Applications of Array Pointers (1A)

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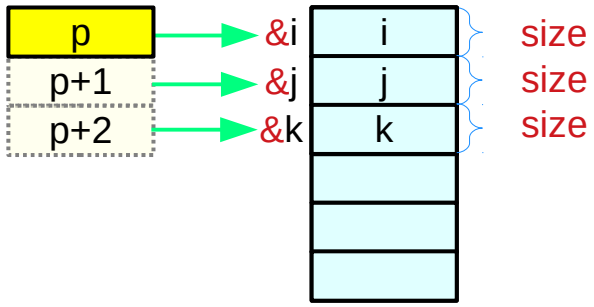
2/22/21

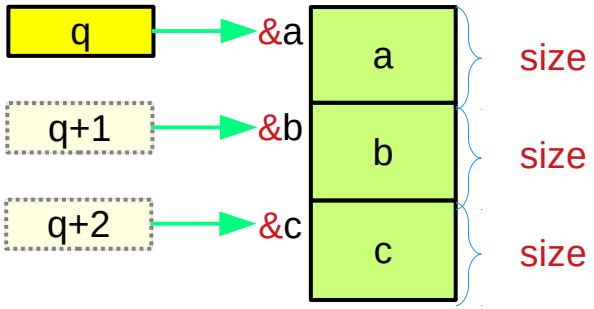
Virtual Array Pointers in Multi-dimensional Arrays

Pointers to various data types

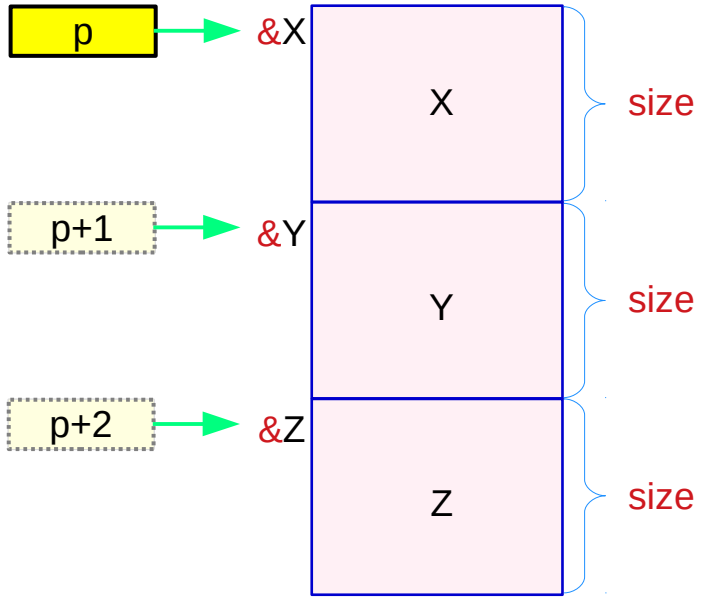
int *p; **int i, j, k;**



double *q; **double a, b, c;**



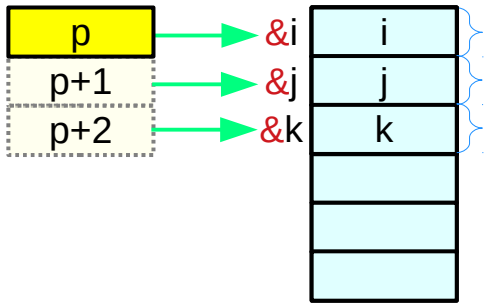
T *p; **T X, Y, Z;**



pointer abstract data

Pointers to primitive data

int *p; **int i, j, k;** **sizeof(int) = 4 bytes**



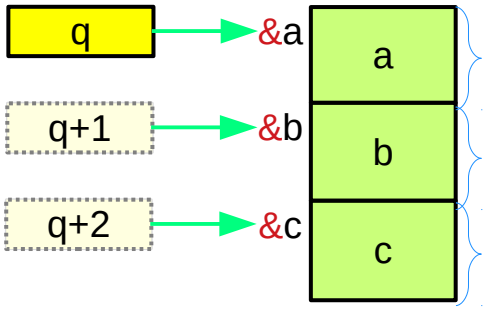
size
size
size

= sizeof(i) = sizeof(*p)
= sizeof(j) = sizeof(*(p+1))
= sizeof(k) = sizeof(*(p+2))

