Laurent Series and z-Transform - Geometric Series Causality A

20180606

2 formulas of z

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$

$\Big\updownarrow z^{-1}$

② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$

$$\boxed{\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{\;z^{-1}\;} \quad \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\,\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5} - \frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1} - \frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z} - \frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2} + \frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2} + \frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$f(z), g(z)$: causal form of $\boxed{\text{Laurent}}$ series

$$\frac{\text{nominator polynomial of } z}{\text{denominator polynomial of } z}$$

$f(z^{-1}), g(z^{-1})$: anti-causal form of $\boxed{\text{Laurent}}$ series

$$\frac{\text{nominator polynomial of } z^{-1}}{\text{denominator polynomial of } z^{-1}}$$

$X(z), Y(z)$: causal form of $\boxed{z\text{- Trans}}$

$$\frac{\text{nominator polynomial of } z^{-1}}{\text{denominator polynomial of } z^{-1}}$$

$X(z^{-1}), Y(z^{-1})$: anti-causal form of $\boxed{z\text{- Trans}}$

$$\frac{\text{nominator polynomial of } z}{\text{denominator polynomial of } z}$$

2 formulas

$$\boxed{\frac{1}{z - p}} \qquad \boxed{\frac{1}{z^{-1} - p}}$$

2 representations each

$$\boxed{\frac{1}{z - p}} \qquad \boxed{-\frac{p^{-1}}{1 - p^{-1} z}} \triangleq f(z) = X(z^{-1})$$

$$\| \qquad \|$$

$$\boxed{\frac{z^{-1}}{1 - p z^{-1}}} \triangleq Y(z) = g(z^{-1})$$

$$\boxed{\frac{1}{z^{-1} - p}} \qquad \boxed{\frac{z}{1 - p z}} \triangleq g(z) = Y(z^{-1})$$

$$\| \qquad \|$$

$$\boxed{-\frac{p^{-1}}{1 - p^{-1} z^{-1}}} \triangleq X(z) = f(z^{-1})$$

$$\begin{array}{lcccl} z\text{ polynomials} & f(z) & & g(z) & z\text{ polynomials} \\ & \Vert & & \Vert & \\ z^{-1}\text{ polynomials} & Y(z) & & X(z) & z^{-1}\text{ polynomials} \end{array}$$

$$\begin{array}{lcccccccl} z\text{ polynomials} & X(z^{-1}) & = & f(z) & & g(z) & = & Y(z^{-1}) & z\text{ polynomials} \\ & \Vert & & \Vert & & \Vert & & \Vert & \\ z^{-1}\text{ polynomials} & g(z^{-1}) & = & Y(z) & & X(z) & = & f(z^{-1}) & z^{-1}\text{ polynomials} \end{array}$$

$\boxed{\text{Laurent}}$ $f(z), g(z)$: causal , $f(z^{-1}), g(z^{-1})$: anti-causal

$\boxed{\text{z- Trans}}$ $X(z), Y(z)$: causal, $X(z^{-1}), Y(z^{-1})$: anti-causal

$$\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} = f(z) \longleftrightarrow -(p^{-1})^{n+1} \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} &
\end{tikzcd}$$

$X(z^{-1})$

$\|$

$g(z^{-1})$

$Y(z) \longleftrightarrow (p)^{n-1}$

$$\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} = f(z^{-1}) \\
\boxed{\dfrac{z}{1-pz}} &
\end{tikzcd}$$

$X(z) \longleftrightarrow -(p^{-1})^{n+1}$

$\|$

$g(z) \longleftrightarrow (p)^{n-1}$

$Y(z^{-1})$

2 formulas of z

2 representations

$f(z),\ g(z)$

$f(z^{-1}),\ g(z^{-1})$

$X(z),\ Y(z)$

$X(z^{-1}),\ Y(z^{-1})$

* Simple Pole Forms

$p \qquad \boxed{\dfrac{1}{z-p}} \quad \xleftrightarrow{z^{-1}} \quad \boxed{\dfrac{1}{z^{-1}-p}} \qquad p^{-1}$

