# K-Map (2B)

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9/25/13

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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# K-Map 4 variables (1)

index

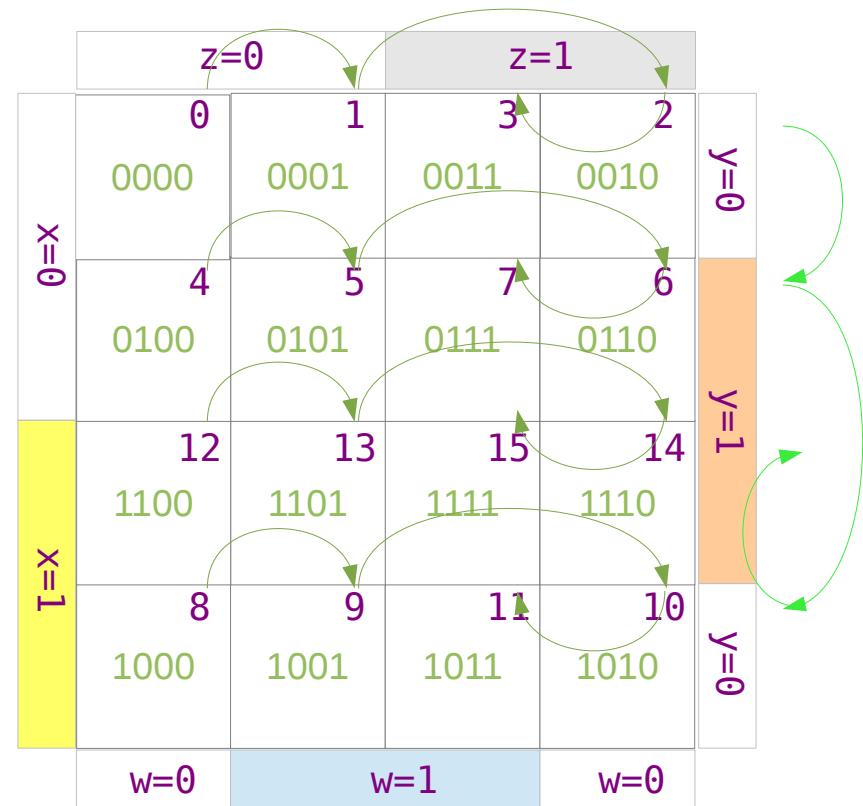| | | | | | minterms |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}\,\bar{w}$ |
| 1 | 0 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,\bar{z}\,w$ |
| 2 | 0 | 0 | 1 | 0 | $\bar{x}\,\bar{y}\,z\,\bar{w}$ |
| 3 | 0 | 0 | 1 | 1 | $\bar{x}\,\bar{y}\,z\,w$ |
| 4 | 0 | 1 | 0 | 0 | $\bar{x}\,y\,\bar{z}\,\bar{w}$ |
| 5 | 0 | 1 | 0 | 1 | $\bar{x}\,y\,\bar{z}\,w$ |
| 6 | 0 | 1 | 1 | 0 | $\bar{x}\,y\,z\,\bar{w}$ |
| 7 | 0 | 1 | 1 | 1 | $\bar{x}\,y\,z\,w$ |
| 8 | 1 | 0 | 0 | 0 | $x\,\bar{y}\,\bar{z}\,\bar{w}$ |
| 9 | 1 | 0 | 0 | 1 | $x\,\bar{y}\,\bar{z}\,w$ |
| 10 | 1 | 0 | 1 | 0 | $x\,\bar{y}\,z\,\bar{w}$ |
| 11 | 1 | 0 | 1 | 1 | $x\,\bar{y}\,z\,w$ |
| 12 | 1 | 1 | 0 | 0 | $x\,y\,\bar{z}\,\bar{w}$ |
| 13 | 1 | 1 | 0 | 1 | $x\,y\,\bar{z}\,w$ |
| 14 | 1 | 1 | 1 | 0 | $x\,y\,z\,\bar{w}$ |
| 15 | 1 | 1 | 1 | 1 | $x\,y\,z\,w$ |

| z=0 | | z=1 | |
|---|---|---|---|
| w=0 | w=1 | | w=0 |

| | | 00 | 01 | 11 | 10 |
|---|---|---|---|---|---|
| 00 | x=0 / y=0 | 0<br>0000 | 1<br>0001 | 3<br>0011 | 2<br>0010 |
| 01 | y=1 | 4<br>0100 | 5<br>0101 | 7<br>0111 | 6<br>0110 |
| 11 | x=1 | 12<br>1100 | 13<br>1101 | 15<br>1111 | 14<br>1110 |
| 10 | y=0 | 8<br>1000 | 9<br>1001 | 11<br>1011 | 10<br>1010 |

# K-Map 4 variables (2)

index | | | | | minterms
--- | --- | --- | --- | --- | ---
0 | 0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}\,\bar{w}$
1 | 0 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,\bar{z}\,w$
2 | 0 | 0 | 1 | 0 | $\bar{x}\,\bar{y}\,z\,\bar{w}$
3 | 0 | 0 | 1 | 1 | $\bar{x}\,\bar{y}\,z\,w$
4 | 0 | 1 | 0 | 0 | $\bar{x}\,y\,\bar{z}\,\bar{w}$
5 | 0 | 1 | 0 | 1 | $\bar{x}\,y\,\bar{z}\,w$
6 | 0 | 1 | 1 | 0 | $\bar{x}\,y\,z\,\bar{w}$
7 | 0 | 1 | 1 | 1 | $\bar{x}\,y\,z\,w$
8 | 1 | 0 | 0 | 0 | $x\,\bar{y}\,\bar{z}\,\bar{w}$
9 | 1 | 0 | 0 | 1 | $x\,\bar{y}\,\bar{z}\,w$
10 | 1 | 0 | 1 | 0 | $x\,\bar{y}\,z\,\bar{w}$
11 | 1 | 0 | 1 | 1 | $x\,\bar{y}\,z\,w$
12 | 1 | 1 | 0 | 0 | $x\,y\,\bar{z}\,\bar{w}$
13 | 1 | 1 | 0 | 1 | $x\,y\,\bar{z}\,w$
14 | 1 | 1 | 1 | 0 | $x\,y\,z\,\bar{w}$
15 | 1 | 1 | 1 | 1 | $x\,y\,z\,w$

