# State Monad (3D)

Young Won Lim
11/8/17

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice.

# Based on

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

# Type Synonyms

```
type String = [Char]

phoneBook :: [(String,String)]
```

```
type PhoneBook = [(String,String)]

phoneBook :: PhoneBook
```

```
type PhoneNumber = String
type Name = String
type PhoneBook = [(Name,PhoneNumber)]

phoneBook :: PhoneBook
```

```
phoneBook =
    [("betty","555-2938")
    ,("bonnie","452-2928")
    ,("patsy","493-2928")
    ,("lucille","205-2928")
    ,("wendy","939-8282")
    ,("penny","853-2492")
    ]
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

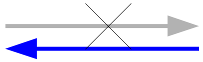
# Record Syntax (named field)

```
data Configuration = Configuration
    { username          :: String
    , localHost         :: String
    , currentDir        :: String
    , homeDir           :: String
    , timeConnected     :: Integer
    }
```

```
username :: Configuration -> String                  -- accessor function  (automatic)
localHost :: Configuration -> String
-- etc.

changeDir :: Configuration -> String -> Configuration        -- update function
changeDir cfg newDir =
    if directoryExists newDir                   -- make sure the directory exists
        then cfg { currentDir = newDir }
        else error "Directory does not exist"
```

https://en.wikibooks.org/wiki/Haskell/More_on_datatypes

# **newtype** and **data**

**data**  **newtype**

Data can only be replaced with newtype
**if** the type has exactly *one constructor* with exactly *one field* inside it.

It ensures that the trivial **wrapping** and **unwrapping**
of the single field is eliminated by the **compiler**.

simple wrapper types such as **State** are usually defined with **newtype**.

**type** : used for type synonyms

```
newtype State s a = State { runState :: s -> (s, a) }
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# **newtype** examples

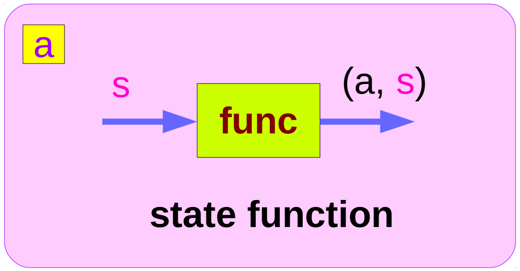
```
newtype Fd = Fd CInt
-- data Fd = Fd CInt would also be valid

-- newtypes can have deriving clauses just like normal types
newtype Identity a = Identity a
  deriving (Eq, Ord, Read, Show)

-- record syntax is still allowed, but only for one field
newtype State s a = State { runState :: s -> (s, a) }

-- this is *not* allowed:
-- newtype Pair a b = Pair { pairFst :: a, pairSnd :: b }
-- but this is:
– data Pair a b = Pair { pairFst :: a, pairSnd :: b }
-- and so is this:
newtype NPair a b = NPair (a, b)
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# The state function

The Haskell type **State** describes **functions**
that take a state
and return both a result and an updated state,
which are given back in a tuple.

The **state function** is wrapped by a data type definition
which comes along with a **runState accessor**
no need for pattern matching




**p** :: **State** s a

**one-field record**

```
newtype State s a = State { runState :: s -> (a, s) }
```

**accessor function**

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Type **State**

```
newtype State s a = State { runState :: s -> (a, s) }
```

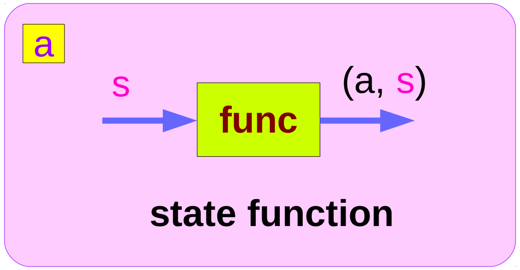
s : the type of the state,

a : the type of the produced result

s -> (a, s) : function type

**State** String,
**State** Int,
**State** SomeLargeDataStructure,
and so forth.

Calling the type **State** is arguably a bit of a misnomer
because the **wrapped value** is not the state itself
but a **state processor** (**accessor function**: **runState**)




**p** :: **State** s a

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# State Packages

Control.Monad.**Trans**.**State**, **transformers** package. (focused here)

Control.Monad.**State**, **mtl** package.

Control.Monad.**State.Lazy**, **mtl** package.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# The "**state**" function

Control.Monad.**Trans**.**State**, **transformers** package. (focused here)

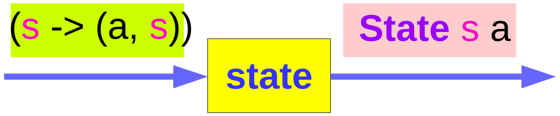
**no State constructor**

but a "**state" function**

```
state :: (s -> (a, s)) -> State s a
```

Control.Monad.**State**, **mtl** package

Implements the State in somewhat different way

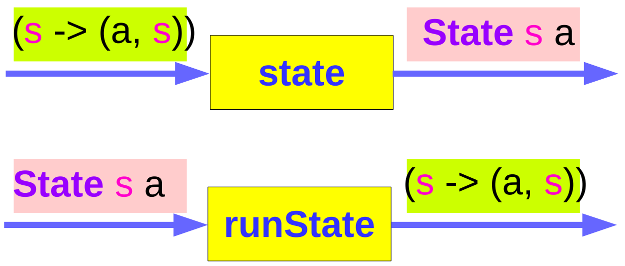



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# runState function

**State** is a record with only one element,
whose type is a function (:: s -> (a, s))

**runState** converts a value of type **State** s a
to a function of this type (:: s -> (a, s))



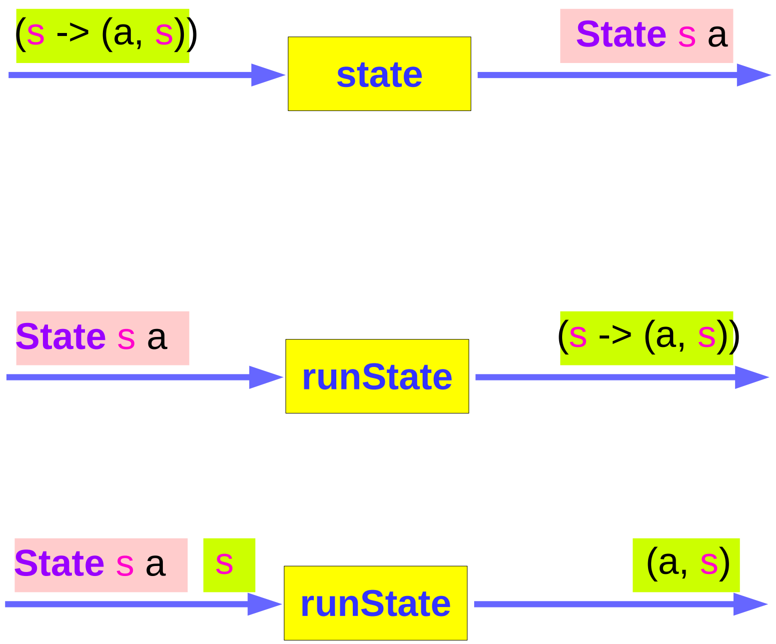

ghci> :t **runState**
**runState** :: **State** s a -> s -> (a, s)

Every time you apply **runState** to the value of type **State** s a,
the result is a function of type s -> (a, s).

**newtype State** s a = **State { runState** :: s -> (a, s) **}**

https://stackoverflow.com/questions/3240947/understanding-haskell-accessor-functions

# state & runState function




runState :: State s a -> s -> (a, s)

runState :: State s a -> s -> (a, s)

newtype State s a = State { runState :: s -> (a, s) }

https://stackoverflow.com/questions/3240947/understanding-haskell-accessor-functions

# Instantiating a State Monad

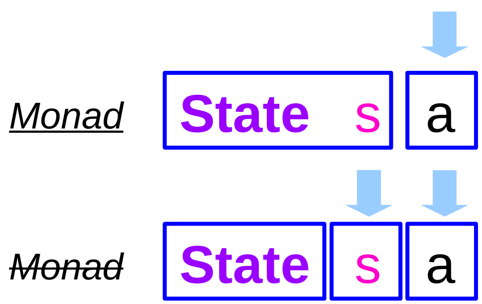
**<u>wrap</u> a function type** and give it a **<u>name</u>**.  s -> (a, s)

**State** s can be made a *<u>Monad instance</u>*, for every type s

the *<u>Monad instance</u>* is **State** s, and <u>not</u> just **State**

(**State** can't be made an instance of Monad,
as it takes <u>two</u> type parameters, rather than <u>one</u>.)

**State** String,
**State** Int,
**State** SomeLargeDataStructure,
and so forth.




