

# Statistical Inference Overview

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1 Based on

2 Overview

- Statistical Inference
- Types of Hypothesis Tests

## "Understanding Statistics in the Behavioral Sciences" R. R. Pagano

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# Populations and Samples (1)

- **population**: everything in the group that we want to learn about.
- **sample**: a part of the population.
- Examples of populations and a sample from those populations:

Population	Sample
All of the people in Germany	500 Germans
All of the customers of Netflix	300 Netflix customers
Every car manufacturer	Tesla, Toyota, BMW, Ford

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)

## Populations and Samples (2)

- For good statistical analysis, the **sample** needs to be as similar as possible to the population.
- If they are similar enough, we say that the **sample** is representative of the population.
- The **sample** is used to make conclusions about the whole **population**.

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)

## Populations and Samples (3)

- If the **sample** is not similar enough to the whole **population**, the **conclusions** could be useless.
- Many words have specific meanings in statistics.
- The word **population** normally refers to a group of people.
- In statistics, it is any specific group that we are interested in learning about.

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)

- Using data analysis and statistics to make conclusions about a population is called **statistical inference**.
- The main types of statistical inference are:
  - **Estimation**
  - **Hypothesis testing**

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)

# Estimation (1)

- Statistics from a sample are used to **estimate** population parameters.
- *The most likely* value is called a point estimate.
- There is always **uncertainty** when estimating.

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)



## Estimation (2)

- The uncertainty is often expressed as **confidence intervals** defined by a *likely* lowest and highest value for the **parameter**.
- An example could be a **confidence interval** for the number of bicycles a Dutch person owns:
  - The **average number** of bikes a Dutch person owns is between 3.5 and 6.

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)

# Hypothesis Testing (1)

- a method to check if a claim about a population is true.
- checks how likely it is that a **hypothesis** is true is based on the sample data.
- there are different types of hypothesis testing.

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)

# Hypothesis Testing (2)

- the steps of the test depends on:
  - Type of data (categorical or numerical)
  - If you are looking at:
    - a single group
    - comparing one group to another
    - comparing the same group before and after a change

[https://www.w3schools.com/statistics/statistics\\_statistical\\_inference.php](https://www.w3schools.com/statistics/statistics_statistical_inference.php)

# Hypothesis Testing (3)

- a **hypothesis** is a claim about a **population parameter**
- a **hypothesis test** is a formal procedure to check if a **hypothesis** is true or not.
- examples of claims that can be checked:
  - the average height of people in Denmark is more than 170 cm.
  - the share of left handed people in Australia is not 10%.
  - The average income of dentists is less the average income of lawyers.

[https://www.w3schools.com/statistics/statistics\\_hypothesis\\_testing.php](https://www.w3schools.com/statistics/statistics_hypothesis_testing.php)

# Steps of Hypothesis Testing

- 1 *State* your research hypothesis as a **null hypothesis** ( $H_0$ ) and **alternate hypothesis** and ( $H_a$  or  $H_1$ ).
- 2 *Collect data* in a way designed to test the hypothesis.
- 3 *Perform* an appropriate statistical test
- 4 *Decide* whether to reject or fail to reject your **null hypothesis**
- 5 *Present* the findings in your results and discussion section.

<https://www.scribbr.com/statistics/hypothesis-testing/>

# Statistical tests (1)

- **Statistical tests** are used in **hypothesis testing**.

They can be used to:

- determine whether a predictor variable has a statistically significant relationship with an outcome variable.
- estimate the difference between two or more groups.

<https://www.scribbr.com/statistics/statistical-tests/>

## Statistical tests (2)

- **Statistical tests** assume a null hypothesis of no relationship or no difference between groups. Then they determine whether the observed data fall outside of the range of values predicted by the null hypothesis.

