Moments

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Moments about the origin

Definition

the moments about the origin of the random varialbe X

$$m_n = E[X^n] = \int_{-\infty}^{+\infty} x^n f_X(x) dx$$

$$g(X) = X^n \qquad n = 0, 1, 2, \cdots$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x) dx$$

Central Moments

Definitions

the moments about the mean value of X

$$\mu_n = E[(X - \overline{X})^n] = \int_{-\infty}^{+\infty} (x - \overline{X})^n f_X(x) dx$$

$$g(X) = (X - \overline{X})^n \qquad n = 0, 1, 2, \cdots$$

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x) f_X(x) dx$$

Variance

Definitions

the second central moments μ_2

$$\sigma_{x^2} = \mu_2 = E[(X - \overline{X})^2] = \int_{-\infty}^{+\infty} (x - \overline{X})^2 f_X(x) dx$$

the standard deviation σ_X

Computing σ_X^2

Definitions

the second central moments μ_2

$$\sigma_X^2 = E[X^2 - 2\overline{X}X + X^2] = E[X^2] - \overline{X}E[X] + \overline{X}^2$$

$$= E[X^2] - \overline{X}^2 = m_2 - m_1^2$$

Skew

Definition

the 3rd momen

$$\mu_3 = E[(X - \overline{X})^3]$$

the measure of the asymmetry of $f_X(x)$ about $x = \overline{X} = m_1$
the skew of the density function
if a density is symmetric aobut $x = \overline{X}$
it has a zero skew and
$\mu_n = 0$ for all odd value of n

Coefficients of skewness

Definition

the 3rd moment $\mu_3 = E[(X - \overline{X})^3]$
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