$t = \infty$



$i_L(\infty) = I_s$
$v_C(\infty) = 0$

② find $x(0^+)$, $\dot{x}(0^+)$

continuous v_c . $v_c(0^-) = v_c(0^+)$

continuous i_L $i_L(0^-) = i_L(0^+)$

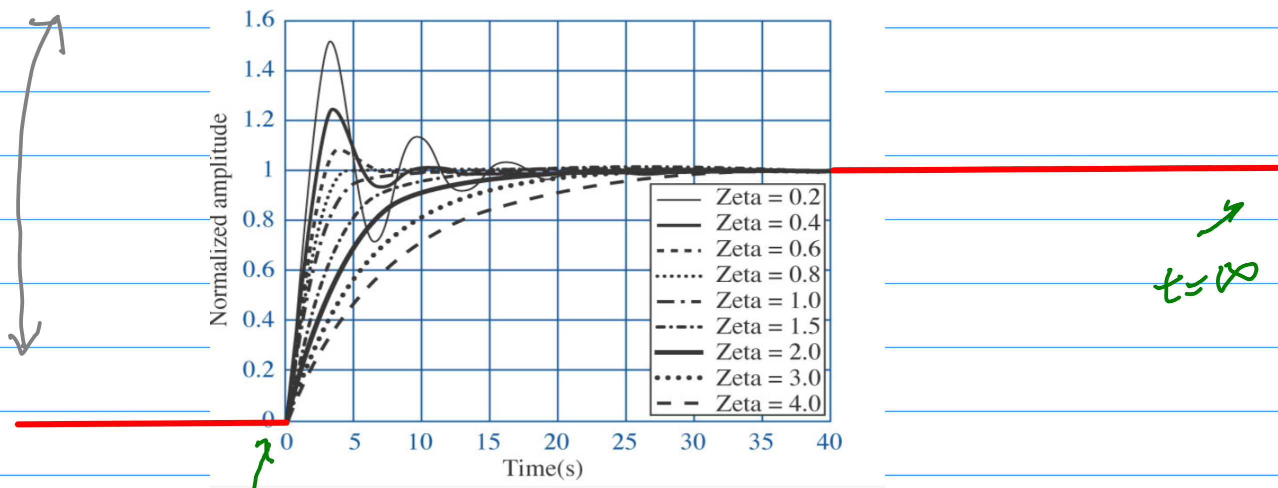
ON

$t=0$ $t=\infty$

0^- 0^+

$x(0^-) \neq x(0^+)$ may be continuous $x(\infty)$

$\dot{x}(0^+)$ discontinuous

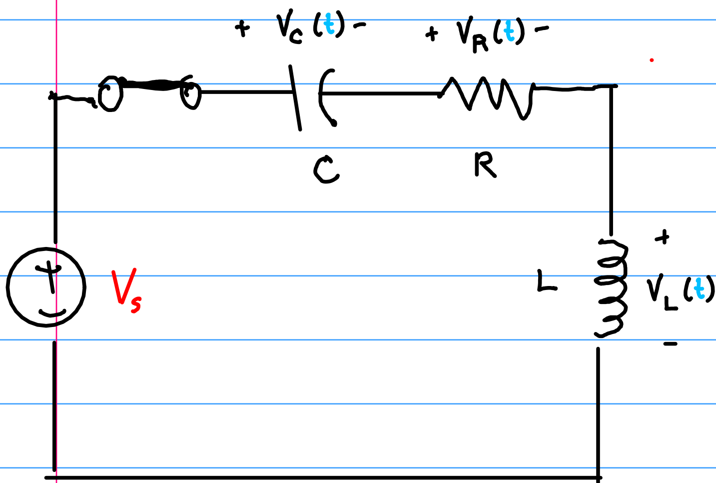


$t=\infty$

$t=0^-$

step2

$t=0^+$



문제에서

$$\begin{cases} v_C(0^-) = 5 = v_C(0^+) \\ i_L(0^-) = 0 = i_L(0^+) \end{cases}$$

continuous

$$i_L'(0^+) = ?$$

$t=0^+$ KVL

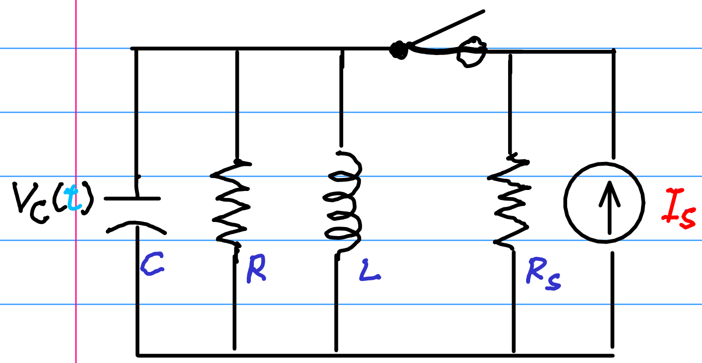
$$V_S - v_C(0^+) - R\,i_L(0^+) - v_L(0^+) = 0$$