$\dfrac{z^{-1}}{z^{-1}} \cdot \quad \cdot \dfrac{z}{z} \qquad\qquad \dfrac{z}{z} \cdot \quad \cdot \dfrac{z^{-1}}{z^{-1}}$

$p \qquad \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \quad \xleftrightarrow{z^{-1}} \quad \boxed{\dfrac{z}{1-pz}} \qquad p^{-1}$

* Geometric Series Forms

$n \geq 0 \qquad |z| < p \qquad\qquad |z| > p^{-1} \qquad n \geq 0$

$\boxed{-(p^{-1})^{n+1}}$ causal f(z) $\quad \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \quad \xleftrightarrow{z^{-1}} \quad \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \quad$ causal X(z) $\boxed{-(p^{-1})^{n+1}}$

$\cdot \dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}} \qquad\qquad \dfrac{p}{p}\dfrac{z}{z} \cdot$

causal X(z) $\boxed{(p)^{n-1}} \quad \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \quad \xleftrightarrow{z^{-1}} \quad \boxed{\dfrac{z}{1-pz}} \quad \boxed{(p)^{n-1}}$ causal f(z)

$n \geq 1 \qquad |z| > p \qquad\qquad |z| < p^{-1} \qquad n \geq 1$

Laurent Series

$$f(z) \quad (|z| < \rho) \quad \leftrightarrow \quad a_n \quad (n \geq 0)$$

$$f(z^{-1}) \quad (|z| > \rho^{-1}) \quad \leftrightarrow \quad a_{-n} \quad (n < 1)$$

$$f(z) \quad (|z| > \rho) \quad \leftrightarrow \quad -a_n \quad (n < 0)$$

$$f(z^{-1}) \quad (|z| < \rho^{-1}) \quad \leftrightarrow \quad -a_{-n} \quad (n \geq 1)$$

Ⓐ $f(z)$ for $\boxed{|z|<p}$, $g(z)$ for $\boxed{|z|<p^{-1}}$ Laurent S

Geometric Series Forms

$|z|<p$

p $\boxed{f(z)} = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$ $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$

$\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$ $\boxed{\dfrac{z}{1-pz}} = \boxed{g(z)}$ p^{-1}