```
newtype State s a = State { runState :: s -> (a, s) }

instance Monad (State s) where
    return implementation
    (>>=) implementation
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Common implementation of **return** and **>>=**

**instance** **Monad** (**State** s) where

many different **State** monads,
one for each possible type of state -

**State** String,

**State** Int,

**State** SomeLargeDataStructure,

and so forth.

one implementation of

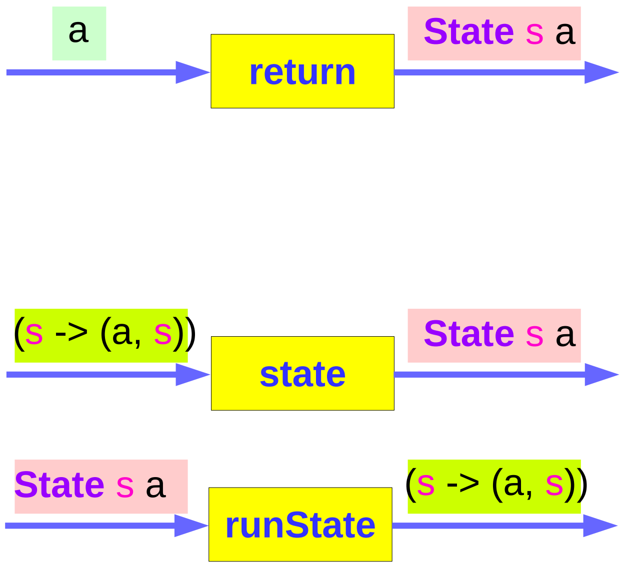
**return** and
(**>>=**);

can handle these different (**State** s) monads
according to different choices of s.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# return method

```
instance Monad (State s) where

return :: a -> State s a
return x = state ( \s -> (x, s) )        ⇒  State s a
```



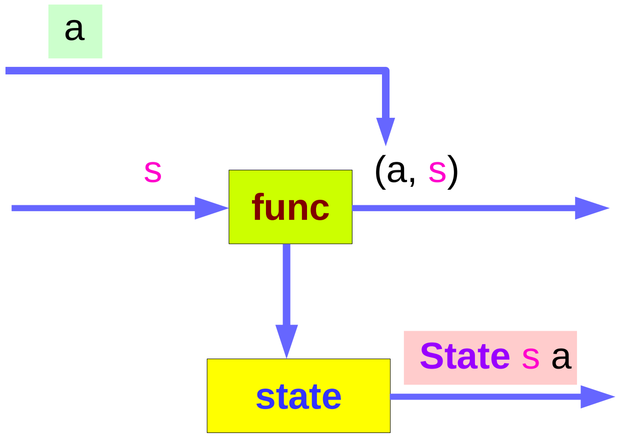

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# return method

```
instance Monad (State s) where

return :: a -> State s a
return x = state ( \s -> (x, s) )
```

⇒ State s a


giving a value (x) to **return** results in a **state processor** function which takes a state (s) and returns it unchanged (s), together with value x we want to be returned.
Finally, the function is wrapped up by **state.**




https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# State Monad Examples – return

```
runState (return 'X') 1

  ('X',1)
```

return
set the result value but leave the state unchanged.

```
return 'X' :: State Int Char
runState (return 'X') :: Int -> (Char, Int)
initial state = 1 :: Int
final value = 'X' :: Char
final state = 1 :: Int
result = ('X', 1) :: (Char, Int)
```