<https://www.scribbr.com/statistics/statistical-tests/>

# Statistical tests (3)

- **Statistical tests** work by calculating a **test statistic**
  - a number that describes how much the relationship between variables in your test differs from the **null hypothesis** of no relationship
- then calculates a **p value** (probability value).  
the **p-value** estimates how likely it is that you would see the difference described by the **test statistic** if the **null hypothesis** of no relationship were true.

<https://www.scribbr.com/statistics/statistical-tests/>



## Statistical tests (4)

- If the value of the **test statistic** is more extreme than the statistic calculated from the **null hypothesis**, then you can infer a statistically significant relationship between the **predictor** and **outcome** variables
- If the value of the **test statistic** is less extreme than the one calculated from the **null hypothesis**, then you can infer no statistically significant relationship between the **predictor** and **outcome** variables.

<https://www.scribbr.com/statistics/statistical-tests/>

# Statistical tests (5)

- You can perform **statistical tests** on data that have been collected in a statistically valid manner
  - either through an experiment,
  - or through observations made using probability sampling methods.
- For a **statistical test** to be valid, your sample size needs to be large enough to approximate the true distribution of the population being studied.
- To determine which **statistical test** to use, you need to know:
  - whether your data meets certain assumptions.
  - the types of variables that you're dealing with.

<https://www.scribbr.com/statistics/statistical-tests/>

# Statistical assumptions (1)

- **Statistical tests** make some common assumptions about the data they are testing:
  - Independence of observations (a.k.a. no autocorrelation):
  - Homogeneity of variance:
  - Normality of data:

<https://www.scribbr.com/statistics/statistical-tests/>

## Statistical assumptions (2)

- Independence of observations (a.k.a. no autocorrelation):  
The observations/variables you include in your test are not related (for example, multiple measurements of a single test subject are not independent, while measurements of multiple different test subjects are independent).
  - Homogeneity of variance: the variance within each group being compared is similar among all groups. If one group has much more variation than others, it will limit the test's effectiveness.
  - Normality of data: the data follows a normal distribution (a.k.a. a bell curve). This assumption applies only to quantitative data.

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# Types of data (1)

- **Data** is a specific measurement of a **variable**  
it is the value you record in your data sheet.  
Data is generally divided into two categories:
  - 1 **Quantitative data** represents amounts
  - 2 **Categorical data** represents groupings

<https://www.scribbr.com/statistics/statistical-tests/>

## Types of data (2)

- a **variable** that contains **quantitative data** is a **quantitative variable**;
- a **variable** that contains **categorical data** is a **categorical variable**.
- Each of these types of variables can be broken down into further types.

<https://www.scribbr.com/statistics/statistical-tests/>



# Quantitative variables

- When you collect quantitative data, the numbers you record represent real amounts that can be added, subtracted, divided, etc. There are two types of quantitative variables: discrete and continuous.

<https://www.scribbr.com/statistics/statistical-tests/>

# Catagorical variables

- Categorical variables represent groupings of some kind. They are sometimes recorded as numbers, but the numbers represent categories rather than actual amounts of things.
- There are three types of categorical variables: binary, nominal, and ordinal variables.

<https://www.scribbr.com/statistics/statistical-tests/>

# Independent and dependent variables

- Experiments are usually designed to find out what effect one variable has on another – in our example, the effect of salt addition on plant growth.
- You manipulate the independent variable (the one you think might be the cause) and then measure the dependent variable (the one you think might be the effect) to find out what this effect might be.
- You will probably also have variables that you hold constant (control variables) in order to focus on your experimental treatment.

<https://www.scribbr.com/statistics/statistical-tests/>

# The Null and Alternative Hypothesis

- Hypothesis testing is based on making two different claims about a population parameter.
- The null hypothesis ( $H_0$ ) and the alternative hypothesis ( $H_1$ ) are the claims.
- The two claims needs to be mutually exclusive, meaning only one of them can be true.
- The alternative hypothesis is typically what we are trying to prove.
- For example, we want to check the following claim:
  - "The average height of people in Denmark is more than 170 cm."

