

# Random Noise

Young W Lim

January 22, 2020

Copyright (c) 2018 Young W. Lim. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 3.0 Unported" license.



Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi



# System Evaluation Using Random Noise

$N$  Gaussian random variables

## Definition

$$R_{XX}(\tau) = \left(\frac{N_0}{2}\right) \delta(t)$$

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} \left(\frac{N_0}{2}\right) \delta(t - \xi) h(\xi) d\xi$$

$$= \left(\frac{N_0}{2}\right) \delta(\tau)$$

$$h(\tau) \cong \left(\frac{2}{N_0}\right) R_{XX}(\tau)$$

# System Evaluation Using Random Noise

$N$  Gaussian random variables

## Definition

$$h(\tau) \cong \left( \frac{2}{N_0} \right) R_{XX}(\tau)$$

$$\tilde{h}(\tau) \cong \left( \frac{2}{N_0} \right) \hat{R}_{XX}(\tau)$$

# Total average power

$N$  Gaussian random variables

## Definition

$$R_{YY} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \frac{N_0}{2} \right) |H(\omega)|^2 d\omega$$

$$R_{YY} = \frac{N_0}{2\pi} \int_0^{\infty} |H(\omega)|^2 d\omega$$

# Noise Bandwidth

$N$  Gaussian random variables

## Definition

$$|H_I(\omega)|^2 = \begin{cases} |H(0)|^2 & |\omega| < W_N \\ 0 & |\omega| > W_N \end{cases}$$

$$\frac{N_0}{2\pi} \int_0^{\infty} |H(\omega)|^2 d\omega = \frac{N_0}{2\pi} \int_0^{W_N} |H(0)|^2 d\omega = \frac{N_0 |H(0)|^2 W_N}{2\pi}$$

$$W_N = \frac{\int_0^{\infty} |H(\omega)|^2 d\omega}{|H(0)|^2}$$

# Noise Bandwidth - bandpass

$N$  Gaussian random variables

## Definition

$$W_N = \frac{\int_0^\infty |H(\omega)|^2 d\omega}{|H(\omega_0)|^2}$$

$$P_{YY} = \frac{N_0 |H(\omega_0)|^2 W_N}{2\pi}$$



# Resistive (Thermal) Noise Source

$N$  Gaussian random variables

## Definition

$$\overline{e_n^2(t)} = \frac{2kTRd\omega}{\pi}$$

$$\overline{i_n^2(t)} = \overline{e_n^2(t)/R^2} = \frac{2kTd\omega}{\pi R}$$

$$dN_L = \frac{\overline{e_n^2(t)}R_L}{(R + R_L)^2} = \frac{2kTRR_Ld\omega}{\pi(R + R_L)^2}$$

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R} = \frac{kTd\omega}{2\pi}$$

# Effective Noise Temperature

$N$  Gaussian random variables

## Definition

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R_o(\omega)}$$

$$\overline{e_n^2(t)} = 2kT_s R_o(\omega) \frac{d\omega}{\pi}$$

$$dN_{as} = kT_s \frac{d\omega}{2\pi}$$

# Antenna as a Noise Source

$N$  Gaussian random variables

## Definition

$$dN_{as} = kT_d \frac{d\omega}{2\pi}$$

# Available Power Gain

$N$  Gaussian random variables

## Definition

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R_s}$$

$$dN_{aos} = \frac{\overline{e_o^2(t)}}{4R_o}$$

$$G_a = \frac{dN_{aos}}{dN_{as}} = \frac{R_s \overline{e_o^2(t)}}{R_o \overline{e_s^2(t)}}$$

# Available Power Gain of Cascade System

$N$  Gaussian random variables

## Definition

$$G_m = \frac{dN_{m,aos}}{dN_{m,as}} = \frac{R_{m,s} \overline{e_{m,o}^2(t)}}{R_{m,o} \overline{e_{m,s}^2(t)}}$$

$$G_a = \prod_{m=1}^M G_m$$

# Equivalent Input Noise Temperature

$N$  Gaussian random variables

## Definition

$$dN_{aos} = G_a dN_{as} = G_a k T_s \frac{d\omega}{2\pi}$$

$$\Delta N_{as} = G_a k T_e \frac{d\omega}{2\pi}$$

$$T_e = T_{c1} + \frac{T_{c2}}{G_1} + \frac{T_{c3}}{G_1 G_2} + \dots + \frac{T_{cM}}{G_1 G_2 \dots G_{M-1}}$$