≠ sizeof(p)
≠ sizeof(p+1)
≠ sizeof(p+2)

pointer size
4 or 8 bytes

double *q; **double a, b, c;** **sizeof(double) = 8 bytes**



size
size
size

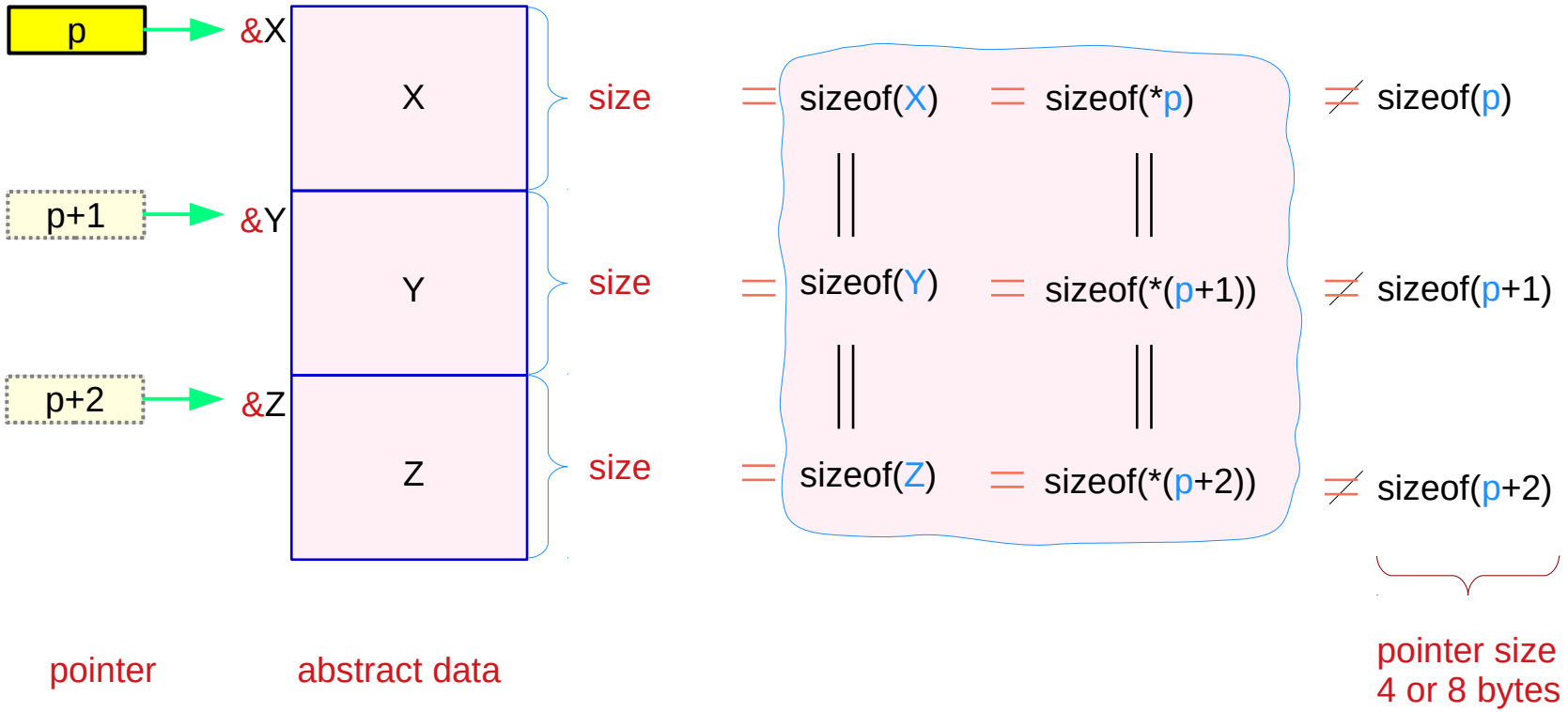
= sizeof(a) = sizeof(*q)
= sizeof(b) = sizeof(*(q+1))
= sizeof(c) = sizeof(*(q+2))

