# Digital Signal Octave Codes (0B)

- Aliasing and Folding Frequecies

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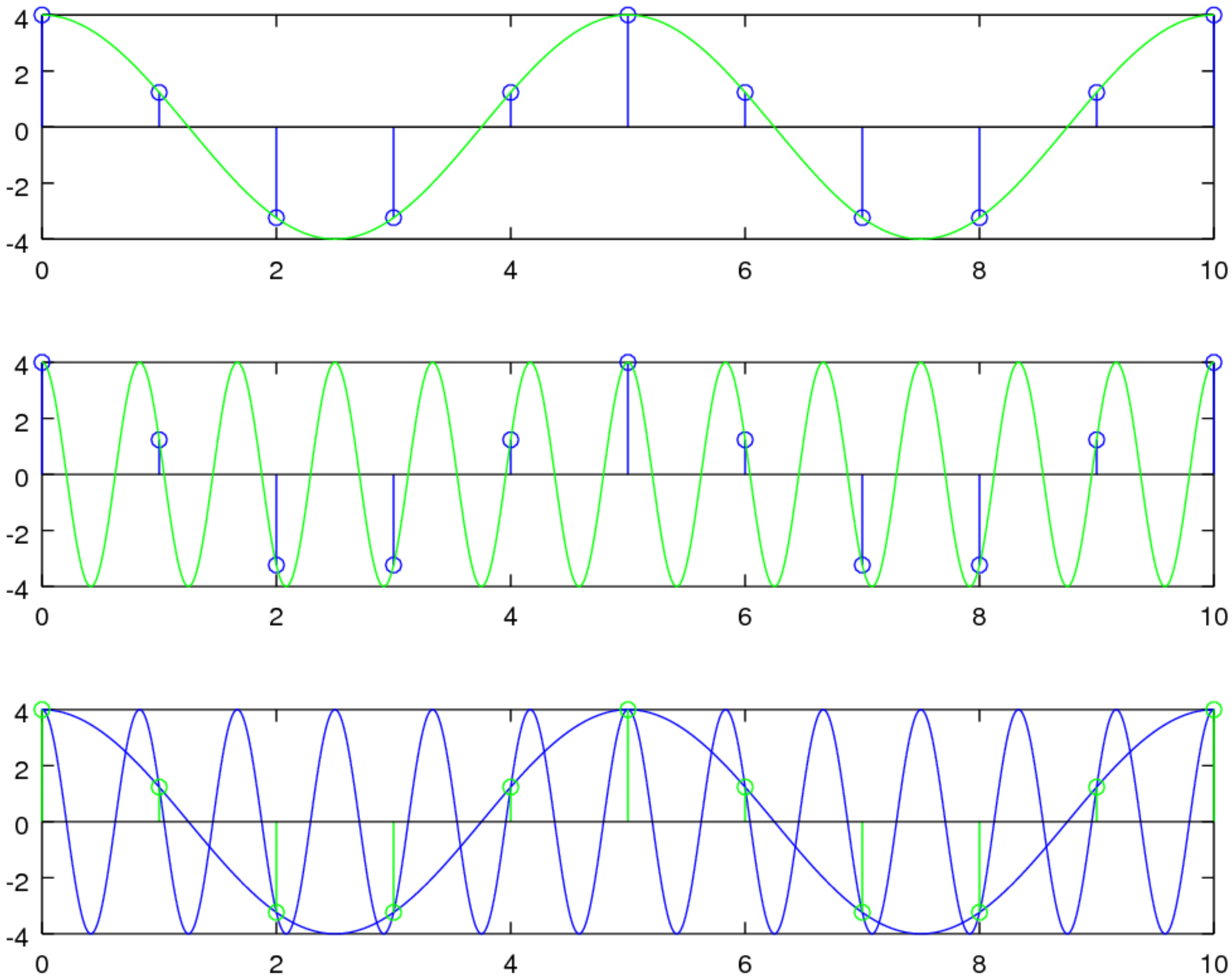
7/13/17

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Based on
M.J. Roberts, Fundamentals of Signals and Systems
S.K. Mitra, Digital Signal Processing : a computer-based approach $2^{nd}$ ed
S.D. Stearns, Digital Signal Processing with Examples in MATLAB

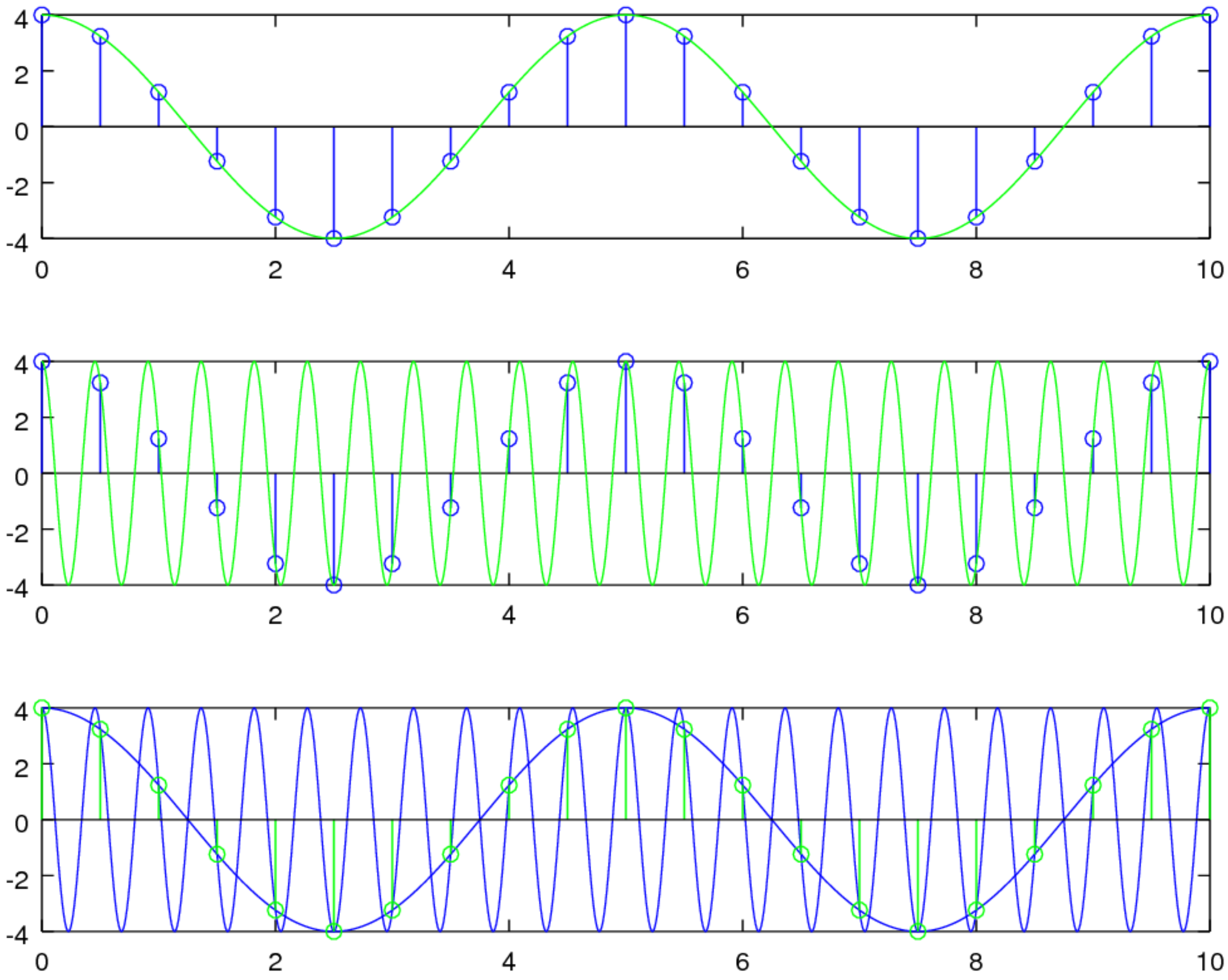
# Aliasing Condition Examples




```
clf
n = [0:1:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(6/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);

subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Condition Examples



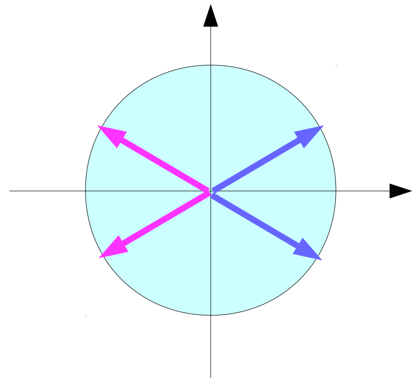

