

Moments

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

1 Definitions of Moments

Moments about the origin

Definition

the moments about the origin of the random variable X

$$m_n = E[X^n] = \int_{-\infty}^{+\infty} x^n f_X(x) dx$$

$$g(X) = X^n \quad n = 0, 1, 2, \dots$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x) dx$$

Central Moments

Definitions

the moments about the mean value of X

$$\mu_n = E[(X - \bar{X})^n] = \int_{-\infty}^{+\infty} (x - \bar{X})^n f_X(x) dx$$

$$g(X) = (X - \bar{X})^n \quad n = 0, 1, 2, \dots$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x) dx$$

Variance

Definitions

the second central moments μ_2

$$\sigma_{X^2} = \mu_2 = E[(X - \bar{X})^2] = \int_{-\infty}^{+\infty} (x - \bar{X})^2 f_X(x) dx$$

the standard deviation σ_X

Computing σ_X^2

Definitions

the second central moments μ_2

$$\begin{aligned}\sigma_X^2 &= E[X^2 - 2\bar{X}X + \bar{X}^2] = E[X^2] - \bar{X}E[X] + \bar{X}^2 \\ &= E[X^2] - \bar{X}^2 = m_2 - m_1^2\end{aligned}$$

Skew

Definition

the 3rd momen

$$\mu_3 = E[(X - \bar{X})^3]$$

the measure of the asymmetry of $f_X(x)$ about $x = \bar{X} = m_1$

the skew of the density function

if a density is symmetric about $x = \bar{X}$

it has a zero skew and

$\mu_n = 0$ for all odd value of n

Coefficients of skewness

Definition

the 3rd moment $\mu_3 = E[(X - \bar{X})^3]$

the measure of the asymmetry of $f_X(x)$ about $x = \bar{X} = m_1$

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