# K-Map 3 variables (1)

index | | | | minterms
0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}$
1 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,z$
2 | 0 | 1 | 0 | $\bar{x}\,y\,\bar{z}$
3 | 0 | 1 | 1 | $\bar{x}\,y\,z$
4 | 1 | 0 | 0 | $x\,\bar{y}\,\bar{z}$
5 | 1 | 0 | 1 | $x\,\bar{y}\,z$
6 | 1 | 1 | 0 | $x\,y\,\bar{z}$
7 | 1 | 1 | 1 | $x\,y\,z$

x    y    z

| | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 3 | 2 |
| 1 | 4 | 5 | 7 | 6 |

| | y=0 | y=1 | | |
|---|---|---|---|---|
| | z=0 | z=1 | | z=0 |
| x=0 | 0 | 1 | 3 | 2 |
| x=1 | 4 | 5 | 7 | 6 |

# K-Map 3 variables (2)

index                    minterms

| 0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}$ |
| 1 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,z$ |
| 2 | 0 | 1 | 0 | $\bar{x}\,y\,\bar{z}$ |
| 3 | 0 | 1 | 1 | $\bar{x}\,y\,z$ |
| 4 | 1 | 0 | 0 | $x\,\bar{y}\,\bar{z}$ |
| 5 | 1 | 0 | 1 | $x\,\bar{y}\,z$ |
| 6 | 1 | 1 | 0 | $x\,y\,\bar{z}$ |
| 7 | 1 | 1 | 1 | $x\,y\,z$ |

$\bar{x}\,\bar{y}$

$\bar{x}\,y$

$x\,\bar{y}$

$x\,y$

$$\bar{x}\,\bar{y}\,\bar{z} \;+\; \bar{x}\,\bar{y}\,z \;=\; \bar{x}\,\bar{y}(\bar{z}+z) \;=\; \bar{x}\,\bar{y}$$

a group of 2 minterms

$$x \quad y \quad \cancel{z}$$

$$\Downarrow$$

$$2^1$$
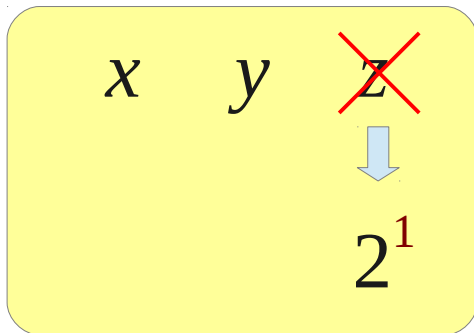
| | y=0 | | y=1 | |
|---|---|---|---|---|
| | z=0 | z=1 | | z=0 |

|  | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| x=0   0 | 0 $\bar{x}\,\bar{y}$ | 1 | 3 $\bar{x}\,y$ | 2 |
| x=1   1 | 4 $x\,\bar{y}$ | 5 | 7 $x\,y$ | 6 |

# K-Map 3 variables (3)

index

minterms

| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |

$\bar{x}\,\bar{y}\,\bar{z}$ ——— $\bar{x}\,\bar{z}$     $\bar{x}\,\bar{y}\,\bar{z} + \bar{x}\,y\,\bar{z} = \bar{x}\,\bar{z}(\bar{y}+y) = \bar{x}\,\bar{z}$

$\bar{x}\,\bar{y}\,z$ ——— $\bar{x}\,z$

$\bar{x}\,y\,\bar{z}$

$\bar{x}\,y\,z$

a group of 2 minterms

$x\,\bar{y}\,\bar{z}$ ——— $x\,\bar{z}$

$x\,\bar{y}\,z$ ——— $x\,z$

$x\,y\,\bar{z}$

$x\,y\,z$

$x \quad \cancel{y} \quad z$

$\Downarrow$

$2^1$

| | | y=0 | | y=1 | |
| | | z=0 | z=1 | z=0 | |

| | | 00 | 01 | 11 | 10 |
|---|---|---|---|---|---|
| x=0 | 0 | 0 $\bar{x}\,\bar{z}$ | 1 $\bar{x}\,z$ | 3 | 2 |
| x=1 | 1 | 4 $x\,\bar{z}$ | 5 $x\,z$ | 7 | 6 |

# K-Map 3 variables (4)

index

minterms

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

0    $\bar{x}\,\bar{y}\,\bar{z}$     $\bar{y}\,\bar{z}$     $\bar{x}\,\bar{y}\,\bar{z} + x\,\bar{y}\,\bar{z} = \bar{y}\,\bar{z}(\bar{x}+x) = \bar{y}\,\bar{z}$

1    $\bar{x}\,\bar{y}\,z$     $\bar{y}\,z$

2    $\bar{x}\,y\,\bar{z}$     $y\,\bar{z}$

3    $\bar{x}\,y\,z$     $y\,z$     a group of 2 minterms

4    $x\,\bar{y}\,\bar{z}$

5    $x\,\bar{y}\,z$

6    $x\,y\,\bar{z}$

7    $x\,y\,z$

$\cancel{x} \quad y \quad z$

$\Downarrow$

$2^1$

| | y=0 | | y=1 | |
|---|---|---|---|---|
| | z=0 | z=1 | | z=0 |

| | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **x=0**   0 | 0   $\bar{y}\,\bar{z}$ | 1   $\bar{y}\,z$ | 3   $y\,z$ | 2   $y\,\bar{z}$ |
| **x=1**   1 | 4 | 5 | 7 | 6 |