$|z|<p^{-1}$

Ⓑ $f(z^{-1})$ for $\boxed{|z|>p^{-1}}$, $g(z^{-1})$ for $\boxed{|z|>p}$

Geometric Series Forms

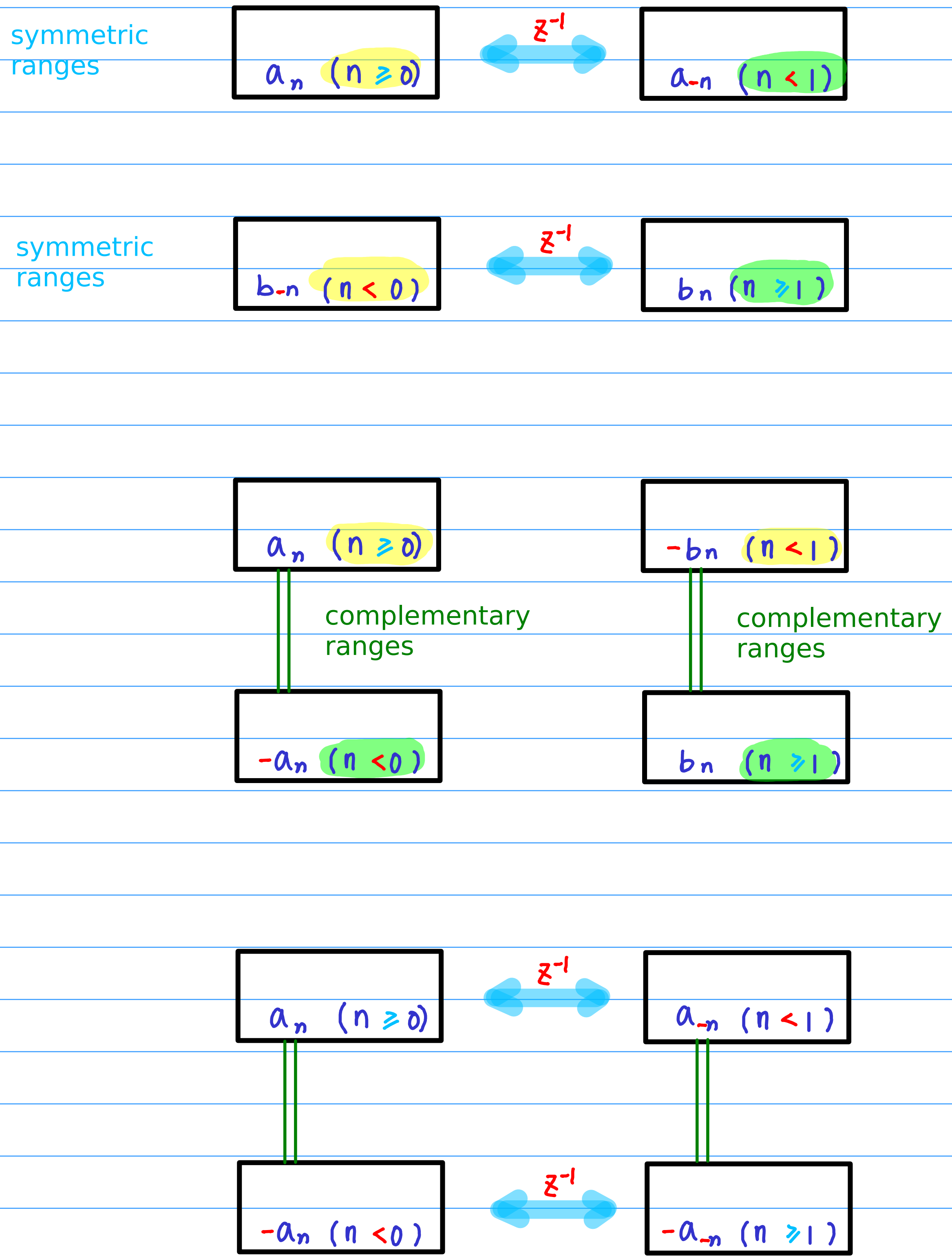
$|z|<p$ $|z|>p^{-1}$

$f(z) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \xleftrightarrow{z^{-1}} \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} = \boxed{f(z^{-1})}$

$\boxed{g(z^{-1})} = \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \xleftrightarrow{z^{-1}} \boxed{\dfrac{z}{1-pz}} = g(z)$

$|z|>p$ $|z|<p^{-1}$

Laurent Series $\boxed{a_n} \leftrightarrow f(z)$



Laurent Series $a_n \leftrightarrow \boxed{f(z)}$

ROC's with reciprocal poles

$$f(z)\ (|z|<p) \xleftrightarrow{z^{-1}} f(z^{-1})\ (|z^{-1}|<p)$$

ROC's with reciprocal poles

$$g(z^{-1})\ (|z^{-1}|<p^{-1}) \xleftrightarrow{z^{-1}} g(z)\ (|z|<p^{-1})$$

$f(z)\ (|z|<p)$

complementary ROC

$g(z^{-1})\ (|z|>p)$

$f(z^{-1})\ (|z|>p^{-1})$

complementary ROC

$g(z)\ (|z|<p^{-1})$

$$f(z)\ (|z|<p) \xleftrightarrow{z^{-1}} f(z^{-1})\ (|z|>p^{-1})$$

$$f(z)\ (|z|>p) \xleftrightarrow{z^{-1}} f(z^{-1})\ (|z|<p^{-1})$$

Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$

Geometric Series Forms

$$|z| < p \qquad\qquad\qquad\qquad |z| > p^{-1}$$

$$f(z) = \boxed{-\frac{p^{-1}}{1 - p^{-1} z}} \xleftrightarrow{z^{-1}} \boxed{-\frac{p^{-1}}{1 - p^{-1} z^{-1}}} = f(z^{-1})$$