[https://www.w3schools.com/statistics/statistics\\_hypothesis\\_testing.php](https://www.w3schools.com/statistics/statistics_hypothesis_testing.php)

# Summary (1) comparing means

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tests

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- **one-sample** test    comparing sample mean, population mean
  - **two-sample** test    comparing two independent sample means
  - **paired** test        comparing two related sample means
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tests

test conditions

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- **t-test**            1. when the **population variance** is *known*  
                          2. when the **sample size** is *large*
  - **z-test**            1, when the **population variance** is *unknown*  
                          2. the **sample size** is *small*
- 

<https://www.qualitygurus.com/common-types-of-hypothesis-tests/>

## Summary (2) comparing means

one sample z-test	<u>sample mean</u> , <u>population mean</u> <u>known population var</u> / <u>large sample size</u>
one sample t-test	<u>sample mean</u> , <u>population mean</u> <u>unknown population var</u> / <u>small sample size</u>
two sample z-test	two <i>independent</i> <u>sample means</u> <u>known population var</u> / <u>large sample size</u>
two sample t-test	two <i>independent</i> <u>sample means</u> <u>unknown population var</u> / <u>small sample size</u>
paired t-test	two <i>related</i> <u>sample means</u> <u>unknown population var</u> / <u>small sample size</u>

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## Summary (3) comparing proportions

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one sample **proportion test**     sample proportion, population proportion  
when  $np \geq 10$  and  $n(1 - p) \geq 10$

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two sample **proportion test**     two *independent* sample proportions  
when  $np \geq 10$  and  $n(1 - p) \geq 10$

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test conditions

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the normal approximation is used

when both  $np \geq 10$  and  $n(1 - p) \geq 10$

(data should have at least 10 "successes" and at least 10 "failures" )

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## Summary (4)

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compare variances between

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sample variance, known population variance      Chi-square test

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two independent sample variances      F-test

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observed frequencies, expected frequencies      goodness of fit test

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observed frequencies, expected frequencies      contingency tables

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means of three or more independent samples      ANOVA (Analysis of Variance)

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# Tests for Comparing Means (1)

- One-sample **z-test**:
  - used to compare the **mean** of a sample to a known population mean
  - used when the population variance is known, or the sample size is *large* ( $n > 30$ ).
- Two-sample **z-test**:
  - used to compare the **means** of two independent samples.
  - used when the population variances are known, or the sample sizes are *large* ( $n > 30$ ).

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# Tests for Comparing Means (2)

- One-sample **t-test**:
  - used to compare the **mean** of a sample to a known population mean.
  - used when the population variance is unknown, and the sample size is *small* ( $n < 30$ ).
- Two-sample **t-test**:
  - used to compare the **means** of two independent **samples**.
  - used when the population variances are unknown, and the sample sizes are *small* ( $n < 30$ ).

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# Tests for Comparing Means (3)

- Paired **t-test**:
  - used to compare the **means** of two *related* samples, such as the before and after measurements of the same group of subjects.
  - used when the population **variances** are unknown, and the sample size is *small* ( $n < 30$ ).

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# Tests for Comparing Proportions (1)

- Let us consider the parameter  $p$  of the **population proportion**
  - eg) we might want to know the **proportion** of males within a total population of adults when we conduct a survey.
- A **test of proportion** will assess whether or not a **sample** from a **population** represents the true **proportion** of the entire **population**

<https://online.stat.psu.edu/statprogram/reviews/statistical-concepts/proportions>

# Examples of proportions

- an example
  - newborn babies are more likely to be boys than girls.
  - a random sample found 13,173 boys were born among 25,468 newborn children
  - the **sample proportion** of boys was 0.5172 ( $= \frac{13173}{25468}$ )
  - is this sample evidence that the birth of boys is more common than the birth of girls in the entire population? ( $0.5172 > 0.4828$ )

<https://online.stat.psu.edu/statprogram/reviews/statistical-concepts/proportions>

