Logic Background (1A)

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Techniques in Artificial Intelligence - courses - MIT

https://courses.csail.mit.edu/6.825/

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Proposition : etymology

From Old French, from Latin prōpositiō

("a proposing, design, theme, case").

The content of an assertion

that may be taken as being **true** or **false**

and is considered abstractly without reference to

the linguistic sentence that constitutes the assertion.

Predicate : etymology

From Middle French predicate (French prédicat),

from post-classical Late Latin praedicatum ("thing said of a subject"),

a noun use of the neuter past participle of praedicare ("proclaim")

From Latin predicātus, perfect passive participle of praedicō,

from prae + dicō ("declare, proclaim"), from dicō ("say, tell").

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Proposition

In Aristotelian logic a proposition is

a particular kind of sentence,

one which affirms or denies

a **predicate** of a subject.

In formal logic a proposition is considered as

objects of a formal language.

A formal language begins with different types of symbols.

Predicate

(grammar) The part of the sentence (or clause)

which states something about

the subject or the object of the sentence.

<div style="border:1px solid">

The dog   barked very loudly

subject   *predicate*

</div>

(logic) A term of a statement,

where the statement may be true or false

depending on whether the thing referred to

by the values of the statement's variables

has the property signified by that (predicative) term.

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Propositional Logic

Propositional logic includes only
- operators and
- propositional constants

as symbols in its language.

The propositions in this language are
- propositional constants

  considered atomic propositions
- composite propositions

  recursive application of operators to propositions

Predicate Logic

Predicate logic include
- variables,
- operators,
- predicate and
- function symbols, and
- quantifiers

as symbols in their languages.

Logic

A Formal Language

Syntax – legal expressions

Semantics – the meaning of legal expressions

Proof System – a way of manipulating syntactic expressions

to get another syntactic expressions

Multiple Percepts → Conclusions

Current State, Operators → Next State Properties

Propositional Logic

Sentences (WFFs : Well Formed Formulas)

T and F are sentences

Propositional variables are sentences (A, B, C, …)

If A and B are sentences, the followings are also sentences

(A), ¬A, A∧B, A∨B, A→B, A↔B

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Precedence of Connectives

¬   highest

∧

∨

⇒

⇔   lowest

A∨B∧C   =   A∨(B∧C)

A∧B ⇒ C∨D   =   (A∧B) ⇒ (C∨D)

A ⇒ B∨C ⇔ D   =   (A ⇒ (B∨C)) ⇔ D

Semantics

Meaning of a sentence : **true** or **false**

Interpretation : an assignment of **true** or **false**

to the propositional variables

$\vDash_i \varphi$: Sentence φ is true in the interpretation **i**

$\nvDash_i \varphi$: Sentence φ is true in the interpretation **i**

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Semantic Rules

$\vDash_i \varphi$: Sentence φ is true in the interpretation **i**

$\vDash_i \varphi$: Sentence φ is true in the interpretation **i**

$\vDash_i$ **T**   for all **i**

$\nvDash_i$ **F**   for all **i**

$\vDash_i$ **T**   for all **i**

Since **i** is a mapping from variables to truth values,

Look P up in **i** and return the truth value assigned to P

$\vDash_i \neg\varphi$ iff $\nvDash_i \varphi$

$\vDash_i \varphi \wedge \psi$ iff $\vDash_i \varphi$ **and** $\vDash_i \psi$

$\vDash_i \varphi \vee \psi$ iff $\vDash_i \varphi$ **or** $\vDash_i \psi$

$\vDash_i$ P   iff   **i**(P) = **T**

Semantics

Meaning of a sentence : **true** or **false**

Interpretation : an assignment of **true** or **false**
to the propositional variables (P, Q, R)

Interpretations **i** PQR=**TTF** **sentence** $P \land Q \land \neg R$ ➡ **true** $\models_i P \land Q \land \neg R$