$$V_S - v_C(0^+) - R\,i_L(0^+) - L\frac{di_L(0^+)}{dt} = 0$$

$$\frac{di_L(0^+)}{dt} = \frac{1}{L}\Big(\underset{2V}{\underset{\parallel}{V_S}} - \underset{5}{\underset{\parallel}{v_C(0^+)}} - \underset{0}{\underset{\parallel}{R\,i_L(0^+)}}\Big)$$

step2

$t = 0^+$



$i_L(0^-) = 0 = i_L(0^+)$

$v_C(0^-) = 0 = v_C(0^+)$

$v_C'(0^+) = ?$

$$I_s - \frac{v_c(t)}{R_s} - i_L(t) - \frac{v_c(t)}{R} - C\frac{dv_c(t)}{dt} = 0$$

$$I_s - \cancel{\frac{v_c(0^+)}{R_s}}^{\,0} - \cancel{i_L(0^+)}^{\,0} - \cancel{\frac{v_c(0^+)}{R}}^{\,0} - C\frac{dv_c(0^+)}{dt} = 0$$

$$\frac{dv_c(0^+)}{dt} = \frac{1}{C} I_s$$

③ find 2nd order Diff Eq.

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt} + 2\frac{\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = K f(t)}$$

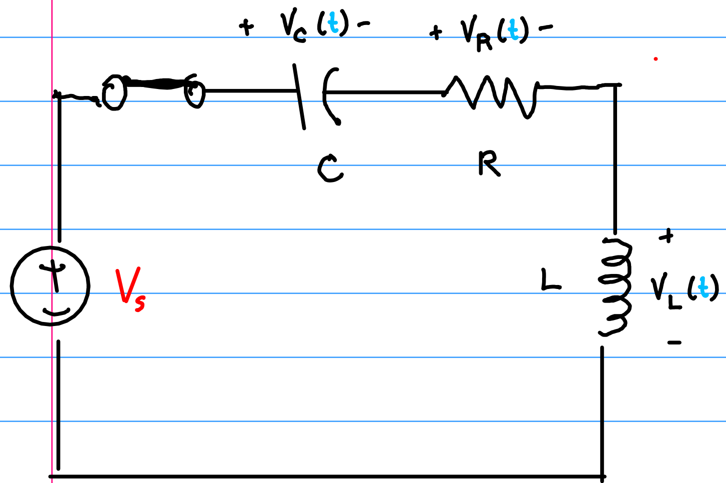
$t>0 \Rightarrow$ switch "on"

$$x(t) = \begin{cases} v_c(t) \leftarrow \text{parallel same volt.} \\ i_L(t) \leftarrow \text{series same current} \end{cases}$$

$$LC\frac{d^2 i_L}{dt} + R_1\frac{di_L}{dt} + 1\,i_L(t) = 0$$

$$LC\frac{d^2V_c}{dt^2} + L\left(\frac{R_s+R}{R_sR}\right)\frac{dV_c}{dt} + 1\,v_c(t) = 0$$

$\boxed{t > 0}$



$$V_s - v_C(t) - R\cdot i_L(t) - L\frac{di_L}{dt} = 0$$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_L(t)\,dt$$

$$\frac{d}{dt}\left\{ V_s - \frac{1}{C}\int_{-\infty}^{t} i_L(t)\,dt - R\cdot i_L(t) - L\frac{di_L}{dt} = 0 \right\}$$

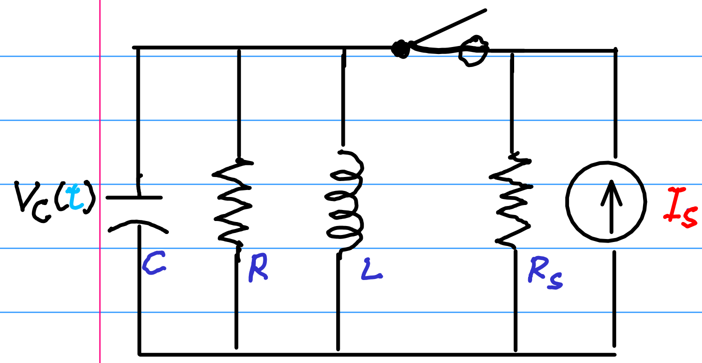
상수

$$-\frac{1}{C}\, i_L(t) - R\cdot\frac{di_L}{dt} - L\frac{d^2 i_L}{dt^2} = 0$$

$$i_L(t) + RC\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt} = 0$$

$$LC\frac{d^2 i_L}{dt} + RC\frac{di_L}{dt} + 1\, i_L(t) = 0$$

$t > 0$



$$L\frac{di_L}{dt} = v_L$$

$$i_L = \frac{1}{L}\int v_L\,dt$$

KCL

$$I_s - \frac{V_c(t)}{R_s} - i_L(t) - \frac{V_c(t)}{R} - C\frac{dV_c(t)}{dt} = 0$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{\infty} v_c(t)\,dt$$

$$\frac{d}{dt}\left\{ I_s - \frac{V_c(t)}{R_s} - \frac{1}{L}\int_{-\infty}^{\infty} v_c(t)\,dt - \frac{V_c(t)}{R} - C\frac{dV_c(t)}{dt} = 0 \right\}$$

