Random Noise

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Definition

$$R_{XX}(\tau) = \left(\frac{N_0}{2}\right)\delta(t)$$

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} \left(\frac{N_0}{2}\right)\delta(t-\xi)h(\xi)d\xi$$

$$= \left(\frac{N_0}{2}\right)\delta(\tau)$$

$$h(\tau) \cong \left(\frac{2}{N_0}\right)R_{XX}(\tau)$$

Definition

$$h(\tau) \cong \left(\frac{2}{N_0}\right) R_{XX}(\tau)$$

$$\widetilde{h}(\tau) \cong \left(\frac{2}{N_0}\right) \widehat{R}_{XX}(\tau)$$

Definition

$$R_{YY} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{N_0}{2} \right) |H(\omega)|^2 d\omega$$

$$R_{YY} = \frac{N_0}{2\pi} \int_{0}^{\infty} |H(\omega)|^2 d\omega$$

Definition

$$|H_I(\omega)|^2 = \begin{cases} |H(0)|^2 & |\omega| < W_N \\ 0 & |\omega| > W_N \end{cases}$$

$$\frac{N_0}{2\pi}\int_0^\infty |H(\omega)|^2 d\omega = \frac{N_0}{2\pi}\int_0^{W_N} |H(0)|^2 d\omega = \frac{N_0 |H(0)|^2 W_N}{2\pi}$$

$$W_N = \frac{\int_0^\infty |H(\omega)|^2 d\omega}{|H(0)|^2}$$

Definition

$$W_N = \frac{\int_0^\infty |H(\omega)|^2 d\omega}{|H(\omega_0)|^2}$$

$$P_{YY} = \frac{N_0 |H(\omega_0)|^2 W_N}{2\pi}$$

Definition

$$\overline{e_n^2(t)} = \frac{2kTRd\omega}{\pi}$$

$$\overline{i_n^2(t)} = \overline{e_n^2(t)/R^2} = \frac{2kTd\omega}{\pi R}$$

$$dN_L = \frac{\overline{e_n^2(t)}R_L}{(R+R_L)^2} = \frac{2kTRR_Ld\omega}{\pi(R+R_L)^2}$$

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R} = \frac{kTd\omega}{2\pi}$$

Definition

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R_o(\omega)}$$

$$\overline{e_n^2(t)} = 2kT_s R_o(\omega)\frac{d\omega}{\pi}$$

$$dN_{as} = kT_s \frac{d\omega}{2\pi}$$

Definition

$$dN_{as} = kT_d \frac{d\omega}{2\pi}$$

Definition

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R_s}$$

$$dN_{aos} = \frac{\overline{e_o^2(t)}}{4R_o}$$

$$G_a = \frac{dN_{aos}}{dN_{as}} = \frac{R_s \overline{e_o^2(t)}}{R_o \overline{e_s^2(t)}}$$

Definition

$$G_m = \frac{dN_{m,aos}}{dN_{m,as}} = \frac{R_{m,s}\overline{e^2_{m,o}(t)}}{R_{m,o}\overline{e^2_{m,s}(t)}}$$

$$G_a = \prod_{m=1}^{M} G_m$$

Definition

$$dN_{aos} = G_a dN_{as} = G_a k T_s \frac{d\omega}{2\pi}$$

$$\Delta N_{as} = G_a k T_e \frac{d\omega}{2\pi}$$

$$T_e = T_{c1} + \frac{T_{c2}}{G_1} + \frac{T_{c3}}{G_1 G_2} + \cdots + \frac{T_{cM}}{G_1 G_2 \cdots G_{M-1}}$$

Definition

$$\overline{F}_{op} = \frac{N_{ao}}{N_{aos}}$$

$$N_{aos} = \frac{k}{2\pi} \int_0^\infty T_s G_a d\omega$$

$$N_{ao} = \int_0^\infty dN_{ao} = \int_0^\infty F_{op} dN_{aos} = \frac{k}{2\pi} \int_0^\infty F_{op} T_s G_a d\omega$$

$$F_{op} = \frac{\int_0^\infty F_{op} T_s G_a d\omega}{\int_0^\infty T_s G_a d\omega}$$

Definition

$$F_{op} = \frac{\int_0^\infty F_{op} T_s G_a d\omega}{\int_0^\infty T_s G_a d\omega}$$

$$F_o = \frac{\int_0^\infty F_o G_a d\omega}{\int_0^\infty G_a d\omega}$$

Definition

$$dN_{ao} = G_a k(\overline{T}_s + \overline{T}_c)\frac{d\omega}{2\pi}$$

$$N_{ao} = \int_0^{\infty} dN_{ao} = \frac{k}{2\pi}\int_0^{\infty} G_a(\overline{T}_s + \overline{T}_c)d\omega$$

$$N_{ao} = \frac{k}{2\pi}(\overline{T}_s + \overline{T}_c)\int_0^{\infty} G_a d\omega$$

Definition

$$\overline{T}_s = \frac{\int_0^\infty T_s G_a d\omega}{\int_0^\infty G_a d\omega}$$

$$\overline{T}_c = \frac{\int_0^\infty T_c G_a d\omega}{\int_0^\infty G_a d\omega}$$

Average Noise Temperature (3)

N Gaussian random variables

Definition

$$\overline{F}_o = 1 + \frac{\overline{T}_c}{\overline{T}_o}$$

$$\overline{F}_{op} = 1 + \frac{\overline{T}_e}{\overline{T}_s}$$

$$\overline{F}_o = 1 + \frac{\overline{T}_s}{\overline{T}_o} \left(\overline{F}_{op} - 1 \right)$$

$$\overline{F}_{op} = 1 + \frac{T_o}{\overline{T}_s} \left(\overline{F}_o - 1 \right)$$

Definition

$$N_{ao} = \frac{k}{2\pi} \left(\overline{T}_s + \overline{T}_c \right) \int_0^{\infty} G_a d\omega$$

$$N_{ao} = \frac{k}{2\pi} \left(\overline{T}_s + \overline{T}_c \right) G_a(\omega_0) \frac{\int_0^{\infty} G_a(\omega) d\omega}{G_a(\omega_0)}$$

$$W_N = \frac{\int_0^{\infty} G_a(\omega) d\omega}{G_a(\omega_0)}$$

$$N_{ao} = \frac{k}{2\pi} \left(\overline{T}_s + \overline{T}_c \right) G_a(\omega_0) W_N$$

Definition

$$\overline{T}_c = T_L(L-1)$$

$$\overline{F}_o = 1 + \frac{\overline{T}_L}{\overline{T}_o}(L-1)$$

$$\overline{F}_{op} = 1 + \frac{T_L}{\overline{T}_s}(L-1)$$

Definition

$$N_{ao} = k\left(T_a + T_L(L-1) + \overline{T}_R L\right)\frac{G_R(\omega_0)W_N}{L(2\pi)}$$

$$N_{ao} = k\overline{T}_{sys}\frac{G_R(\omega_0)W_N}{L(2\pi)}$$

$$\overline{T}_{sys} = T_a + T_L(L-1) + \overline{T}_R L$$

$$\overline{F}_{op} = 1 + \frac{T_L}{\overline{T}_a}(L-1) + \frac{\overline{T}_R}{\overline{T}_a}L$$