Angle Recoding 2. Wu
2. AR (Angle Recode)

20180831 Sat

① AR [Hu]

skip certain micro rotations .

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$\mu(i) = 0 \rightarrow$ skip

desire to minimize

$$\boxed{\sum_{i=0}^{N} |\mu(i)|}$$

so that the total number of
CORDIC iterations can be minimized

Angle Recoding ← Multiplier Recoding

angle recoding method for efficient
implementation of the CORDIC algorithm
Hu & Naganathan, ISCAS 89

Greedy algorithm

the angle quantization error

$$\boxed{\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)}$$

skipping rotations

$\mu(i) = 0$

null operations

$$\begin{cases} x(i+1) = x(i) \\ y(i+1) = y(i) \end{cases}$$

keep the iteration number N the same

in the AQ perspective, remove these steps

the iteration number N becomes N'

$N' \Leftarrow$ the number of subangles N_A of EAS

$\theta(0) = \theta$, $\{\mu(i) = 0,\ 0 \leqslant i \leqslant N-1\}$, $k = 0$.

repeat until $|\theta(k)| < a(N-1)\ D_0$

choose i_k, $0 \leqslant i_k \leqslant N-1$

$$\big|\,|\theta(k)| - a(i_k)\big| = \min_{0 \leqslant i \leqslant N-1} \big|\,|\theta(k)| - a(i_k)\big|$$

$$\theta(k+1) = \theta(k) - \mu(i_k)\, a(i_k)$$

$$\mu(i_k) = \mathrm{Sign}(\theta(k))$$

AQ & AR

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \qquad N \qquad AR$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}\left(\alpha(j) \cdot 2^{-S(j)}\right) \right] \qquad N' \qquad AQ$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j) \right] \qquad N'$$

$N' \triangleq \sum_{i=0}^{N-1} |\mu(i)|$ the effective transition number

count non-zero rotation

$S(j) \in \{0, 1, \cdots, N-1\}$ the rotational sequence

determines the micro-rotation angle

in the j-th iteration

$\alpha(j) \in \{-1, 0, +1\}$ the directional sequence

$\{-1, +1\}$ controls the direction of

(after sampling) the j-th micro-rotation of $a(S(j))$

$\tilde{\theta}(j)$ the jth micro-rotation angle

$$\tilde{\theta}(j) = \tan^{-1}\left(\alpha(j) \cdot 2^{-S(j)}\right)$$

AR essentially tries to approximate θ with the combination of selected angle elements from a pre-defined **elementary angle set** (EAS).

the EAS consists of **all** possible values of $\tilde{\theta}(j)$'s

the EAS S_1 in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\},\ s^* \in \{0, 1, \cdots, N-1\} \}$$

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$

$$= \theta - \left[\sum_{j=0}^{N'} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right]$$

$$= \theta - \left[\sum_{j=0}^{N'} \tilde{\theta}(j) \right]$$

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

AR performs AQ of the target angle θ

the sub-angle θ_i becomes $\tilde{\theta}(i) = \tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$

N_A N'

EAS S_1 AR

elementary angle   value
r(1)   = atan(-2^{-0})
r(2)   = atan(-2^{-1})
r(3)   = atan(-2^{-2})
r(4)   = atan(-2^{-3})
r(5)   = atan(-2^{-4})
r(6)   = atan(-2^{-5})
r(7)   = atan(-2^{-6})
r(8)   = atan(-2^{-7})
N_A r(9)   = atan(0   )
r(10) = atan(2^{-7})
r(11) = atan(2^{-6})   N conventional CORDIC
r(12) = atan(2^{-5})
r(13) = atan(2^{-4})
r(14) = atan(2^{-3})
r(15) = atan(2^{-2})
r(16) = atan(2^{-1})
r(17) = atan(2^{-0})

```
>> mu = [1, 0, 0, -1, 0, 0, -1, -1, 0, 0, 0, 1, 0, 0, 0, 1]
>> length(mu)
ans =  16
>> s =[ 0: 15]
>> atan(1)
ans =  0.78540
>> pi/4
ans =  0.78540
>> sum(atan(2.^(-s)) .* mu)
ans =  0.63813
>> 13 * pi / 32
ans =  1.2763
```

i	Conventional CORDIC	only $\{-1, +1\}$		skip allowed $\{-1, 0, +1\}$
0	$S(0) = 0$	$\alpha(0) = +1$	$S(0) = 0$	$\alpha(0) = +1$
1	$S(1) = 1$	$\alpha(1) = -1$	$S(1) = 1$	$\alpha(1) = 0$
2	$S(2) = 2$	$\alpha(2) = +1$	$S(2) = 2$	$\alpha(2) = 0$
3	$S(3) = 3$	$\alpha(3) = +1$	$S(3) = 3$	$\alpha(3) = -1$
4	$S(4) = 4$	$\alpha(4) = -1$	$S(4) = 4$	$\alpha(4) = 0$
5	$S(5) = 5$	$\alpha(5) = +1$	$S(5) = 5$	$\alpha(5) = 0$
6	$S(6) = 6$	$\alpha(6) = -1$	$S(6) = 6$	$\alpha(6) = -1$
7	$S(7) = 7$	$\alpha(7) = -1$	$S(7) = 7$	$\alpha(7) = -1$
8	$S(8) = 8$	$\alpha(8) = +1$	$S(8) = 8$	$\alpha(8) = 0$
9	$S(9) = 9$	$\alpha(9) = -1$	$S(9) = 9$	$\alpha(9) = 0$
10	$S(10) = 10$	$\alpha(10) = -1$	$S(10) = 10$	$\alpha(10) = 0$
11	$S(11) = 11$	$\alpha(11) = +1$	$S(11) = 11$	$\alpha(11) = +1$
12	$S(12) = 12$	$\alpha(12) = +1$	$S(12) = 12$	$\alpha(12) = 0$
13	$S(13) = 13$	$\alpha(13) = -1$	$S(13) = 13$	$\alpha(13) = 0$
14	$S(14) = 14$	$\alpha(14) = +1$	$S(14) = 14$	$\alpha(14) = 0$
15	$S(15) = 15$	$\alpha(15) = +1$	$S(15) = 15$	$\alpha(15) = +1$

i				Angle Recode	removing skipped angle
0	$S(0)=0$	$\alpha(0)=+1$	0	$S(0)=0$	$\alpha(0)=+1$
1	$S(1)=1$	$\alpha(1)=0$	1		
2	$S(2)=2$	$\alpha(2)=0$	2		
3	$S(3)=3$	$\alpha(3)=-1$	3	$S(1)=3$	$\alpha(1)=-1$
4	$S(4)=4$	$\alpha(4)=0$	4		
5	$S(5)=5$	$\alpha(5)=0$	5		
6	$S(6)=6$	$\alpha(6)=-1$	6	$S(2)=6$	$\alpha(2)=-1$
7	$S(7)=7$	$\alpha(7)=-1$	7	$S(3)=7$	$\alpha(3)=-1$
8	$S(8)=8$	$\alpha(8)=0$	8		
9	$S(9)=9$	$\alpha(9)=0$	9		
10	$S(10)=10$	$\alpha(10)=0$	10		
11	$S(11)=11$	$\alpha(11)=+1$	11	$S(4)=11$	$\alpha(4)=+1$
12	$S(12)=12$	$\alpha(12)=0$	12		
13	$S(13)=13$	$\alpha(13)=0$	13		
14	$S(14)=14$	$\alpha(14)=0$	14		
$W-1=15$	$S(15)=15$	$\alpha(15)=+1$	15	$S(5)=15$	$\alpha(5)=+1$
				$N'-1$	

i	Conventional CORDIC	only $\{-1, +1\}$	Angle Recode	skip allowed $\{-1, 0, +1\}$
0	$S(0) = 0$	$\alpha(0) = +1$	$S(0) = 0$	$\alpha(0) = +1$
1	$S(1) = 1$	$\alpha(1) = -1$	$S(1) = 3$	$\alpha(1) = -1$
2	$S(2) = 2$	$\alpha(2) = +1$	$S(2) = 6$	$\alpha(2) = -1$
3	$S(3) = 3$	$\alpha(3) = +1$	$S(3) = 7$	$\alpha(3) = -1$
4	$S(4) = 4$	$\alpha(4) = -1$	$S(4) = 11$	$\alpha(4) = +1$
5	$S(5) = 5$	$\alpha(5) = +1$	$S(5) = 15$	$\alpha(5) = +1$
6	$S(6) = 6$	$\alpha(6) = -1$		
7	$S(7) = 7$	$\alpha(7) = -1$		
8	$S(8) = 8$	$\alpha(8) = +1$		
9	$S(9) = 9$	$\alpha(9) = -1$		
10	$S(10) = 10$	$\alpha(10) = -1$		
11	$S(11) = 11$	$\alpha(11) = +1$		
12	$S(12) = 12$	$\alpha(12) = +1$		
13	$S(13) = 13$	$\alpha(13) = -1$		
14	$S(14) = 14$	$\alpha(14) = +1$		
15	$S(15) = 15$	$\alpha(15) = +1$		

W

N'