Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Probability Distribution
one function transformation

Definition

The probability distribution of $Y = g(X_1, ..., X_N)$

$$F_Y(y) = P\{Y \leq y\} = P\{g(X_1, ..., X_N) \leq y\}$$

this probability is associated with all points in the $(x_1, ..., x_N)$ hyperspace that map such that $g(X_1, ..., X_N) \leq y$ for any y
integrate all such points according to

$$F_Y(y) = P\{g(X_1, ..., X_N) \leq y\}$$

$$= \int \cdots \int_{g(X_1, ..., X_N) \leq y} f_{X_1, \cdots, X_N}(x_1, \cdots, x_N) dx_1 \cdots dx_N$$

Probability Density
one function transformation

Definition

The probability density of $Y = g(X_1, ..., X_N)$

$$f_Y(y) = \frac{dF_Y(y)}{dy}$$

$$= \frac{d}{dy} \underset{g(X_1,...,X_N) \le y}{\int \cdots \int} f_{X_1,\cdots,X_N}(x_1, \cdots, x_N) dx_1 \cdots dx_N$$

Multiple Function Transformation(1)

multiple function transformation

Definition

$$Y_i = T_i(X_1, ..., X_N)$$

$$X_j = T_j^{-1}(Y_1, \cdots, Y_N)$$

$$\int \cdots \int_{R_X} f_{X_1,\cdots,X_N}(x_1, \cdots, x_N)\, dx_1 \cdots dx_N$$

$$= \int \cdots \int_{R_Y} f_{Y_1,\cdots,Y_N}(y_1, \cdots, y_N)\, dy_1 \cdots dy_N$$

Multiple Function Transformation(2)

multiple function transformation

Definition

$$J = \begin{vmatrix} \frac{\partial T_1^{-1}}{\partial Y_1} & \cdots & \frac{\partial T_1^{-1}}{\partial Y_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial T_N^{-1}}{\partial Y_1} & \cdots & \frac{\partial T_N^{-1}}{\partial Y_N} \end{vmatrix}$$

$$\underset{R_X}{\int \cdots \int} f_{X_1,\cdots,X_N}(x_1, \cdots, x_N) dx_1 \cdots dx_N$$

$$= \underset{R_Y}{\int \cdots \int} f_{X_1,\cdots,X_N}(x_1 = T_1^{-1}, \cdots, x_N = T_N^{-1}) dy_1 \cdots dy_N$$

$$f_{Y_1,\cdots,Y_N}(y_1, \cdots, y_N) = f_{X_1,\cdots,X_N}(x_1 = T_1^{-1}, \cdots, x_N = T_N^{-1})|J|$$