Background (1A)

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CORDIC Background

1. CORDIC FAQ, G. R. Griffin, www.dspguru.com/info/faqs/cordic2.htm

Complex Multiplication

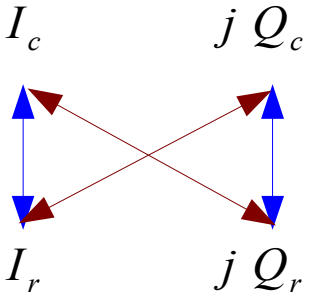
Given Complex Value $C = I_c + j\,Q_c$

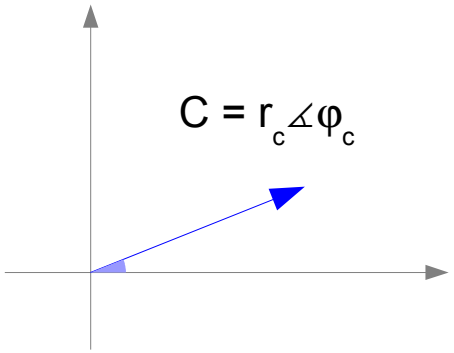
Rotation Value $R = I_r + j\,Q_r$

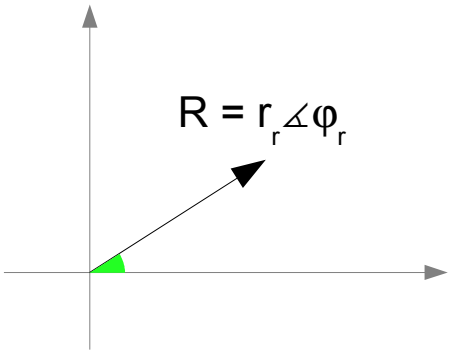
Rotated Complex Value $C' = I_c' + j\,Q_c'$

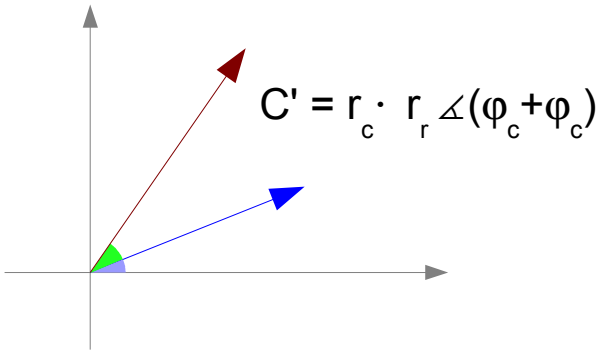
$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot (I_r + j\,Q_r) \\ &= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r) \end{aligned}$$



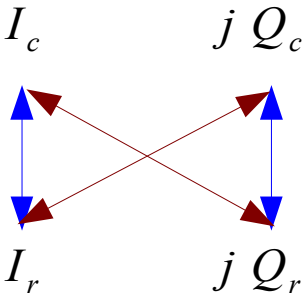






Adding / Subtracting Phase

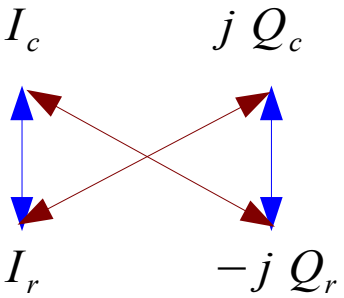
To **add** R' phase to C



$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot (I_r + j\,Q_r) \\ &= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r) \end{aligned}$$

To **sub** R' phase to C

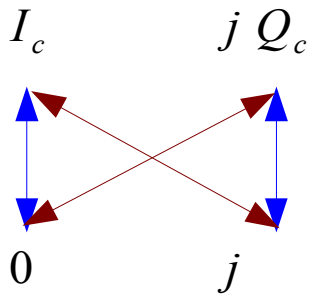


$$C' = C \cdot R^*$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot (I_r - j\,Q_r) \\ &= (I_c I_r + Q_c Q_r) + j\,(Q_c I_r - I_c Q_r) \end{aligned}$$

Adding / Subtracting 90 Degrees

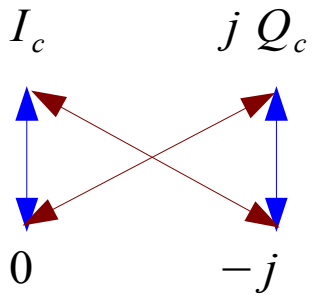
To add R' phase to C



$$C' = C \cdot (+j)$$

$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 + j)$$
$$= (-Q_c) + j\,(I_c)$$