# K-Map 3 variables (5)

index

minterms

| 0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}$ |
| 1 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,z$ |
| 2 | 0 | 1 | 0 | $\bar{x}\,y\,\bar{z}$ |
| 3 | 0 | 1 | 1 | $\bar{x}\,y\,z$ |
| 4 | 1 | 0 | 0 | $x\,\bar{y}\,\bar{z}$ |
| 5 | 1 | 0 | 1 | $x\,\bar{y}\,z$ |
| 6 | 1 | 1 | 0 | $x\,y\,\bar{z}$ |
| 7 | 1 | 1 | 1 | $x\,y\,z$ |

$\bar{x}$

$x$

a group of 4 minterms

$x$ ~~$y$~~ ~~$z$~~

$2^2$

| | y=0 | | y=1 | |
|---|---|---|---|---|
| | z=0 | z=1 | | z=0 |

| | | 00 | 01 | 11 | 10 |
|---|---|---|---|---|---|
| x=0 | 0 | 0 | 1 | 3 | 2 |
| | | | | $\bar{x}$ | |
| x=1 | 1 | 4 | 5 | 7 | 6 |
| | | | | $x$ | |

# K-Map 3 variables (5)

index | | | | minterms
--- | --- | --- | --- | ---

| index | | | | minterms |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}$ |
| 1 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,z$ |
| 2 | 0 | 1 | 0 | $\bar{x}\,y\,\bar{z}$ |
| 3 | 0 | 1 | 1 | $\bar{x}\,y\,z$ |
| 4 | 1 | 0 | 0 | $x\,\bar{y}\,\bar{z}$ |
| 5 | 1 | 0 | 1 | $x\,\bar{y}\,z$ |
| 6 | 1 | 1 | 0 | $x\,y\,\bar{z}$ |
| 7 | 1 | 1 | 1 | $x\,y\,z$ |

$\bar{y}$

$y$

a group of 4 minterms

$\cancel{x} \quad y \quad \cancel{z}$

$\Downarrow$

$2^2$

| | y=0 | | y=1 | |
|---|---|---|---|---|
| | z=0 | z=1 | z=1 | z=0 |

| | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| x=0  0 | 0 | 1 | 3 | 2 |
| | | $\bar{y}$ | | $y$ |
| x=1  1 | 4 | 5 | 7 | 6 |

# K-Map 3 variables (5)

index         minterms

| index | | | | minterms |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}$ |
| 1 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,z$ |
| 2 | 0 | 1 | 0 | $\bar{x}\,y\,\bar{z}$ |
| 3 | 0 | 1 | 1 | $\bar{x}\,y\,z$ |
| 4 | 1 | 0 | 0 | $x\,\bar{y}\,\bar{z}$ |
| 5 | 1 | 0 | 1 | $x\,\bar{y}\,z$ |
| 6 | 1 | 1 | 0 | $x\,y\,\bar{z}$ |
| 7 | 1 | 1 | 1 | $x\,y\,z$ |

$\bar{z}$

$z$

a group of 4 minterms

$$\cancel{x} \quad \cancel{y} \quad z$$

$$2^2$$

| y=0 | | y=1 | |
|---|---|---|---|
| z=0 | z=1 | | z=0 |

|  | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| x=0   0 | 0 | 1 | 3 | 2 |
| x=1   1 | 4 | 5 | 7 | 6 |
| | $\bar{z}$ | $z$ | | |

# K-Map, minterms, and Maxterms

index                          minterms

| index | | | | minterms |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $\bar{x}\,\bar{y}\,\bar{z}$ |
| 1 | 0 | 0 | 1 | $\bar{x}\,\bar{y}\,z$ |
| 2 | 0 | 1 | 0 | $\bar{x}\,y\,\bar{z}$ |
| 3 | 0 | 1 | 1 | $\bar{x}\,y\,z$ |
| 4 | 1 | 0 | 0 | $x\,\bar{y}\,\bar{z}$ |
| 5 | 1 | 0 | 1 | $x\,\bar{y}\,z$ |
| 6 | 1 | 1 | 0 | $x\,y\,\bar{z}$ |
| 7 | 1 | 1 | 1 | $x\,y\,z$ |