$$\cdot \frac{p}{p}\frac{z^{-1}}{z^{-1}} \qquad\qquad\qquad\qquad \cdot \frac{p}{p}\frac{z}{z}$$

$$g(z^{-1}) = \boxed{\frac{z^{-1}}{1 - p z^{-1}}} \xleftrightarrow{z^{-1}} \boxed{\frac{z}{1 - p z}} = g(z)$$

$$|z| > p \qquad\qquad\qquad\qquad |z| < p^{-1}$$

$$\boxed{\begin{matrix} f(z) \ (|z| < p) \\ a_n \ (n \geq 0) \end{matrix}} \xleftrightarrow{z^{-1}} \boxed{\begin{matrix} f(z^{-1}) \ (|z^{-1}| < p) \\ a_{-n} \ (n < 1) \end{matrix}} \boxed{\begin{matrix} g(z) \ (|z| > p^{-1}) \\ -b_n \ (n < 1) \end{matrix}}$$

$$\boxed{\begin{matrix} f(z) \ (|z| > p) \\ -a_n \ (n < 0) \end{matrix}} \boxed{\begin{matrix} g(z^{-1}) \ (|z^{-1}| < p^{-1}) \\ b_{-n} \ (n < 0) \end{matrix}} \xleftrightarrow{z^{-1}} \boxed{\begin{matrix} g(z) \ (|z| < p^{-1}) \\ b_n \ (n \geq 1) \end{matrix}}$$

Laurent Series using only $\boxed{a_n} \leftrightarrow \boxed{f(z)}$

$$
\begin{tikzcd}
 & \boxed{\begin{array}{ll} f(z) & (|z|<p) \\ a_n & (n \geq 0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{array}{ll} f(z^{-1}) & (|z^{-1}|<p) \\ a_{-n} & (n<1) \end{array}} \arrow[d, equal] & \boxed{\begin{array}{ll} g(z) & (|z|>p^{-1}) \\ -b_n & (n<1) \end{array}} \\
\boxed{\begin{array}{ll} f(z) & (|z|>p) \\ -a_n & (n<0) \end{array}} & \boxed{\begin{array}{ll} g(z^{-1}) & (|z^{-1}|<p^{-1}) \\ b_{-n} & (n<0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{ll} g(z) & (|z|<p^{-1}) \\ b_n & (n \geq 1) \end{array}} &
\end{tikzcd}
$$

$$a_{-n} = -b_n \qquad\qquad -a_{-n} = b_n$$

$$
\begin{tikzcd}
\boxed{\begin{array}{ll} f(z) & (|z|<p) \\ a_n & (n \geq 0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{array}{ll} f(z^{-1}) & (|z|>p^{-1}) \\ a_{-n} & (n<1) \end{array}} \arrow[d, equal] \\
\boxed{\begin{array}{ll} f(z) & (|z|>p) \\ -a_n & (n<0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{ll} f(z^{-1}) & (|z|<p^{-1}) \\ -a_{-n} & (n \geq 1) \end{array}}
\end{tikzcd}
$$

Z - Transform

$X(z^{-1})\ (|z| < p) \leftrightarrow x_{-n}\ (n < 1)$

$X(z)\ (|z| > p^{-1}) \leftrightarrow x_n\ (n \geqslant 0)$

$X(z^{-1})\ (|z| > p) \leftrightarrow -x_{-n}\ (n \geq 1)$

$X(z)\ (|z| < p^{-1}) \leftrightarrow -x_n\ (n < 0)$

(A) $X(z)$ for $\boxed{|z| > p^{-1}}$, $Y(z)$ for $\boxed{|z| > p}$ z-Transform

Geometric Series Forms

$$\boxed{-\frac{p^{-1}}{1 - p^{-1} z}} \qquad \overset{|z| > p^{-1}}{\boxed{-\frac{p^{-1}}{1 - p^{-1} z^{-1}}}} = \boxed{X(z)} \quad p^{-1}$$

$$p \quad \boxed{Y(z)} = \overset{|z| > p}{\boxed{\frac{z^{-1}}{1 - p z^{-1}}}} \qquad \boxed{\frac{z}{1 - p z}}$$