≠ sizeof(q)
≠ sizeof(q+1)
≠ sizeof(q+2)

pointer size
4 or 8 bytes

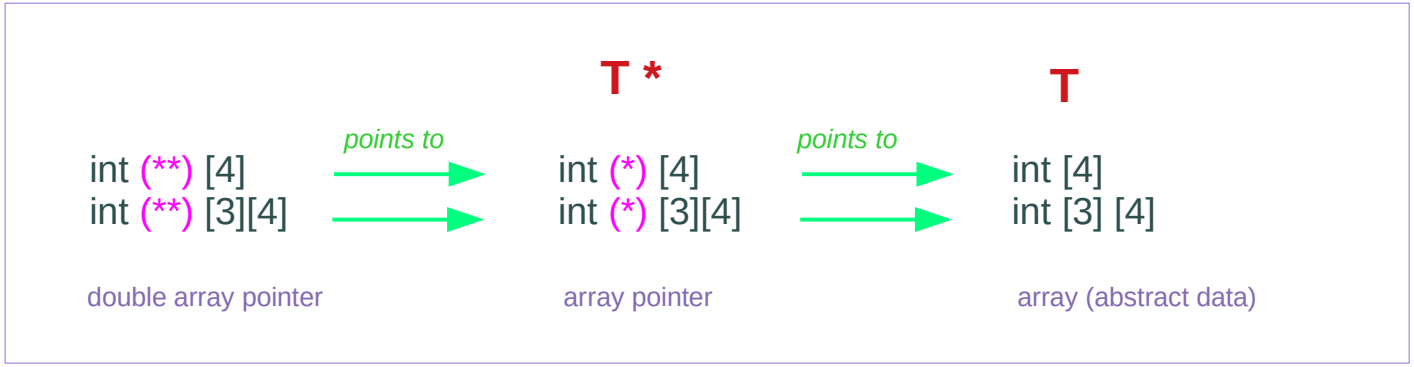
Pointers to abstract data

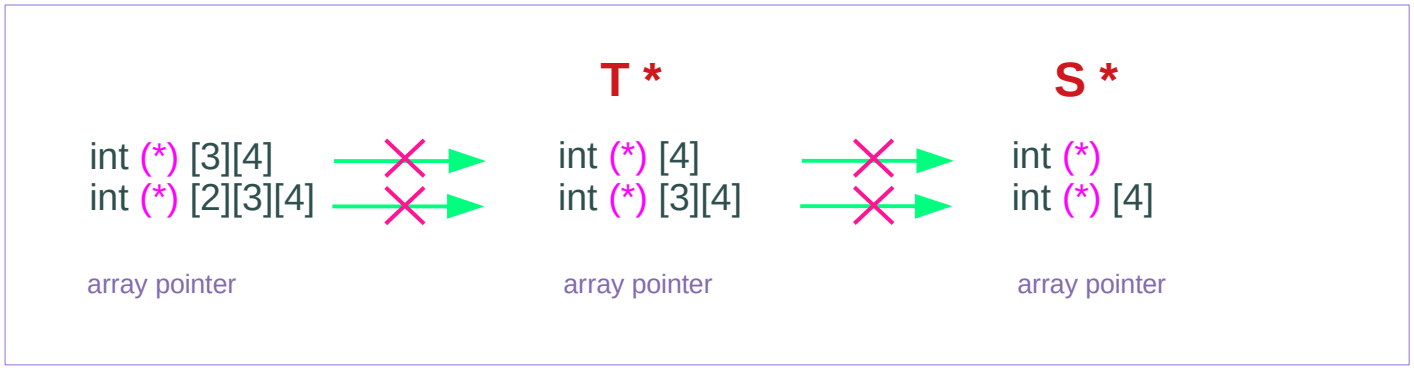
T *p; **T X, Y, Z;** **sizeof(T)**



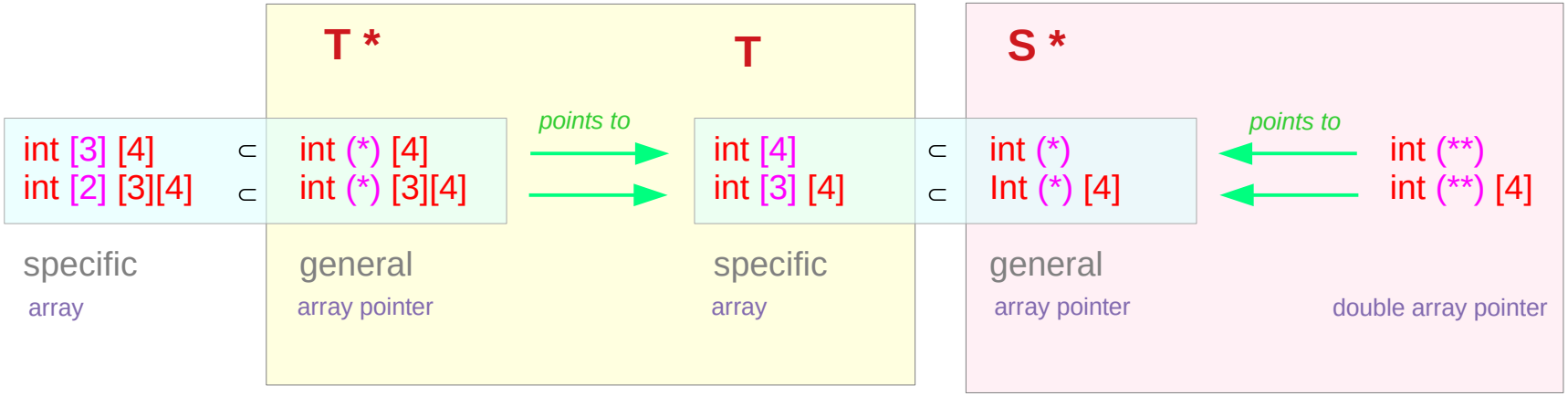
type -------- array
value -------- start address
increment size -------- size