x   y   z

|     | 00 | 01 | 11 | 10 |
|-----|----|----|----|----|
|     | 0  | 1  | 3  | 2  |
| 0   | $m_0$ | $m_1$ | $m_3$ | $m_2$ |
|     | 4  | 5  | 7  | 6  |
| 1   | $m_4$ | $m_5$ | $m_7$ | $m_6$ |
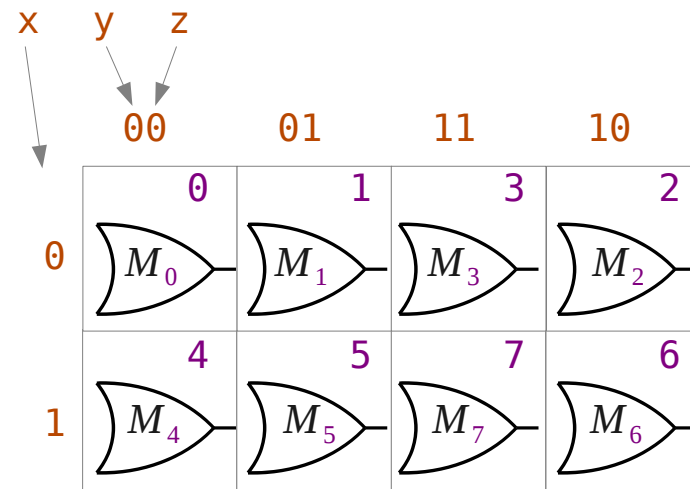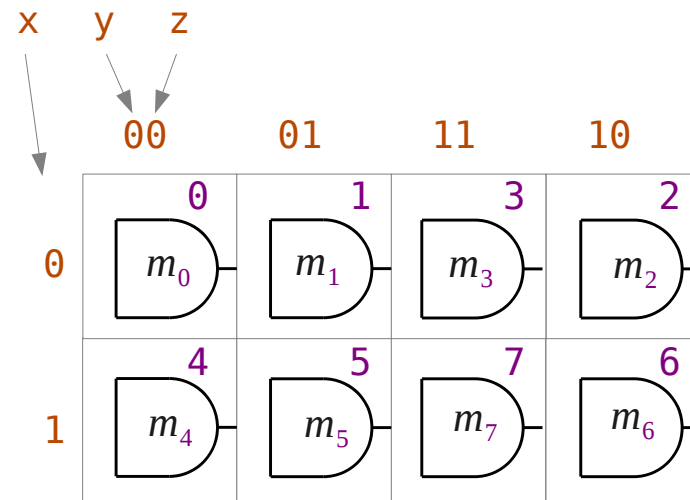
Each rectangle is associated with a minterm or a maxterm which represents a particular input variable conditions.

In this table, output function value is overlaid

x   y   z

|     | 00 | 01 | 11 | 10 |
|-----|----|----|----|----|
|     | 0  | 1  | 3  | 2  |
| 0   | $M_0$ | $M_1$ | $M_3$ | $M_2$ |
|     | 4  | 5  | 7  | 6  |
| 1   | $M_4$ | $M_5$ | $M_7$ | $M_6$ |

# Boolean Function with minterms

x y z F

|   | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| → 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| → 3 | 0 | 1 | 1 | 1 |
| → 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |

**1**

**1**

**1**

**1**

$F$

|  | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 $m_0$ | 1 $m_1$ | 3 $m_3$ | 2 $m_2$ |
| 1 | 4 $m_4$ | 5 $m_5$ | 7 $m_7$ | 6 $m_6$ |

F=1 when $\bar{x}z = 1$

an overlaid table     y

a simplified function

$$F = \bar{x}z + x\bar{y}\bar{z}$$

|  | 0 | 1 | 3 | 2 |
|---|---|---|---|---|
|  |  | 1 | 1 |  |
|  | 4 | 5 | 7 | 6 |
|  | 1 |  |  |  |

x

z

F=1 when $x\bar{y}\bar{z} = 1$
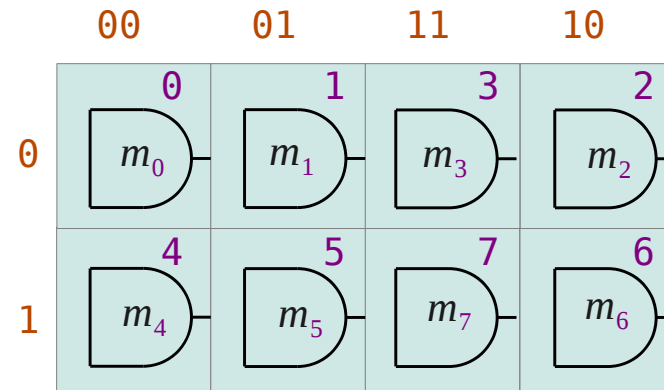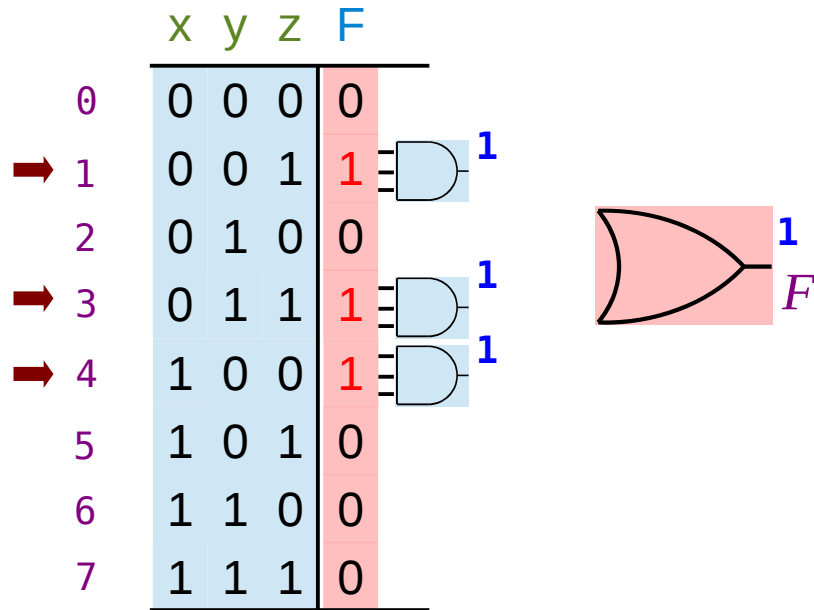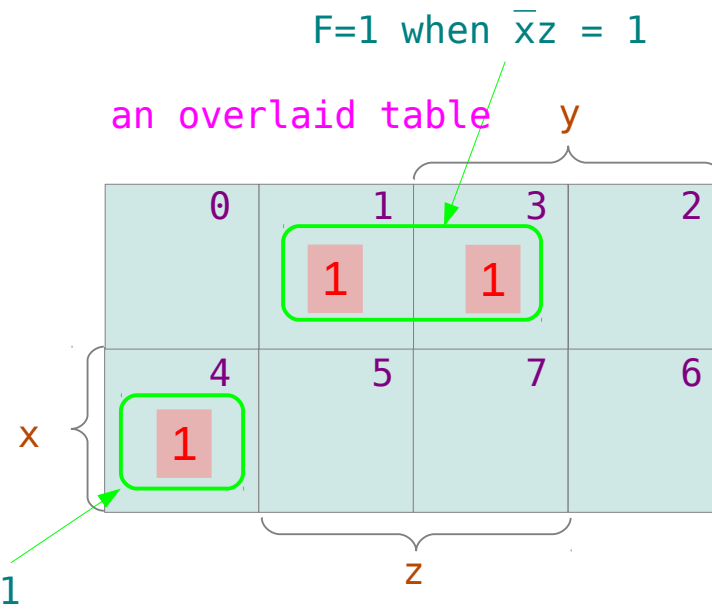
# Boolean Function with Maxterms