https://wiki.haskell.org/State_Monad

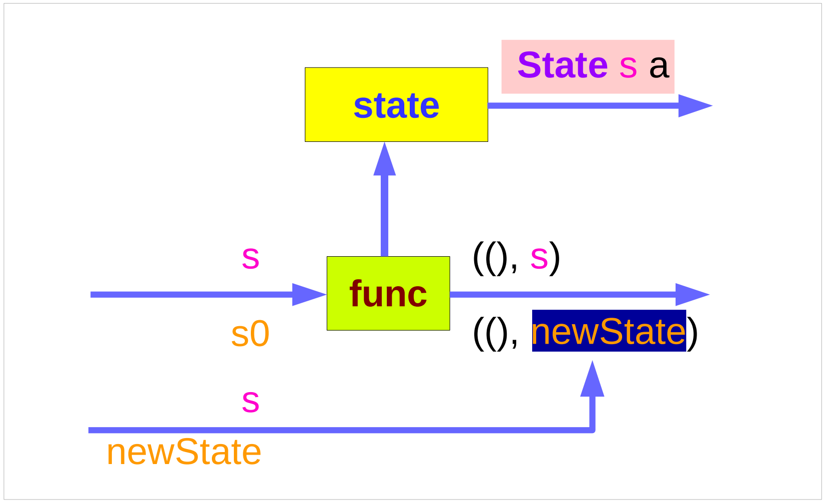
# Setting and Getting the State

**put ::** s **->** **State** s a

**put** s **::** **State** s a

**put** newState = **state** **$** _ -> ((), newState)

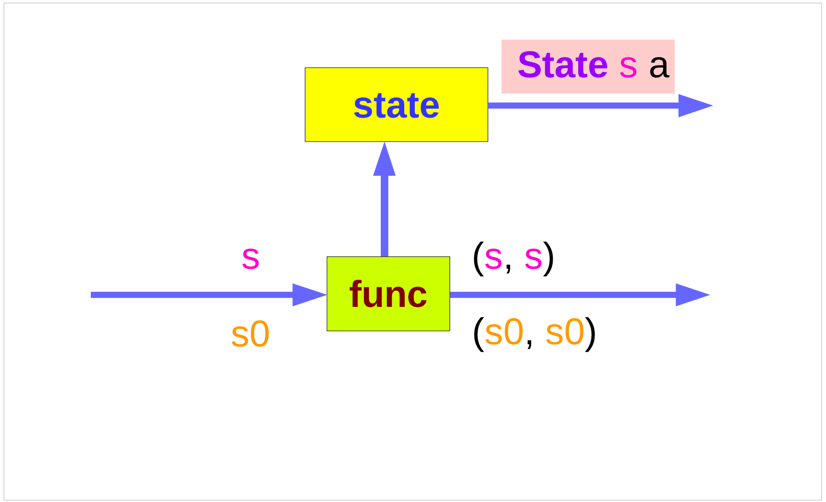
-- setting a state to newState




**get ::** **State** s s

**get** = **state** **$** \s -> (s, s)

-- getting the current state s



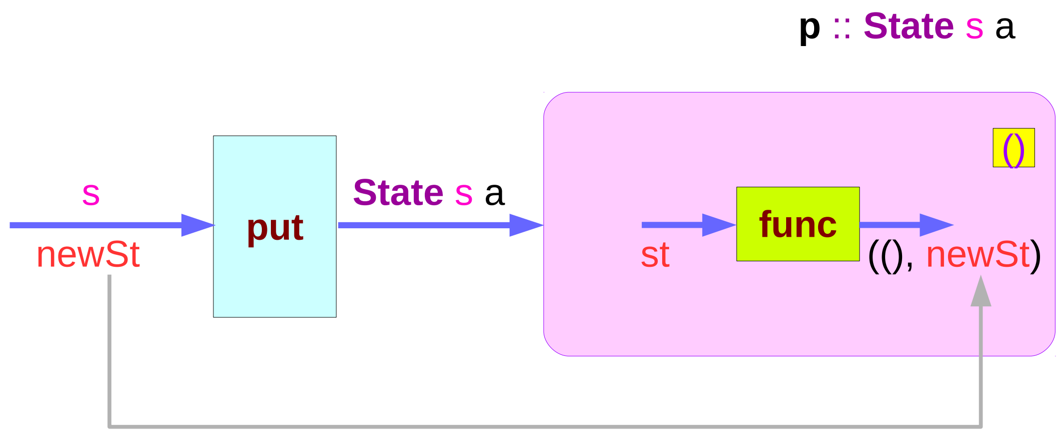

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# put and get

put :: s -> State s a

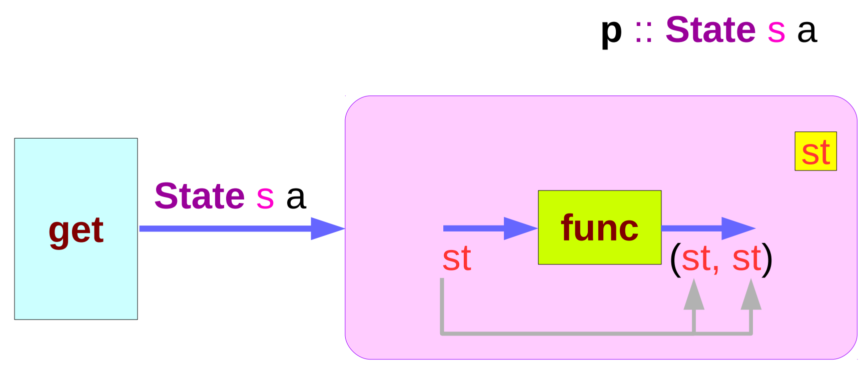
put s :: State s a

put newSt = state $ _ -> ((), newSt)




get :: State s s

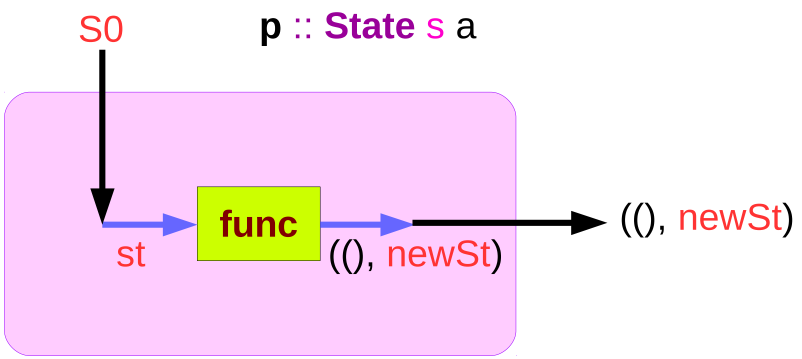
get = state $ \s -> (s, s)



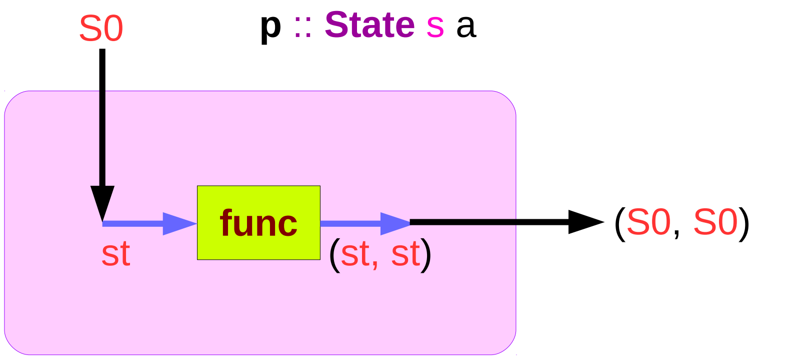

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# runState put and runState get

```
put :: s -> State s a
put newSt = state $  \_ -> ((), newSt)

runState (put newSt) S0

((), newSt)
```



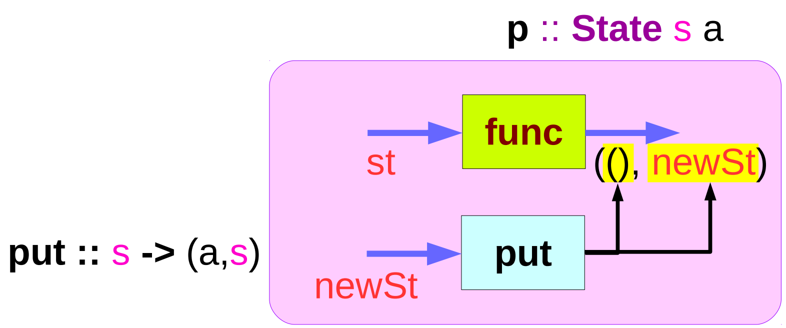

```
get :: State s s
get = state $  \s -> (s, s)

runState (get) S0

(S0, S0)
```



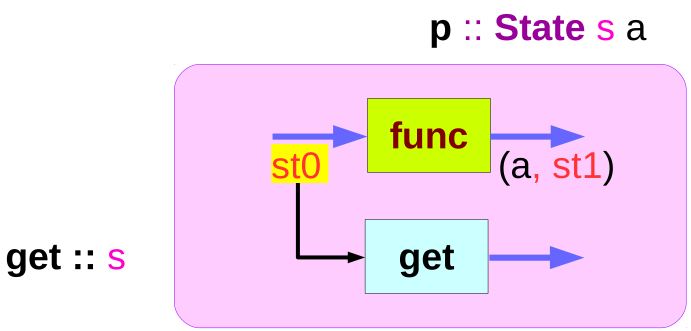

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# put and get viewed as inside functions

```
put :: s -> State s a
put newSt = state $ \_ -> ((), newSt)
```




```
get :: State s s
get = state $ \s -> (s, s)
```




https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Inside the state monad

Whenever **sc** is a **s**tateful **c**omputation
**sc** can be directly assigned to x, **inside** the state monad,

```
x <- sc
```

the result of the stateful computation **sc** is assigned to x
(like **evalState** is called with an initial state).

In order to check the current state, you can do

```
s <- get
```

and s will have the value of the current state.

https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell

# Inside Functions and runState Functions

Most monads are equipped with some "*run*" functions
such as **runState**, **execState**, and so forth.