# Tests for Comparing Proportions (2-1)

- examples involved testing whether a single population **proportion**  $p$  equals some value .
- different examples of testing whether one population **proportion** equals a second population **proportion**

<https://online.stat.psu.edu/stat415/lesson/9/9.4>

# Tests for Comparing Proportions (2-2)

- Additionally, most of our examples thus far have involved
  - left-tailed tests in which the **alternative hypothesis** involved
  - right-tailed tests in which the **alternative hypothesis** involved
- Here, let's consider an example that tests the equality of two **proportions** against the **alternative** that they are not equal

<https://online.stat.psu.edu/stat415/lesson/9/9.4>

## Tests for Comparing Proportions (2-3)

- Time magazine reported the result of a telephone poll of 800 adult Americans.
- the question posed of the Americans who were surveyed was: "Should the federal tax on cigarettes be raised to pay for health care reform?"
- the results of the survey were:

Non-smokers	Smokers
$n_1 = 605$	$n_2 = 195$
$y_1 = 351$ said yes	$y_2 = 41$ said yes
$\hat{p}_1 = \frac{351}{605} = 0.58$	$\hat{p}_2 = \frac{41}{195} = 0.21$

<https://online.stat.psu.edu/stat415/lesson/9/9.4>



# Tests for Comparing Proportions (2-4)

- If  $p_1$  = the proportion of the non-smoker population who reply "yes"
- and  $p_2$  = the proportion of the smoker population who reply "yes,"
- then we are interested in testing the null hypothesis:  
 $H_0 : p_1 = p_2$   
against the alternative hypothesis:  
 $H_A : p_1 \neq p_2$
- Before we can actually conduct the hypothesis test, we'll have to derive the appropriate test statistic.

<https://online.stat.psu.edu/stat415/lesson/9/9.4>

# Tests for Comparing Proportions (2-5)

- The overall sample proportion is:

$$\hat{p} = \frac{41+351}{195+605} = \frac{392}{800} = 0.49$$

- that implies then that the test statistic for testing:

$$H_0 : p_1 = p_2 \text{ versus } H_A : p_1 \neq p_2$$

is:

$$Z = \frac{(0.58 - 0.21) - 0}{\sqrt{0.49(0.51)\left(\frac{1}{195} + \frac{1}{605}\right)}} = 8.9$$

<https://online.stat.psu.edu/stat415/lesson/9/9.4>

# Tests for Comparing Proportions (3)

- One-sample **proportion test** :
  - used to compare the **proportion** of a sample to a known population proportion.
  - the normal approximation is used when both  $np \geq 10$  and  $n(1 - p) \geq 10$  (data should have at least 10 "successes" and at least 10 "failures" ) (in some books, it is 5)

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# Tests for Comparing Proportions (4)

- Two-sample **proportion test** :
  - used to compare the proportions of two independent samples.
  - the normal approximation is used  
when both  $np \geq 10$  and  $n(1 - p) \geq 10$   
(data should have at least 10 "successes" and at least 10 "failures" )  
(in some books, it is 5)

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## Tests for Comparing Proportions (5)

- Time magazine reported the result of a telephone poll of 800 adult Americans.
- The question posed of the Americans who were surveyed was:  
"Should the federal tax on cigarettes be raised to pay for health care reform?"

The results of the survey were:

<https://online.stat.psu.edu/stat415/lesson/9/9.4>

## Tests for Comparing Proportions (6)

Non-Smokers	Smokers
$n_1 = 605$	$n_2 = 195$
$y_1 = 351$ said "yes"	$y_2 = 41$ said "yes"
$\hat{p}_1 = \frac{351}{605} = 0.58$	$\hat{p}_2 = \frac{41}{195} = 0.21$

- Is there sufficient evidence at the  $\alpha$ , say, to conclude that the two populations - smokers and non-smokers - differ significantly with respect to their opinions?

<https://online.stat.psu.edu/stat415/lesson/9/9.4>

## Tests for Comparing Proportions (7)

- Errr. . . . that Z-value is off the charts, so to speak. Let's go through the formalities anyway making the decision first using the rejection region approach, and then using the P-value approach. Putting half of the rejection region in each tail, we have:
- That is, we reject the null hypothesis if or if . We clearly reject , since 8.99 falls in the "red zone," that is, 8.99 is (much) greater than 1.96. There is sufficient evidence at the 0.05 level to conclude that the two populations differ with respect to their opinions concerning imposing a federal tax to help pay for health care reform.

<https://online.stat.psu.edu/stat415/lesson/9/9.4>

## Tests for Comparing Proportions (8)

- That is, the P-value is less than 0.0001. Because, we reject the null hypothesis. Again, there is sufficient evidence at the 0.05 level to conclude that the two populations differ with respect to their opinions concerning imposing a federal tax to help pay for health care reform.
- Thankfully, as should always be the case, the two approaches. . . . the critical value approach and the P-value approach. . . lead to the same conclusion

<https://online.stat.psu.edu/stat415/lesson/9/9.4>



# Tests for Comparing Variance

- **Chi-square test** for variance :
  - used to compare the **variance** of a sample to a known population variance
- **F-test** for variance :
  - used to compare the **variances** of two *independent samples*

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# Other Common Tests (1)

- **Goodness of fit** test :
- used to determine whether a sample fits a *specific* **distribution**.
- used to compare the observed frequencies of a *categorical variable* to the expected frequencies under a *particular* **distribution**.

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## Other Common Tests (2)

- Testing for **independence of two attributes** (**Contingency Tables**) :
- used to determine whether there is a relationship between two *categorical variables*.
- often used in the form of a **chi-square** test, which compares the observed frequencies in a **contingency table** to the expected frequencies under the assumption of independence.

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## Other Common Tests (3)

- ANOVA (Analysis of Variance) :
- used to compare the **means** of three or more independent samples.
- used to determine whether there is a significant difference between the **means** of the groups.

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# One-sample z-test

- used to test a hypothesis about the *population mean*
- based on the assumption that the sample is drawn from a **normally distributed** population.
  - the **null hypothesis**  
the *population mean* is equal to a specific value
  - the **alternative hypothesis**  
the *population mean* is not equal to that value

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# Two-sample z-test

- based on the assumption that both samples are drawn from **normally distributed** populations with equal **variances**.
- the two-sample **z-test** requires that the population **standard deviations** be known or that the sample **sizes** be *large* (30 or more),
  - the **null hypothesis**  
the **means** of the two samples are equal
  - the **alternative hypothesis**  
the **means** are not equal

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# One-sample t-test

- used to test a hypothesis about the *population mean*
- based on the assumption that the sample is drawn from a **normally distributed** population
  - the **null hypothesis**  
the *population mean* is equal to a specific value
  - the **alternative hypothesis**  
the *population mean* is not equal to that value

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# Two-sample t-test

- based on the assumption that the samples are drawn from populations with **normal distributions**.
- the two-sample **t-test** that the population **standard deviations** need not be known or that the sample **sizes** need not be large (30 or more),
  - the **null hypothesis**  
the **means** of the two samples are equal
  - the **alternative hypothesis**  
the **means** are not equal

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# Paired t-test

- used to test a hypothesis about the difference between the means of the two samples
- based on the assumption that the differences between the pairs are **normally distributed**
- In a dependent two-sample t-test (a paired t-test), the samples in the two groups being compared are *related* in some way.
  - the **null hypothesis**  
there is no difference between the means of the two samples
  - the **alternative hypothesis**  
there is a difference between the means

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## Two proportions **z-test**

- used to test a hypothesis about the difference between the proportions of the two samples and
- based on the assumption that the samples are drawn from populations with a **normal distribution**
  - the **null hypothesis** :  
there is no difference between the proportions of the two samples
  - the **alternative hypothesis** :  
there is a difference between the proportion

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