| x | y | z | F | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | |
| 1 | 0 | 0 | 1 | |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |

$F$

|  0  |  1  |  3  |  2  |
|-----|-----|-----|-----|
| $M_0$ | $M_1$ | $M_3$ | $M_2$ |
|  4  |  5  |  7  |  6  |
| $M_4$ | $M_5$ | $M_7$ | $M_6$ |

F=0 when x+z=0

an overlaid table    y

a simplified function

$$F = (\bar{x} + \bar{z})(\bar{y} + z)(x + z)$$

|  0  |  1  |  3  |  2  |
|-----|-----|-----|-----|
|  0  |     |     |  0  |
|  4  |  5  |  7  |  6  |
|     |  0  |  0  |  0  |

x

z

F=0 when $\bar{x}+\bar{z}=0$        F=0 when $\bar{y}+z=0$
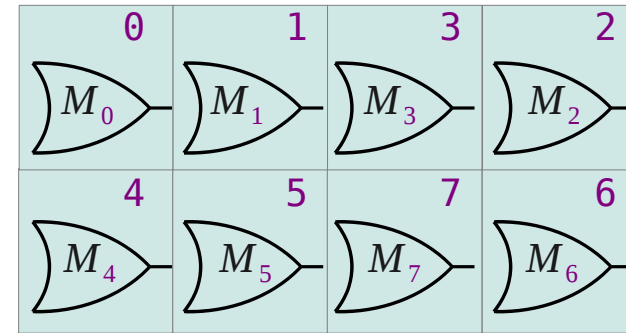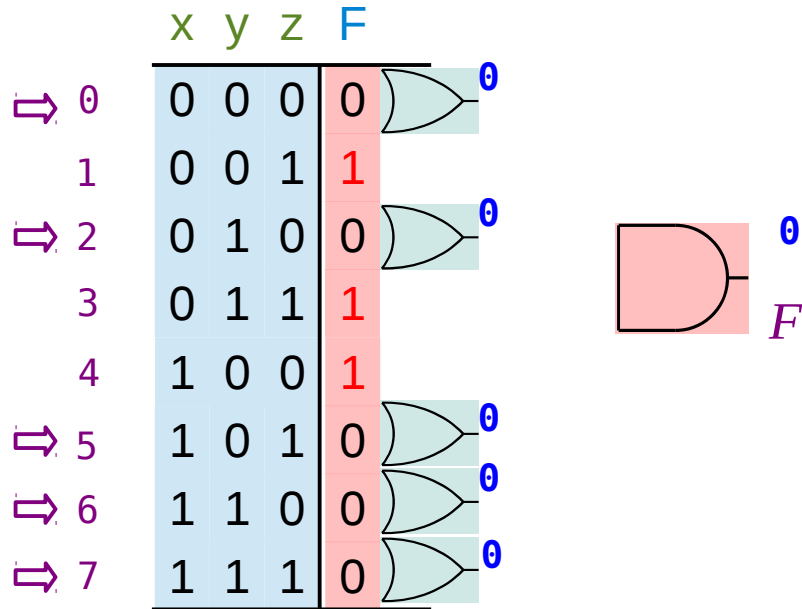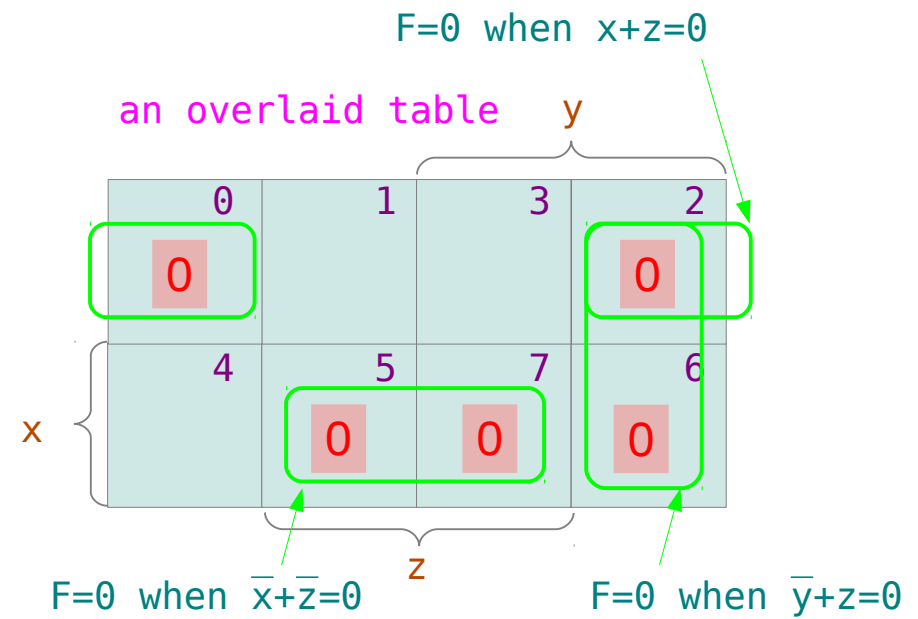
# Implicant

