

Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

1 Joint Characteristic Functions

Joint Characteristic Function

two random variables

Definition

The joint characteristic function of two random variables X and Y is given by

$$\Phi_{X,Y}(\omega_1, \omega_2) = E \left[e^{j\omega_1 X + j\omega_2 Y} \right]$$

where ω_1 and ω_2 are real numbers. An equivalent form is

$$\Phi_{X,Y}(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x, y) e^{j\omega_1 x + j\omega_2 y} dx dy$$

Joint Characteristic Function and Fourier Transform

two random variables

Definition

the 2-dimension Fourier transform of $f_{X,Y}(x,y)$ when signs of ω_1 and ω_2 are reversed

$$\Phi_{X,Y}(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y) e^{j\omega_1 x + j\omega_2 y} dx dy$$

the inverse Fourier transform

$$f_{X,Y}(x,y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{X,Y}(\omega_1, \omega_2) e^{-j\omega_1 x - j\omega_2 y} d\omega_1 d\omega_2$$

Marginal Characteristic Functions

two random variables

Definition

Marginal characteristic functions are

$$\Phi_X(\omega_1) = \Phi_{X,Y}(\omega_1, 0) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) e^{j\omega_1 x} dx$$

$$\Phi_Y(\omega) = \Phi_{X,Y}(0, \omega_2) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) e^{j\omega_2 y} dy$$

