Isomorphic Graph (8A)

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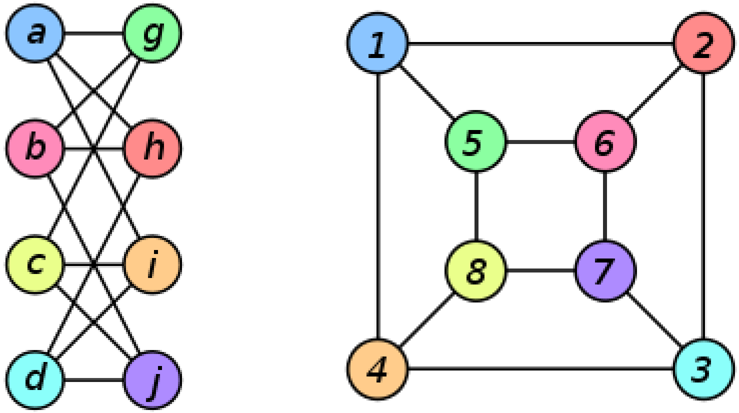
Graph Isomorphism

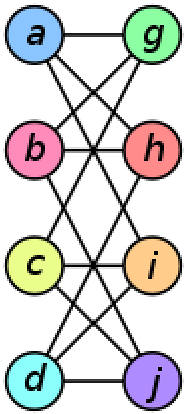
The two graphs shown below are **isomorphic**, despite their different looking drawings.



f(a) = 1
f(b) = 6
f(c) = 8
f(d) = 3
f(g) = 5
f(h) = 2
f(i) = 4
f(j) = 7

https://en.wikipedia.org/wiki/Graph_isomorphism

Graph G_1 and its Adjacency Matrix

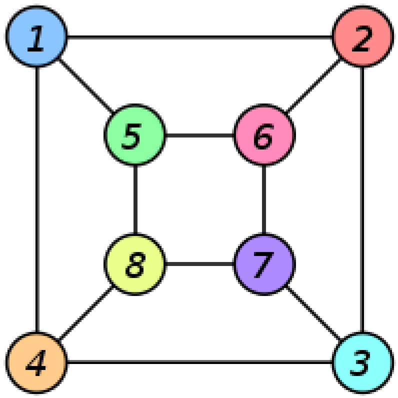


	a	b	c	d	g	h	i	j
a	0	0	0	0	1	1	1	0
b	0	0	0	0	1	1	0	1
c	0	0	0	0	1	0	1	1
d	0	0	0	0	0	1	1	1
g	1	1	1	0	0	0	0	0
h	1	1	0	1	0	0	0	0
i	1	0	1	1	0	0	0	0
j	0	1	1	1	0	0	0	0

https://en.wikipedia.org/wiki/Graph_isomorphism

Graph G_2 and its Adjacency Matrix

	1	2	3	4	5	6	7	8
1	0	1	0	1	1	0	0	0
2	1	0	1	0	0	1	0	0
3	0	1	0	1	0	0	1	0
4	1	0	1	0	0	0	0	1
5	1	0	0	0	0	1	0	1
6	0	1	0	0	1	0	1	0
7	0	0	1	0	0	1	0	1
8	0	0	0	1	1	0	1	0