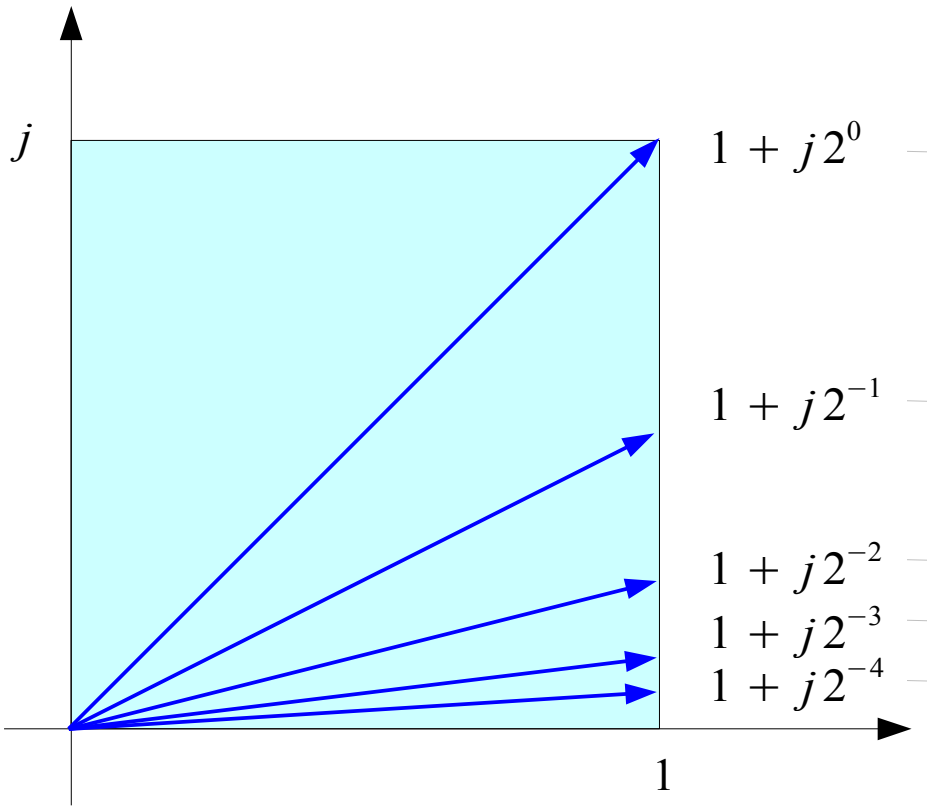
To sub R' phase to C



$$C' = C \cdot (-j)$$

$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 - j)$$
$$= (Q_c) + j\,(-I_c)$$

Elementary Angle: $\tan^{-1}(K)$



$$\theta_L = \tan^{-1}(2^{-L}) = \tan^{-1}(K)$$

$$\theta_0 = \tan^{-1}(2^0) = 45.00000$$

$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505$$

$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624$$

$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502$$

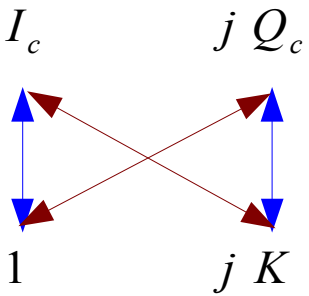
$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633$$

Represent arbitrary angle θ

in terms of $\pm\theta_0, \pm\theta_1, \pm\theta_2, \pm\theta_3, \cdots, \pm\theta_L, \cdots$ $\left(K = \frac{1}{2^L}, \quad L = 0,1,2,\cdots\right)$

Adding / Subtracting $\tan^{-1}(K)$

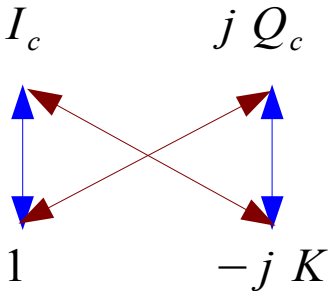
To add R' phase to C



$$C' = C \cdot (1 + jK)$$

$$\begin{aligned} I_c' + jQ_c' &= (I_c + jQ_c) \cdot \boxed{(1 + jK)} \\ &= (I_c - KQ_c) + j(Q_c + KI_c) \\ &= (I_c - 2^{-L}Q_c) + j(Q_c + 2^{-L}I_c) \end{aligned}$$

To sub R' phase to C



$$C' = C \cdot (1 - jK)$$

$$\begin{aligned} I_c' + jQ_c' &= (I_c + jQ_c) \cdot \boxed{(1 - jK)} \\ &= (I_c + KQ_c) + j(Q_c - KI_c) \\ &= (I_c + 2^{-L}Q_c) + j(Q_c - 2^{-L}I_c) \end{aligned}$$

$$\theta_L = \tan^{-1}(K) = \tan^{-1}(2^{-L})$$

$$K = \frac{1}{2^L}, \quad L = 0,1,2,\cdots$$

Phase and Magnitude of 1 + jK (1)

Cumulative Magnitude

L	$K = \frac{1}{2^L}$	$R = 1 + jK$	*Phase of R*	*Magnitude of R*	*CORDIC Gain*
0	1.0	1 + j1.0	45°	1.41421356	1.414213562
1	0.5	1 + j0.5	26.56505°	1.11803399	1.581138830
2	0.25	1 + j0.25	14.03624°	1.03077641	1.629800601
3	0.125	1 + j0.125	7.12502°	1.00778222	1.642484066
4	0.0625	1 + j0.0625	3.57633°	1.00195122	1.645688916
5	0.03125	1 + j0.03125	1.78991°	1.00048816	1.646492279
6	0.015625	1 + j0.015625	0.89517°	1.00012206	1.646693254
7	0.007813	1 + j0.007813	0.44761°	1.00003052	1.646743507
...	...	...	...	...	...
					1.647

$$R = 1 + jK \xrightarrow[L = 0,1,2,\cdots]{K = 1/2^L} \sqrt{1^2 + K^2} \quad > 1.0$$

Phase and Magnitude of 1 + jK (2)

Represent arbitrary angle θ

in terms of $\pm\theta_{0,}\ \pm\theta_{1,}\ \pm\theta_{2,}\ \pm\theta_{3,}\ \cdots,\ \pm\theta_{L},\ \cdots$ $\left(K = \frac{1}{2^L},\quad L = 0,1,2,\cdots\right)$

Binary Search ➡ Shift and Add ➡ No multiplier

$$\begin{aligned}
\theta_0 &= \tan^{-1}(2^0) = \\
\theta_1 &= \tan^{-1}(2^{-1}) = \\
\theta_2 &= \tan^{-1}(2^{-2}) = \\
\theta_3 &= \tan^{-1}(2^{-3}) = \\
\theta_4 &= \tan^{-1}(2^{-4}) = \\
\theta_5 &= \tan^{-1}(2^{-5}) = \\
\theta_6 &= \tan^{-1}(2^{-6}) = \\
\theta_7 &= \tan^{-1}(2^{-7}) =
\end{aligned}$$

Phase of R

45°
26.56505°
14.03624°
7.12502°
3.57633°
1.78991°
0.89517°
0.44761°
...

$\approx \frac{1}{2}$ $\approx \frac{1}{2}$ $\approx \frac{1}{2}$ $\approx \frac{1}{2}$ $\approx \frac{1}{2}$ $\approx \frac{1}{2}$ $\approx \frac{1}{2}$

> 92°

[-180, +180]

$R = 0 \pm j$

[-90, +90]

4 or more rotations

$R = 1 \pm jK$

Phase and Magnitude of 1 + jK (3)

$R = 1 \pm jK$	*Magnitude of R*	$\sqrt{1^2 + K^2}$ > 1.0	*Cumulative Magnitude* *CORDIC Gain*
$R_0 = 1 \pm j\,(1/2^0)$	$\sqrt{1^2 + 1^2}$ =	1.41421356	1.414213562
$R_1 = 1 \pm j\,(1/2^1)$	$\sqrt{1^2 + (1/2)^2}$ =	1.11803399	1.581138830
$R_2 = 1 \pm j\,(1/2^2)$	$\sqrt{1^2 + (1/2^2)^2}$ =	1.03077641	1.629800601
$R_3 = 1 \pm j\,(1/2^3)$	$\sqrt{1^2 + (1/2^3)^2}$ =	1.00778222	1.642484066
$R_4 = 1 \pm j\,(1/2^4)$	$\sqrt{1^2 + (1/2^4)^2}$ =	1.00195122	1.645688916
$R_5 = 1 \pm j\,(1/2^5)$	$\sqrt{1^2 + (1/2^5)^2}$ =	1.00048816	1.646492279
$R_6 = 1 \pm j\,(1/2^6)$	$\sqrt{1^2 + (1/2^6)^2}$ =	1.00012206	1.646693254
$R_7 = 1 \pm j\,(1/2^7)$	$\sqrt{1^2 + (1/2^7)^2}$ =	1.00003052	1.646743507
		...	...

Iteration

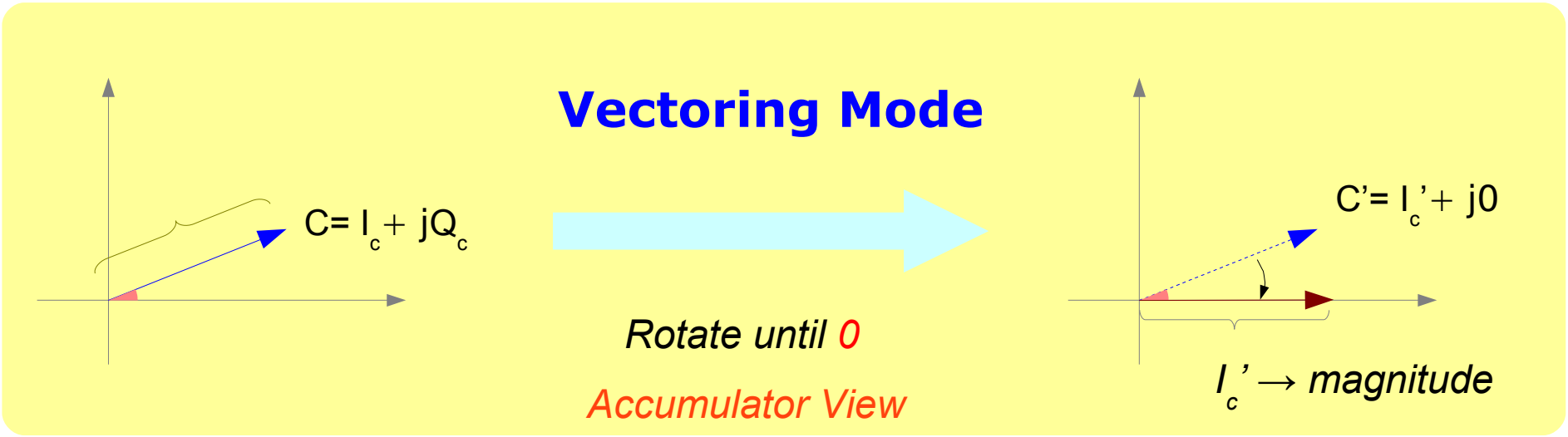
1.647

The actual CORDIC Gain depends on the number of iterations

For each iteration, the corresponding θ_i *is always used (either* $+\theta_i$ *or* $-\theta_i$ *)*

The magnitude is growing → for correction multiply by 0.6073 = 1 / 1.647

Calculating Magnitude (1)



Each iteration, the magnitude is increased by $\sqrt{1^2 + K^2}$

CORDIC Gain (cumulative gain)

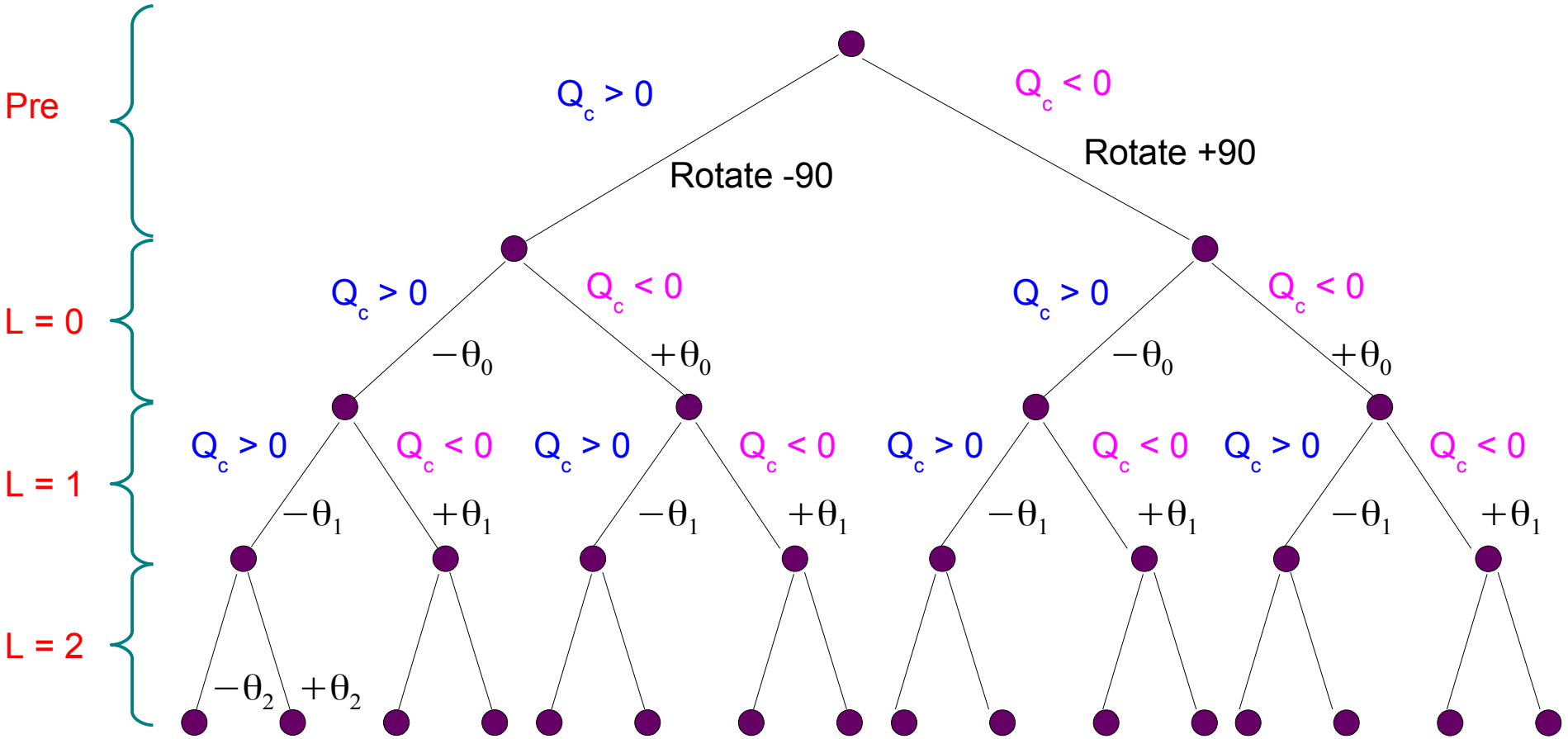
$$\simeq 1.647 \;=\; 0.607^{-1}$$

To compensate, multiply by 0.607

Can't perform this gain adjustment by simple shift and add

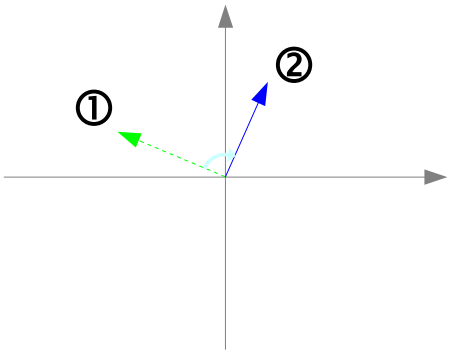
- *This adjustment can be done in other part of a system*
- *When relative magnitude is enough*

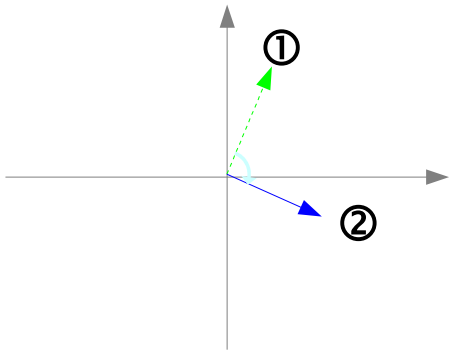
Calculating Magnitude (2)

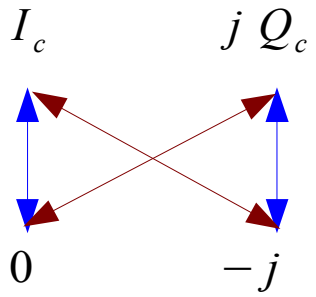


Multiplication of ±j

Positive Phase ($Q_c > 0$) ➡ *Rotate by –90 degrees*

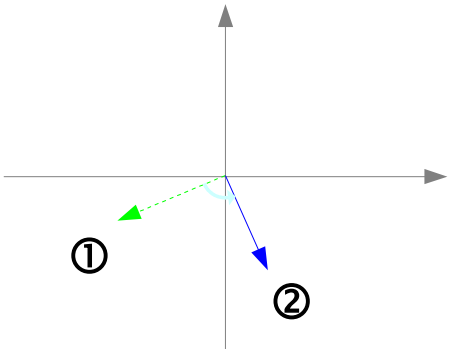


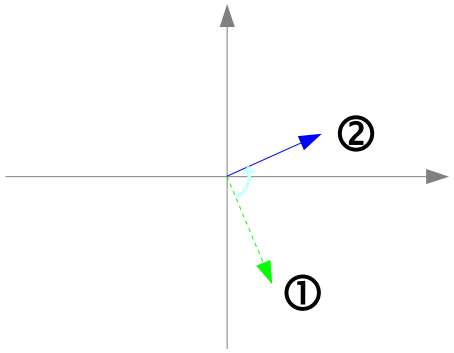


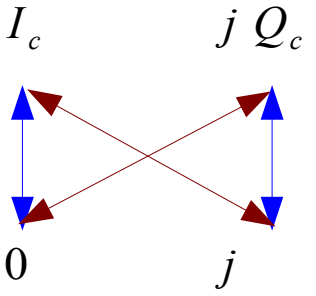


$$(Q_c) + j\,(-I_c)$$

Negative Phase ($Q_c < 0$) ➡ *Rotate by +90 degrees*





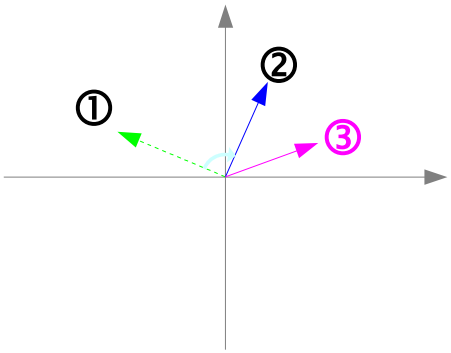


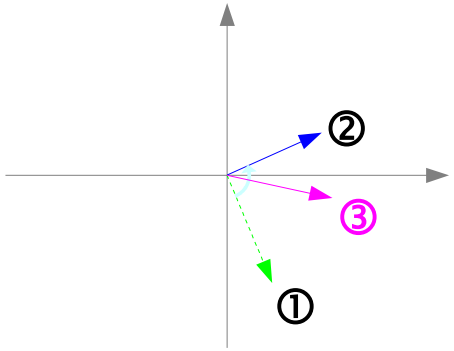
$$(-Q_c) + j\,(I_c)$$

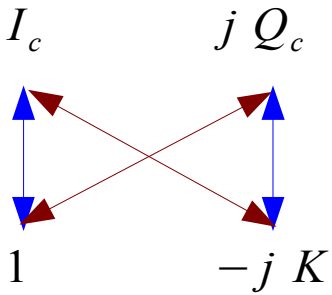
Resulting Phase ➡ [-90, +90]

Multiplication of 1 ± jK

Positive Phase ($Q_c > 0$) ➡ *Rotate by* $1 - j\,K$

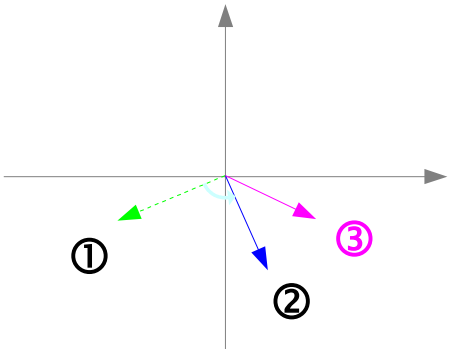


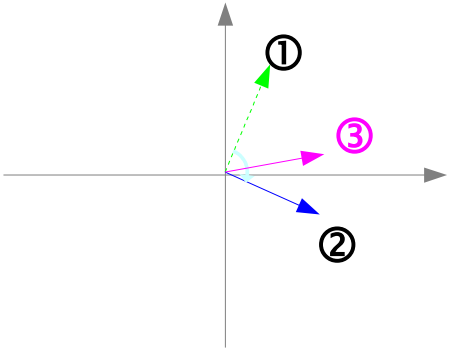


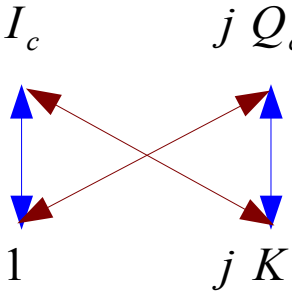


$$(I_c + 2^{-L} Q_c) + j\,(Q_c - 2^{-L} I_c)$$

Negative Phase ($Q_c < 0$) ➡ *Rotate by* $1 + j\,K$



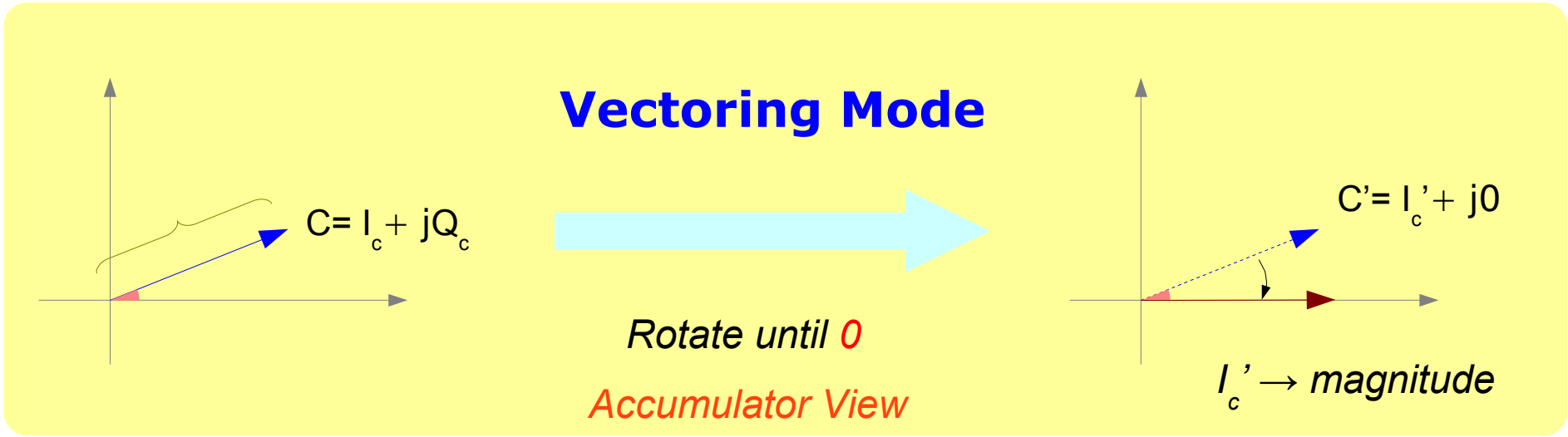




$$(I_c - 2^{-L} Q_c) + j\,(Q_c + 2^{-L} I_c)$$

After iterations, the result ➡ $I_c + j\,0$

Calculating Phase



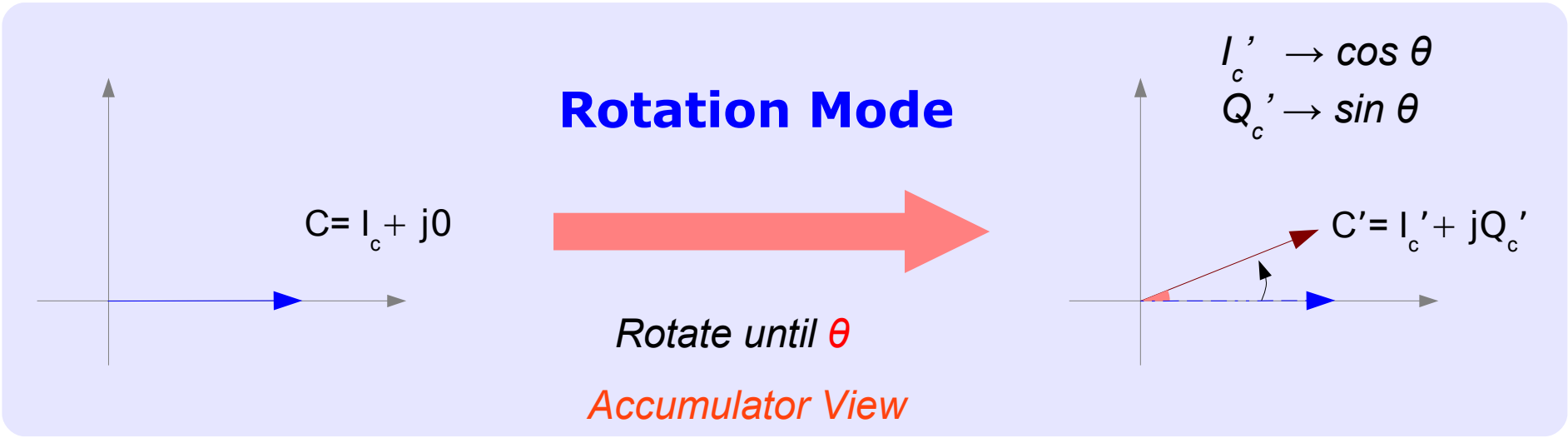
$$Phase\ of\ R$$

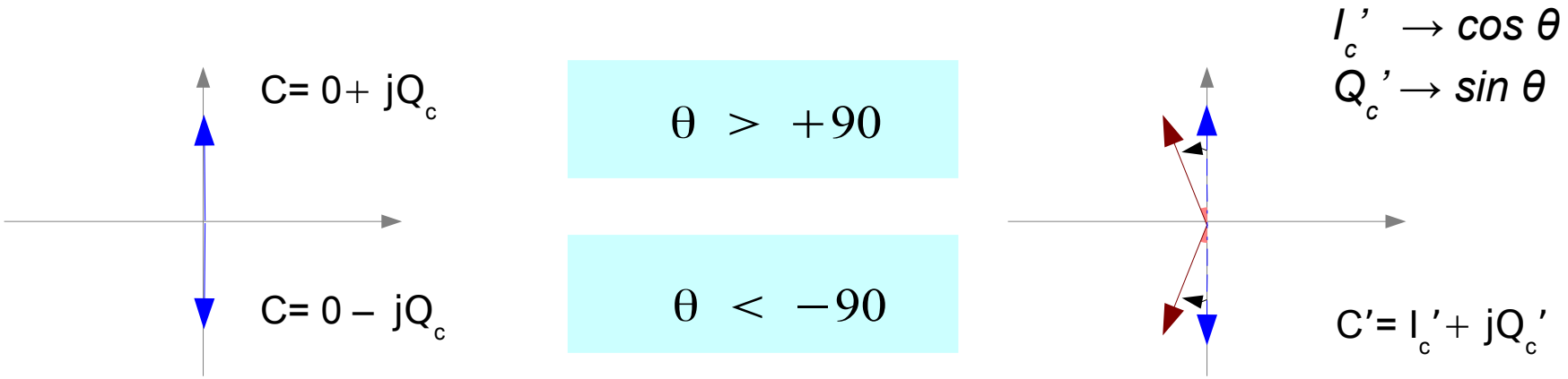
$$\theta_0 = \tan^{-1}(2^0) = 45°$$
$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505°$$
$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624°$$
$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502°$$
$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633°$$
$$\theta_5 = \tan^{-1}(2^{-5}) = 1.78991°$$
$$\cdots$$

$$\theta \quad \pm\theta_0 \pm\theta_1 \pm\theta_2 \pm\theta_3 \cdots = 0$$

Accumulate each rotating angles then negate the result

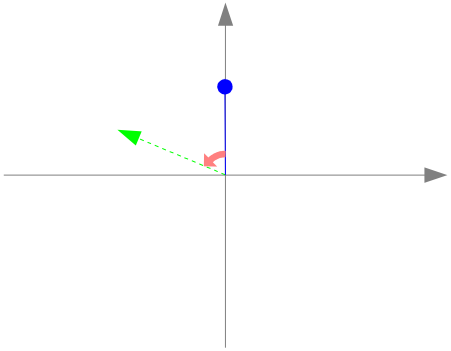
Calculating Sine and Cosine (1)



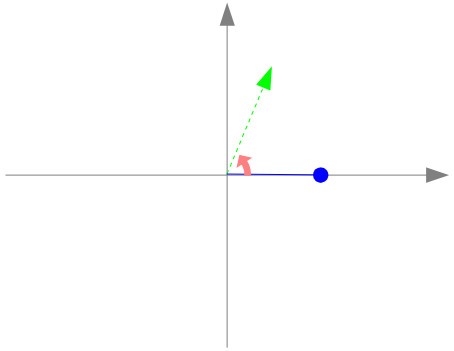


Calculating Sine and Cosine (3)

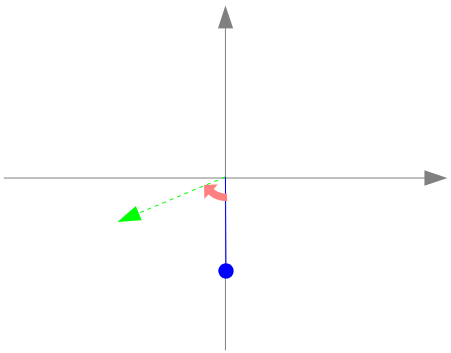
Θ > +90 ➡ *starting from* *0 + j1*



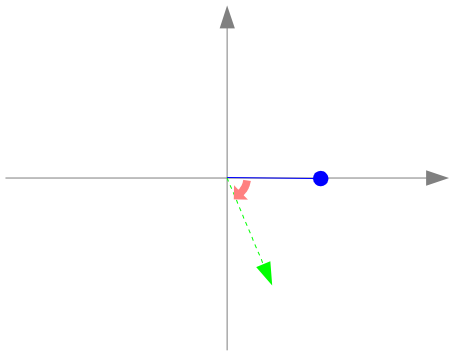
Θ < +90 ➡ *starting from* *1 + j0*



Θ < –90 ➡ *starting from* *0 – j1*



Θ > –90 ➡ *starting from* *1 – j0*



Initialize the accumulate rotation ➡ -90, +90, 0

Calculating Sine and Cosine (3)

In each iteration

(Θ – *the accumulated rotation*) *< 0*

➡ *then add the next angle*

(Θ – *the accumulated rotation*) *> 0*

➡ *then subtract the next angle*

The final

I_c ➡ *cosine*

Q_c ➡ *sine*

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
[4] CORDIC FAQ, www.dspguru.com