F takes the value 1, whenever P equals 1

$$P \quad \overset{\text{implies}}{\Rightarrow} \quad F$$

assuming *P* is a product term in a sum of products

$$F(x,y,z,w) \;=\; xy + yz + w$$

General,
Reduced

$$(xy = 1) \quad \Rightarrow \quad (f=1)$$
$$(xyz = 1) \quad \Rightarrow \quad (f=1)$$
$$(xyzw = 1) \quad \Rightarrow \quad (f=1)$$

$xy \qquad \Rightarrow \quad F(x,y,z,w) \;=\; xy + yz + w$

$xyz \qquad \Rightarrow \quad F(x,y,z,w) \;=\; xy + yz + w$

$xyzw \qquad \Rightarrow \quad F(x,y,z,w) \;=\; xy + yz + w$

implicants

# Implicant

implies

$$P \quad \Rightarrow \quad F$$
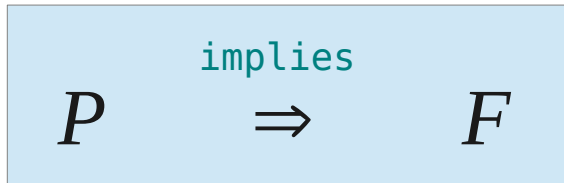
assuming *P* is a product term in a sum of products

$$F(x,y,z,w) = xy + yz + w$$

general, reduced

$(x=1, y=1)$

$(x=1, y=1, z=1)$

$(x=1, y=1, z=1, w=1)$

$$xy \quad \Rightarrow \quad F(x,y,z,w) = xy + yz + w \qquad (f=1)$$

$$xyz \quad \Rightarrow \quad F(x,y,z,w) = xy + yz + w \qquad (f=1)$$

$$xyzw \quad \Rightarrow \quad F(x,y,z,w) = xy + yz + w \qquad (f=1)$$

input conditions that will make F = 1

# Prime Implicant

**Prime Implicant**: An implicant that is <u>minimal</u>

The removal of any literal from P results in a non-implicant for F

**Prime Implicant**

general, reduced

$(x=1,\ y=1)$        $xy$    $\Rightarrow$    $F(x,y,z,w) = xy + yz + w$

$(x=1,\ y=1,\ z=1)$    $xyz$    $\Rightarrow$    $F(x,y,z,w) = xy + yz + w$

$(x=1,\ y=1,\ z=1,\ w=1)$    $xyzw$    $\Rightarrow$    $F(x,y,z,w) = xy + yz + w$

$xyzw \Rightarrow F$       $xyzw \Rightarrow F$       $xyzw \Rightarrow F$

$yzw \Rightarrow F$       $xy\ w \Rightarrow F$       $xyz \Rightarrow F$

$zw \Rightarrow F$       $xy \Rightarrow F$       $yz \Rightarrow F$

$w \Rightarrow F$

A process of expanding terms

general, reduced

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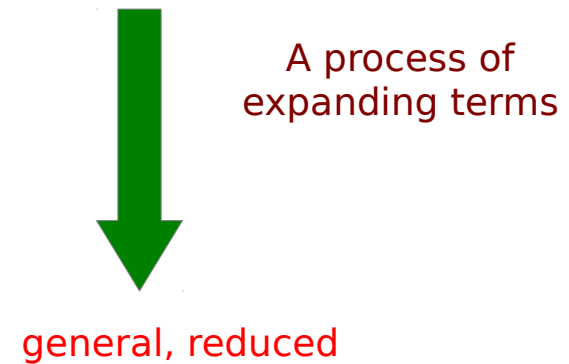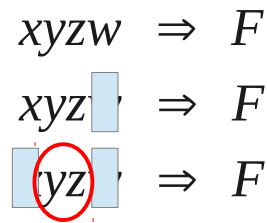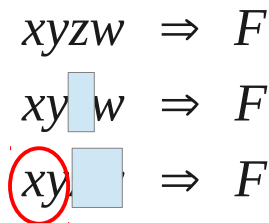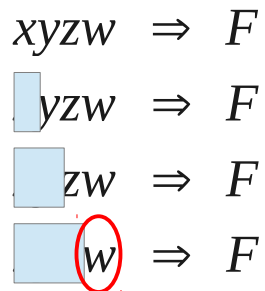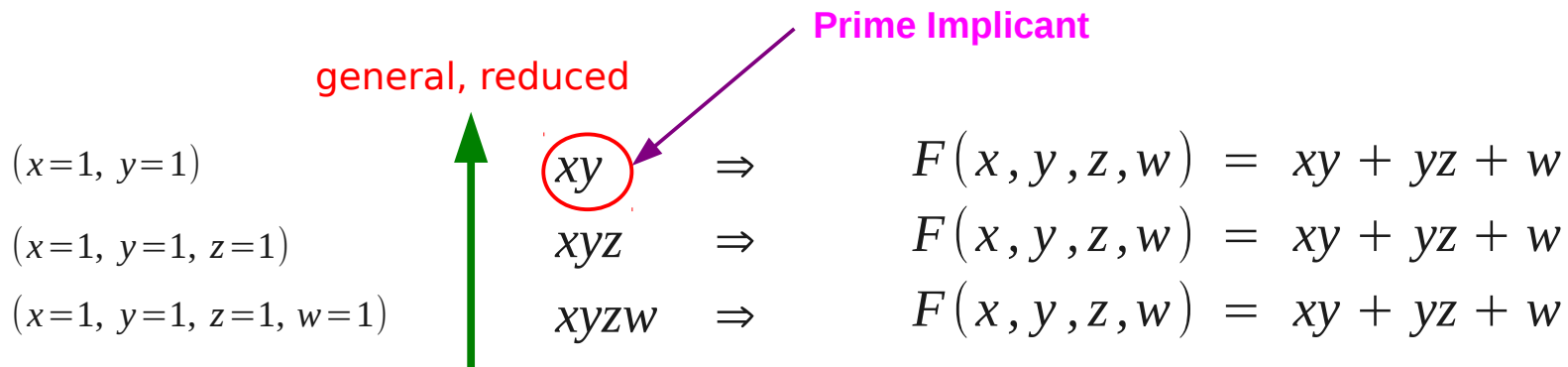
# Expanding Terms

