By Matthew Raspanti

Chapter 13

GENERAL RELATIVITY

The General Principle of Relativity

Special Relativity stemmed from the work of several people. It was motivated by the definite need to reconcile Newton's and Maxwell's laws. The General Theory of Relativity, instead, rests almost exclusively on the insight of one man, Einstein, who was determined to pursue to the limit the implications of a philosophical principle.

With his Special Theory of Relativity, Einstein had shown that both Maxwell's equations and Newton's modified equations of motion are valid regardless of the particular inertial frame from which one happens to observe the universe. As a result, neither Stan nor Moe could detect being in uniform motion.

Einstein embarked now on an even bolder venture: to show that the laws of physics can be stated in a form that is equally valid not only for *inertial* frames of reference (which move with straight uniform motion), but also for *any* frame of reference, *whatever* its motion (for instance, linearly accelerating frames, or rotating frames). Einstein saw no reason why inertial frames should have a privileged status. It is the same universe, he felt, whatever the reference frame we observe it from.

This is the philosophical principle that motivated Einstein: it is called the *General Principle of Relativity*. It implies, however, that it should be impossible to detect any motion of one's own frame of reference - uniform or otherwise. Einstein's venture seemed doomed from the start because of the well familiar jerking effect that accompanies any change in speed and/or direction of motion. There appears to be something absolute about acceleration. A reevaluation of gravity, however, gave a way out.

To explain the motions of bodies, earthly as well as heavenly, Newton had proposed a gravitational force of attraction which, instantly, even across enormous distances, can act on two bodies. Instantaneous action at a distance, however, is incompatible with the Special Theory of Relativity, which asserts that no physical influence can travel faster than the speed of light. Inspired by Maxwell's theory of electromagnetic fields, instead of a force of gravity, Einstein postulated a gravitational field that propagates at the speed of light.

Gravitational vs. Inertial Mass

There was something about Newton's force of gravity that had puzzled physicists from the start. The term "mass" that appears in Newton's law of gravitational force could be replaced by the more specific term "gravitational mass". Similarly, the term "charge" that appears in Coulomb's law of electrostatic force could be replaced by the term "electrical mass". Finally, in Newton's law of motion (force = mass x acceleration), the term "mass" could be replaced by the more specific term "inertial mass".

Whether the force applied to a body is from a gravitational or an electrical source, it is the *inertial* mass of the body that determines the resulting acceleration. There did not seem to be any reason why either the gravitational or the electrical mass should equal the inertial mass, and yet the gravitational mass does.

In Newtonian mechanics, this equivalence of inertial and gravitational mass had been regarded as just a strange coincidence. Einstein was the first to recognize that this "coincidence" indicated something peculiar about the nature of the gravitational force. His interpretation of the situation led to the second fundamental principle of his General Theory of Relativity: the Principle of Equivalence.

The Principle of Equivalence

As previously noted, Newton's laws of motion are valid only with respect to an inertial frame of reference. To achieve the ideal conditions required for a truly inertial frame, we need to place ourselves in some region of outer space so far removed from stars and other appreciable masses that we can assume no gravity force that would cause acceleration.

Let's consider now another of Einstein's thought experiments. This time, instead of a train, we have a very special elevator, which he described as "a spacious chest resembling a [windowless] room with an observer inside who is equipped with [measuring] apparatus". We will call this observer Eliot as a way of remembering that he is the guy in the elevator. Let us imagine this elevator in the depths of intergalactic space far from all matter, where we will also place our friend Stan and his inertial reference frame.

If the elevator were drifting in empty space with constant velocity, it too would be an inertial reference frame. Stan and Eliot would disagree about each other's clocks and yardsticks, but not about their interpretation of physical phenomena.

Let us assume, instead, that somebody is pulling on a rope attached to a hook in the middle of the elevator's roof, applying a constant force. As a result, the elevator is moving "upwards" with uniformly accelerated motion. We have now a *non-inertial* reference frame. With the right force, we can cause a constant acceleration equal to 1g, i.e., equal to Galileo's acceleration of gravity near the Earth's surface (about 32 feet per second every second).

In his elevator accelerating at 1g, Eliot feels his normal weight, as he can verify on a scale that is part of his measuring equipment. In fact, everything feels quite normal. If he drops an apple he is holding in his hand, he will see it fall with an acceleration g. Even if he drops a much heavier object, he sees it too fall with exactly the same acceleration. If he drops both objects at the same time, both will hit the floor at the same time. From all that he can see inside his windowless laboratory, he concludes that his elevator is at rest in a gravitational field just like Earth's.

From his inertial reference frame, Stan gives a totally different interpretation of things. The elevator is moving upwards with uniformly accelerated motion, pushing on Eliot's feet and carrying him with it. As long as Eliot holds on to the apple, the apple is accelerating along. When Eliot lets go of the apple, he stops applying a force. By Newton's law of inertia, the apple continues to move upwards at constant speed, but soon enough the floor of the accelerating elevator catches up with it and starts pushing it along. A second object released at the same time will continue to move upward at the same uniform speed, until it too is hit by the floor at exactly the same time as the apple. Eliot does not realize what is really happening - he has invented a fictitious force of gravity to explain things and retain his illusion of being at rest.

This dual interpretation of the same phenomena is made possible by the fact that gravitational mass and inertial mass are equal. If, for instance, the inertial mass of a body were twice its gravitational mass, in his no-gravity environment, Eliot would measure twice his normal weight. He would be able to tell that things are different.

We should not conclude, however, that the existence of a gravitational field is always only an apparent one. Inside an elevator that is accelerating in empty space, the forces of pseudo gravity are all equal and parallel to the direction of motion. Inside an elevator standing on the surface of the Earth, the forces of gravity vary from point to point, and are all converging toward the center of the Earth. The differences, however, are imperceptible within the small space of the elevator.

These considerations led Einstein to state his Principle of Equivalence as follows:

Within a small space, there is no measurable difference between the effects of a gravitational field, and the effects that result from the acceleration of one's reference frame.

The Principle of Equivalence tells us that what Eliot observes in his elevator accelerating in no-gravity intergalactic space, is the same as what he would observe if his elevator were at rest in a gravity field. From this principle, Einstein derived important insights that guided him in the development of a mathematical theory.

If a pulse of light were to enter at some point near the top of the accelerating elevator and proceed in a direction perpendicular to the motion of the elevator, it would appear to Eliot to follow some downward curved path and leave the elevator at some point lower than the point of entry. Using the Equivalence Principle, Einstein could predict then that light would be bent in the vicinity of a large mass such as the Sun.

Einstein predicted also that time is affected by a gravity field. He imagined Eliot experimenting with two identical clocks: one near the bottom of the elevator, the other near the top. Einstein showed that Eliot would conclude that the clock on top runs faster than the clock on the bottom.

Note that, when we were following the antics of Stan and Moe, we were comparing identical clocks on separate inertial frames in relative motion. Stan and Moe were claiming that each other's clock ran slower. Now we are talking of two identical clocks at rest on the *same* non-inertial frame, and, according to Eliot, the top clock runs faster than the bottom clock.

Applying the Principle of Equivalence, Einstein could predict that, in the presence of a gravity field, a clock at a higher level would run faster than a clock at a lower level. This has been verified experimentally. In the 1960's, for instance, researchers found that clocks in the mile-high city of Boulder, Colorado, gained about 15 billionths of a second a day compared with clocks near sea level in Washington, D.C. Near a massive star, time dilation would be much more pronounced.

Special Relativity told us that time is affected by motion. Now, General Relativity tells us that time is also affected by gravity. More speed slows down a clock, and so does more gravity.

<u>"Help me, Marcel, or I'll Go Crazy!"</u>

Confusing as things might have been with special relativity, at least everything made sense on any particular inertial "platform", where clocks and yardsticks gave consistent measurements. Two people on the same platform could agree on measurements of time and distances. It was only with somebody on another inertial frame that disagreements arose.

When Einstein started considering non-inertial frames moving in various ways, he faced a totally bewildering situation. It is no longer possible to define a single length or a single time that has the same meaning everywhere, even within a single reference frame. From one point to another, a yardstick shrinks or expands, a clock runs faster or slower. Add the fact that the mass of a body changes with speed, and you can appreciate the overwhelming difficulties Einstein was confronted with, while trying to develop a mathematical theory of gravity that would hold in any frame of reference. The problem was stretching his mathematical skills to the limit. Once again, rescue came from his friend Marcel Grossman, to whom he wrote in desperation, "Help me, Marcel, or I'll go crazy!"

Marcel, who was by now a professor of mathematics, introduced Einstein to the work of the German mathematician Bernhard Riemann, who, without any particular application in mind, had developed just the mathematical tools Einstein needed.

Riemann's work was about "non-Euclidean" geometry, the geometry of curved surfaces and spaces. This field was extensively studied by Riemann and others during the 19th century. Until then, the only geometry known was the one built upon Euclid's Elements, which was considered the "true" geometry of space.

If, instead of a plane, we consider some other type of surface, say, a sphere, the rules of geometry change completely. On a plane, for instance, the angles of any triangle always add to 180 degrees. The angles of a triangle drawn on a sphere, instead, always add to more than 180 degrees. A triangle drawn on a sphere can have each of its angles equal to 90 degrees.

On any kind of surface - spherical, pear-shaped or whatever - given two points A and B, there is a path of shortest distance between them, which is called the "geodesic" between the two points. On a plane, a geodesic is a segment of a straight line. On the surface of a sphere, a geodesic is an arc of a circle, whose center coincides with the center of the sphere. Each type of surface has its own geometry and its own kind of geodesic.

Einstein's Theory of Gravity

Einstein was interested in a geometry of curved space because he speculated that the presence of matter causes the warping of a 4-dimensional space-time, whose geometry becomes non-Euclidean. The gravitational interaction between two bodies arises from the geometry of space itself.

A helpful 2-dimensional analogy is to think of space-time as if it were a huge very flexible rubber sheet that is stretched tightly across a frame, like a cosmic trampoline. Imagine now massive bodies (such as stars) placed here and there on the trampoline. Pressing down on the surface of the rubber sheet, each star will warp it causing a depression, called a "gravity well". (You may have seen a rigid model of a gravity well in a science museum, where you can release a coin or a small steel ball, and watch it go around and around down to the bottom of the well.) Smaller masses will have a smaller warping effect. Where there is little significant mass, the surface is essentially flat.

A planet orbiting around a star moves the way it does on the cosmic trampoline simply because it is following the warped contour of the surface, not because some Newtonian force of gravity is acting on it from a distance.

A comet moving toward a star will descend into the star's gravity well. It may remain trapped in the well, orbiting along its sides forever. If the comet is fast enough, however, instead of being trapped, it will emerge from the well and continue its journey, somewhat deflected from its original path.

Where there is no mass, the geometry of space is Euclidean and Newton's law of inertia holds. Here, without the intervention of any force, a body moves on its own with constant speed along a *straight* line. Einstein generalized Newton's law of inertia to state that a body moves on its own along a *geodesic* line. Within a gravitational field, however, geodesic lines are no longer straight lines, but are still paths of minimum distance.

For Newton, forceless motion by inertia was natural and did not require an explanation. For Einstein, a body falling or orbiting under the influence of gravity is also in a forceless natural state of motion, but in a warped space-time, in which it moves along a geodesic line.

In Newton's theory, space and time were distinct, independent entities. As a universal clock ticked away, space was a passive, immutable stage on which the phenomena of physics unfolded without either affecting, or being affected by, the stage.

Special relativity combined space and time into a single entity, space-time, which became the new passive, immutable stage of physics. With general relativity, instead, space-time becomes an integral part of the whole process. The presence of matter causes space-time to warp; the warping of space-time, in turn, affects the motion of matter. The stage and the play mutually affect one another. As the eminent physicist John Wheeler put it very succinctly, matter tells space how to curve, and space tells matter how to move.

Confirmations of General Relativity

Only in the presence of huge masses can we expect any measurable differences between the predictions of general relativity and those of Newton's gravitational theory.

Consequently, most experimental tests of general relativity have been based upon astronomical observations. As previously mentioned, Eddington's expedition to Africa provided confirmation of Einstein's prediction that light too travels along geodesic lines and, therefore, will be bent in the vicinity of a large mass such as the Sun.

Another confirmation of the theory was provided by Einstein's ability to account for a peculiarity in the orbit of Mercury, the planet that is closest to the Sun. According to Kepler, a planet should retrace the same elliptical orbit again and again, always coming back to the same "perihelion", the point on its orbit closest to the Sun. In the case of Mercury, evidence accumulated during the 19th century revealed a discrepancy that could not be completely explained in Newtonian terms. Instead of remaining stationary with respect to the stars, Mercury's elliptical orbit as a whole slowly rotates.

What is observed is a small advance of Mercury's perihelion by 543 seconds of arc (about one seventh of a degree) per century. Taking into consideration the gravitational pull of the Sun and of all the known planets, Newtonian mechanics could account only for an advance of about 500 seconds of arc per century. Einstein's theory can account for the remaining 43 seconds. For the orbits of the other planets, Einstein's theory gives the same results as Newton's.

The general theory of relativity is widely accepted as the most satisfactory explanation for the gravitational interaction. It has achieved its most dramatic success in the field of cosmology, the study of the universe as a whole.