(B) $X(z^{-1})$ for $\boxed{|z| < p}$, $Y(z^{-1})$ for $\boxed{|z| < p^{-1}}$

Geometric Series Forms

$$\boxed{X(z^{-1})} = \overset{|z| < p}{\boxed{-\frac{p^{-1}}{1 - p^{-1} z}}} \xleftrightarrow{z^{-1}} \overset{|z| > p^{-1}}{\boxed{-\frac{p^{-1}}{1 - p^{-1} z^{-1}}}} = X(z)$$

$$Y(z) = \underset{|z| > p}{\boxed{\frac{z^{-1}}{1 - p z^{-1}}}} \xleftrightarrow{z^{-1}} \underset{|z| < p^{-1}}{\boxed{\frac{z}{1 - p z}}} = \boxed{Y(z^{-1})}$$

z - Transform $\boxed{x_n} \leftrightarrow X(z)$

symmetric ranges

$x_{-n}\ (n < 1) \xleftrightarrow{z^{-1}} x_n\ (n \geq 0)$

symmetric ranges

$y_n\ (n \geq 1) \xleftrightarrow{z^{-1}} y_{-n}\ (n < 0)$

$-y_n\ (n < 1)$

complementary ranges

$y_n\ (n \geq 1)$

$x_n\ (n \geq 0)$

complementary ranges

$-x_n\ (n < 0)$

$x_{-n}\ (n < 1) \xleftrightarrow{z^{-1}} x_n\ (n \geq 0)$

$-x_{-n}\ (n \geq 1) \xleftrightarrow{z^{-1}} -x_n\ (n < 0)$

z - Transform $x_n \leftrightarrow \boxed{X(z)}$

ROC's with reciprocal poles

$X(z^{-1})\ (|z^{-1}| > p^{-1}) \xleftrightarrow{z^{-1}} X(z)\ (|z| > p^{-1})$

ROC's with reciprocal poles

$Y(z)\ (|z| > p) \xleftrightarrow{z^{-1}} Y(z^{-1})\ (|z^{-1}| > p)$

$Y(z)\ (|z| < p)$

complementary ROC

$Y(z)\ (|z| > p)$

$X(z)\ (|z| > p^{-1})$

complementary ROC

$X(z)\ (|z| < p)$

$X(z^{-1})\ (|z| < p) \xleftrightarrow{z^{-1}} X(z)\ (|z| > p^{-1})$

$X(z^{-1})\ (|z| > p) \xleftrightarrow{z^{-1}} X(z)\ (|z| < p)$

Z- Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

Geometric Series Forms

$$
\begin{tikzcd}[column sep=large, row sep=large]
\underset{}{\overset{|z|<p}{X(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}}}} \arrow[rr, leftrightarrow, "z^{-1}"] \arrow[d, equal, "\cdot\frac{p}{p}\frac{z^{-1}}{z^{-1}}"] & & \overset{|z|>p^{-1}}{\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = X(z)} \arrow[d, equal, "\cdot\frac{p}{p}\frac{z}{z}"'] \\
\underset{|z|>p}{Y(z) = \boxed{\frac{z^{-1}}{1-pz^{-1}}}} \arrow[rr, leftrightarrow, "z^{-1}"] & & \underset{|z|<p^{-1}}{\boxed{\frac{z}{1-pz}} = Y(z^{-1})}
\end{tikzcd}
$$

$$
\begin{tikzcd}[column sep=small, row sep=large]
\boxed{\begin{array}{c} Y(z)\ (|z|<p) \\ -y_n\ (n<1) \end{array}} & \boxed{\begin{array}{c} X(z^{-1})\ (|z^{-1}|>p^{-1}) \\ x_{-n}\ (n<1) \end{array}} \arrow[d, equal] \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{c} X(z)\ (|z|>p^{-1}) \\ x_n\ (n\geq 0) \end{array}} \arrow[d, equal] & \\
 & \boxed{\begin{array}{c} Y(z)\ (|z|>p) \\ y_n\ (n\geq 1) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{c} Y(z^{-1})\ (|z^{-1}|>p) \\ y_{-n}\ (n<0) \end{array}} & \boxed{\begin{array}{c} X(z)\ (|z|<p^{-1}) \\ -x_n\ (n<0) \end{array}}
\end{tikzcd}
$$

z-Transform using only $\boxed{x_n} \leftrightarrow \boxed{X(z)}$

$$
\begin{tikzcd}
\boxed{\begin{array}{l} Y(z)\ (|z|<p) \\ -y_n\ (n<1) \end{array}} & \boxed{\begin{array}{l} X(z^{-1})\ (|z^{-1}|>p^{-1}) \\ x_{-n}\ (n<1) \end{array}} \arrow[d, equal] \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{l} X(z)\ (|z|>p^{-1}) \\ x_n\ (n\geq 0) \end{array}} \arrow[d, equal] & \\
 & \boxed{\begin{array}{l} Y(z)\ (|z|>p) \\ y_n\ (n\geq 1) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{l} Y(z^{-1})\ (|z^{-1}|>p) \\ y_{-n}\ (n<0) \end{array}} & \boxed{\begin{array}{l} X(z)\ (|z|<p^{-1}) \\ -x_n\ (n<0) \end{array}}
\end{tikzcd}
$$

$$x_{-n} = -y_n \qquad\qquad -x_{-n} = y_n$$

$$
\begin{tikzcd}
\boxed{\begin{array}{l} X(z^{-1})\ (|z|<p) \\ x_{-n}\ (n<1) \end{array}} \arrow[d, equal] \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{l} X(z)\ (|z|>p^{-1}) \\ x_n\ (n\geq 0) \end{array}} \arrow[d, equal] \\
\boxed{\begin{array}{l} X(z^{-1})\ (|z|>p) \\ -x_{-n}\ (n\geq 1) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{l} X(z)\ (|z|<p^{-1}) \\ -x_n\ (n<0) \end{array}}
\end{tikzcd}
$$

Laurent Series and z-Transform

with the same ROC

$$
\begin{tikzcd}
 & \boxed{\begin{array}{c} f(z)\ \ (|z|<p) \\ a_n\ \ (n \geq 0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{array}{c} f(z^{-1})\ (|z^{-1}|<p) \\ a_{-n}\ \ (n<1) \end{array}} \arrow[d, equal] & \boxed{\begin{array}{c} g(z)\ \ (|z|>p^{-1}) \\ -b_n\ \ (n<1) \end{array}} \\
\boxed{\begin{array}{c} f(z)\ (|z|>p) \\ -a_n\ \ (n<0) \end{array}} & \boxed{\begin{array}{c} g(z^{-1})\ (|z^{-1}|<p^{-1}) \\ b_{-n}\ \ (n<0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{c} g(z)\ (|z|<p^{-1}) \\ b_n\ \ (n \geq 1) \end{array}} &
\end{tikzcd}
$$

$$
\begin{tikzcd}
\boxed{\begin{array}{c} Y(z)\ (|z|<p) \\ -y_n\ \ (n<1) \end{array}} & \boxed{\begin{array}{c} X(z^{-1})\ (|z^{-1}|>p^{-1}) \\ x_{-n}\ \ (n<1) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{array}{c} X(z)\ (|z|>p^{-1}) \\ x_n\ \ (n \geq 0) \end{array}} \arrow[d, equal] & \\
 & \boxed{\begin{array}{c} Y(z)\ \ (|z|>p) \\ y_n\ \ (n \geq 1) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{c} Y(z^{-1})\ (|z^{-1}|>p) \\ y_{-n}\ \ (n<0) \end{array}} & \boxed{\begin{array}{c} X(z)\ \ (|z|<p^{-1}) \\ -x_n\ \ (n<0) \end{array}}
\end{tikzcd}
$$

$\boxed{\text{Laurent Series}}$ and $\boxed{z\text{-Transform}}$ $a_n,\ x_n$

with the same ROC

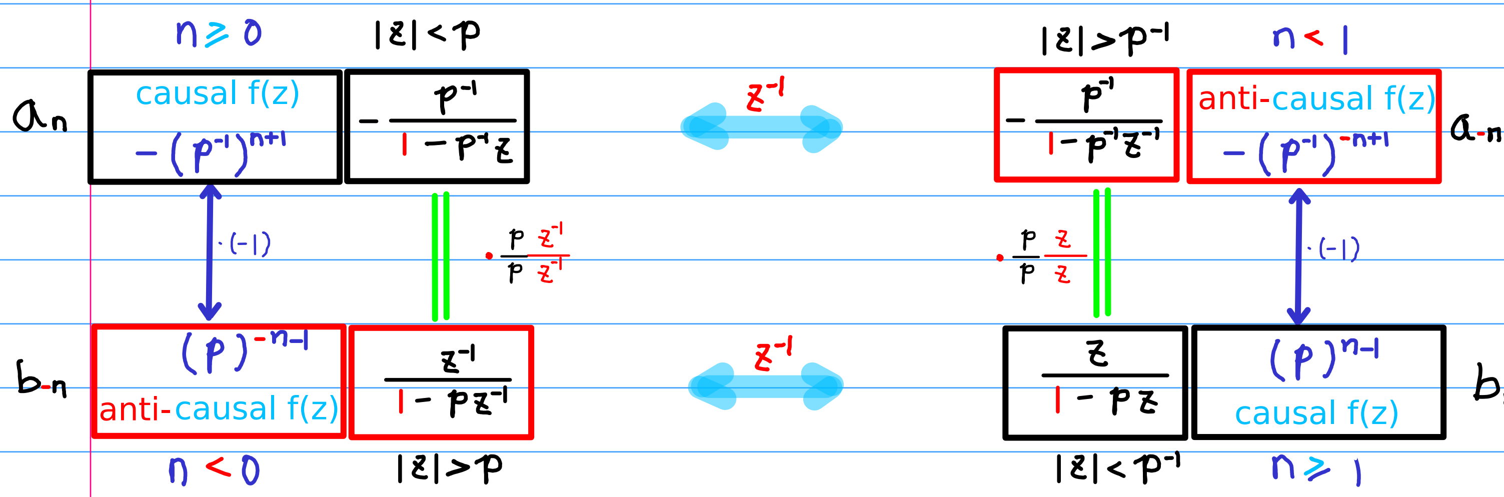
$$\begin{tikzcd}
\boxed{\begin{array}{ll} f(z) & (|z|<p) \\ a_n & (n \geq 0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{array}{ll} f(z^{-1}) & (|z|>p^{-1}) \\ a_{-n} & (n<1) \end{array}} \arrow[d, equal] \\
\boxed{\begin{array}{ll} f(z) & (|z|>p) \\ -a_n & (n<0) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{ll} f(z^{-1}) & (|z|<p^{-1}) \\ -a_{-n} & (n \geq 1) \end{array}}
\end{tikzcd}$$

$a_n = x_{-n}$ $\qquad\qquad$ $a_{-n} = x_n$

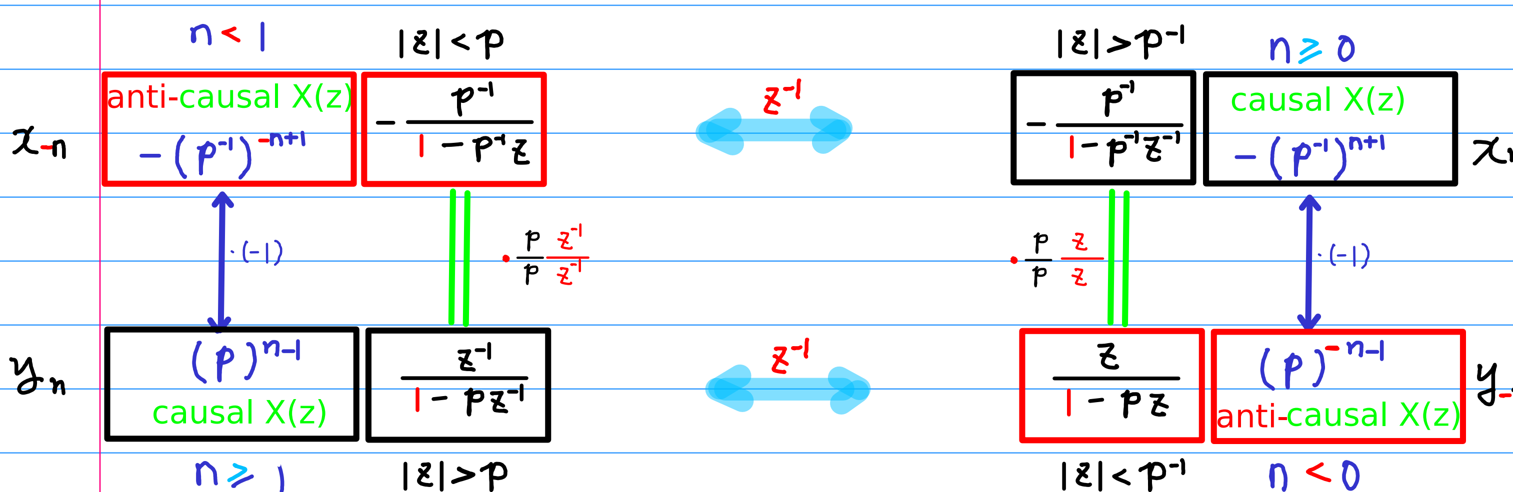
$$\begin{tikzcd}
\boxed{\begin{array}{ll} X(z^{-1}) & (|z|<p) \\ x_{-n} & (n<1) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{array}{ll} X(z) & (|z|>p^{-1}) \\ x_n & (n \geq 0) \end{array}} \arrow[d, equal] \\
\boxed{\begin{array}{ll} X(z^{-1}) & (|z|>p) \\ -x_{-n} & (n \geq 1) \end{array}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{array}{ll} X(z) & (|z|<p^{-1}) \\ -x_n & (n<0) \end{array}}
\end{tikzcd}$$

Laurent Series and z-Transform Examples

with the same ROC



z-Transform



$\boxed{\text{Laurent Series}}$ and $\boxed{z\text{-Transform}}$

$$f(z) \ (|z| < p) \leftrightarrow a_n \ (n \geq 0)$$
$$X(z^{-1}) \ (|z| < p) \leftrightarrow x_{-n} \ (n < 1)$$

$$f(z^{-1}) \ (|z| > p^{-1}) \leftrightarrow a_{-n} \ (n < 1)$$
$$X(z) \ (|z| > p^{-1}) \leftrightarrow x_n \ (n \geq 0)$$

$$f(z) \ (|z| > p) \leftrightarrow -a_n \ (n < 0)$$
$$X(z^{-1}) \ (|z| > p) \leftrightarrow -x_{-n} \ (n \geq 1)$$

$$f(z^{-1}) \ (|z| < p^{-1}) \leftrightarrow -a_{-n} \ (n \geq 1)$$
$$X(z) \ (|z| < p^{-1}) \leftrightarrow -x_n \ (n < 0)$$

$f(z) \quad (|z| < \rho) \quad \longleftrightarrow \quad a_n \quad (n \geq 0)$

the same ROC

complementary ranges

$X(z^{-1}) \quad (|z| < \rho) \quad \longleftrightarrow \quad x_{-n} \quad (n < 1)$