But, frequent calling such functions inside the monad
shows that the functionality of the monad does not fully exploited

```
s0 <- get                     -- Read state
let (a,s') = runState s s0    -- Pass state to 's', get new state
put s'                        -- Save new state
```

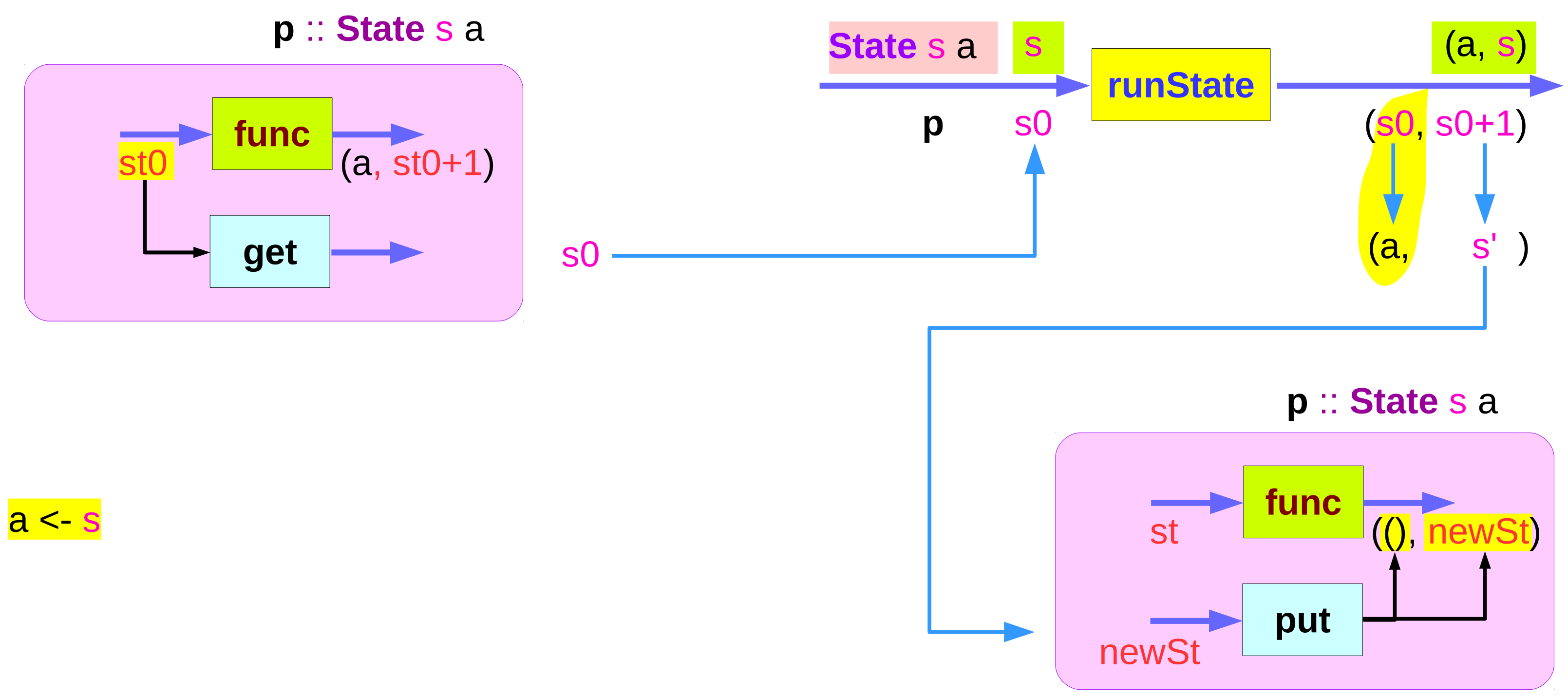

```
a <- s
```

https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell

# Redundant computation examples

```
s0 <- get
let (a,s') = runState p s0
put s'
```

-- Read state
-- Pass state to **p**, get new state
-- Save new state




https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell

# Inside function examples

```
collectUntil :: (s -> Bool) -> State s a -> State s [a]
collectUntil f s = step
  where
    step = do a <- s
              liftM (a:) continue
    continue = do s' <- get
                  if f s' then return [] else step
```

```
simpleState = state (\x -> (x,x+1))

*Main> evalState (collectUntil (>10) simpleState) 0
[0,1,2,3,4,5,6,7,8,9,10]
```

https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell

# liftM

**liftM** :: (**Monad** m) => (a -> b) -> m a -> m b
**mapM** :: (**Monad** m) => (a -> m b) -> [a] -> m [b]

**liftM** lifts a function of type a -> b to a monadic counterpart.
**mapM** applies a function which yields a monadic value to a list of values, yielding list of results embedded in the monad.

```
> liftM (map toUpper) getLine
Hallo
"HALLO"

> :t mapM return "monad"
mapM return "monad" :: (Monad m) => m [Char]
```

https://stackoverflow.com/questions/5856709/what-is-the-difference-between-liftm-and-mapm-in-haskell

# mapM

```
> :t mapM return "monad"
mapM return "monad" :: (Monad m) => m [Char]
```

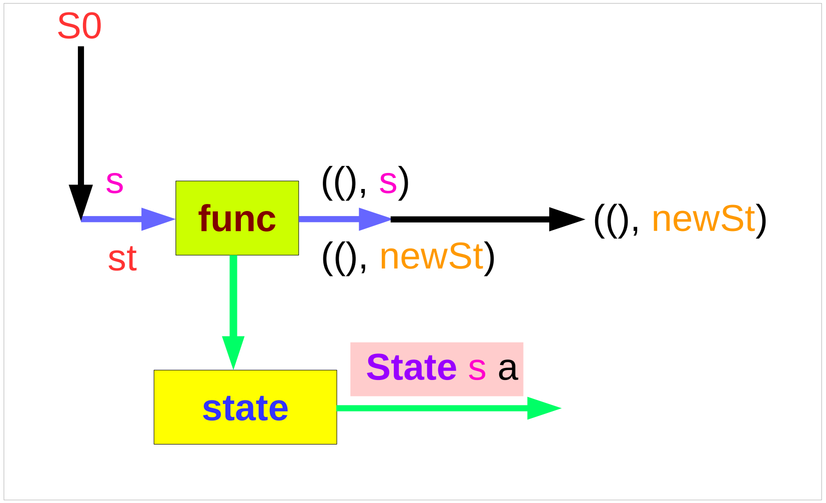
```
> map (x -> [x+1]) [1,2,3]
[[2],[3],[4]]
```

```
> mapM (x -> [x+1]) [1,2,3]
[[2,3,4]]
```

https://stackoverflow.com/questions/5856709/what-is-the-difference-between-liftm-and-mapm-in-haskell

# Setting the State

**put ::** s **->** **State** s a
**put** newSt = **state** **$** _ -> ((), newSt)




Given a wanted state newState,
**put** generates a **state processor**
which ignores whatever the state it receives,
and gives back the state we originally provided to put.
the same state

Since we don't care about the result (a) of this processor
(all we want to do is to change the state),
the first element of the tuple will be (),
the **universal placeholder value**.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

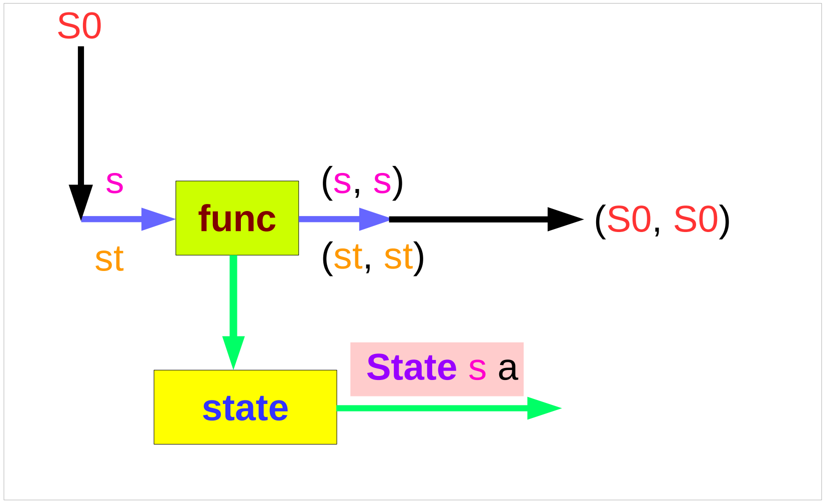
# Getting the State

**get ::** **State** s s

**get** = **state** **$** \s -> (s, s)

The resulting **state processor** gives back the state st
it is given in both as a result and as a state.