$$xyzw \Rightarrow F$$

$$yzw \Rightarrow F$$

$$zw \Rightarrow F$$

$$w \Rightarrow F$$

$$xyzw \Rightarrow F$$

$$xy \; w \Rightarrow F$$

$$xy \Rightarrow F$$

$$xyzw \Rightarrow F$$

$$xyz \Rightarrow F$$

$$yz \Rightarrow F$$
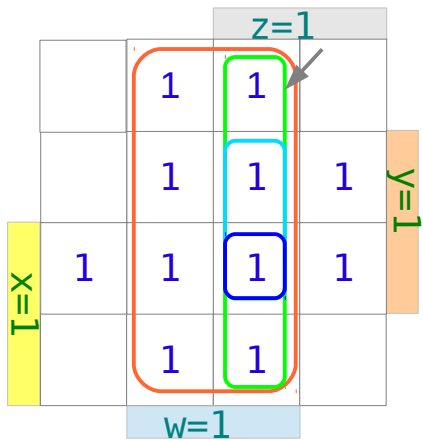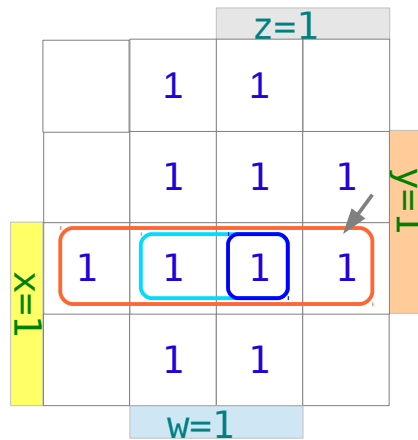


Prime Implicant $\quad w \Rightarrow F$

Prime Implicant $\quad xy \Rightarrow F$
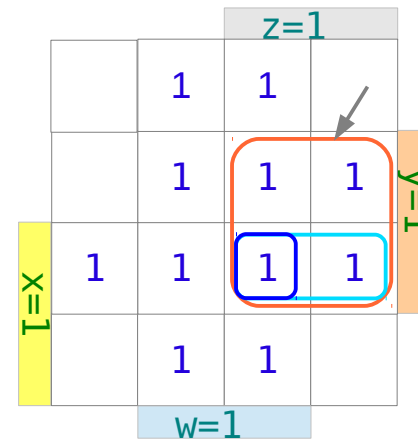
Prime Implicant $\quad yz \Rightarrow F$

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# Prime Implicant Example
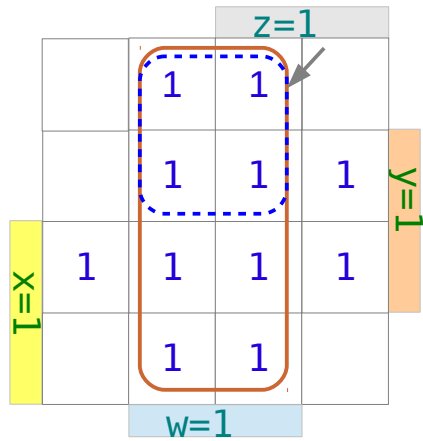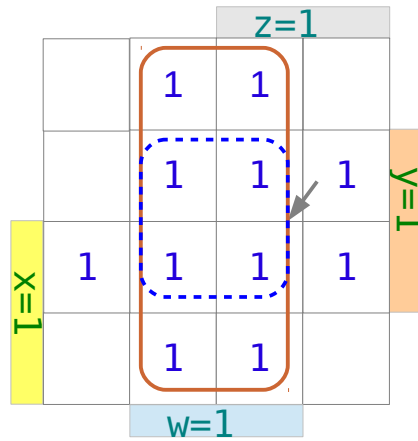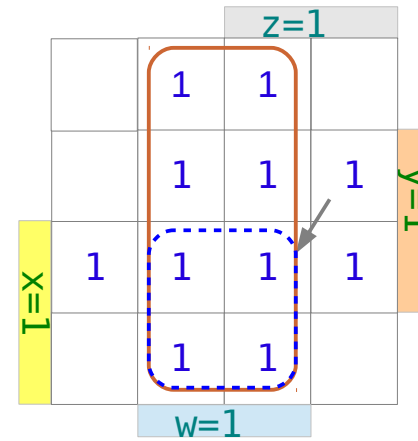
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# Prime Implicants