Array pointer and array types





Specific array types v.s. general array pointer types



Array pointers have augmented dimensions

```
typedef int (*T1) [4];
typedef int (*T1) [3][4];
```

```
int (*) [4]
int (*) [3][4]
general
```

```
typedef int T2[4];
typedef int T2[3][4];
```

```
int [4]
int [3] [4]
specific
```

```
T1 a;
T2 b;
```

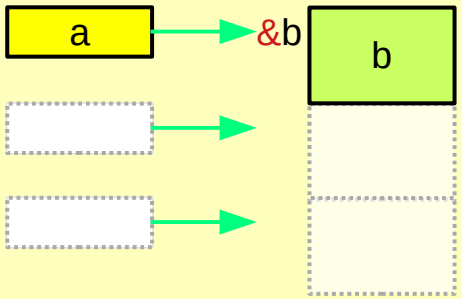
T1 is a pointer type
T2 is an array type
T1 has one more dimension than T2

```
 a = &b;
*a =  b;
```

a references **b**

b is the dereference of **a**

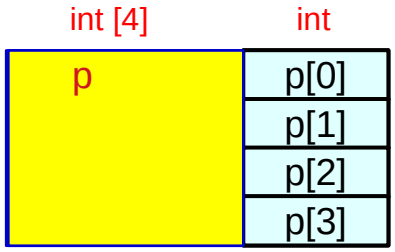
```
  (a+1) = ?
* (a+1) = ?

  (a+2) = ?
* (a+2) = ?
```



Virtual pointers in an array of integers

int p[3];

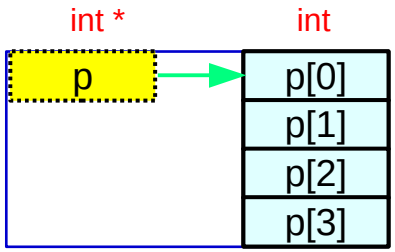
p is an abstract data (array)



p is the name of an array
p has the size of the whole array

p has an array type (abstract data)