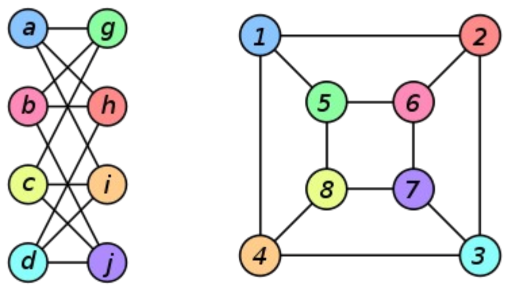


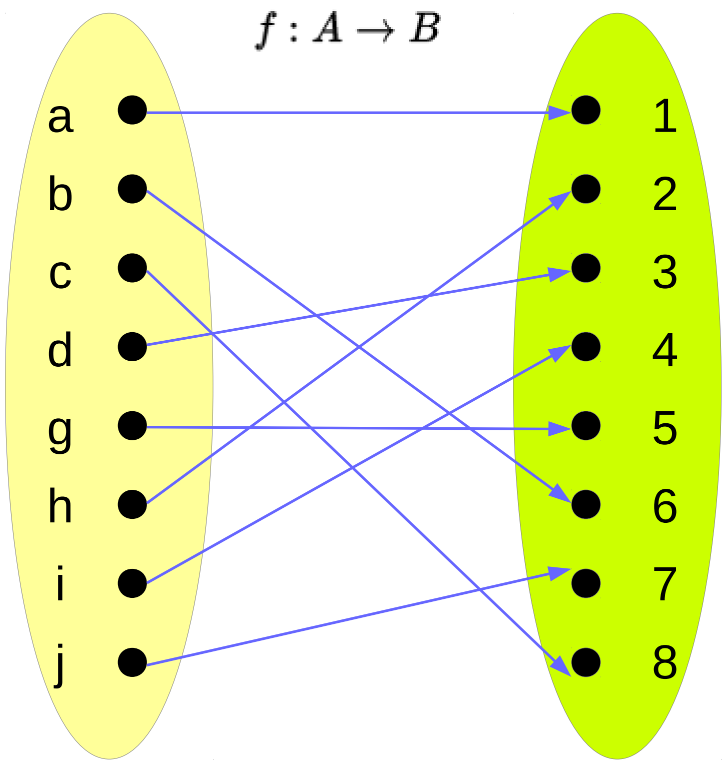
edge-preserving bijection

structure-preserving bijection.

https://en.wikipedia.org/wiki/Graph_isomorphism

Bijection Mapping f





		1	6	8	3	5	2	4	7
		a	b	c	d	g	h	i	j
1	a	0	0	0	0	1	1	1	0
6	b	0	0	0	0	1	1	0	1
8	c	0	0	0	0	1	0	1	1
3	d	0	0	0	0	0	1	1	1
5	g	1	1	1	0	0	0	0	0
2	h	1	1	0	1	0	0	0	0
4	i	1	0	1	1	0	0	0	0
7	j	0	1	1	1	0	0	0	0

Converting the Adjacency Matrix

permuting the rows and columns

	1	6	8	3	5	2	4	7
1	0	0	0	0	1	1	1	0
6	0	0	0	0	1	1	0	1
8	0	0	0	0	1	0	1	1
3	0	0	0	0	0	1	1	1
5	1	1	1	0	0	0	0	0
2	1	1	0	1	0	0	0	0
4	1	0	1	1	0	0	0	0
7	0	1	1	1	0	0	0	0

Adjacency Matrix of G_1

	1	2	3	4	5	6	7	8
1	0	1	0	1	1	0	0	0
2	1	0	1	0	0	1	0	0
3	0	1	0	1	0	0	1	0
4	1	0	1	0	0	0	0	1
5	1	0	0	0	0	1	0	1
6	0	1	0	0	1	0	1	0
7	0	0	1	0	0	1	0	1
8	0	0	0	1	1	0	1	0

Adjacency Matrix of G_2

Converting the Adjacency Matrix

	1	6	8	3	5	2	4	7
1	0	0	0	0	1	1	1	0
6	0	0	0	0	1	1	0	1
8	0	0	0	0	1	0	1	1
3	0	0	0	0	0	1	1	1
5	1	1	1	0	0	0	0	0
2	1	1	0	1	0	0	0	0
4	1	0	1	1	0	0	0	0
7	0	1	1	1	0	0	0	0

G_1 adjacency matrix
after maping

	1	2	3	4	5	6	7	8
1	0	1	0	1	1	0	0	0
6	0	1	0	0	1	0	1	0
8	0	0	0	1	1	0	1	0
3	0	1	0	1	0	0	1	0
5	1	0	0	0	0	1	0	1
2	1	0	1	0	0	1	0	0
4	1	0	1	0	0	0	0	1
7	0	0	1	0	0	1	0	1