$w$

$xy$

$yz$
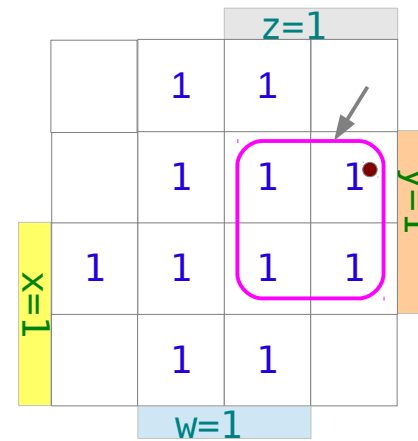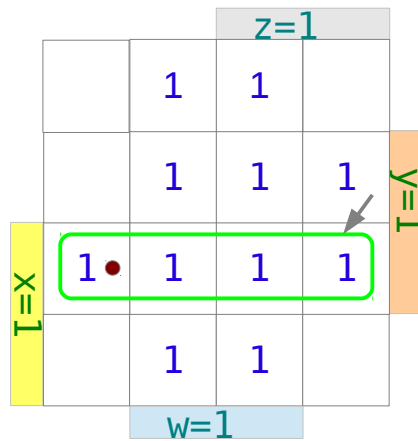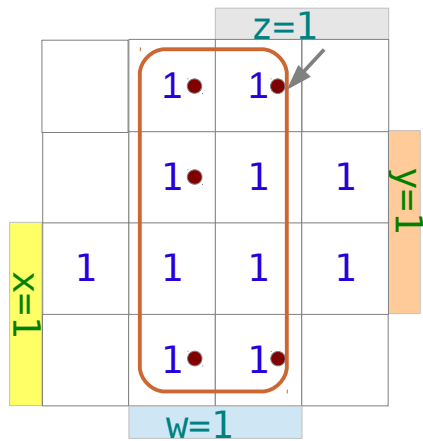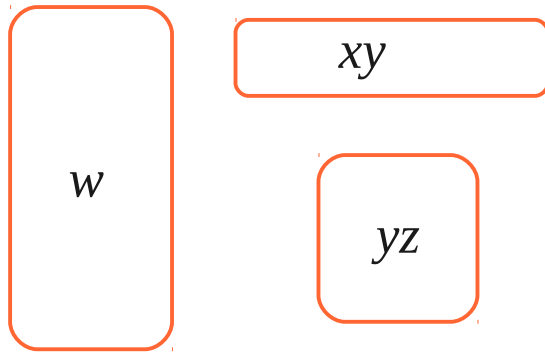
Three Prime Implicants
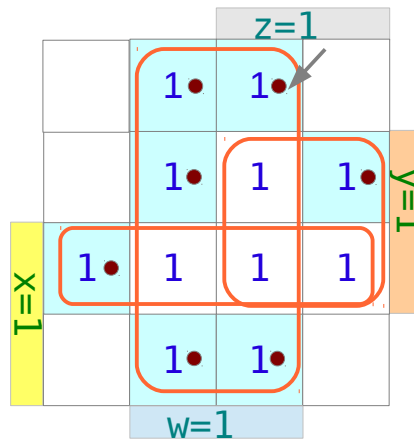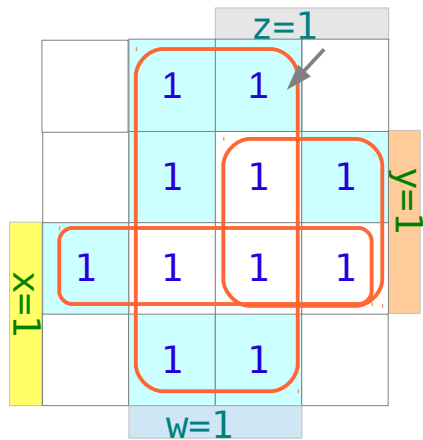


some minterms belong to only <u>one</u> prime implicant

Those prime implicants containing any such minterm is called an **essential prime implicant**

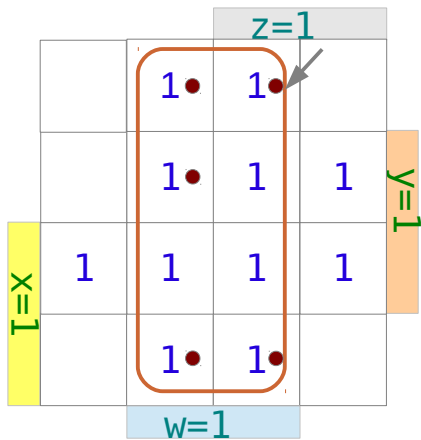In this example, all three prime implicants are essential

# Essential Prime Implicant

**Essential Prime Implicant**:
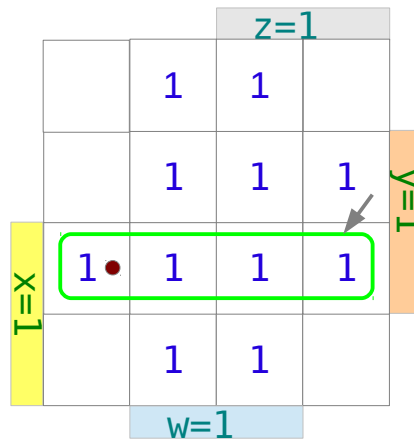*prime implicants*
    that cover an *output of the function*
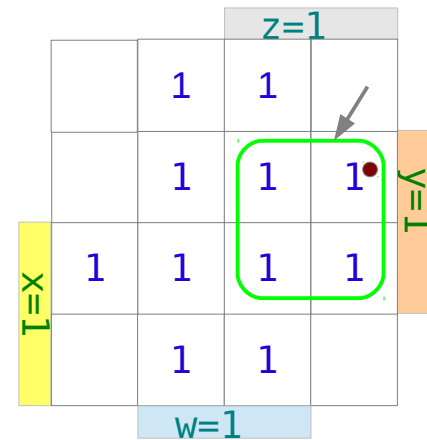        that no combination of other implicants is able to cover



Essential Prime Implicant      Essential Prime Implicant      Essential Prime Implicant

# 2's Complement

# Decimal to Binary (1)

# Decimal to Binary (2)

# Laplace Equation

# Laplace Equation

# Laplace Equation

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## References

[1]  http://en.wikipedia.org/
[2]  http://planetmath.org/
[3]  M.L. Boas, "Mathematical Methods in the Physical Sciences"

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9/25/13