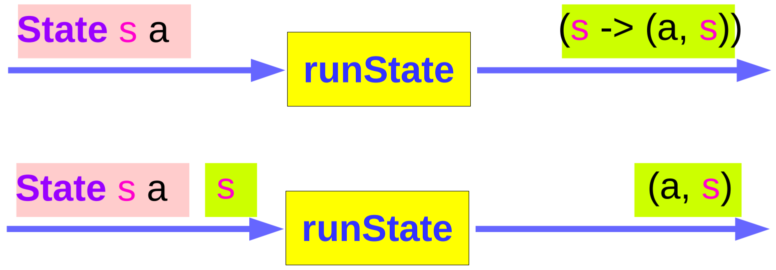
That means the state will remain unchanged,
and that a copy of it will be made available
for us to manipulate.




https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

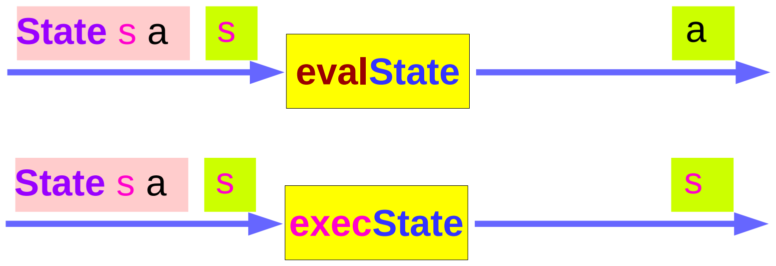
# **evalState** and **execState**

**runState**
unwrap the **State** s a value
to get the actual **state processing function**
which is then applied to some initial state.




Given a **State** s a and an initial state s,
**evalState** only the result value
**execState** just the new state.

```
evalState :: State s a -> s -> a
evalState p s = fst (runState p s)

execState :: State s a -> s -> s
execState p s = snd (runState p s)
```




https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# State Monad Examples – **get**

```
runState get 1

  (1,1)
```

get
set the result value to the state and leave the state unchanged.

Comments:

**get** :: **State** Int Int
**runState get** :: Int -> (Int, Int)
initial state = 1 :: Int
final value = 1 :: Int
final state = 1 :: Int

**get :: State** s s
**get** = **state $** \s -> (s, s)

https://wiki.haskell.org/State_Monad

# State Monad Examples – **put**

```
runState (put 5) 1

  ((),5)
```

put
set the result value to () and set the state value.

Comments:

```
put 5 :: State Int ()
runState (put 5) :: Int -> ((),Int)
initial state = 1 :: Int
final value = () :: ()
final state = 5 :: Int
```

```
put :: s -> State s a
put newState = state $ \_ -> ((), newState)
```

https://wiki.haskell.org/State_Monad

# Put and get in mtl packages

Return leaves the state unchanged and sets the result:
-- ie: (**return** 5)   1 -> (5,1)

**return** :: a -> **State** s a
**return** x s = (x,s)

Get leaves state unchanged and sets the result to the state:
-- ie:  **get**  1 -> (1,1)

**get** :: **State** s s
**get** s = (s,s)

Put sets the result to () and sets the state:
-- ie: (**put** 5)   1 -> ((),5)

**put** :: s -> **State** s ()
**put** x s = ((),x)

https://wiki.haskell.org/State_Monad

# Unwrapped Implementation Examples (1)

Return leaves the state unchanged and sets the result:
-- ie: (**return** 5)   1 -> (5,1)

**return** :: a -> **State** s a
**return** x s = (x,s)

Get leaves state unchanged and sets the result to the state:
-- ie:  **get**   1 -> (1,1)

**get** :: **State** s s
**get** s = (s,s)

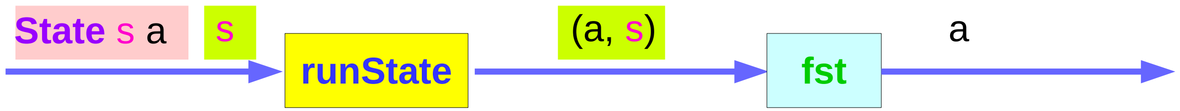
Put sets the result to () and sets the state:
-- ie: (**put** 5)   1 -> ((),5)

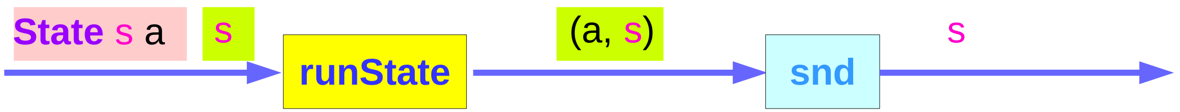
**put** :: s -> **State** s ()
**put** x s = ((),x)

https://wiki.haskell.org/State_Monad

# Unwrapped Implementation Examples (1)

```
evalState :: State s a -> s -> a
evalState act = fst . runState act

execState :: State s a -> s -> s
execState act = snd . runState act
```




https://wiki.haskell.org/State_Monad

# Unwrapped Implementation Examples (2)

```
modify :: (s -> s) -> State s ()
modify f = do { x <- get; put (f x) }

gets :: (s -> a) -> State s a
gets f = do { x <- get; return (f x) }
```

```
runState (modify (+1)) 1
   ((),2)

runState (gets (+1)) 1
   (2,1)
```

```
evalState (gets (+1)) 1
   2

execState (gets (+1)) 1
   1
```

**get** & **put** :
functions inside the State monad

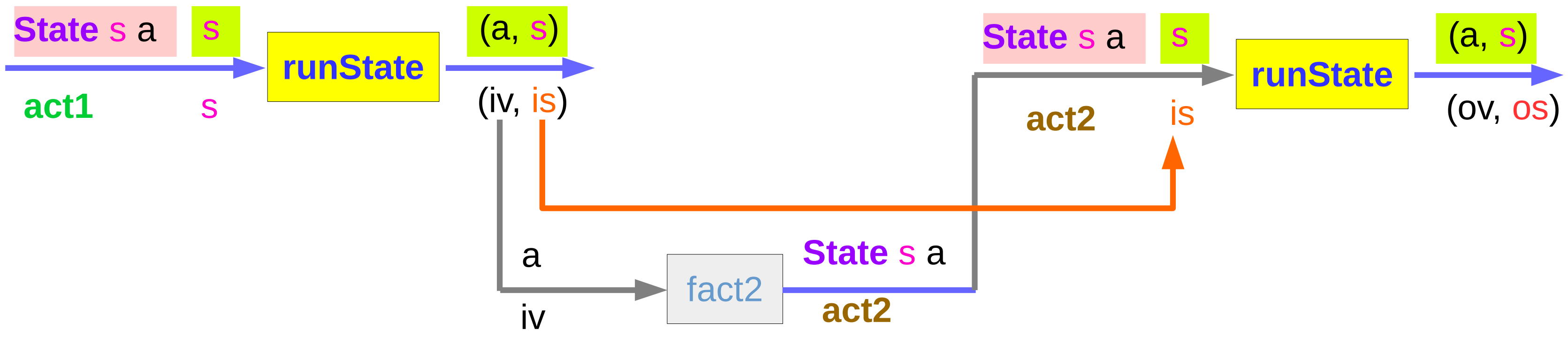
**get** :: s

**put** :: s -> (a, s)

https://wiki.haskell.org/State_Monad

# Unwrapped Implementation Examples (3)

```
(>>=) :: State s a -> (a -> State s b) -> State s b
(act1 >>= fact2) s = runState act2 is
   where (iv, is) = runState act1 s
         act2 = fact2 iv
```




https://wiki.haskell.org/State_Monad

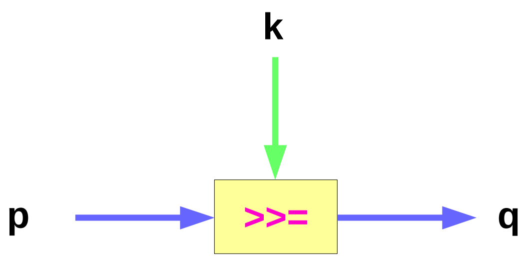
# Function type of >>=

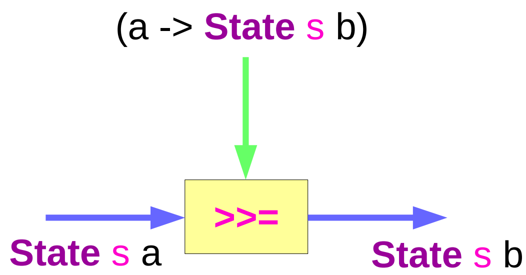
```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
```

p :: State s a

k :: (a -> State s b)

| State s a | -> | (a -> State s b) | -> | State s b |
|---|---|---|---|---|
| p | | k | | p >>= k |