상수

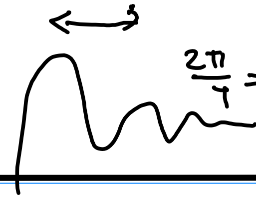
0

$$\left(\frac{1}{R_s} + \frac{1}{R}\right)\frac{dV_c}{dt} + \frac{1}{L}v_c(t) + C\frac{d^2V_c}{dt^2} = 0$$

$$LC\frac{d^2V_c}{dt^2} + L\left(\frac{R_s + R}{R_s R}\right)\frac{dV_c}{dt} + 1\,v_c(t) = 0$$

④ Find ω_n & ζ

natural freq — damping ratio



$\frac{2\pi}{T} = \omega_n$

$$\boxed{\frac{1}{\omega_n^2}\frac{d^2x(t)}{dt} + \frac{2\zeta}{\omega_n}\frac{dx(t)}{dt} + 1\,x(t) = Kf(t)}$$

$$LC\frac{d^2 i_L}{dt} + RC\frac{di_L}{dt} + 1\,i_L(t) = 0$$

$$\omega_n = \sqrt{\frac{1}{LC}} = 1000$$

$$\zeta = RC\frac{\omega_n}{2} = 2.5$$ Overdamping

$$LC\frac{d^2 v_c}{dt^2} + L\left(\frac{R_s + R}{R_s R}\right)\frac{dv_c}{dt} + 1\,v_c(t) = 0$$

$$\omega_n = \sqrt{\frac{1}{LC}} = 560$$

$$\zeta = L(\;)\frac{\omega_n}{2} = 1$$ Critical damping

⑤ find $x(t) = x_h(t) + x_p(t)$ $x(\infty)$

ζ 에 따라서

$\zeta > 1$ $D > 0$ 2 distinct real roots m_1, m_2

$x_h(t) \Leftarrow$ $\boxed{x_h(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t}}$

$\zeta = 1$ $D = 0$ repeated real root m_1

$x_h(t) \Leftarrow$ $\boxed{x_h(t) = C_1 e^{m_1 t} + C_2 t \cdot e^{m_2 t}}$

$\zeta < 1$ $D < 0$ 2 complex conjugate root $m_1 = \alpha + j\beta$, $m_2 = \alpha - j\beta$

$x_h(t) \Leftarrow$

$$\boxed{\begin{aligned} x_h(t) &= C_1 e^{(\alpha + j\beta)t} + C_2 e^{(\alpha - j\beta)t} \\ &= e^{\alpha t}\left(C_3 \cos(\beta t) + C_4 \sin(\beta t)\right) \end{aligned}}$$

(a) $\zeta > 1$

$$x(t) = x_N(t) + x_F(t)$$

$$\boxed{\begin{aligned} &\doteq \alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t} \\ &+ \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t} \\ &+ x(\infty) \end{aligned}}$$

$t \geq 0$

(b) $\zeta = 1$

$$x(t) = x_N(t) + x_F(t)$$

$$\boxed{\begin{aligned} &\doteq \alpha_1 e^{(-\zeta\omega_n)t} \\ &+ \alpha_2 t e^{(-\zeta\omega_n)t} \\ &+ x(\infty) \end{aligned}}$$

(c) $\zeta < 1$

$$x(t) = x_N(t) + x_F(t)$$

$$\boxed{\begin{aligned} &\doteq \alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t} \\ &+ \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t} \\ &+ x(\infty) \end{aligned}}$$

⑥ find α_1 & α_2

initial condition

$$\begin{cases} i_L(0^+) \\ \dot{i}_L(0^+) \end{cases} \qquad \begin{cases} v_C(0^+) \\ \dot{v}_C(0^+) \end{cases}$$

Step 2

ⓐ $\zeta > 1$

$$\left.\begin{aligned} x(t) &= \alpha_1 e^{(-\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n)t} \\ &+ \alpha_2 e^{(-\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n)t} \\ &+ x(\infty) \end{aligned}\right\} \quad x'(t)\Big|_{t=0^+}$$

$t \geq 0$

ⓑ $\zeta = 1$

$$\begin{aligned} x(t) &= \alpha_1 e^{(-\zeta\omega_n)t} \Rightarrow x'(t)\Big|_{t=0^+} \\ &+ \alpha_2 t e^{(-\zeta\omega_n)t} \\ &+ x(\infty) \end{aligned}$$

ⓒ $\zeta < 1$

$$\begin{aligned} x(t) &= \alpha_1 e^{(-\zeta\omega_n + j\sqrt{1-\zeta^2}\,\omega_n)t} \\ &+ \alpha_2 e^{(-\zeta\omega_n - j\sqrt{1-\zeta^2}\,\omega_n)t} \\ &+ x(\infty) \end{aligned}$$