# Average Operating Noise Figure

$N$  Gaussian random variables

## Definition

$$\bar{F}_{op} = \frac{N_{ao}}{N_{aos}}$$

$$N_{aos} = \frac{k}{2\pi} \int_0^{\infty} T_s G_a d\omega$$

$$N_{ao} = \int_0^{\infty} dN_{ao} = \int_0^{\infty} F_{op} dN_{aos} = \frac{k}{2\pi} \int_0^{\infty} F_{op} T_s G_a d\omega$$

$$F_{op} = \frac{\int_0^{\infty} F_{op} T_s G_a d\omega}{\int_0^{\infty} T_s G_a d\omega}$$

# Average Standard Noise Figure

$N$  Gaussian random variables

## Definition

$$F_{op} = \frac{\int_0^{\infty} F_{op} T_s G_a d\omega}{\int_0^{\infty} T_s G_a d\omega}$$

$$F_o = \frac{\int_0^{\infty} F_o G_a d\omega}{\int_0^{\infty} G_a d\omega}$$



# Average Noise Temperature (1)

$N$  Gaussian random variables

## Definition

$$dN_{ao} = G_a k (\bar{T}_s + \bar{T}_c) \frac{d\omega}{2\pi}$$

$$N_{ao} = \int_0^\infty dN_{ao} = \frac{k}{2\pi} \int_0^\infty G_a (\bar{T}_s + \bar{T}_c) d\omega$$

$$N_{ao} = \frac{k}{2\pi} (\bar{T}_s + \bar{T}_c) \int_0^\infty G_a d\omega$$

# Average Noise Temperature (2)

$N$  Gaussian random variables

## Definition

$$\bar{T}_s = \frac{\int_0^\infty T_s G_a d\omega}{\int_0^\infty G_a d\omega}$$

$$\bar{T}_c = \frac{\int_0^\infty T_c G_a d\omega}{\int_0^\infty G_a d\omega}$$

# Average Noise Temperature (3)

$N$  Gaussian random variables

## Definition

$$\bar{F}_o = 1 + \frac{\bar{T}_c}{\bar{T}_o}$$

$$\bar{F}_{op} = 1 + \frac{\bar{T}_e}{\bar{T}_s}$$

$$\bar{F}_o = 1 + \frac{\bar{T}_s}{\bar{T}_o} (\bar{F}_{op} - 1)$$

$$\bar{F}_{op} = 1 + \frac{\bar{T}_o}{\bar{T}_s} (\bar{F}_o - 1)$$

# Average Noise Temperature (4)

$N$  Gaussian random variables

## Definition

$$N_{ao} = \frac{k}{2\pi} (\bar{T}_s + \bar{T}_c) \int_0^\infty G_a d\omega$$

$$N_{ao} = \frac{k}{2\pi} (\bar{T}_s + \bar{T}_c) G_a(\omega_0) \frac{\int_0^\infty G_a(\omega) d\omega}{G_a(\omega_0)}$$

$$W_N = \frac{\int_0^\infty G_a(\omega) d\omega}{G_a(\omega_0)}$$

$$N_{ao} = \frac{k}{2\pi} (\bar{T}_s + \bar{T}_c) G_a(\omega_0) W_N$$

# Attenuator Modeling

$N$  Gaussian random variables

## Definition

$$\bar{T}_c = T_L(L-1)$$

$$\bar{F}_o = 1 + \frac{\bar{T}_L}{\bar{T}_o}(L-1)$$

$$\bar{F}_{op} = 1 + \frac{T_L}{T_s}(L-1)$$

# Example

$N$  Gaussian random variables

## Definition

$$N_{ao} = k (T_a + T_L(L-1) + \bar{T}_R L) \frac{G_R(\omega_0) W_N}{L(2\pi)}$$

$$N_{ao} = k \bar{T}_{sys} \frac{G_R(\omega_0) W_N}{L(2\pi)}$$

$$\bar{T}_{sys} = T_a + T_L(L-1) + \bar{T}_R L$$

$$\bar{F}_{op} = 1 + \frac{T_L}{T_a} (L-1) + \frac{\bar{T}_R}{T_a} L$$