```
clf
n = [0:0.5:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(11/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(11/5)*t);

subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

M.J. Roberts, Fundamentals of Signals and Systems

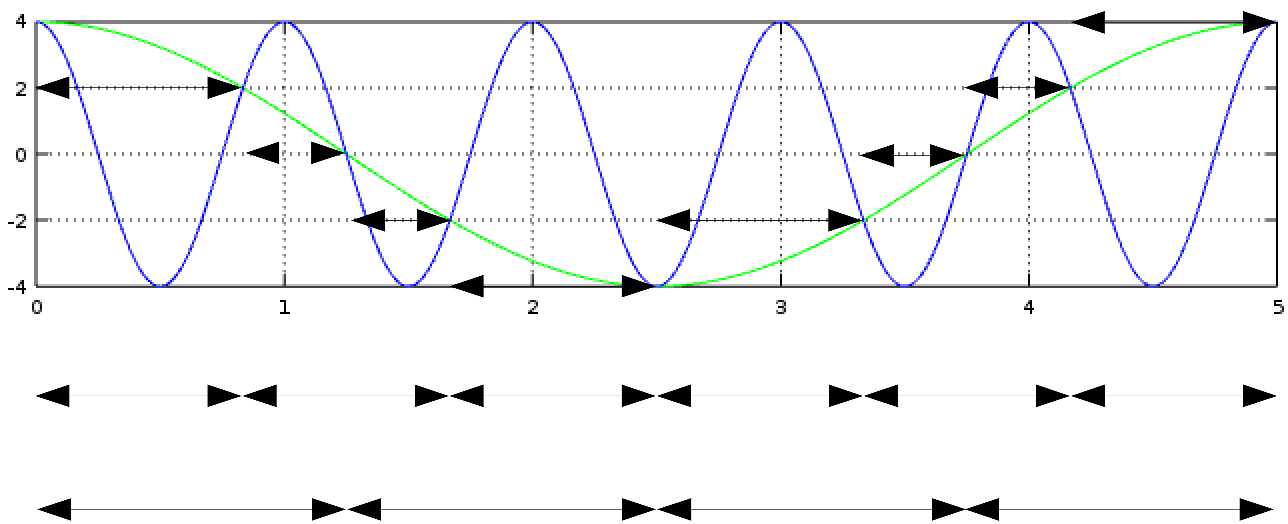
# cos(2πft) & cos(2πft)

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t - \omega_2 t = 2n\pi$$
$$\omega_1 t + \omega_2 t = 2n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = n \end{cases} \qquad \begin{cases} \frac{6}{5}t = n \\ \frac{4}{5}t = n \end{cases}$$




$$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \cdots$$

$$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \cdots$$

M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Condition Examples

| | | | | | |
|---|---|---|---|---|---|
| $\frac{2}{5}t + \frac{1}{5}t = n$ | $\frac{2}{5}t - \frac{1}{5}t = n$ | $\frac{3}{5}t = n$ | $\frac{1}{5}t = n$ | $T_s = \frac{5}{3}$ | $T_s = \frac{5}{1}$ |
| $\frac{3}{5}t + \frac{1}{5}t = n$ | $\frac{3}{5}t - \frac{1}{5}t = n$ | $\frac{4}{5}t = n$ | $\frac{2}{5}t = n$ | $T_s = \frac{5}{4}$ | $T_s = \frac{5}{2}$ |
| $\frac{4}{5}t + \frac{1}{5}t = n$ | $\frac{4}{5}t - \frac{1}{5}t = n$ | $\frac{5}{5}t = n$ | $\frac{3}{5}t = n$ | $T_s = \frac{5}{5}$ | $T_s = \frac{5}{3}$ |
| $\frac{5}{5}t + \frac{1}{5}t = n$ | $\frac{5}{5}t - \frac{1}{5}t = n$ | $\frac{6}{5}t = n$ | $\frac{4}{5}t = n$ | $T_s = \frac{5}{6}$ | $T_s = \frac{5}{4}$ |
| $\frac{6}{5}t + \frac{1}{5}t = n$ | $\frac{6}{5}t - \frac{1}{5}t = n$ | $\frac{7}{5}t = n$ | $\frac{5}{5}t = n$ | $T_s = \frac{5}{7}$ | $T_s = \frac{5}{5}$ |
| $\frac{7}{5}t + \frac{1}{5}t = n$ | $\frac{7}{5}t - \frac{1}{5}t = n$ | $\frac{8}{5}t = n$ | $\frac{6}{5}t = n$ | $T_s = \frac{5}{8}$ | $T_s = \frac{5}{6}$ |
| $\frac{8}{5}t + \frac{1}{5}t = n$ | $\frac{8}{5}t - \frac{1}{5}t = n$ | $\frac{9}{5}t = n$ | $\frac{7}{5}t = n$ | $T_s = \frac{5}{9}$ | $T_s = \frac{5}{7}$ |