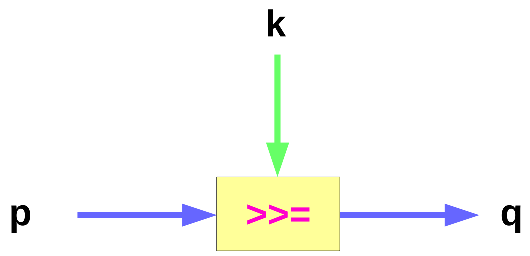


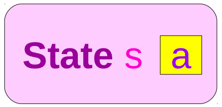

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# 1st and 2nd arguments of >>= :

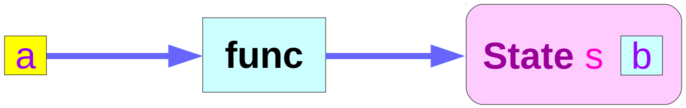
```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
```




p :: State s a




k :: (a -> State s b)




https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

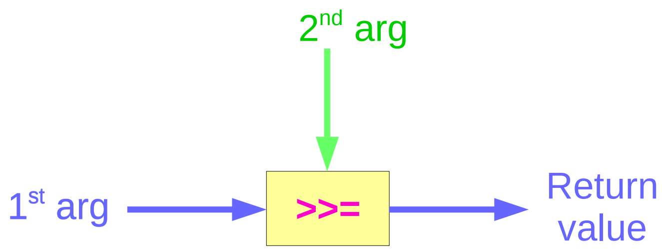
# Binding operator >>=

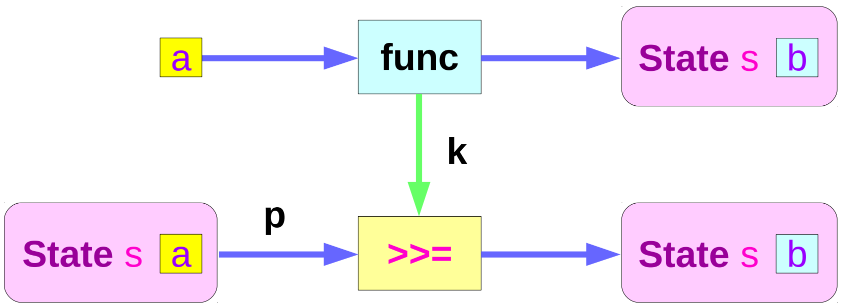
```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
```

p :: State s a — State Monad value

k :: (a -> State s b) — State Monad returning function

p >>= k = q

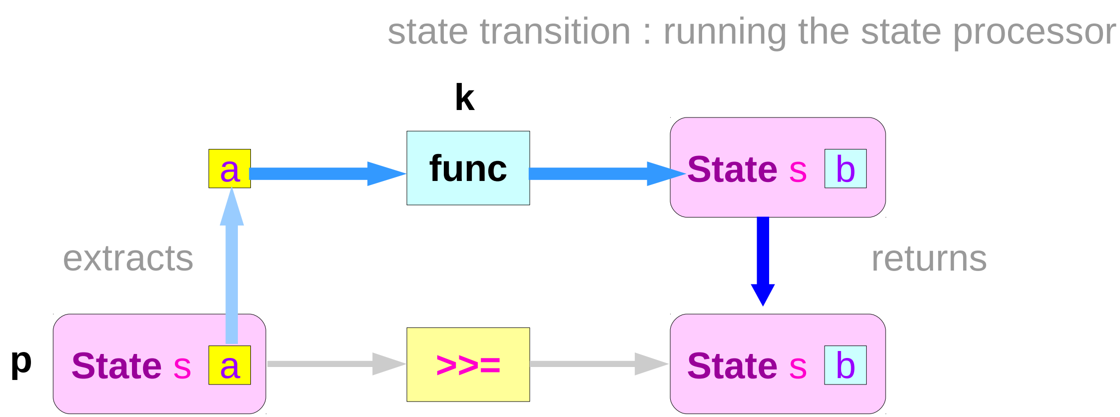



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Conceptual computation flow of >>=

```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
```




https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Three Orthogonal Functions

Thinking of extraction : a slightly misleading intuition.

Nothing is being "extracted" from a monad.

The more *fundamental* definition of a monad
can be stated by three orthogonal functions:

**fmap** :: (a -> b) -> (m a -> m b)
**return** :: a -> m a
**join** :: m (m a) -> m a

m is a monad.

https://stackoverflow.com/questions/15016339/haskell-computation-in-a-monad-meaning

# Three Orthogonal Functions and >>=

**fmap** :: (a -> b) -> (m a -> m b)
**return** :: a -> m a
**join** :: m (m a) -> m a

(a -> b) -> (m a -> m b)
(a -> m b) -> (m a -> m (m b))
(a -> m b) -> (m a -> m b)

how to implement (>>=) with these:

starting with arguments of type m a and a -> m b,

your only option is using **fmap** to get something of type m (m b),

(a -> m b) -> (m a -> m (m b))

after which you can use **join** to *flatten* the nested "layers" to get just m b.

(a -> m b) -> (m a ->m b)

https://stackoverflow.com/questions/15016339/haskell-computation-in-a-monad-meaning

# Monad Law

(a -> b) -> (m a -> m b)
(a -> m b) -> (m a -> m (m b))
(a -> m b) -> (m a -> m b)

**join** :: m (m a) -> m a
nothing is being taken "out" of the monad
as the computation going *deeper* into the monad,
with successive steps being *collapsed* into a single layer of the monad.

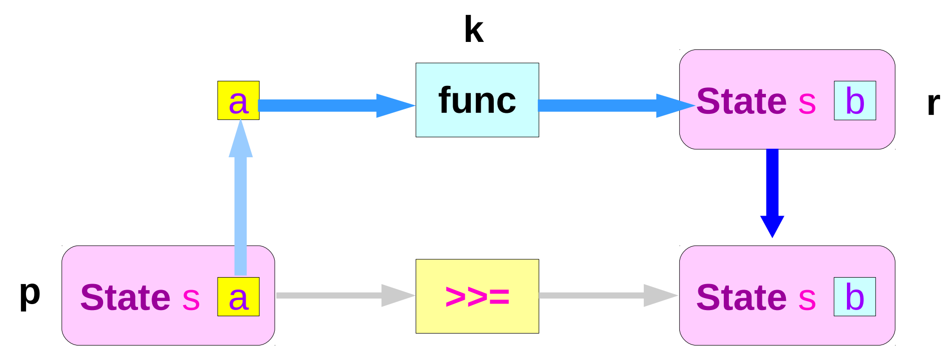
when **join** (m (m a) -> m a) is applied, it doesn't matter
as long as *the nesting order is preserved* (a form of *associativity*) and
that the *monadic* *layer* introduced by **return** does *nothing* (an *identity* value for **join**).

| | | |
|---|---|---|
| Left identity | **return** a **>>=** f | f a |
| Right identity | m **>>= return** | m |
| Associativity | (m **>>=** f) **>>=** g | m **>>=** (\x -> f x **>>** g) |

https://stackoverflow.com/questions/15016339/haskell-computation-in-a-monad-meaning