	1	2	3	4	5	6	7	8
1	0	1	0	1	1	0	0	0
2	1	0	1	0	0	1	0	0
3	0	1	0	1	0	0	1	0
4	1	0	1	0	0	0	0	1
5	1	0	0	0	0	1	0	1
6	0	1	0	0	1	0	1	0
7	0	0	1	0	0	1	0	1
8	0	0	0	1	1	0	1	0

G_2 adjacency matrix
after permuting
rows and columns

Morphism

morphism refers to a structure-preserving **map**
from one mathematical structure *to* another

in set theory, morphisms are functions;
in linear algebra, linear transformations;
in group theory, group homomorphisms;
in topology, continuous functions

In category theory, morphism is a broadly similar idea,
but somewhat more abstract:
the mathematical objects involved need not be sets,
and the relationship between them may be
something more general than a map.

https://en.wikipedia.org/wiki/Morphism

Homomorphism

In **algebra**, a **homomorphism** is a structure-preserving map
between two algebraic structures of the same type
(such as two groups, two rings, or two vector spaces).

The word homomorphism comes from the ancient Greek language:
ὁμός (homos) meaning "same"
and μορφή (morphe) meaning "form" or "shape".

Homomorphisms of vector spaces
are also called linear maps,
and their study is the object of linear algebra.

The concept of **homomorphism** has been generalized,
under the name of **morphism**, to many other structures
that either do not have an underlying set,
or are not algebraic.

https://en.wikipedia.org/wiki/Homomorphism

Isomorphism

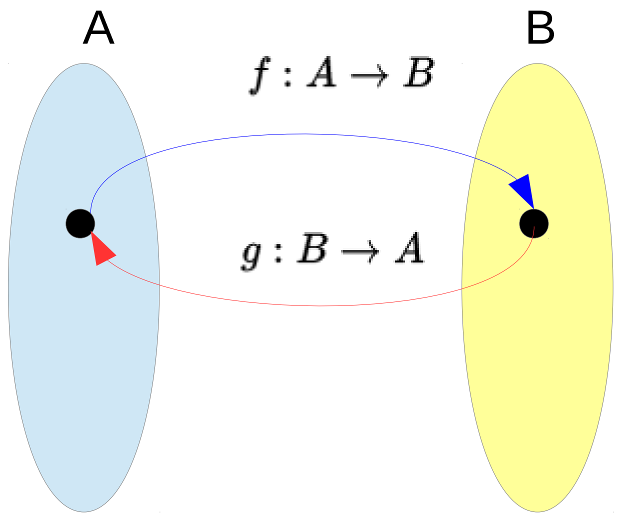
an **isomorphism**
(from the Ancient Greek: ἴσος isos "equal",
and μορφή morphe "form" or "shape")

is a **homomorphism** or **morphism**
(i.e. a mathematical mapping)
that admits an **inverse**.

morphism refers to a structure-preserving **map**
from one mathematical structure *to* another

https://en.wikipedia.org/wiki/Homomorphism

Isomorphism – bijective

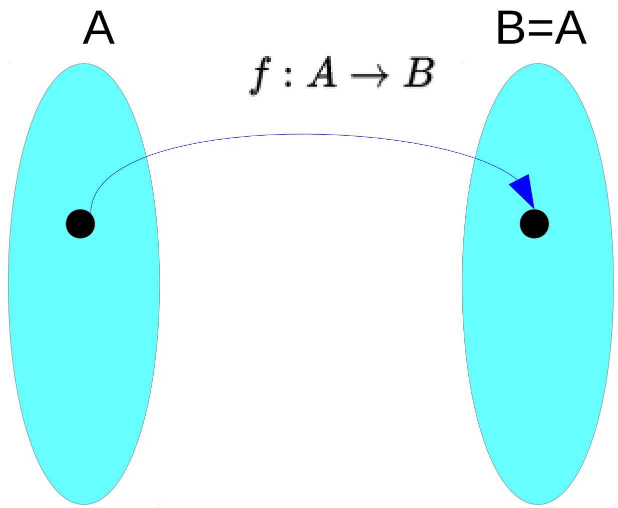


$$f \circ g = \mathrm{Id}_B \qquad \text{and} \qquad g \circ f = \mathrm{Id}_A.$$

An isomorphism between algebraic structures of the same type is commonly defined as a **bijective homomorphism**.

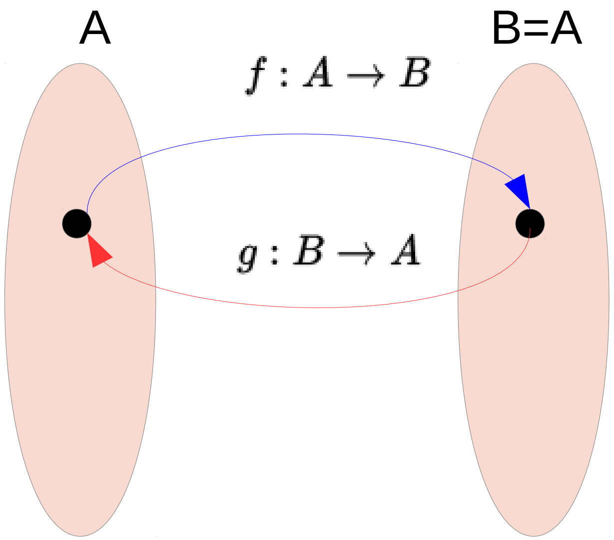
https://en.wikipedia.org/wiki/Homomorphism

Automorphism – (domain = codomain)



An endomorphism is an homomorphism
whose **domain** equals the **codomain**, or, more generally,
a morphism whose source is equal to the target.

https://en.wikipedia.org/wiki/Homomorphism



$$f \circ g = \mathrm{Id}_B \quad \text{and} \quad g \circ f = \mathrm{Id}_A.$$

An **automorphism** is
an **endomorphism** that is also an **isomorphism**.

https://en.wikipedia.org/wiki/Homomorphism

Isomorphism

an **isomorphism**
(from the Ancient Greek: ἴσος isos "equal",
and μορφή morphe "form" or "shape")

is a homomorphism or morphism
(i.e. a mathematical mapping)
that admits an **inverse**.

Two mathematical objects are **isomorphic**
if an isomorphism exists between them.

https://en.wikipedia.org/wiki/Homomorphism

Isomorphism

An **automorphism** is an isomorphism whose source and target coincide.

The interest of isomorphisms lies in the fact
that two isomorphic objects cannot be distinguished
by using only the properties used to define morphisms;

thus isomorphic objects may be considered the same
as long as one considers only these properties and their consequences.

https://en.wikipedia.org/wiki/Homomorphism

Graph Isomorphism

In graph theory, an **isomorphism** of graphs G and H
is a bijection between the vertex sets of G and H

$$f\colon V(G) \to V(H)$$

such that any two vertices u and v of G are adjacent in G
if and only if f(u) and f(v) are adjacent in H.

This kind of bijection is commonly described
as "edge-preserving bijection",

in accordance with the general notion of isomorphism
being a structure-preserving bijection.

https://en.wikipedia.org/wiki/Graph_isomorphism

Graph Isomorphism

If an isomorphism exists between two graphs,
then the graphs are called **isomorphic**
and denoted as $G \simeq H$

In the case when the bijection is
a mapping of a graph onto itself,
i.e., when G and H are one and the same graph,
the bijection is called an **automorphism** of G.

Graph isomorphism is
an equivalence relation on graphs
and as such it partitions the class of all graphs
into equivalence classes.

A set of graphs isomorphic to each other is called
an isomorphism class of graphs.

https://en.wikipedia.org/wiki/Graph_isomorphism

References

[1] http://en.wikipedia.org/
[2]