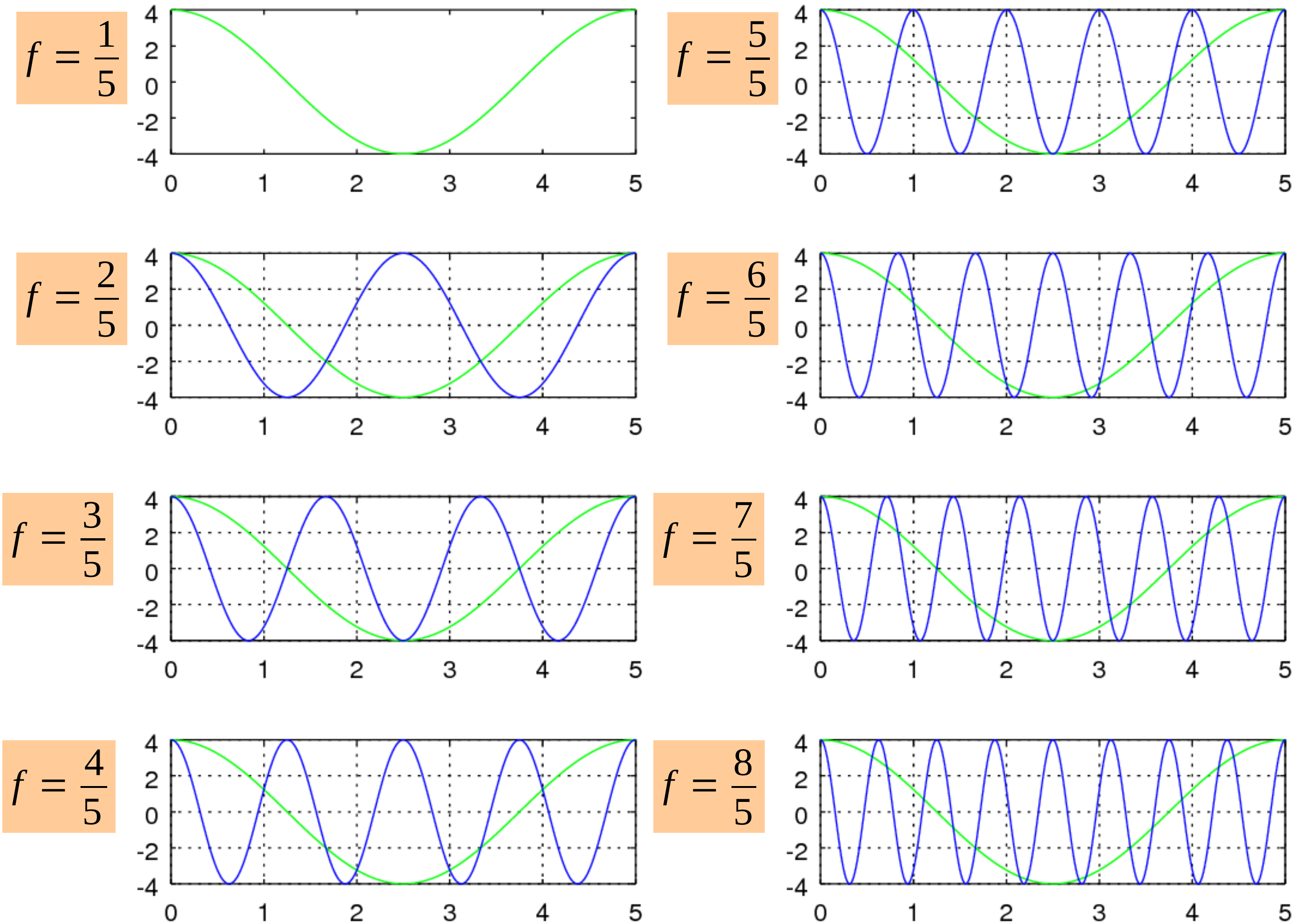
M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Condition Examples

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);

n1 = 0: 5/2 : 5;
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;

y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);

subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');

subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

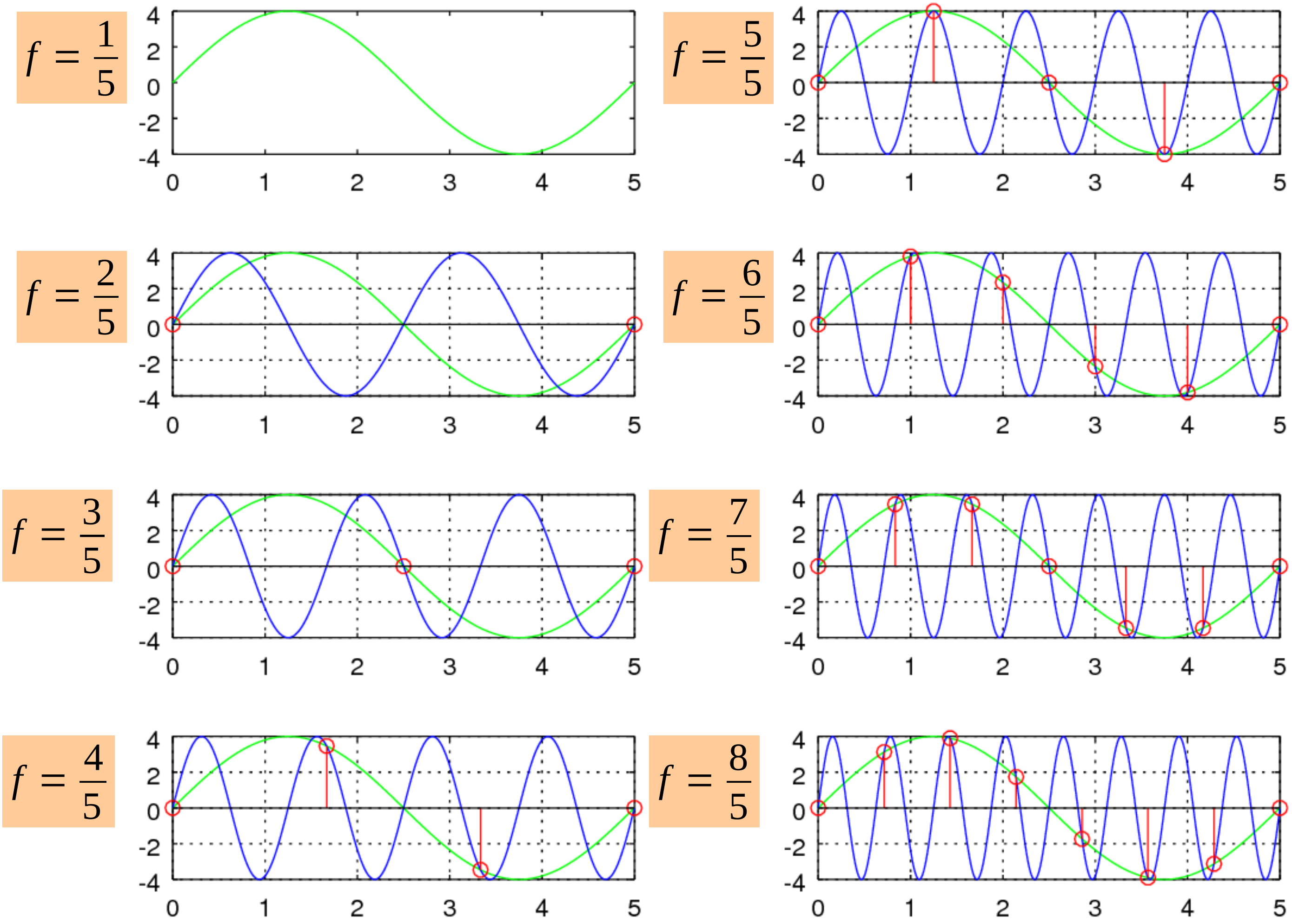
M.J. Roberts, Fundamentals of Signals and Systems