# Applying the state function to **p** and **r**

```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
    p' = runState p          -- p' :: s -> (a, s)
    k' = runState . k        -- k' :: a -> s -> (b, s)
```




```
newtype State s a = State { runState :: s -> (a, s) }

state :: (s -> (a, s)) -> State s a
```

p' :: s -> (a, s)    p' = runState p    State s a    p

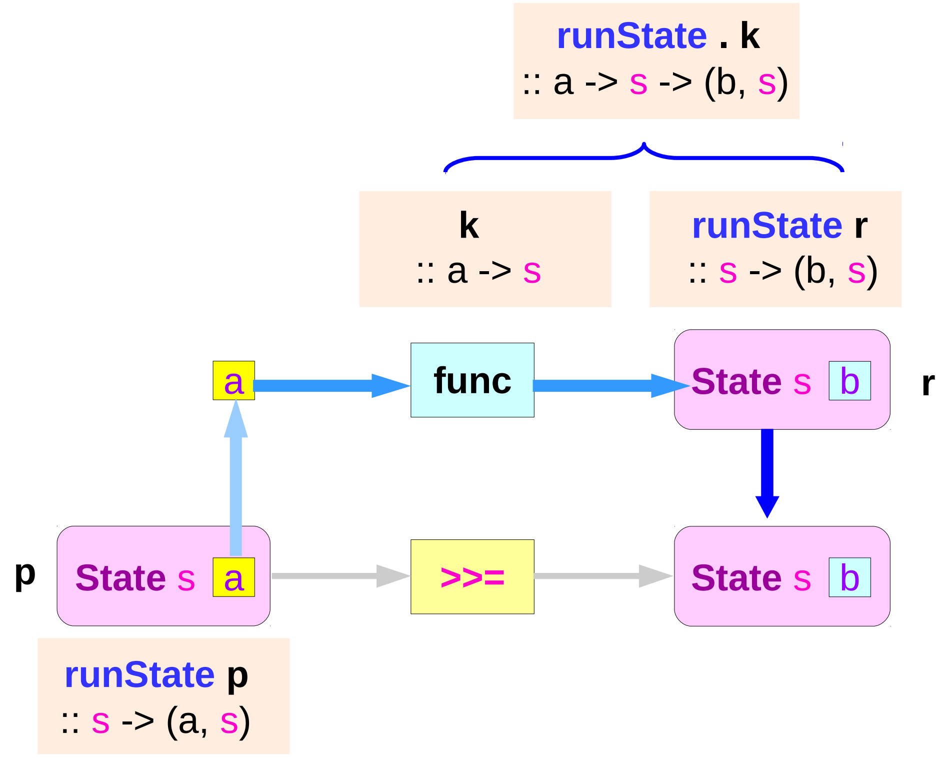
r' :: s -> (b, s)    r' = runState r    State s b    r

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Applying **k** and the state function

```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
    p' = runState p            -- p' :: s -> (a, s)
    k' = runState . k          -- k' :: a -> s -> (b, s)
```

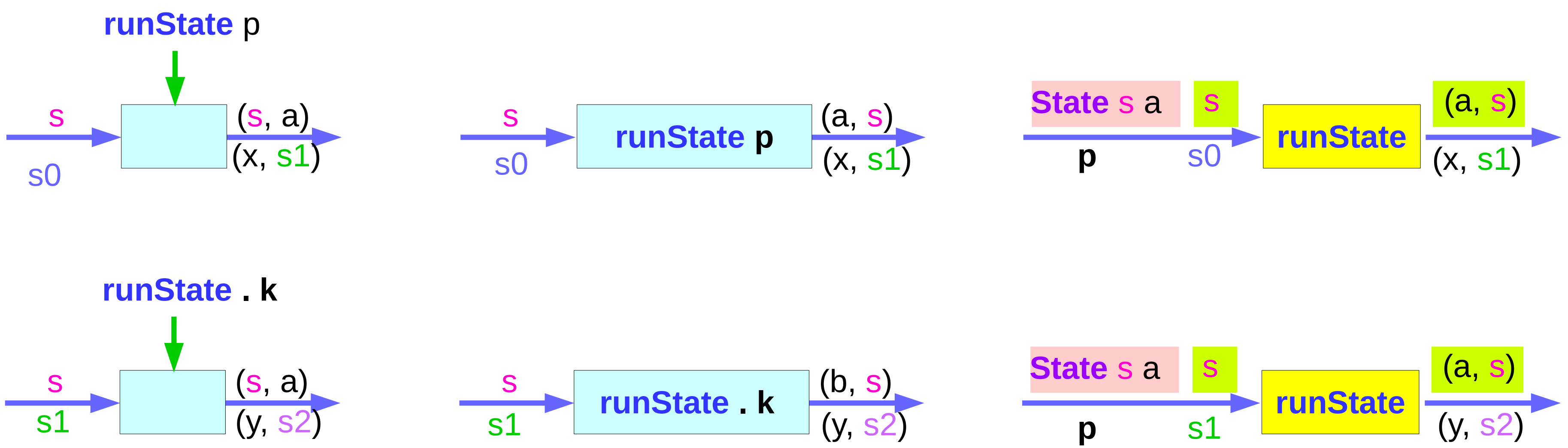
```
newtype State s a = State { runState :: s -> (a, s) }

state :: (s -> (a, s)) -> State s a
```




https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Running the state processor

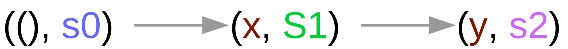
```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
   p' = runState p            -- p' :: s -> (a, s)
   k' = runState . k          -- k' :: a -> s -> (b, s)
```

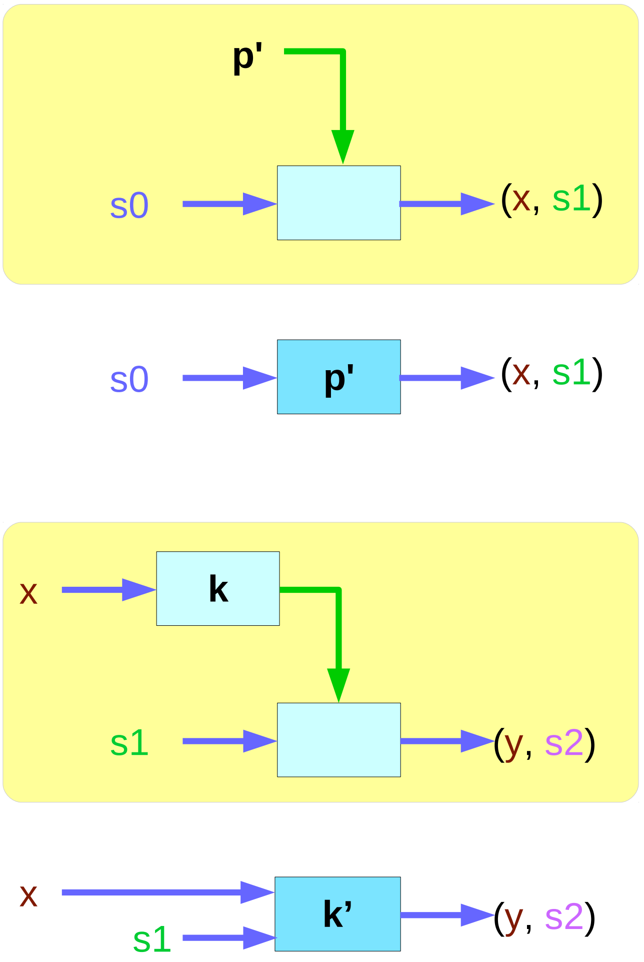



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# State Transition

```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
    p' = runState p             -- p' :: s -> (a, s)
    k' = runState . k           -- k' :: a -> s -> (b, s)
    q' s0 = (y, s2) where       -- q' :: s -> (b, s)
        (x, s1) = p' s0         -- (x, s1) :: (a, s)
        (y, s2) = k' x s1       -- (y, s2) :: (b, s)
    q = state q'
```

