

# Stationarity

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August 20, 2019

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi

# Outline

- 1 First-Order Stationary Processes
- 2 Higher Order Stationary Processes
- 3 Wide Sense Stationarity
- 4 Autocorrelation Function and its Properties

# First Order Stationary

$N$  Gaussian random variables

## Definition

if the first order density function does not change with a shift in time origin

$$f_X(x_1; t_1) = f_X(x_1; t_1 + \Delta)$$

must be true for any time  $t_1$  and any real number  $\Delta$  if  $X(t)$  is to be a first-order stationary

## Consequences of stationarity

### $N$ Gaussian random variables

#### Definition

$f_X(x, t_1)$  is independent of  $t_1$   
the process mean value is a constant

$$m_X(t) = \bar{X} = \text{constant}$$

## the process mean value

$N$  Gaussian random variables

### Definition

$$m_X(t) = \bar{X} = \text{constant}$$

$$m_X(t_1) = \int_{-\infty}^{\infty} xf_X(x; t_1) dx$$

$$m_X(t_2) = \int_{-\infty}^{\infty} xf_X(x; t_2) dx$$

$$m_X(t_1) = m_X(t_1 + \Delta)$$

## Second-Order Stationary Process

$N$  Gaussian random variables

### Definition

if the second order density function does not change with a shift in time origin

$$f_X(x_1, x_2; t_1, t_2) = f_X(x_1, x_2; t_1 + \Delta, t_2 + \Delta)$$

must be true for any time  $t_1, t_2$  and any real number  $\Delta$  if  $X(t)$  is to be a second-order stationary

Auto-correlation function

$$R_{XX}(t, t + \tau) = E[X(t)X(t + \tau)] = R_{XX}(\tau)$$

# $N^{\text{th}}$ -order Stationary Processes

$N$  Gaussian random variables

## Definition

if the second order density function does not change with a shift in time origin

$$f_X(x_1, \dots, x_N; t_1, \dots, t_N) = f_X(x_1, \dots, x_N; t_1 + \Delta, \dots, t_N + \Delta)$$

must be true for any time  $t_1, \dots, t_N$  and any real number  $\Delta$  if  $X(t)$  is to be a second-order stationary



# Wide Sense Stationary Process

$N$  Gaussian random variables

## Definition

$$m_X(t) = \bar{X} = \text{constant}$$

$$E[X(t)X(t+\tau)] = R_{XX}(\tau)$$

# The properties of autocorrelation functions

## $N$ Gaussian random variables

### Definition

$$|R_{XX}(\tau)| \leq R_{XX}(0)$$

$$R_{XX}(-\tau) = R_{XX}(\tau)$$

$$R_{XX}(0) = E[X^2(t)]$$

$$P[|X(t+\tau) - X(t)| > \varepsilon] = \frac{2}{\varepsilon^2} (R_{XX}(0) - R_{XX}(\tau))$$