# Graphs of cos(2π(n/5)t) & cos(2π(1/5)t)



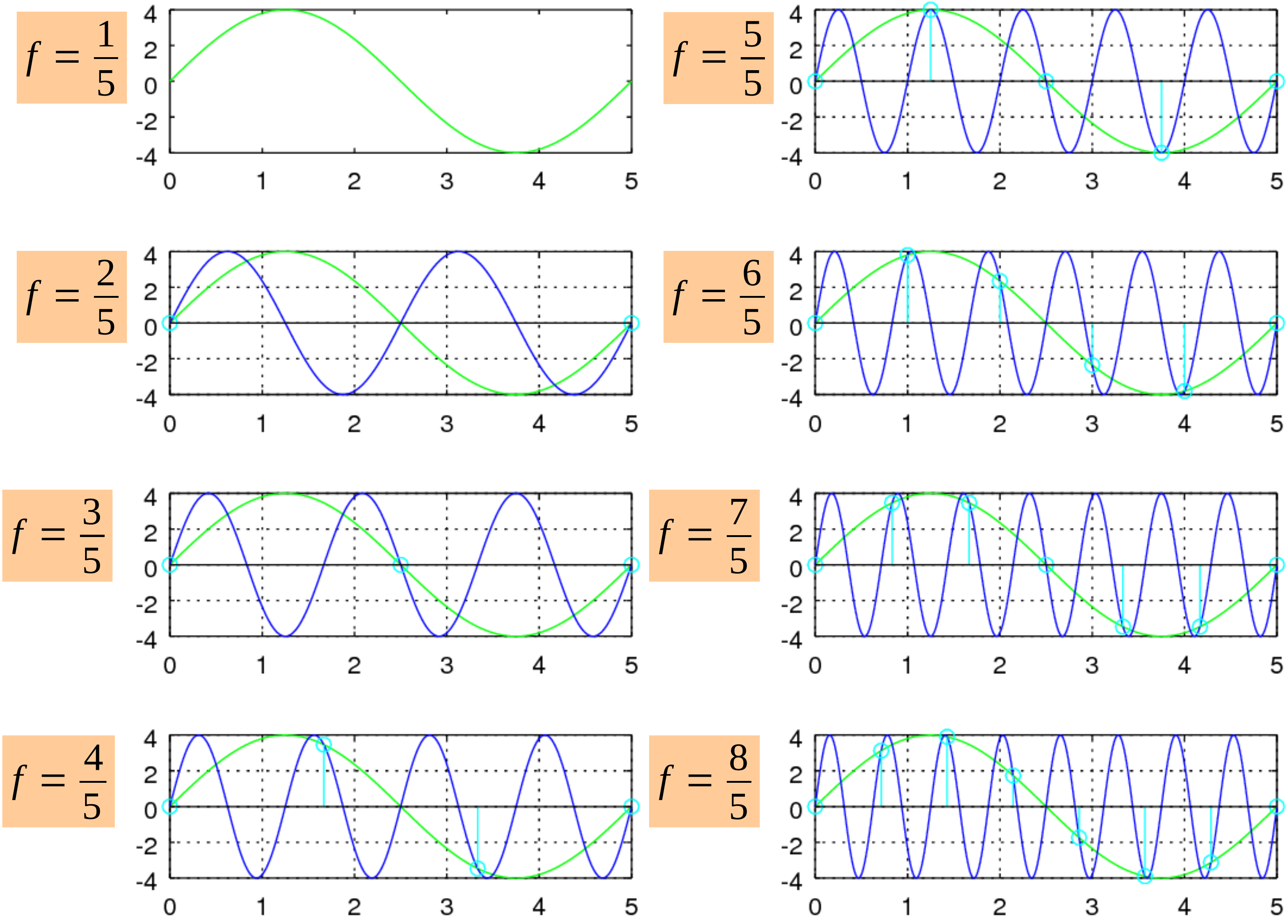

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

# A Set of Roots of cos(2π(n/5)t) = cos(2π(1/5)t)




```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;

y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

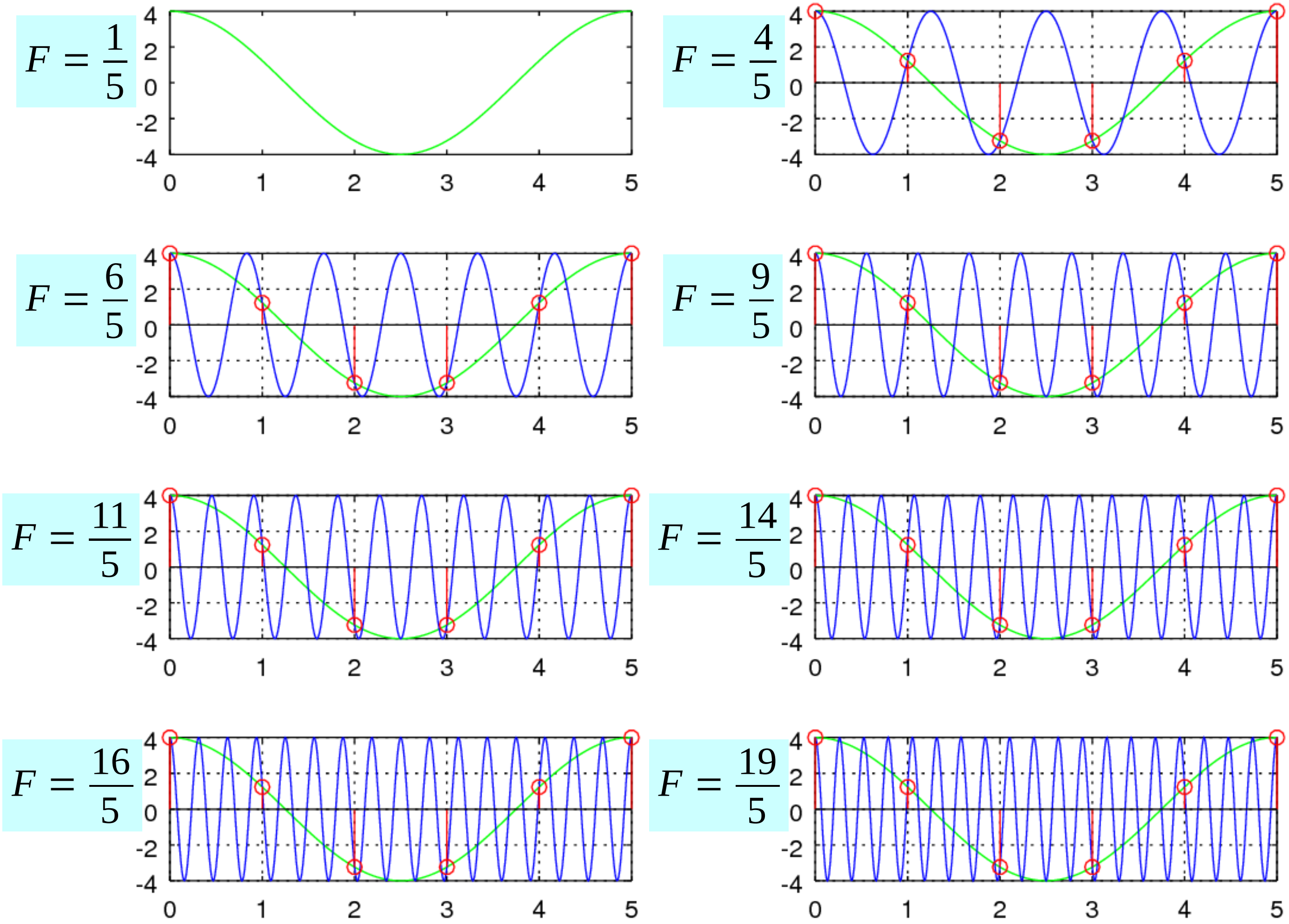
# Another Set of Roots of cos(2π(n/5)t) = cos(2π(1/5)t)




```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;

y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

M.J. Roberts, Fundamentals of Signals and Systems

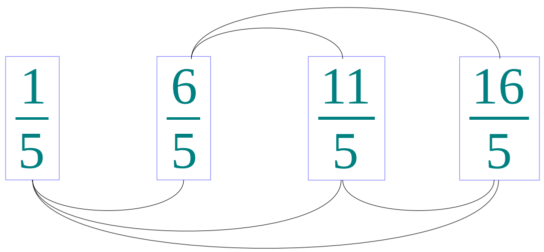
# Aliasing and Folding



# Plotting of Aliasing & Folding Frequencies

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
yt3 = 4*cos(2*pi*(11/5)*t);
yt4 = 4*cos(2*pi*(16/5)*t);
yt5 = 4*cos(2*pi*(4/5)*t);
yt6 = 4*cos(2*pi*(9/5)*t);
yt7 = 4*cos(2*pi*(14/5)*t);
yt8 = 4*cos(2*pi*(19/5)*t);

n1 = 0: 5/5 : 5;
n2 = 0: 5/5 : 5;
n3 = 0: 5/5 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/5 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/5 : 5;
n8 = 0: 5/5 : 5;

y2 = 4*cos(2*pi*(6/5)*n2);
y3 = 4*cos(2*pi*(11/5)*n2);
y4 = 4*cos(2*pi*(16/5)*n2);
y5 = 4*cos(2*pi*(4/5)*n5);
y6 = 4*cos(2*pi*(9/5)*n5);
y7 = 4*cos(2*pi*(14/5)*n5);
y8 = 4*cos(2*pi*(19/5)*n5);


subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n2, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n2, y4, 'r');

subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n5, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n5, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n5, y8, 'r');
```

# Aliasing Frequencies (1)




A+B

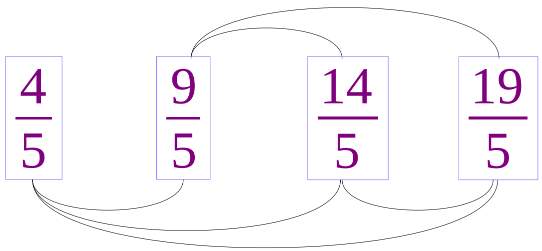
$\frac{7}{5}$ $\frac{12}{5}$ $\frac{17}{5}$ $\frac{22}{5}$ $\frac{27}{5}$ ➡ $\frac{5}{7}$ $\frac{5}{12}$ $\frac{5}{17}$ $\frac{5}{22}$ $\frac{5}{27}$

A-B

$\frac{5}{5}$ $\frac{10}{5}$ $\frac{15}{5}$ ➡ $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ ➡ $1$

M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Frequencies (2)



$\frac{4}{5}$ $\frac{9}{5}$ $\frac{14}{5}$ $\frac{19}{5}$

A+B

$\frac{13}{5}$ $\frac{18}{5}$ $\frac{23}{5}$ $\frac{28}{5}$ $\frac{33}{5}$ $\Rightarrow$ $\frac{5}{13}$ $\frac{5}{18}$ $\frac{5}{23}$ $\frac{5}{28}$ $\frac{5}{33}$

A-B

$\frac{5}{5}$ $\frac{10}{5}$ $\frac{15}{5}$ $\Rightarrow$ $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\Rightarrow$ $1$

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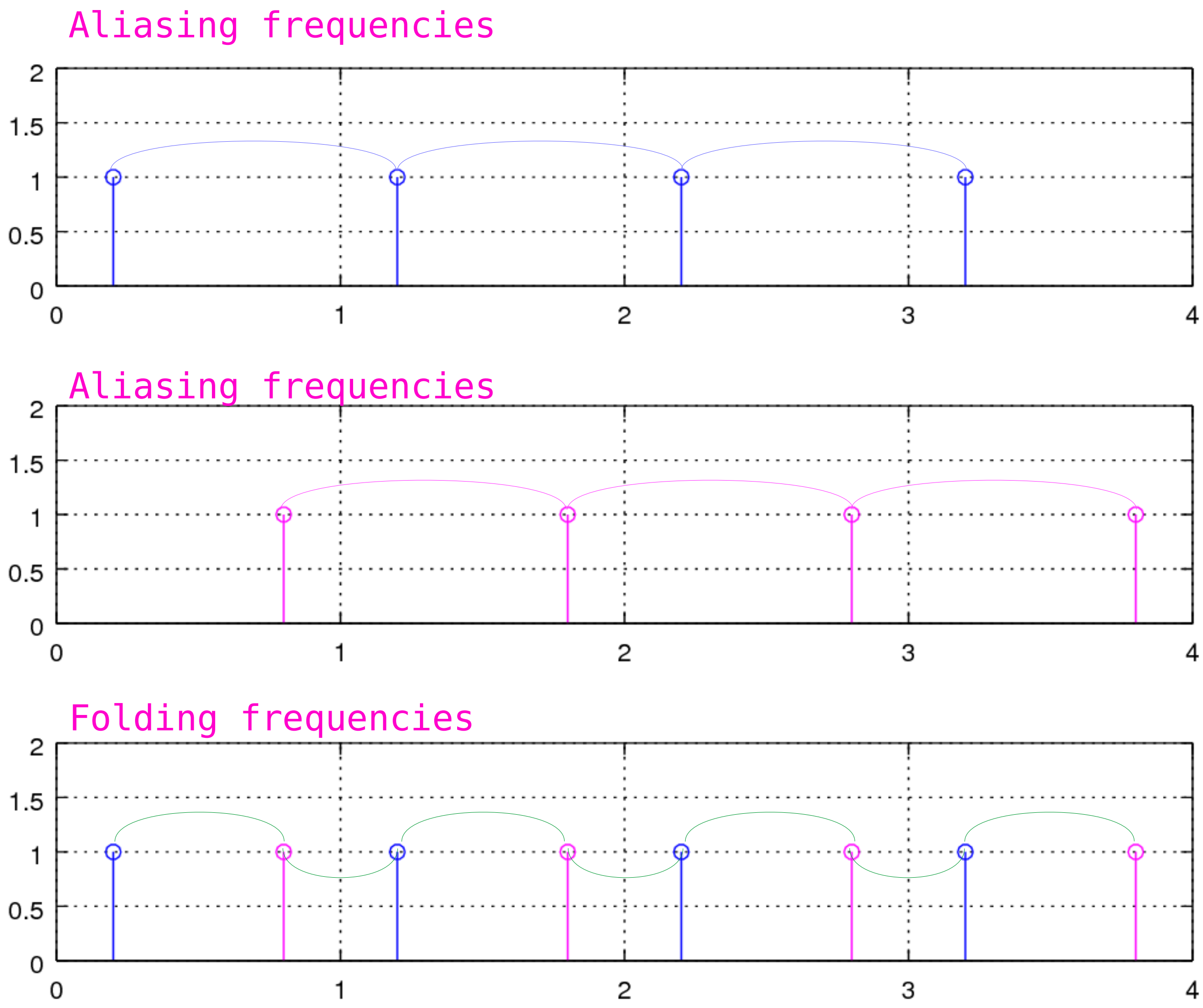
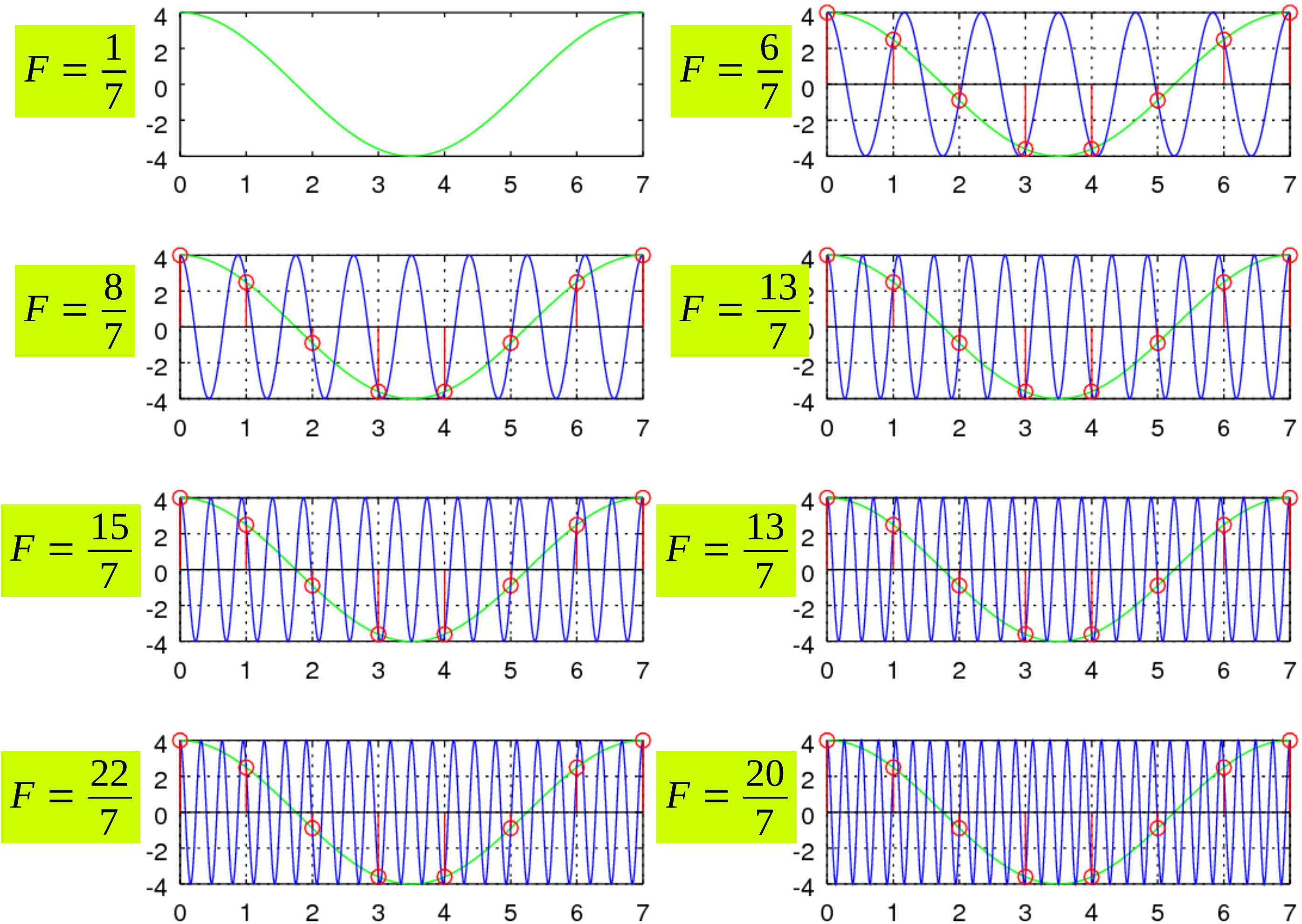
# Aliasing and Folding Frequencies