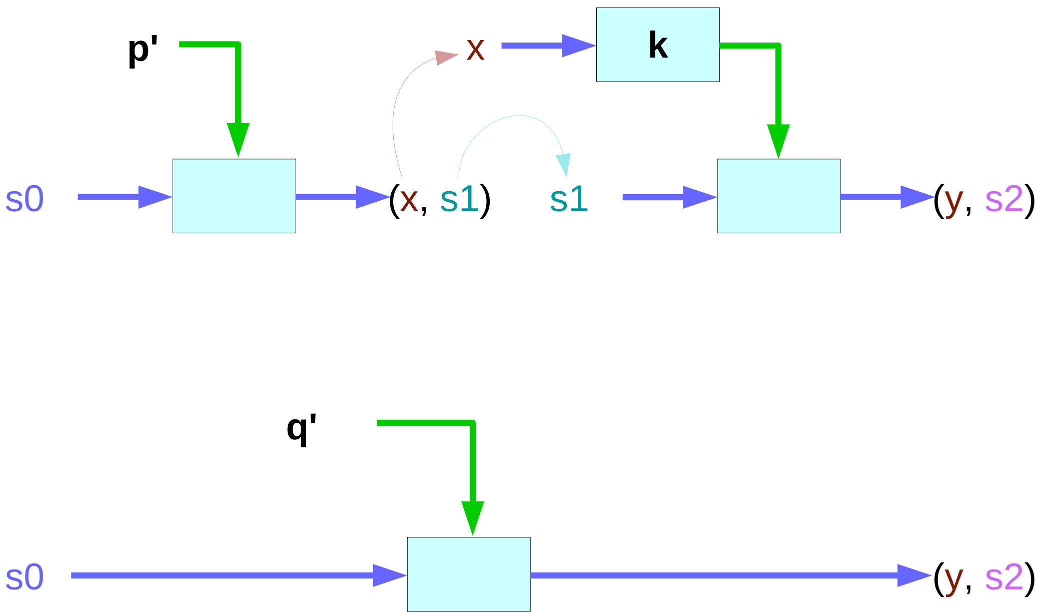



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# State Transition from s0 to s2

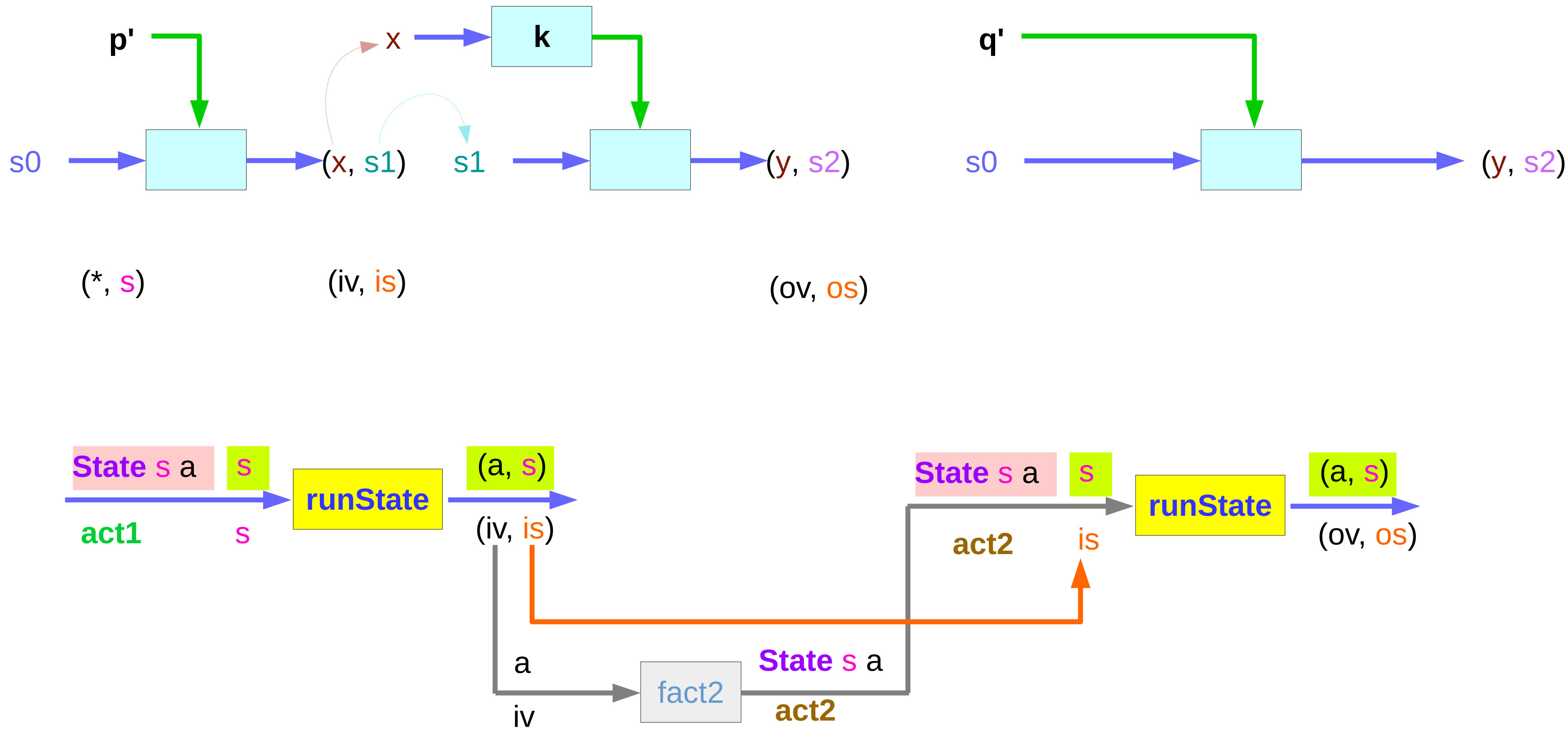
```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
    p' = runState p              -- p' :: s -> (a, s)
    k' = runState . k            -- k' :: a -> s -> (b, s)
    q' s0 = (y, s2) where        -- q' :: s -> (b, s)
        (x, s1) = p' s0          -- (x, s1) :: (a, s)
        (y, s2) = k' x s1        -- (y, s2) :: (b, s)
    q = state q'
```




```
state :: (s -> (a, s)) -> State s a

newtype State s a = State { runState :: s -> (s, a) }
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

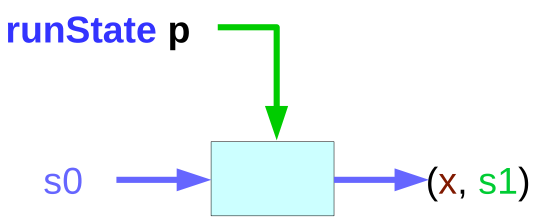
# State Transition from s0 to s2



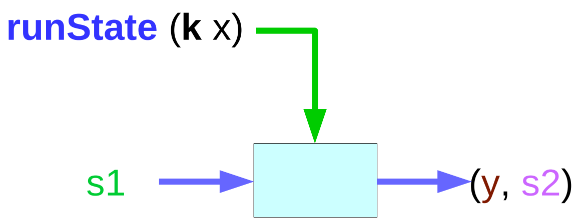

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

# Another implementation of >>=

```
instance Monad (State s) where

(>>=) :: State s a -> (a -> State s b) -> State s b
p  >>= k = state $ \ s0 ->
   let (x, s1) = runState p s0
   in runState (k x) s1
```




-- running the first processor on s0.




-- running the second processor on s1.

```
state ( \ s0 ->   (y, s2) )
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

## References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf