

Fourier Series (H.1)

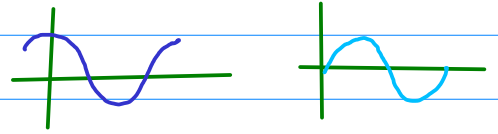
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Taylor Series

$$f(x) = \cos x$$



$$f'(x) = -\sin x$$

$$f'(0) = -\sin 0 = 0$$

$$f^{(2)}(x) = -\cos x$$

$$f^{(2)}(0) = -\cos 0 = -1$$

$$f^{(3)}(x) = +\sin x$$

$$f^{(3)}(0) = +\sin 0 = 0$$

$$f^{(4)}(x) = +\cos x$$

$$f^{(4)}(0) = +\cos 0 = +1$$

$$f^{(5)}(x) = -\sin x$$

$$f^{(5)}(0) = -\sin 0 = 0$$

$$f^{(6)}(x) = -\cos x$$

$$f^{(6)}(0) = -\cos 0 = -1$$

$$f^{(7)}(x) = +\sin x$$

$$f^{(7)}(0) = +\sin 0 = 0$$

$$f^{(8)}(x) = +\cos x$$

$$f^{(8)}(0) = +\cos 0 = +1$$

⋮

⋮

$x=0$ 근방에서 Taylor series

$$f(x) = f(0) + \frac{f^{(1)}(0)}{1!} x^1 + \frac{f^{(2)}(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4$$

$$\cos x = +1 + \frac{0}{1!} x^1 + \frac{-1}{2!} x^2 + \frac{0}{3!} x^3 + \frac{-1}{4!} x^4$$

$$f'(0) = -\sin 0 = 0$$

$$f^{(2)}(0) = -\cos 0 = -1$$

$$f^{(3)}(0) = +\sin 0 = 0$$

$$f^{(4)}(0) = +\cos 0 = +1$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!}$$

0 근방에서

Series (2/4)

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

a given periodic function

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

a_0	+	$a_1 \cos(1x)$		+	$b_1 \sin(1x)$
	+	$a_2 \cos(2x)$		+	$b_2 \sin(2x)$
	+	$a_3 \cos(3x)$		+	$b_3 \sin(3x)$
		\vdots			\vdots
	+	$a_n \cos(nx)$		+	$b_n \sin(nx)$
		\vdots			\vdots

$$\left\{ \begin{array}{l} \cos(1x) \\ \cos(2x) \\ \cos(3x) \\ \vdots \\ \cos(nx) \\ \vdots \end{array} \right\} \quad \left\{ \begin{array}{l} \sin(1x) \\ \sin(2x) \\ \sin(3x) \\ \vdots \\ \sin(nx) \\ \vdots \end{array} \right\}$$

\Downarrow \Downarrow
 Orthogonal Orthogonal
 set set

$$\langle \cos(mx), \cos(nx) \rangle = 0 \quad \langle \sin(mx), \sin(nx) \rangle = 0$$

$$\langle \cos(mx), \sin(nx) \rangle = 0$$

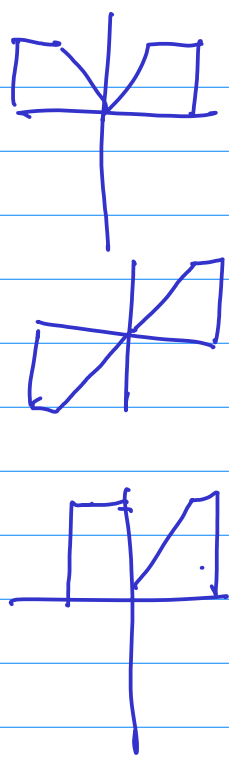
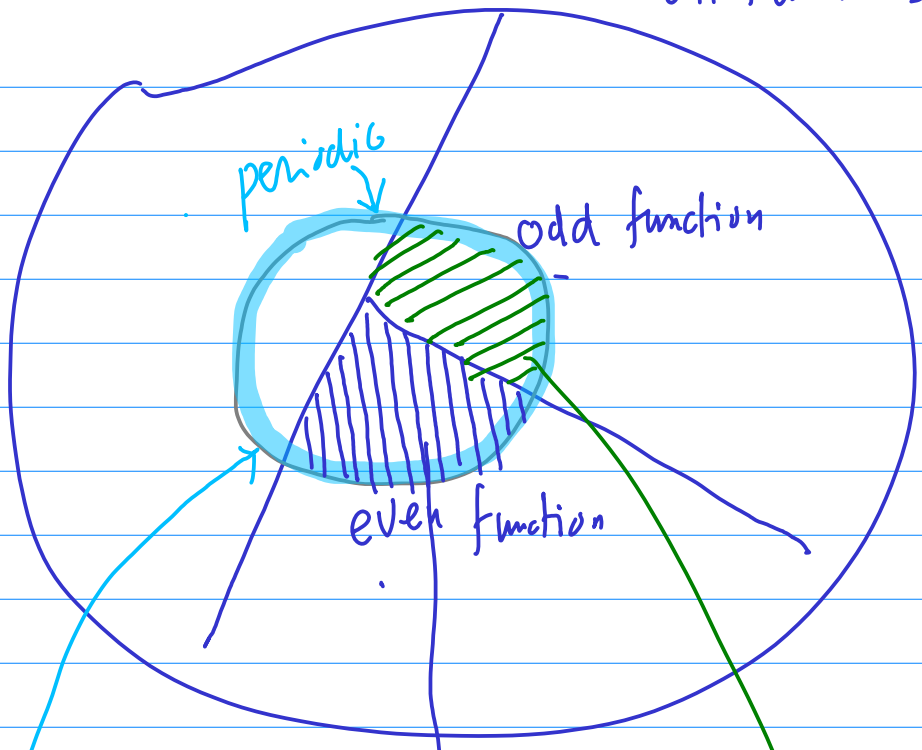
$$\langle \sin(mx), \cos(nx) \rangle = 0$$

$$\langle f(x), g(x) \rangle \triangleq \int_a^b f(x) \cdot g(x) \, dx$$

$$f(x) = a_0 + a_1 \cos(1x) + b_1 \sin(1x) \\ + a_2 \cos(2x) + b_2 \sin(2x) \\ + a_3 \cos(3x) + b_3 \sin(3x) \\ \vdots \\ + a_n \cos(nx) + b_n \sin(nx) \\ \vdots$$

$$\cos(1x) \times f(x) = \left[\begin{array}{rcl} a_0 + a_1 \cos(1x) + b_1 \sin(1x) & \times \cos(1x) \\ + a_2 \cos(2x) + b_2 \sin(2x) \\ + a_3 \cos(3x) + b_3 \sin(3x) \\ \vdots \\ + a_n \cos(nx) + b_n \sin(nx) \\ \vdots \end{array} \right]$$

all functions



Fourier series

$$\left\{ \begin{array}{l} \cos mx \\ \sin nx \end{array} \right\}$$

cosine series

$$\{ \cos mx \}$$

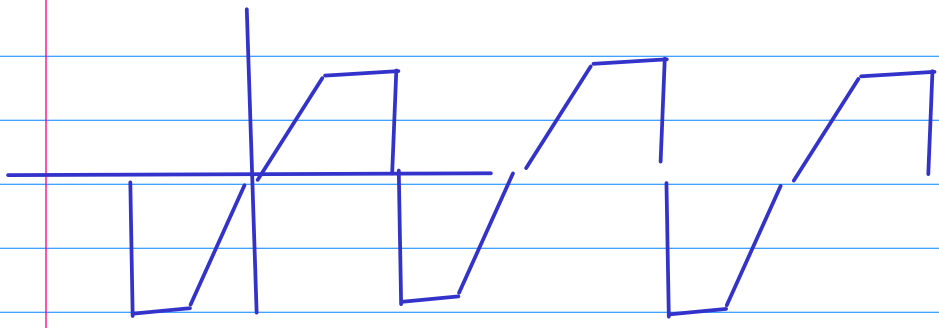
sine series

$$\{ \sin mx \}$$

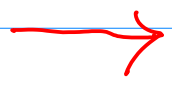
discrete set of values



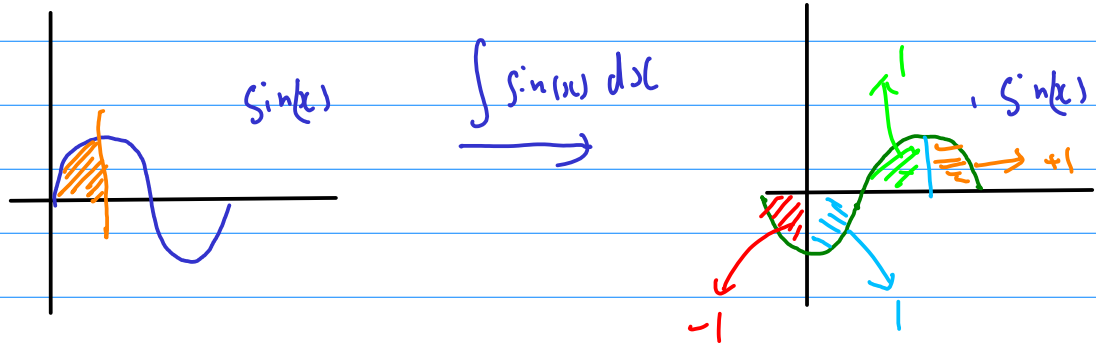
a_n
 b_n
 ~~~~~



continuous function  
 ^  
 periodic



# Fourier Series



$$\int_0^{\frac{\pi}{2}} \sin(x) dx = 1$$

$$\int_0^{2\pi} \sin(x) dx = 0$$

$$\int_0^{2\pi} \sin(2x) dx = 0$$

$$n = 0, 1, 2, \dots \quad \int_{-\pi}^{+\pi} \cos(nx) dx = 0 \quad \int_{-\pi}^{+\pi} \sin(nx) dx = 0$$

$$\int_{-\pi}^{+\pi} \cos(mx) \cos(nx) dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$$\int_{-\pi}^{+\pi} \sin(mx) \sin(nx) dx = \begin{cases} 0 & (m \neq n) \\ \pi & (m = n) \end{cases}$$

$$\cos \alpha \cos \beta = \frac{1}{2} \{ +\cos(\alpha+\beta) + \cos(\alpha-\beta) \}$$

$$\sin \alpha \sin \beta = \frac{1}{2} \{ -\cos(\alpha+\beta) + \cos(\alpha-\beta) \}$$

$$\cos(m x) \cos(n x) = \frac{1}{2} \{ +\cos((m+n)x) + \cos((m-n)x) \}$$

$$\sin(m x) \sin(n x) = \frac{1}{2} \{ -\cos((m+n)x) + \cos((m-n)x) \}$$

$$\frac{\cos(Px)}{p=m+n} \quad \frac{\cos(Qx)}{Q=m-n}$$

$$\int_{-\pi}^{+\pi} \cos(m x) \cos(n x) dx = 0 \quad m \neq n$$

$$\int_{-\pi}^{+\pi} \sin(m x) \sin(n x) dx = 0 \quad m \neq n$$

$$\int_{-\pi}^{+\pi} \cos^2(m x) dx \neq 0 \quad m = n$$

$$\int_{-\pi}^{+\pi} \sin^2(m x) dx \neq 0 \quad m = n$$



$$\cos \alpha \cos \beta = \frac{1}{2} \{ +\cos(\alpha+\beta) + \cos(\alpha-\beta) \}$$

$$\cos \theta \cos \theta = \frac{1}{2} \{ +\cos(2\theta) + \cos(0) \}$$

$$\cos^2 \theta = \frac{1}{2} \{ \cos(2\theta) + 1 \}$$

$$\sin \alpha \sin \beta = \frac{1}{2} \{ -\cos(\alpha+\beta) + \cos(\alpha-\beta) \}$$

$$\sin \theta \sin \theta = \frac{1}{2} \{ -\cos(2\theta) + \cos(0) \}$$

$$\sin^2 \theta = \frac{1}{2} \{ -\cos(2\theta) + 1 \}$$

$$\sin^2 \theta = \frac{1}{2} \{ 1 - \cos(2\theta) \}$$

$$\cos^2 \theta = \frac{1}{2} \{ 1 + \cos(2\theta) \}$$

$$\int_{-\pi}^{+\pi} \cos^2(mx) dx = \int_{-\pi}^{+\pi} \frac{1}{2} \{ 1 - \cos(2mx) \} dx = \pi \quad m = n$$

$$\int_{-\pi}^{+\pi} \sin^2(mx) dx = \int_{-\pi}^{+\pi} \frac{1}{2} \{ 1 + \cos(2mx) \} dx = \pi \quad m = n$$

$$= \left[ \frac{1}{2} x + \text{[scribble]} \right]_{-\pi}^{\pi} = \frac{1}{2} \cdot (2\pi) = \pi$$



$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

|

$$\cos \alpha \sin \beta = \frac{1}{2} \{ +\sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

$$\sin \alpha \cos \beta = \frac{1}{2} \{ +\cos(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$\cos(mx) \sin(nx) = \frac{1}{2} \{ +\sin((m+n)x) - \sin((m-n)x) \}$$

$$\sin(mx) \cos(nx) = \frac{1}{2} \{ +\sin((m+n)x) + \sin((m-n)x) \}$$

$$\int_{-\pi}^{+\pi} \cos(mx) \sin(nx) dx = 0$$

$$\int_{-\pi}^{+\pi} \sin(mx) \cos(nx) dx = 0$$

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\rightarrow \sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

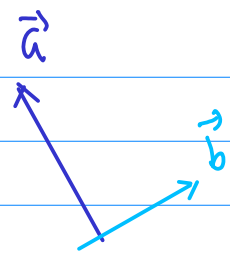
$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos\alpha \sin\beta$$

$$\cos\alpha \sin\beta = \frac{1}{2} \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

# Def Orthogonal Functions

각각

$$\vec{a} \cdot \vec{b} = 0$$



$$(f_1, f_2) = \int_a^b f_1(x) f_2(x) dx$$

function  $f_1$  &  $f_2 \rightarrow$  orthogonal to each other  
on an interval  $[a, b]$

$$\int_{-\pi}^{+\pi} \cos(1 \cdot x) \cos(2x) dx = 0$$

$\cos(1 \cdot x)$  &  $\cos(2x)$   
orthogonal to each other

$$\int_{-\pi}^{+\pi} \sin(1 \cdot x) \sin(2x) dx = 0$$

$\sin(1 \cdot x)$  &  $\sin(2x)$   
orthogonal to each other

# Inner Product

$$\vec{a} \cdot \vec{b}$$

vectors

$$\langle f, g \rangle$$

functions

$$\mathbb{R}^2 \quad \vec{a} = (a_1, a_2)$$
$$\vec{b} = (b_1, b_2)$$

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 = |\vec{a}| |\vec{b}| \cos \theta$$

$$\mathbb{R}^3 \quad \vec{a} = (a_1, a_2, a_3)$$
$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} \cdot \vec{b} = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3$$

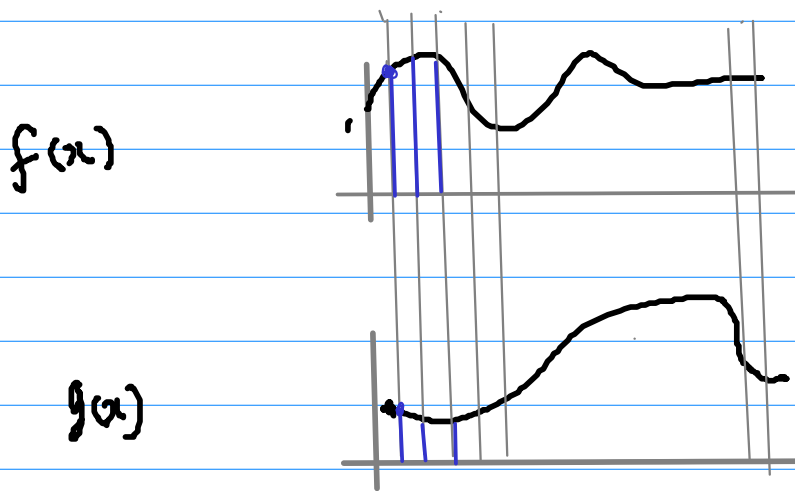
$$\mathbb{R}^n \quad \vec{a} = (a_1, a_2, a_3, \dots, a_n)$$

$$\vec{b} = (b_1, b_2, b_3, \dots, b_n)$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n$$

$$\sum f(x_i), f(x_2), f(x_3), \dots, f(x_n)$$

$$\sum g(x_i), g(x_2), g(x_3), \dots, g(x_n)$$



$$\langle f, g \rangle = \int_{x_1}^{x_2} f(x) \cdot g(x) dx$$

# Orthogonal set

$$\{ \phi_0(x), \phi_1(x), \phi_2(x), \dots \}$$

$$\langle \phi_0(x), \phi_1(x) \rangle = \int_a^b \phi_0(x) \phi_1(x) dx = 0$$

$$\langle \phi_0(x), \phi_2(x) \rangle = \int_a^b \phi_0(x) \phi_2(x) dx = 0$$

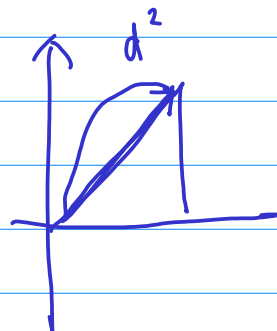
$$\langle \phi_0(x), \phi_3(x) \rangle = \int_a^b \phi_0(x) \phi_3(x) dx = 0$$

$$\langle \phi_m(x), \phi_n(x) \rangle = 0 \quad m \neq n$$

$$\begin{aligned} \langle \phi_m(x), \phi_n(x) \rangle &= \|\phi_m(x)\|^2 \quad m = n \\ &= \|\phi_n(x)\|^2 \end{aligned}$$

$$\vec{a} \cdot \vec{a} = (a_1, a_2) \cdot (a_1, a_2) = a_1^2 + a_2^2$$

$$\|\vec{a}\|^2 = \vec{a} \cdot \vec{a}$$



norm

$$\|\phi_n\|^2 = \langle \phi_n(x), \phi_n(x) \rangle = \int_a^b \phi_n^2(x) dx$$

$$\|\phi_n\|$$

$$= \sqrt{\int_a^b \phi_n^2(x) dx}$$



Orthogonal set

$[-\pi, \pi]$

$\{ \cos(0 \cdot x), \cos(1 \cdot x), \cos(2 \cdot x), \dots \}$

$$\langle \cos(mx), \cos(nx) \rangle = \int_{-\pi}^{+\pi} \cos(mx) \cos(nx) dx$$

$m \neq n$  ||  
0

$\{ \sin(0 \cdot x), \sin(1 \cdot x), \sin(2 \cdot x), \dots \}$

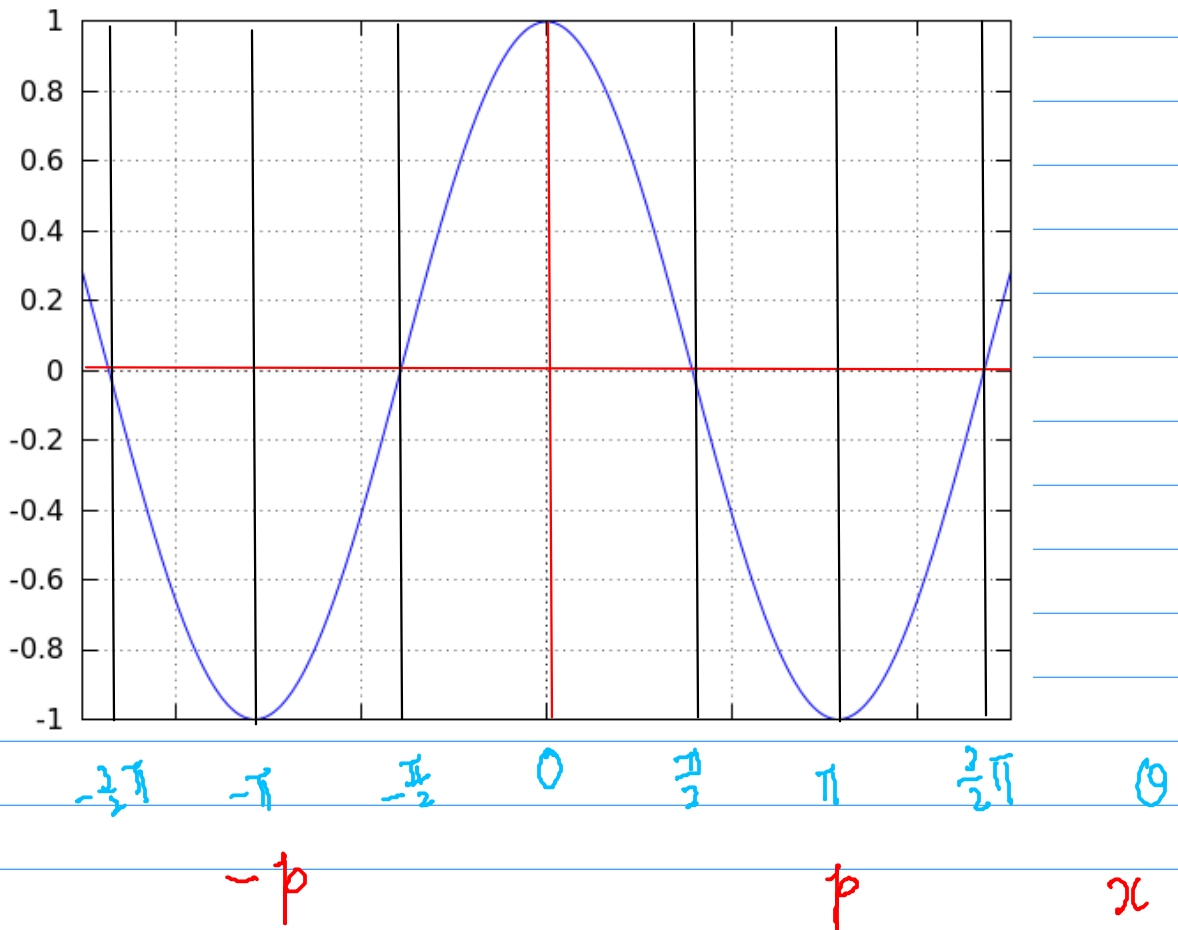
$$\langle \sin(mx), \sin(nx) \rangle = \int_{-\pi}^{+\pi} \sin(mx) \sin(nx) dx$$

$m \neq n$  ||  
0

$\left\{ \begin{array}{l} \cos(0 \cdot x), \cos(1 \cdot x), \cos(2 \cdot x), \dots \\ \sin(0 \cdot x), \sin(1 \cdot x), \sin(2 \cdot x), \dots \end{array} \right\}$

$$\cos\left(\frac{n\pi}{p}x\right) \xrightarrow{n=1} \cos\left(\frac{\pi}{p}x\right) = \cos(\theta)$$

$-\pi \sim \pi$



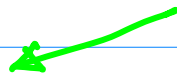
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$a_0$ ?

$a_n$ ?

$b_n$ ?

orthogonal function  
char.



Finding  $a_0$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$$\begin{aligned} \int_{-\pi}^{+\pi} f(x) dx &= \int_{-\pi}^{+\pi} \left( \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx)) \right) dx \\ &= \int_{-\pi}^{+\pi} \frac{a_0}{2} dx + \sum_{n=1}^{\infty} \left( \int_{-\pi}^{+\pi} a_n \cos(nx) + b_n \sin(nx) dx \right) \\ &= \left[ \frac{a_0}{2} x \right]_{-\pi}^{+\pi} = a_0 \pi \end{aligned}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) dx$$

# Finding $a_n$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

change index variables  $k=1, 2, 3, \dots$   $n$  a specific number  $a_n, b_n$

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx))$$

$$f(x) \cos(nx) =$$

$$\frac{a_0}{2} \cos(nx) + \sum_{k=1}^{\infty} (a_k \cos(kx) \cos(nx) + b_k \sin(kx) \cos(nx))$$

$$\int_{-\pi}^{+\pi} f(x) \cos(nx) dx = \int_{-\pi}^{+\pi} \frac{a_0}{2} \cos(nx) dx$$
$$+ \sum_{k=1}^{\infty} \left( \int_{-\pi}^{+\pi} a_k \cos(kx) \cos(nx) + b_k \sin(kx) \cos(nx) dx \right)$$
$$= \pi a_n$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx$$

# Finding $b_n$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

change index variables  $k=1, 2, 3, \dots$   $n$ : a specific num  $a_n, b_n$

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx))$$

$$f(x) \sin(nx) =$$

$$\frac{a_0}{2} \sin(nx) + \sum_{k=1}^{\infty} (a_k \cos(kx) \sin(nx) + b_k \sin(kx) \sin(nx))$$

$$\int_{-\pi}^{+\pi} f(x) \sin(nx) dx = \int_{-\pi}^{+\pi} \frac{a_0}{2} \sin(nx) dx$$

$$\sum_{k=1}^{\infty} \left( \int_{-\pi}^{+\pi} a_k \cos(kx) \sin(nx) + b_k \sin(kx) \sin(nx) dx \right)$$

$$= \pi b_n$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

$$-\pi \leq x \leq \pi$$

$$-p \leq x \leq p$$

## Def Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right) \right)$$

$$a_0 = \frac{1}{p} \int_{-p}^{+p} f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^{+p} f(x) \cos\left(\frac{n\pi}{p} x\right) dx$$

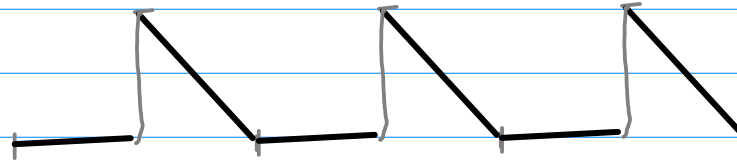
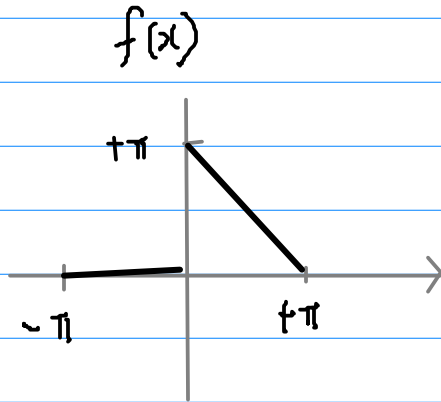
$$b_n = \frac{1}{p} \int_{-p}^{+p} f(x) \sin\left(\frac{n\pi}{p} x\right) dx$$

$$-p \leq x \leq p$$

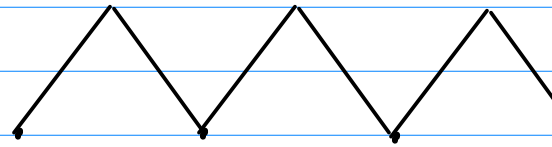
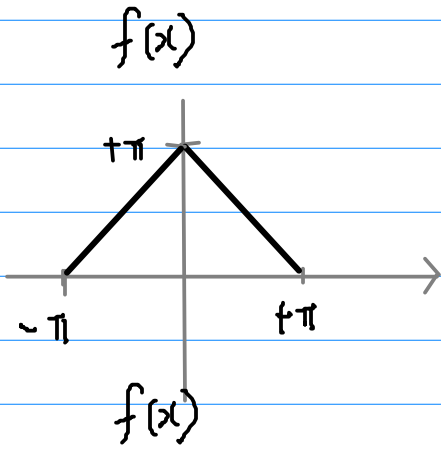


Find  $a_0, a_n, b_n$

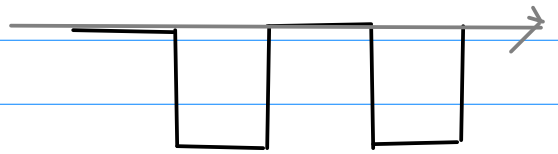
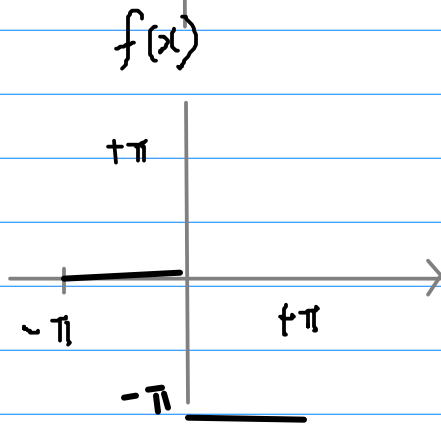
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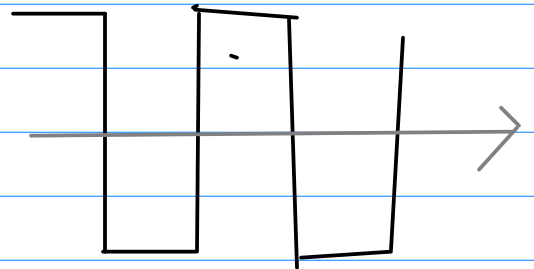
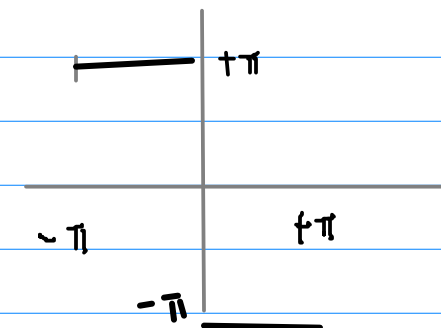
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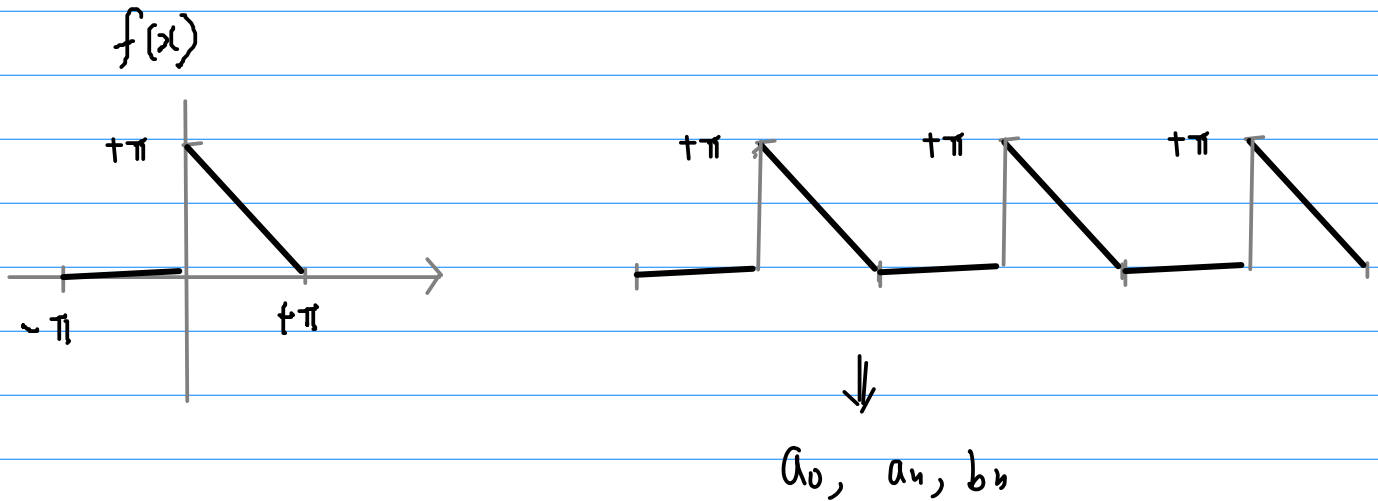
③



④



①



$$f(x) = 0 \quad -\pi < x \leq 0$$

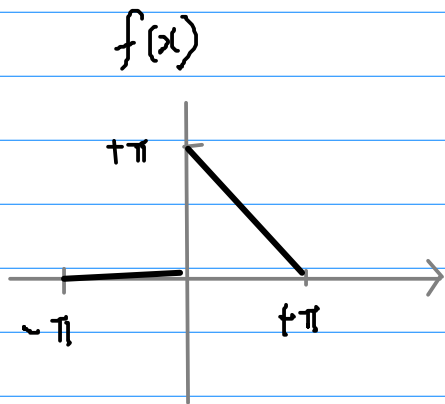
$$f(x) = -x + \pi \quad 0 < x < \pi$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right) \right)$$

$$a_0 = \frac{1}{p} \int_{-p}^{+p} f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^{+p} f(x) \cos\left(\frac{n\pi}{p} x\right) dx$$

$$b_n = \frac{1}{p} \int_{-p}^{+p} f(x) \sin\left(\frac{n\pi}{p} x\right) dx$$



$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ -x + \pi & 0 \leq x < \pi \end{cases}$$

$$\begin{aligned} a_0 &= \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) dx = \frac{1}{\pi} \int_{-\pi}^0 0 dx + \frac{1}{\pi} \int_0^{\pi} (-x + \pi) dx \\ &= \frac{1}{\pi} \int_0^{\pi} -x + \pi dx = \frac{1}{\pi} \left[ -\frac{1}{2}x^2 + \pi x \right]_0^{\pi} = \frac{1}{\pi} \left[ -\frac{\pi^2}{2} + \pi^2 \right] \\ &= \frac{1}{\pi} \cdot \frac{\pi^2}{2} = \left( \frac{\pi}{2} \right) \end{aligned}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos\left(\frac{n\pi}{\pi} x\right) dx$$

$$= \frac{1}{\pi} \int_{-\pi}^0 0 \cdot \cos(nx) dx + \frac{1}{\pi} \int_0^{\pi} \underbrace{(-x + \pi)} \cdot \underbrace{\cos(nx)} dx$$

$$\begin{aligned} \int f g' dx \\ = fg - \int f' g dx \end{aligned}$$

$$\int \cos(nx) dx = \frac{1}{n} \sin(nx)$$

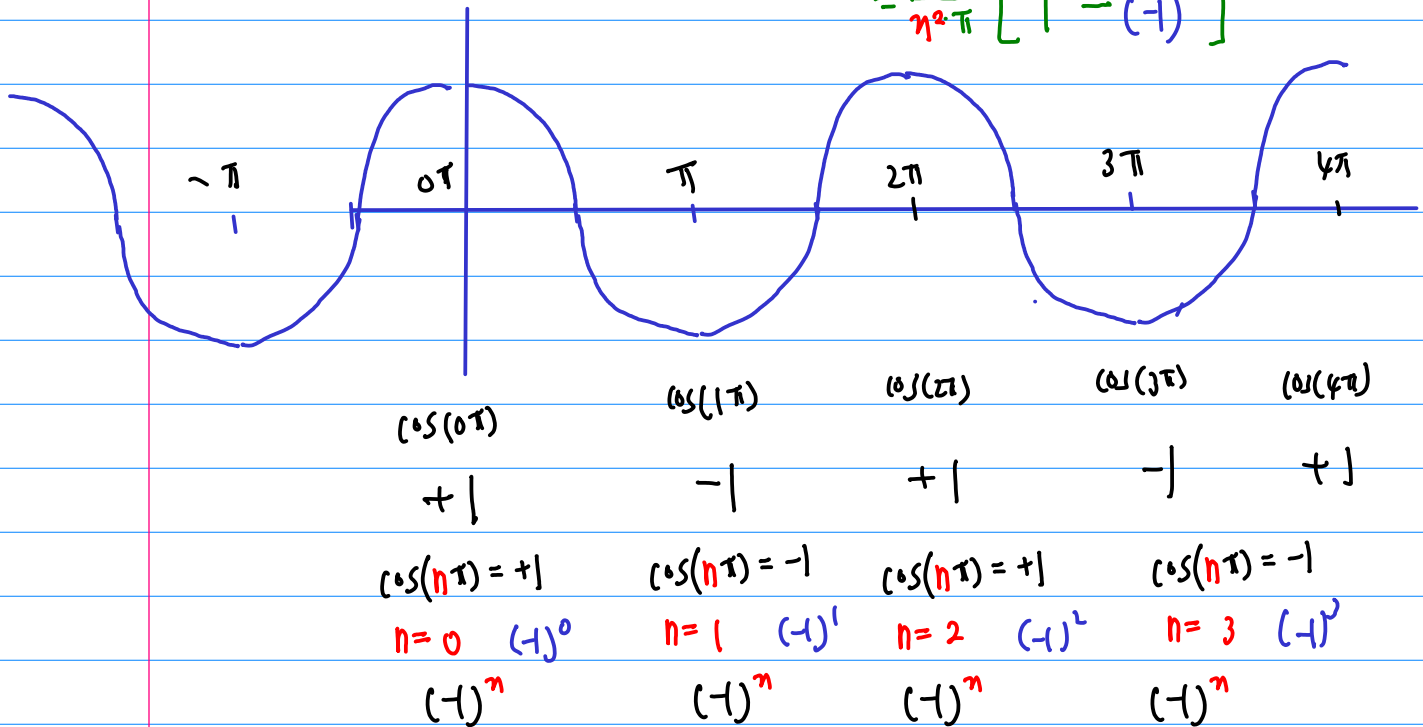
$$= \frac{1}{\pi} \left[ (-x + \pi) \frac{1}{n} \sin(nx) - \int (-1) \frac{1}{n} \sin(nx) dx \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[ (-x + \pi) \frac{1}{n} \sin(nx) - \frac{1}{n^2} \cos(nx) \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[ \int_0^{\pi} \frac{(-x+\pi)}{\eta} \sin(nx) dx - \int_0^{\pi} \frac{1}{\eta^2} \cos(nx) dx \right]$$

$$\begin{aligned} \sin(n\pi) &= 0 \\ \sin(n \cdot 0) &= 0 \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\pi} \left[ \frac{1}{\eta^2} \cos(n\pi) - \frac{1}{\eta^2} \cos(n \cdot 0) \right] \\ &= \frac{1}{\pi} \left[ \frac{1}{\eta^2} (-1)^n - \frac{1}{\eta^2} \right] \\ &= \frac{1}{\eta^2 \pi} [1 - (-1)^n] \end{aligned}$$



$$\boxed{\cos(n\pi) = (-1)^n} \quad n \text{ or integer}$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{+0} 0 \cdot \sin(nx) dx + \frac{1}{\pi} \int_0^{+\pi} (\pi-x) \sin(nx) dx$$

$$\int \sin(nx) dx = -\frac{1}{n} \cos(nx)$$

Integration by parts  $\int f g' dx = f g - \int f' g dx$

$$= \frac{1}{\pi} \left[ (\pi-x) \frac{1}{n} \cos(nx) \right]_0^{\pi} - \int_0^{\pi} (-1) \frac{1}{n} \cos(nx) dx$$

$$= \frac{1}{\pi} \left[ \frac{\pi}{n} - \int_0^{\pi} \frac{1}{n} \cos(nx) dx \right]$$

$$= \frac{1}{\pi} \left[ \frac{\pi}{n} - \left[ \frac{1}{n^2} \sin(nx) \right]_0^{\pi} \right]$$

$$= \frac{1}{n}$$

$$a_0 = \frac{\pi}{2}$$

$$a_n = \frac{1 - (-1)^n}{n^2 \pi}$$

$$b_n = \frac{1}{n}$$

```
(%i1) a(n) := (1 - (-1)^n) / (n^2 * %pi);
```

```
(%o1) a(n) :=  $\frac{1 - (-1)^n}{n^2 \pi}$ 
```

```
(%i2) a(1);
```

```
(%o2)  $\frac{2}{\pi}$ 
```

```
(%i3) a(2);
```

```
(%o3) 0
```

```
(%i4) a(3);
```

```
(%o4)  $\frac{2}{9 \pi}$ 
```

```
(%i5) a(4);
```

```
(%o5) 0
```

```
(%i6) a(5);
```

```
(%o6)  $\frac{2}{25 \pi}$ 
```

```
(%i7) a(6);
```

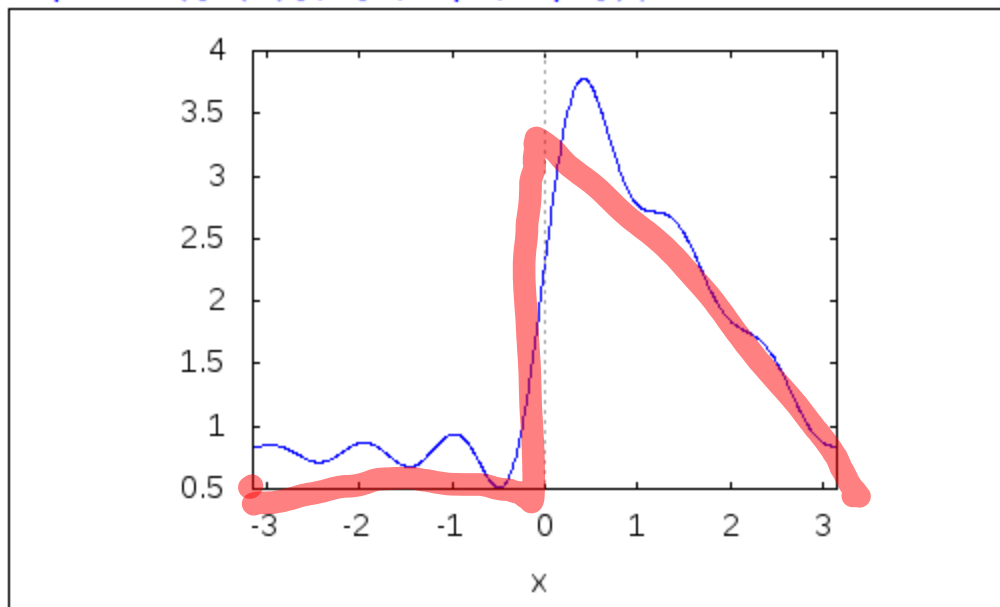
```
(%o7) 0
```

```
(%i27) f(x) := %pi/2
+ a(1)*cos(x) + b(1)*sin(x)
+ a(2)*cos(2*x) + b(2)*sin(2*x)
+ a(3)*cos(3*x) + b(3)*sin(3*x)
+ a(4)*cos(4*x) + b(4)*sin(4*x)
+ a(5)*cos(5*x) + b(5)*sin(5*x)
+ a(6)*cos(6*x) + b(6)*sin(6*x);
```

```
(%o27) f(x) :=  $\frac{\pi}{2} + a(1) \cos(x) + b(1) \sin(x) + a(2) \cos(2x) + b(2) \sin(2x) + a(3) \cos(3x) +$ 
 $b(3) \sin(3x) + a(4) \cos(4x) + b(4) \sin(4x) + a(5) \cos(5x) + b(5) \sin(5x) + a(6) \cos(6x) +$ 
 $b(6) \sin(6x)$ 
```

```
(%i29) wxplot2d([f(x)], [x, -%pi, %pi])$
```

```
(%t29)
```



# Taylor Series (McLaurin Series : $x=0$ )

$$f(x) = f(0) + \frac{f^{(1)}(0)}{1!} x^1 + \frac{f^{(2)}(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4$$

# Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right) \right)$$

$$x \in [-p, +p] \quad \cos\left(\frac{\pi}{p} x\right) \quad \sin\left(\frac{\pi}{p} x\right)$$

$$x \in \left[-\frac{p}{2}, +\frac{p}{2}\right] \quad \cos\left(\frac{2\pi}{p} x\right) \quad \sin\left(\frac{2\pi}{p} x\right)$$

$$x \in \left[-\frac{p}{3}, +\frac{p}{3}\right] \quad \cos\left(\frac{3\pi}{p} x\right) \quad \sin\left(\frac{3\pi}{p} x\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos(nx) + b_n \sin(nx) \right)$$

$$x \in [-\pi, \pi] \quad \cos(1x) \quad \sin(1x) \quad n=1$$

$$x \in \left[-\frac{\pi}{2}, +\frac{\pi}{2}\right] \quad \cos(2x) \quad \sin(2x)$$

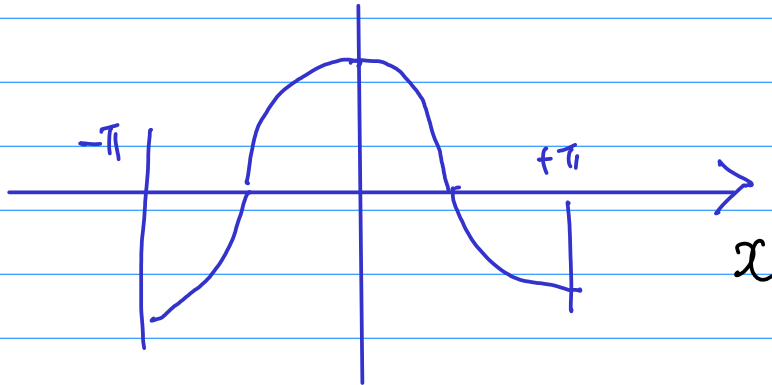
$$x \in \left[-\frac{\pi}{3}, +\frac{\pi}{3}\right] \quad \cos(3x) \quad \sin(3x)$$



$$-\pi \leq x \leq \pi \quad \text{vs.} \quad -p \leq x \leq p$$

$$\cos(\square)$$

$$-\pi \leq \square \leq \pi$$



$$\cos(2x)$$

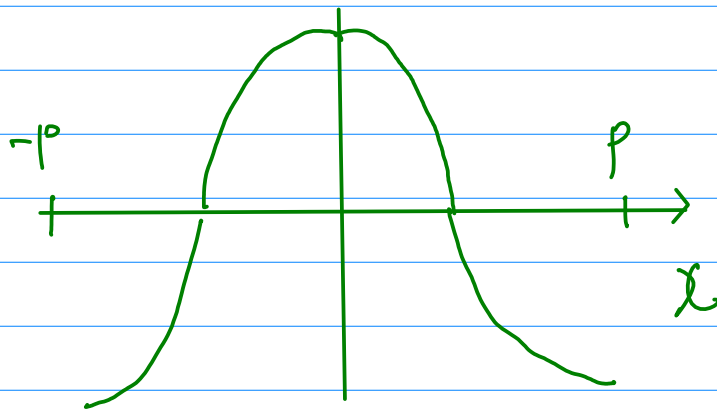
$$-\pi \leq 2x \leq \pi$$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\cos\left(\frac{\pi}{p}x\right)$$

$$-\pi \leq \left(\frac{\pi}{p}x\right) \leq \pi$$

$$-p \leq x \leq p$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$a_0, a_1, a_2, a_3, \dots$   
 $b_1, b_2, b_3, \dots$  ) 알 때  
 $f(x)$  는 알 수 있다.

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

이런 함수  $f(x)$  를 푸리에 계수 구하기.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right) \right)$$

$a_0, a_1, a_2, a_3, \dots$   
 $b_1, b_2, b_3, \dots$  ) 알 때  
 $f(x)$  는 만들 수 있다.

$$a_0 = \frac{1}{p} \int_{-p}^{+p} f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^{+p} f(x) \cos\left[\frac{n\pi}{p} x\right] dx$$

$$b_n = \frac{1}{p} \int_{-p}^{+p} f(x) \sin\left[\frac{n\pi}{p} x\right] dx$$

이런 함수  $f(x)$  를 복리 계수 구하기.



2h  
7

04/2011 (1) F Fourier Series

$$A_0 = \frac{1}{2L} \int_{-L}^L f(x) dx = \frac{1}{2L} \int_{-L}^L (L-x) dx = \frac{1}{2L} \left[ Lx - \frac{1}{2}x^2 \right]_{-L}^L$$

$$= \frac{1}{2L} (2L^2) = L$$

$$A_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L} \int_{-L}^L (L-x) \cos\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L} \int_{-L}^L \underline{L \cos\left(\frac{n\pi x}{L}\right)} - x \cos\left(\frac{n\pi x}{L}\right) dx$$

*odd x even = odd*

$$= \frac{1}{L} \int_{-L}^L -x \cos\left(\frac{n\pi x}{L}\right) dx = 0$$

$$B_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L} \int_{-L}^L (L-x) \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{1}{L} \int_{-L}^L \underline{L \sin\left(\frac{n\pi x}{L}\right)} - x \sin\left(\frac{n\pi x}{L}\right) dx$$

*odd x odd = even*

$$= \frac{1}{L} \int_{-L}^L -x \sin\left(\frac{n\pi x}{L}\right) dx = 0$$

$$\int x \sin(kx) dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$= \frac{1}{L} \int_{-L}^L -x \sin\left(\frac{n\pi x}{L}\right) dx = 0$$

$$= -\frac{1}{L} \int_{-L}^L x \sin\left(\frac{n\pi}{L} x\right) dx$$

$$= \frac{1}{L} \left[ -\frac{L}{n\pi} x \cos\left(\frac{n\pi}{L} x\right) + \left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L} x\right) \right]_{-L}^L$$

$$= -\frac{1}{L} \left[ -\frac{L}{n\pi} \cdot L \cos\left(\frac{n\pi}{L} L\right) + \frac{L}{n\pi} (-L) \cos\left(\frac{n\pi}{L} (-L)\right) \right]$$

$$\left[ \left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L} L\right) - \left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L} (-L)\right) \right]$$

$$= -\frac{1}{L} \left[ -\frac{L}{n\pi} \cdot L \cos\left(\frac{n\pi}{L} L\right) + \frac{L}{n\pi} (-L) \cos\left(\frac{n\pi}{L} (-L)\right) \right]$$

$$= -\frac{1}{L} \left[ -\frac{2L^2}{n\pi} (-1)^n \right] = \frac{2L}{n\pi} (-1)^n$$

$$\int f g' dx = f g - \int f' g dx$$

$$\begin{aligned}\int x \sin(kx) dx &= x \left[ -\frac{1}{k} \cos(kx) \right] - \int \left[ -\frac{1}{k} \cos(kx) \right] dx \\ &= -\frac{1}{k} x \cos(kx) + \frac{1}{k} \int \cos(kx) dx \\ &= -\frac{1}{k} x \cos(kx) + \frac{1}{k} \left[ \frac{1}{k} \sin(kx) \right] \\ &= -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)\end{aligned}$$

$$\int x \sin(kx) dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\left[ F(x) \right]_{-a}^{+a} = F(a) - F(-a)$$



$$\int f g' dx = f g - \int f' g dx$$

$$\begin{aligned}\int x \cos(kx) dx &= x \left[ \frac{1}{k} \sin(kx) \right] - \int \left[ \frac{1}{k} \sin(kx) \right] dx \\ &= \frac{1}{k} x \sin(kx) - \frac{1}{k} \int \sin(kx) dx \\ &= \frac{1}{k} x \sin(kx) + \frac{1}{k} \left[ \frac{1}{k} \cos(kx) \right] \\ &= \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)\end{aligned}$$

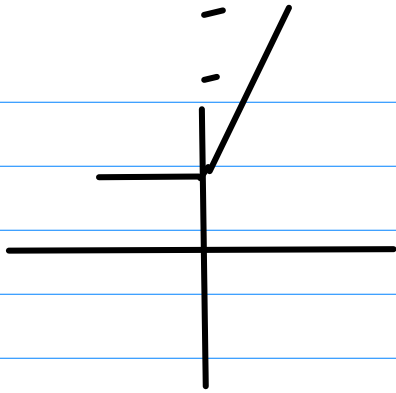
$$\int x \cos(kx) dx = \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

$$\int x \cos(kx) dx = \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

$$\int x \sin(kx) dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\int x^2 \cos(kx) dx = ?$$

$$\int x^2 \sin(kx) dx = ?$$



$$A_0 = \frac{1}{2L} \int_{-L}^L f(x) dx = \frac{1}{2L} \left[ \int_{-L}^0 L dx + \int_0^L 2x dx \right]$$

$$= \frac{1}{2L} [L^2 + L^2] = \frac{2L^2}{2L} = L$$

$$A_n = \frac{1}{L} \int_{-L}^L f(x) \cos(kx) dx = \frac{1}{L} \left[ \int_{-L}^0 L \cos(kx) dx + \int_0^L 2x \cos(kx) dx \right]$$

$$= \frac{1}{L} \left[ \left[ \frac{L}{k} \sin(kx) \right]_{-L}^0 + 2 \left[ \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx) \right]_0^L \right]$$

$$= \frac{1}{L} \left[ \left[ \frac{L}{k} \sin\left(\frac{n\pi}{L}x\right) \right]_{-L}^0 + 2 \left[ \frac{L}{n\pi} \sin\left(\frac{n\pi}{L}x\right) + \left(\frac{L}{n\pi}\right)^2 \cos\left(\frac{n\pi}{L}x\right) \right]_0^L \right]$$

$$= \frac{1}{L} \left[ 2 \left(\frac{L}{n\pi}\right)^2 \left( (-1)^n - 1 \right) \right]$$

$$= \frac{2L}{n^2\pi^2} \left( (-1)^n - 1 \right)$$

$$\cos\left(\frac{n\pi}{L}x\right) - \cos(0)$$

$$B_n = \frac{1}{L} \int_{-L}^L f(x) \sin(kx) dx = \frac{1}{L} \left[ \int_{-L}^0 L \sin(kx) dx + \int_0^L 2x \sin(kx) dx \right]$$

$$= \frac{1}{L} \left[ \left[ -\frac{L}{k} \cos(kx) \right]_{-L}^0 + 2 \left[ -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx) \right]_0^L \right]$$

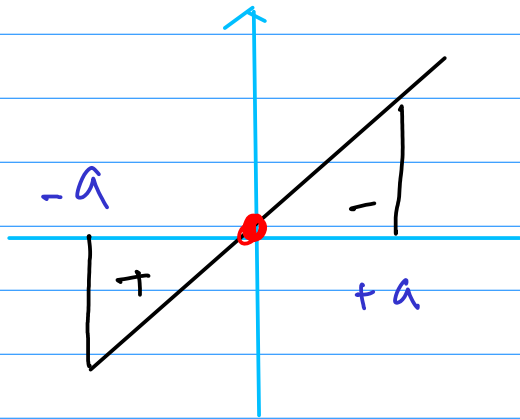
$$= \frac{1}{L} \left[ \left[ -\frac{L^2}{n\pi} \cos\left(\frac{n\pi}{L}x\right) \right]_{-L}^0 + 2 \left[ -\frac{L}{n\pi} x \cos\left(\frac{n\pi}{L}x\right) + \frac{L^2}{(n\pi)^2} \sin\left(\frac{n\pi}{L}x\right) \right]_0^L \right]$$

$$= \frac{1}{L} \left[ \left[ -\frac{L^2}{n\pi} (1 - (-1)^n) + 2 \left[ -\frac{L^2}{n\pi} (-1)^n \right] \right] \right]$$

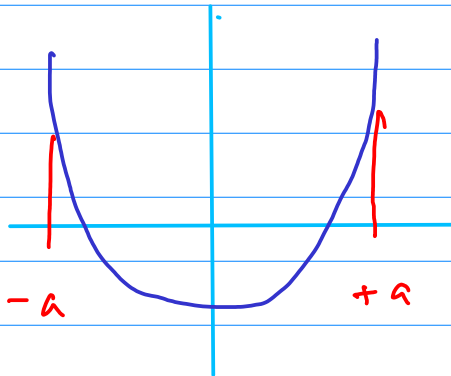
$$= \frac{1}{L} \left[ \frac{L^2}{n\pi} (-1 + (-1)^n) - \frac{2L^2}{n\pi} (-1)^n \right]$$

$$= \frac{1}{L} \left[ \frac{L^2}{n\pi} (-1 + (-1)^n - 2(-1)^n) \right]$$

$$= -\frac{L}{n\pi} (1 + (-1)^n)$$



$$\int_{-a}^{+a} f_{\text{odd}}(x) dx = 0$$



$$\int_{-a}^{+a} f_{\text{even}}(x) dx$$

$$= 2 \int_0^a f_{\text{even}}(x) dx$$

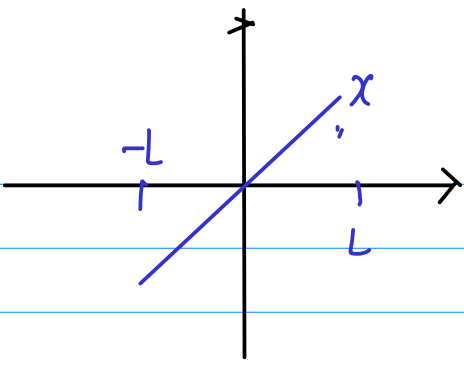
$$\cos(\pi) = -1$$

$$\cos(2\pi) = +1$$

$$\cos(3\pi) = -1$$

$$\cos(4\pi) = +1$$

$$\underline{\cos(n\pi)} = \underline{(-1)^n}$$



$$A_0 = \frac{1}{2L} \int_{-L}^L f(x) dx = \frac{1}{2L} \int_{-L}^L x dx = 0$$

$$A_n = \frac{1}{L} \int_{-L}^L f(x) \cos(kx) dx = \frac{1}{L} \int_{-L}^L \underline{x} \cdot \underline{\cos(kx)} dx = 0$$

$$B_n = \frac{1}{L} \int_{-L}^L f(x) \sin(kx) dx = \frac{1}{L} \int_{-L}^L x \sin(kx) dx$$

$$= \frac{1}{L} \left[ -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx) \right]_{-L}^L$$

$$= \frac{1}{L} \left[ -\frac{L}{n\pi} x \cos\left(\frac{n\pi}{L} x\right) + \left(\frac{L}{n\pi}\right)^2 \sin\left(\frac{n\pi}{L} x\right) \right]_{-L}^L$$

$$= \frac{1}{L} \left[ -\frac{L}{n\pi} L \cos(n\pi) + \underbrace{\left(\frac{L}{n\pi}\right)^2 \sin(n\pi)}_{\rightarrow 0} - \left( -\frac{L}{n\pi} (-L) \cos(-n\pi) + \underbrace{\left(\frac{L}{n\pi}\right)^2 \sin(-n\pi)}_{\rightarrow 0} \right) \right]$$

$$= \frac{1}{L} \left( \frac{L^2}{n\pi} \right) \left( -(-1)^n - (-1)^n \right)$$

$$= -\frac{2L^2}{n\pi} (-1)^n$$







## Laplace Transform

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s) e^{st} ds$$

## Fourier Serie

$$c_n = \frac{1}{2p} \int_{-p}^{+p} f(x) e^{-i\left(\frac{n\pi}{p} x\right)} dx$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\left(\frac{n\pi}{p} x\right)}$$