Interpretations **j** PQR=**FTF** **sentence** $P \land Q \land \neg R$ ➡ **false** $\not\models_j P \land Q \land \neg R$
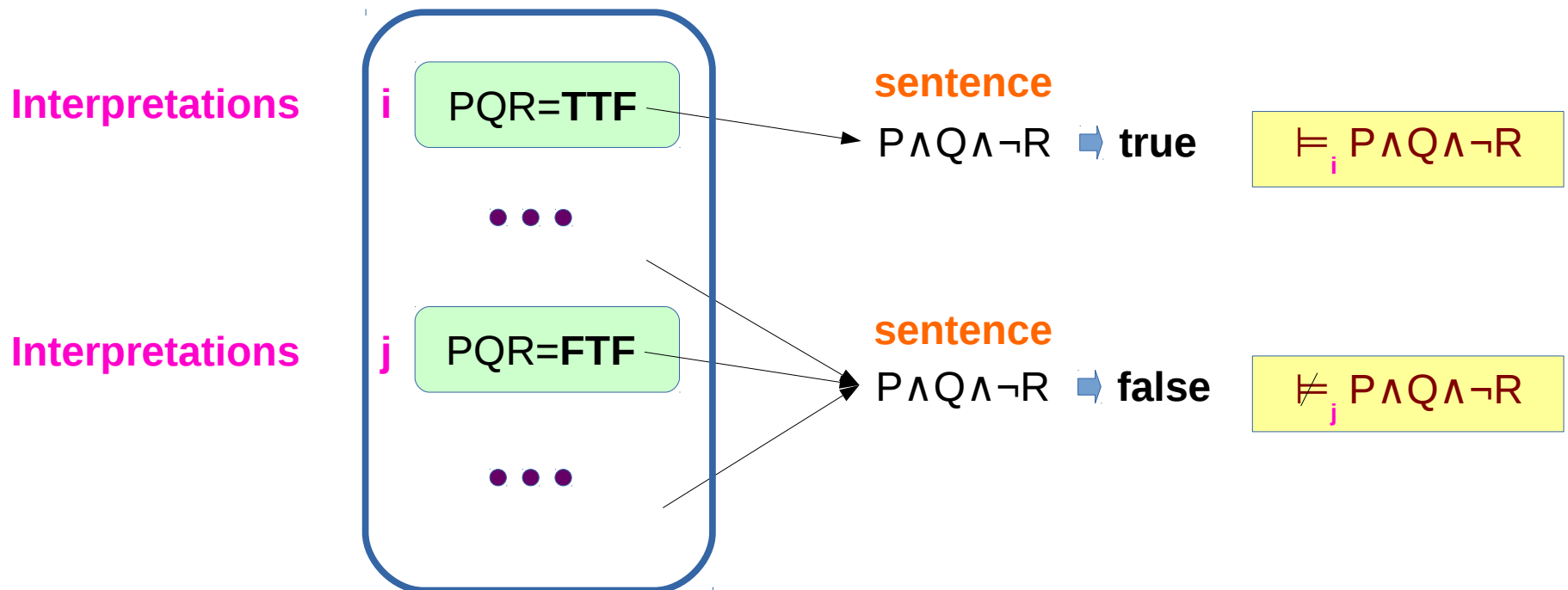
Valid, Satisfiable, Unsatisfiable

A sentence is **valid** iff

Its truth value is **T** in <u>all</u> interpretations

    ex) **T**, ¬**F**, P ∨ ¬P   (**tautology**: ⊤)

A sentence is **satisfiable** iff

Its truth value is **T** in <u>at least one</u> interpretation

    ex) P, **T**, ¬P

A sentence is **unsatisfiable** valid iff

Its truth value is **F** in <u>all</u> interpretations

    ex) **F**, ¬**T**, P∧¬P   (**contradiction**: ⊥)

Models
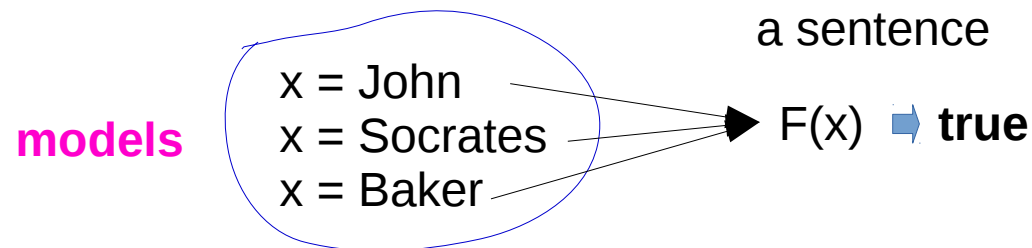
Semantics :

The relationship between sentences and interpretations

There are some set of interpretations → **models** of a sentence
that makes a sentence **true**

An interpretation is a **model** of a sentence
if the sentence is **true** in that interpretation