```
n1 = [1/5, 6/5, 11/5, 16/5];
n2 = [4/5, 9/5, 14/5, 19/5];

y1 = [1, 1, 1, 1];
y2 = [1, 1, 1, 1];

subplot(3, 1, 1)
stem(n1, y1, 'b'); grid on;
axis([0, 4, 0, 2]);

subplot(3, 1, 2)
stem(n2, y2, 'm'); grid on;
axis([0, 4, 0, 2]);

subplot(3, 1, 3)
stem(n1, y1, 'b'); hold on;
stem(n2, y2, 'm'); grid on;
axis([0, 4, 0, 2]);
```

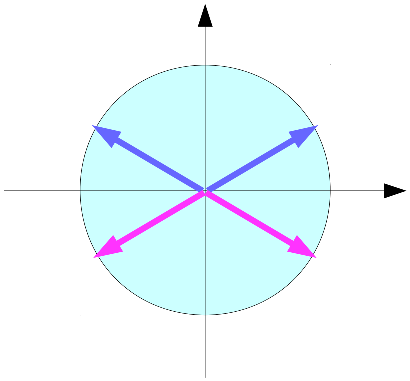
J.H. McClellan, et al., Signal Processing First

# Graphs of cos(2π(n/7)t) & cos(2π(1/7)t)

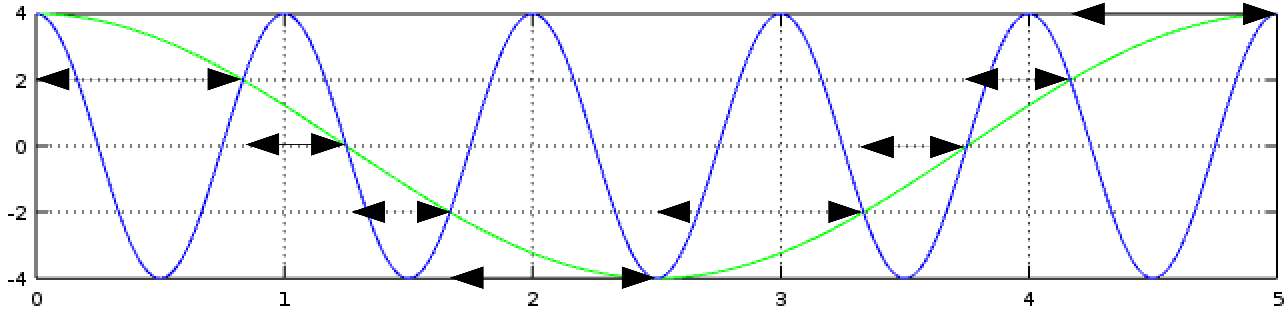


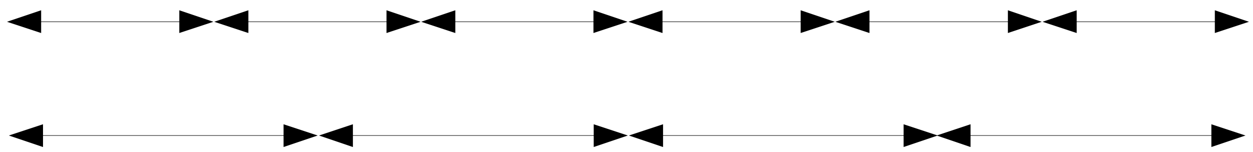
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\omega_1 t - \omega_2 t = 2n\pi$$
$$\omega_1 t + \omega_2 t = n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = \frac{n}{2} \end{cases} \qquad \begin{cases} \frac{6}{5}t = n \\ \frac{4}{5}t = \frac{n}{2} \end{cases}$$



$$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \cdots$$

$$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \cdots$$

M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Condition Examples

| | | | | | |
|---|---|---|---|---|---|
| $\frac{2}{5}t + \frac{1}{5}t = \frac{n}{2}$ | $\frac{2}{5}t - \frac{1}{5}t = n$ | $\frac{3}{5}t = \frac{n}{2}$ | $\frac{1}{5}t = n$ | $T_s = \frac{5}{6}$ | $T_s = 5$ |
| $\frac{3}{5}t + \frac{1}{5}t = \frac{n}{2}$ | $\frac{3}{5}t - \frac{1}{5}t = n$ | $\frac{4}{5}t = \frac{n}{2}$ | $\frac{2}{5}t = n$ | $T_s = \frac{5}{8}$ | $T_s = \frac{5}{2}$ |
| $\frac{4}{5}t + \frac{1}{5}t = \frac{n}{2}$ | $\frac{4}{5}t - \frac{1}{5}t = n$ | $\frac{5}{5}t = \frac{n}{2}$ | $\frac{3}{5}t = n$ | $T_s = \frac{5}{10}$ | $T_s = \frac{5}{3}$ |
| $\frac{5}{5}t + \frac{1}{5}t = \frac{n}{2}$ | $\frac{5}{5}t - \frac{1}{5}t = n$ | $\frac{6}{5}t = \frac{n}{2}$ | $\frac{4}{5}t = n$ | $T_s = \frac{5}{12}$ | $T_s = \frac{5}{4}$ |
| $\frac{6}{5}t + \frac{1}{5}t = \frac{n}{2}$ | $\frac{6}{5}t - \frac{1}{5}t = n$ | $\frac{7}{5}t = \frac{n}{2}$ | $\frac{5}{5}t = n$ | $T_s = \frac{5}{14}$ | $T_s = \frac{5}{5}$ |
| $\frac{7}{5}t + \frac{1}{5}t = \frac{n}{2}$ | $\frac{7}{5}t - \frac{1}{5}t = n$ | $\frac{8}{5}t = \frac{n}{2}$ | $\frac{6}{5}t = n$ | $T_s = \frac{5}{16}$ | $T_s = \frac{5}{6}$ |
| $\frac{8}{5}t + \frac{1}{5}t = \frac{n}{2}$ | $\frac{8}{5}t - \frac{1}{5}t = n$ | $\frac{9}{5}t = \frac{n}{2}$ | $\frac{7}{5}t = n$ | $T_s = \frac{5}{18}$ | $T_s = \frac{5}{7}$ |

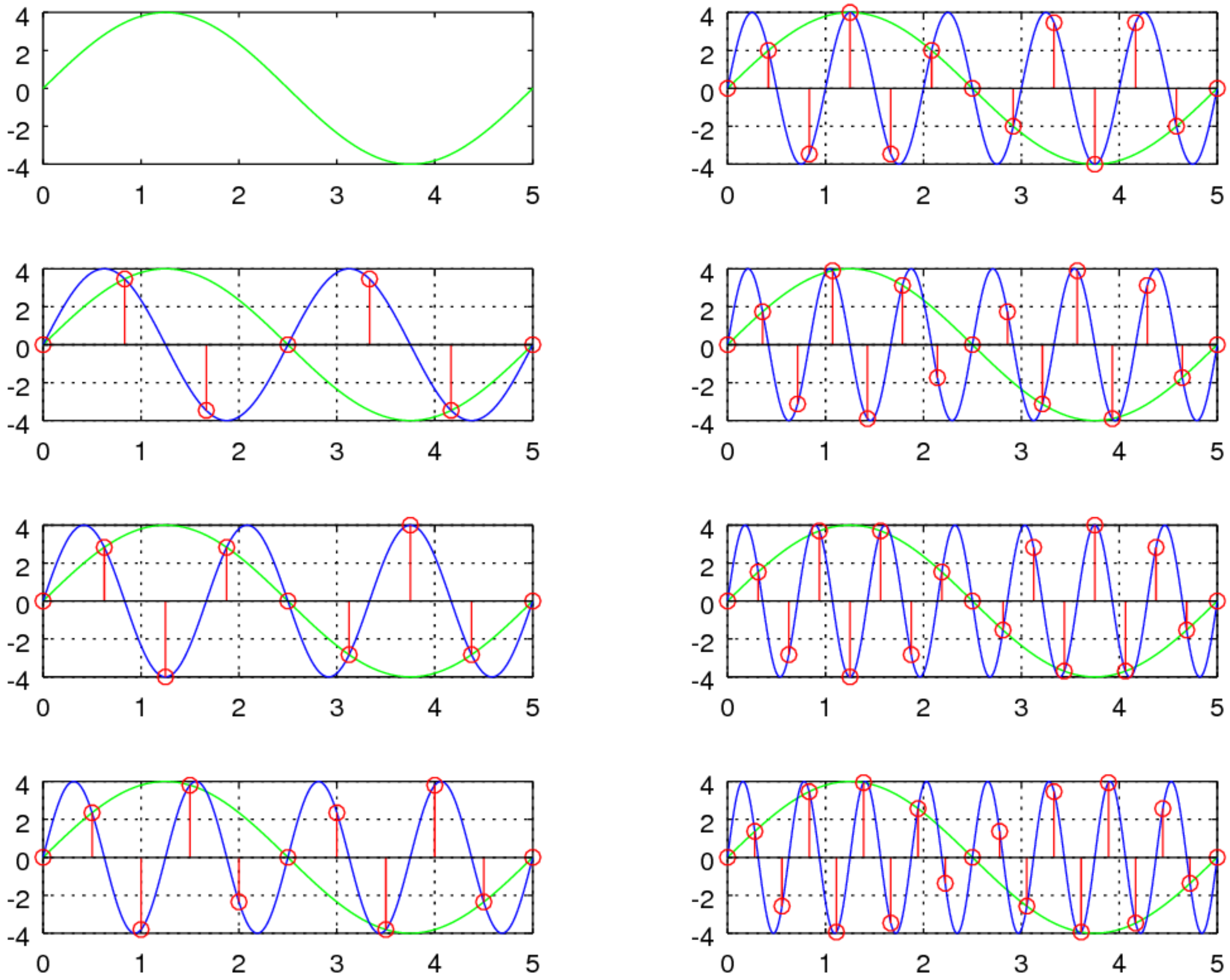
M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Condition Examples

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);

n1 = 0: 5/2 : 5;
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;

y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);

subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');

subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

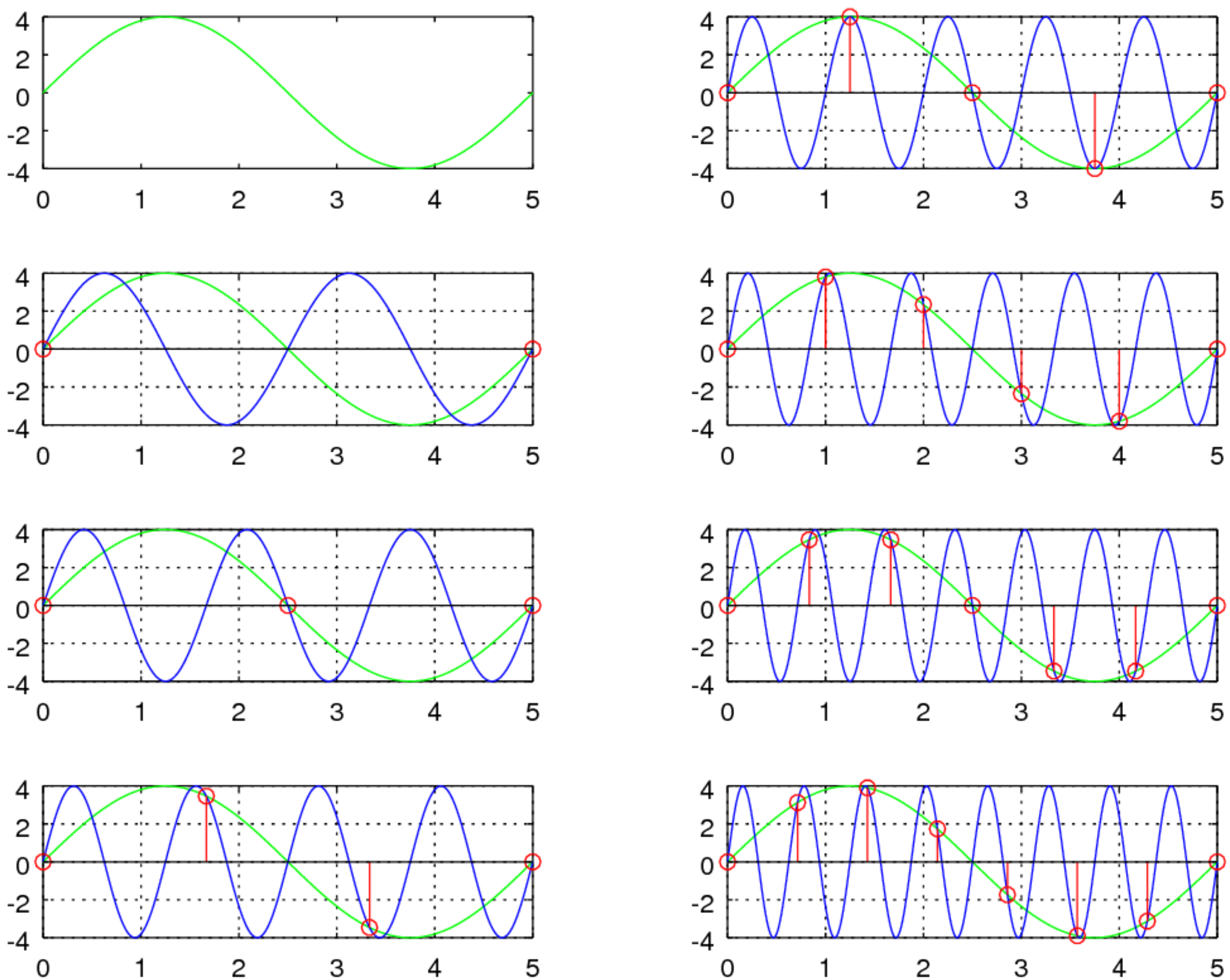
M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Condition Examples




M.J. Roberts, Fundamentals of Signals and Systems

# Aliasing Condition Examples




M.J. Roberts, Fundamentals of Signals and Systems

**References**

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[4] S.J. Orfanidis, Introduction to Signal Processing
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[6] A “graphical interpretation” of the DFT and FFT, by Steve Mann