An interpretation **i** is a **model** of a sentence φ **iff** ⊨$_i$ φ

a sentence

models

x = John
x = Socrates
x = Baker

F(x) ➡ **true**

Models

An interpretation *i* is a **model** of a sentence φ *iff* $\vDash_i$ φ

$$\vDash_i \; \varphi$$

an interpretation   a sentence

interpretation *i*

φ

T

interpretation *i*
is a model
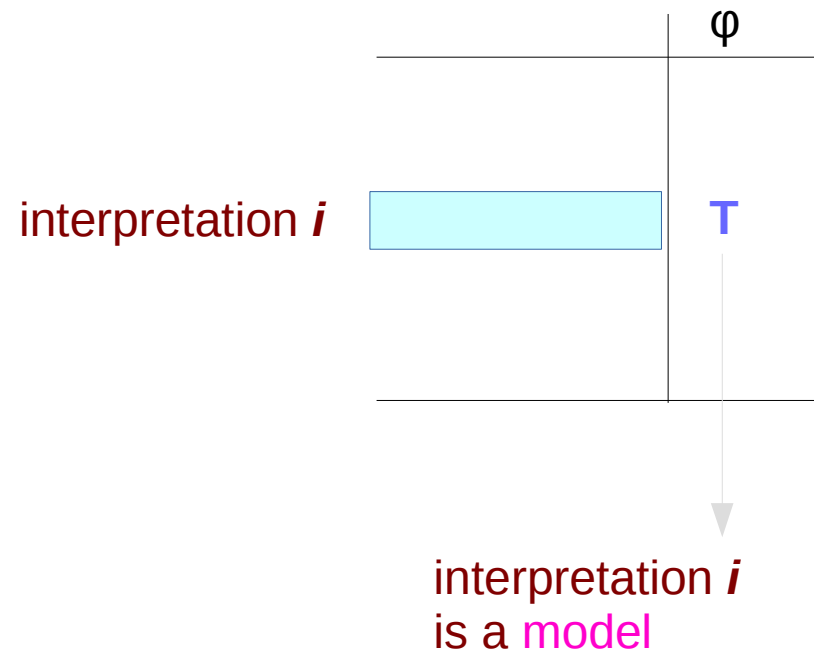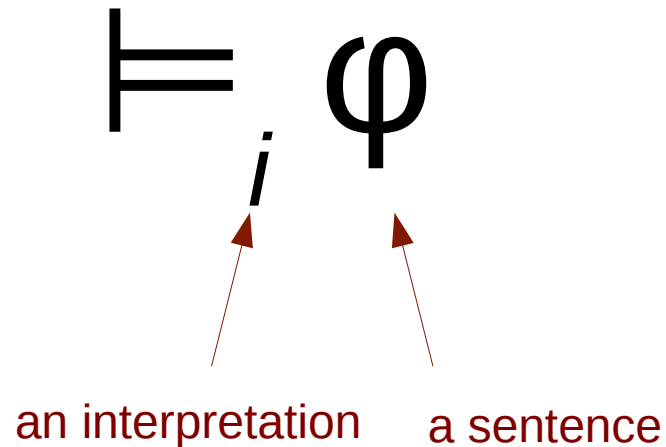
Models and Interpretations

models

8 interpretations

P	Q	R	P∧(Q∨R)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

T	T	T
T	T	F

T	T	T
T	F	T

T	F	T

Entailment

An interpretation i is a **model** of a sentence φ *iff* ⊨$_i$ φ
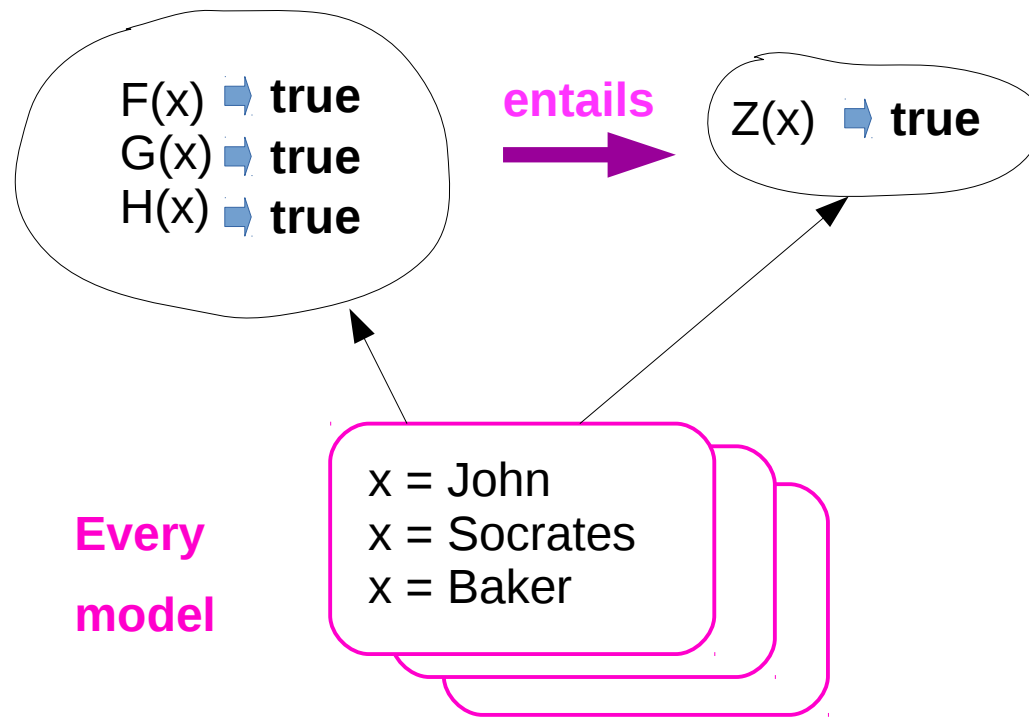
A set of sentences KB **entails** a sentence φ

iff **every** **model** of KB is also a **model** of φ

KB: Knowledge Base
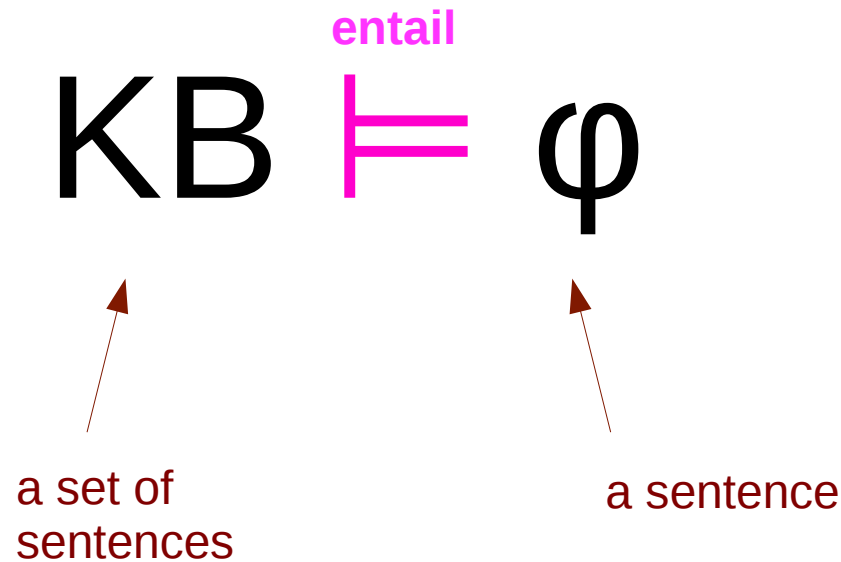   a set of sentences

every model of KB

every model of
  each sentence in KB

the model of a sentence φ

F(x) ➡ **true**
G(x) ➡ **true**
H(x) ➡ **true**

entails

Z(x) ➡ **true**

Every

model
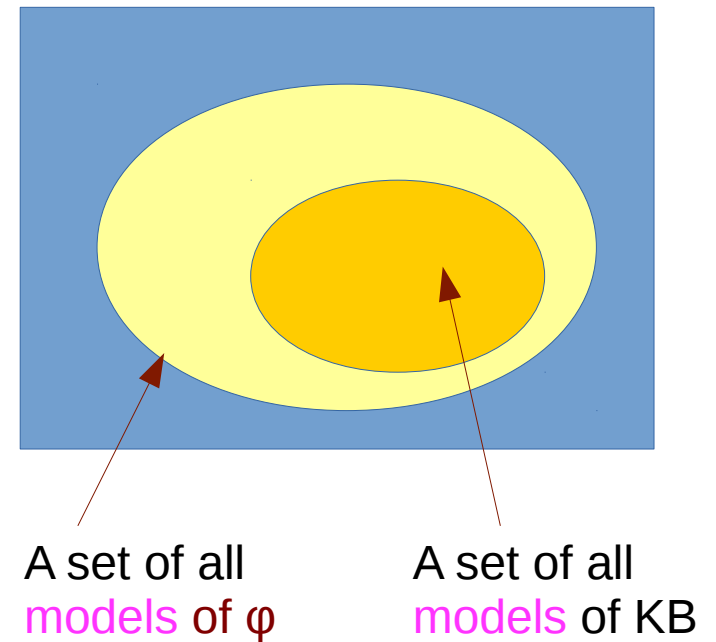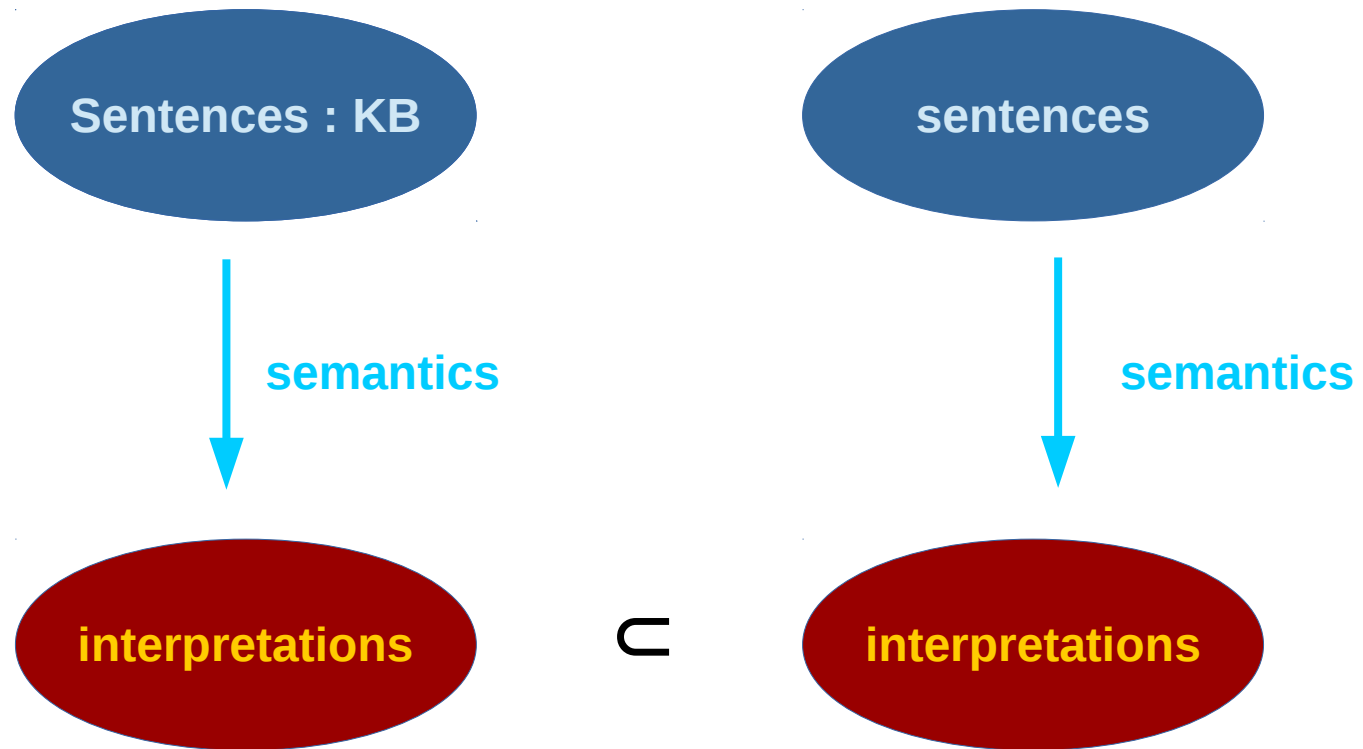
x = John
x = Socrates
x = Baker

Models

A set of sentences KB **entails** a sentence φ

iff <u>every</u> **model** of KB is also a **model** of φ

U: A set of all possible interpretation

entail

$$KB \models \varphi$$

a set of
sentences

a sentence

A set of all
models of φ

A set of all
models of KB

Entailment

References

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