p can also be viewed as a pointer

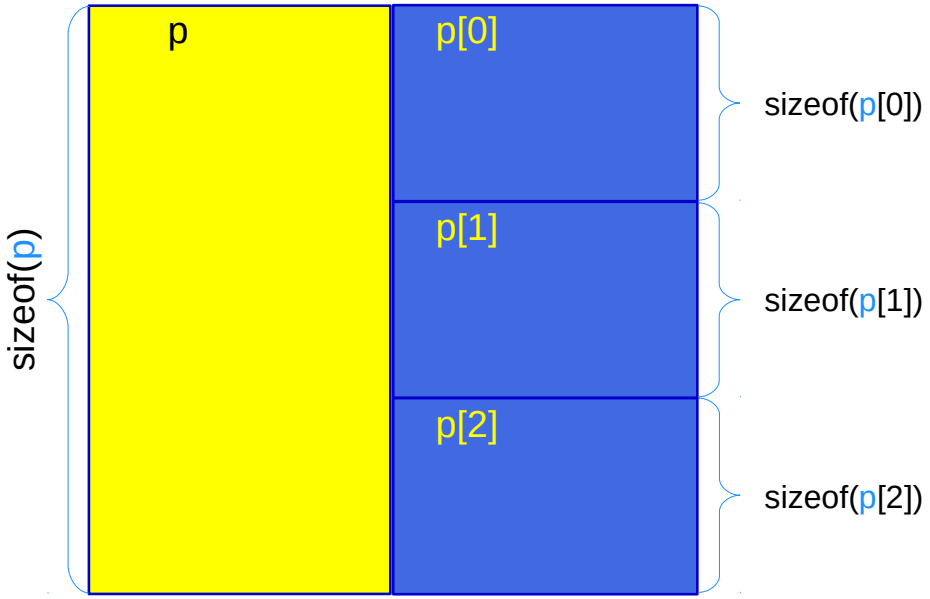


p also has pointer characteristics
p has the value of the starting address

p is a virtual pointer

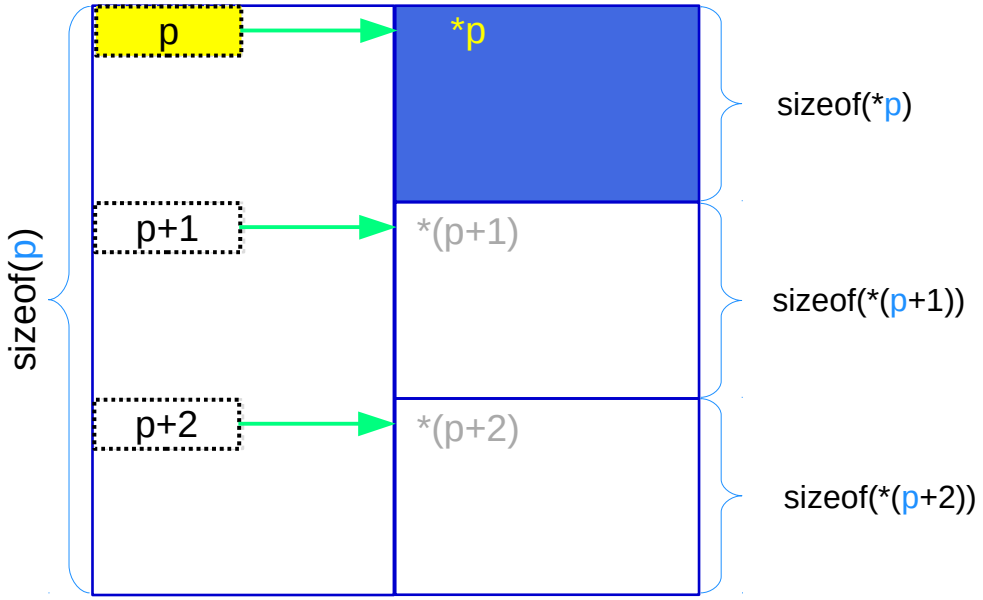
Virtual pointers in an array of abstract data

Abstract data array p



p has an array type (abstract data element)
p is the name of an array
p has the size of the whole array

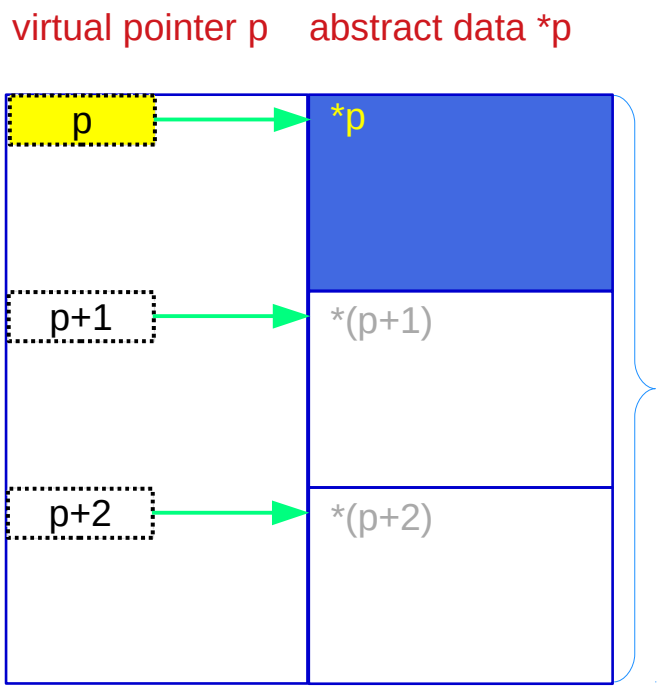
Virtual pointer p

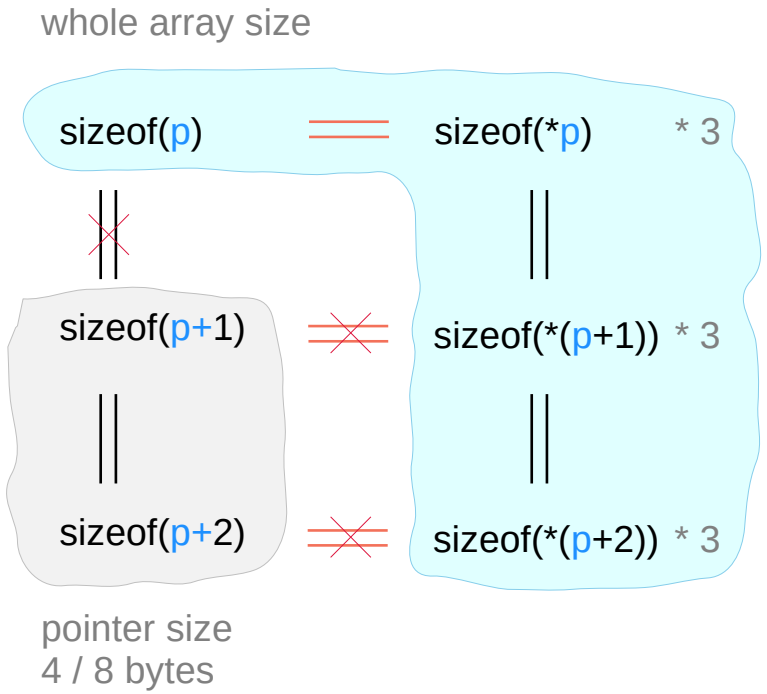


p also has a pointer type
p has the value of the starting address

p is a virtual array pointer

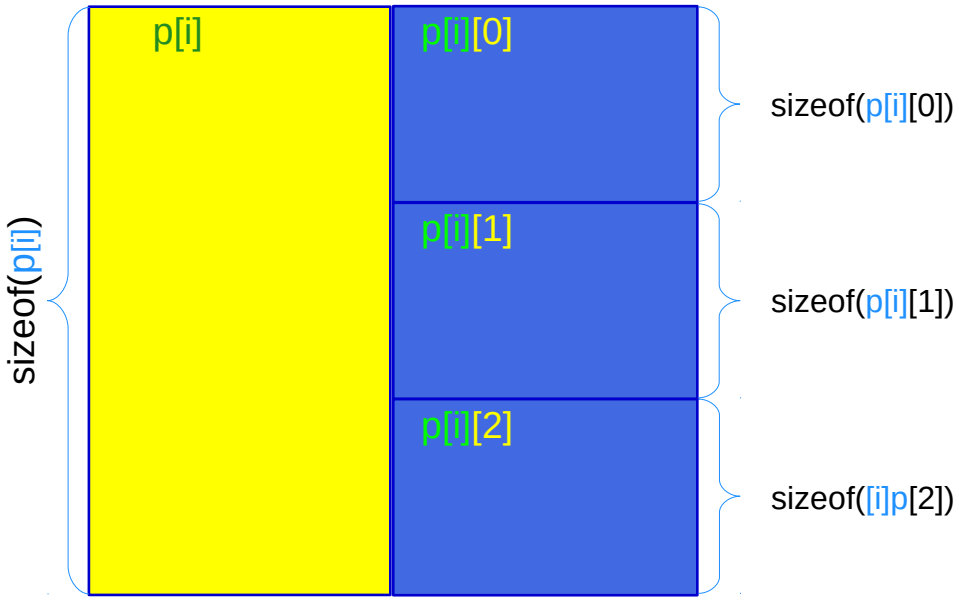
Virtual pointer to abstract data





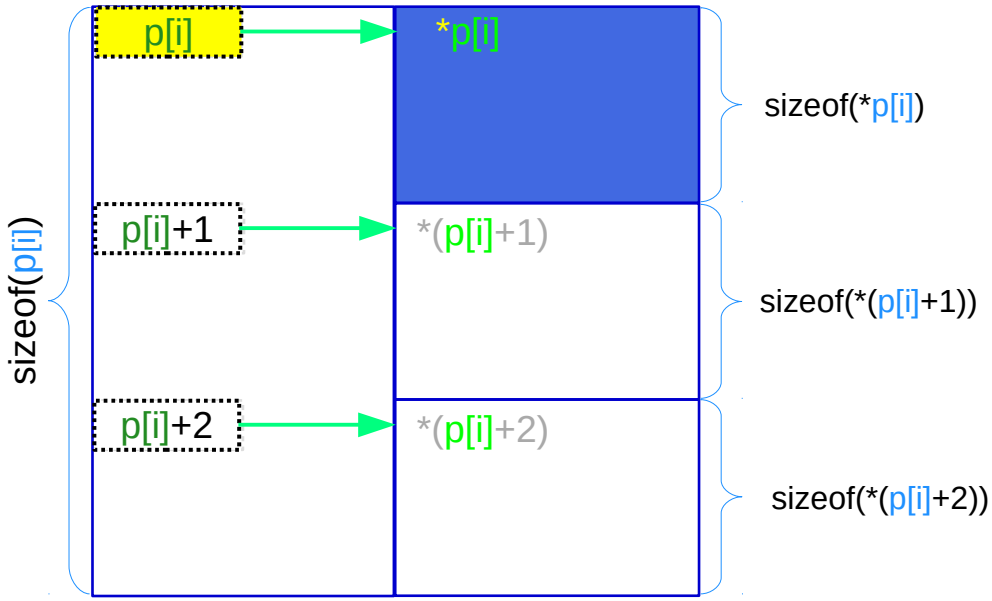
Virtual array pointers in a multi-dimensional array

Abstract data (array) p[i]



p[i] has an array type (abstract data)
p[i] is the name of an array
p[i] has the size of the whole array

Virtual array pointer p[i]



p[i] also has an array pointer type
p[i] has the value of the starting address

p[i] is a virtual array pointer

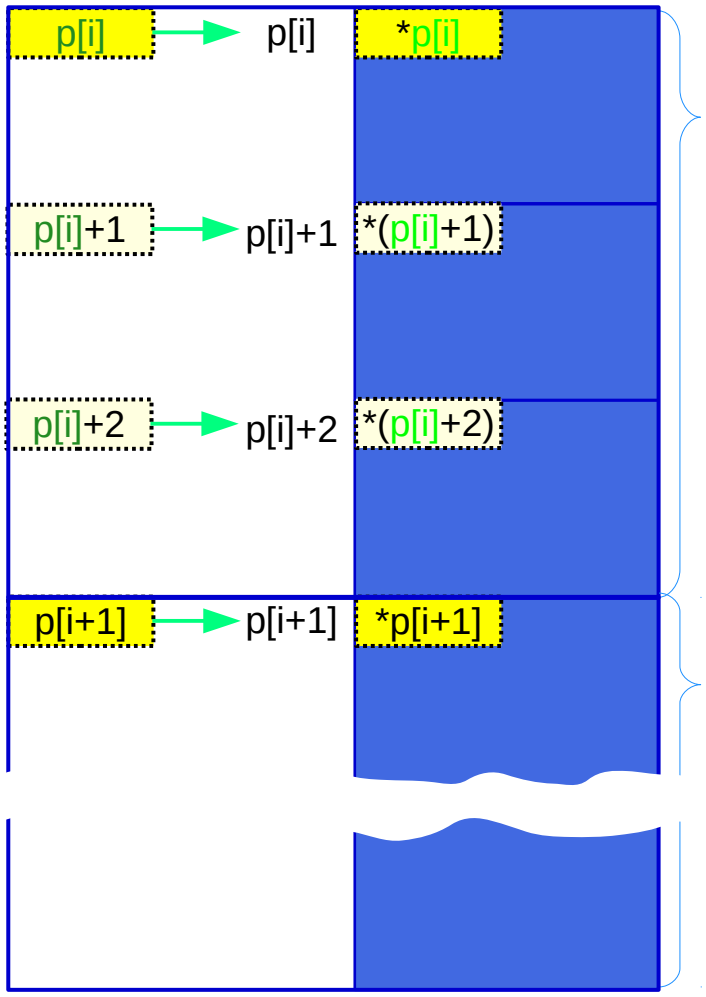
Virtual pointers to a sub array

p[i] :: T *

array pointer type

***p[i], *p[i+1] :: T**

array type



sizeof(p[i])

= sizeof(*p[i]) * N
sizeof(p[i][0]) * N

sizeof(p[i+1])

= sizeof(*p[i+1]) * N
sizeof(p[i+1][0]) * N

size = sizeof(*p[i]) =
sizeof(p[i][0])

||

size = sizeof(*(p[i]+1)) =
sizeof(p[i][1])

||

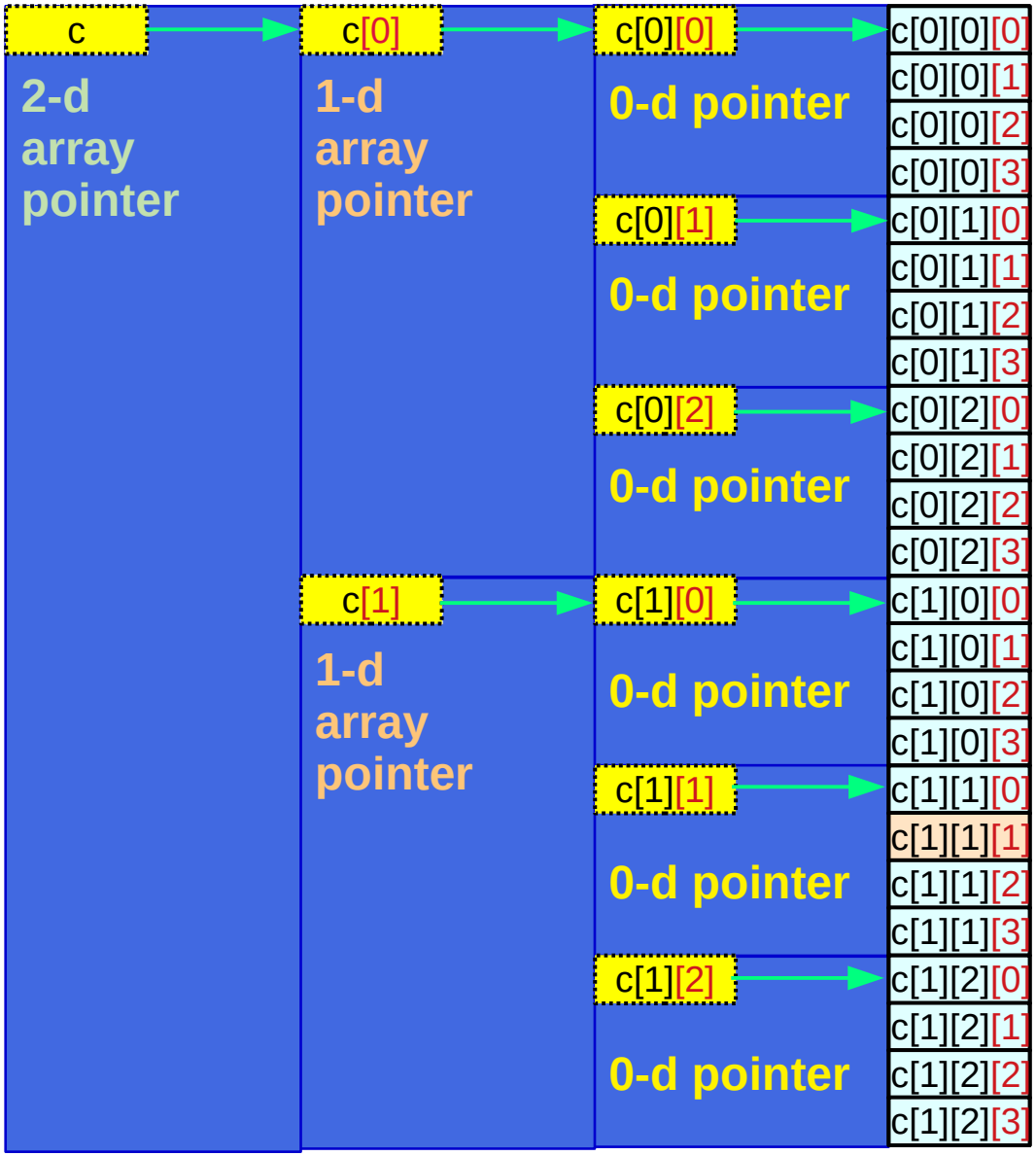
size = sizeof(*(p[i]+2)) =
sizeof(p[i][2])

3-d array structure – pointer representation

```
int c[2][3][4];
```

```
*(*(*(c +i) +j) +k)
```

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order



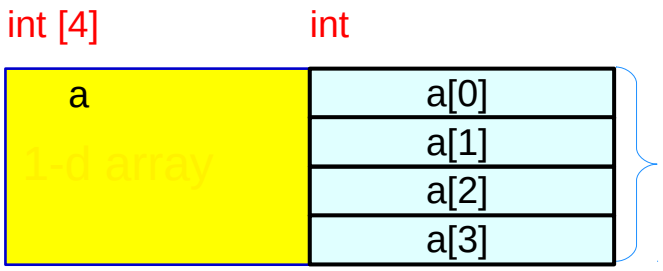
3-d array structure – abstract data representation

int c[2][3][4];

((c [i])[j])[k]

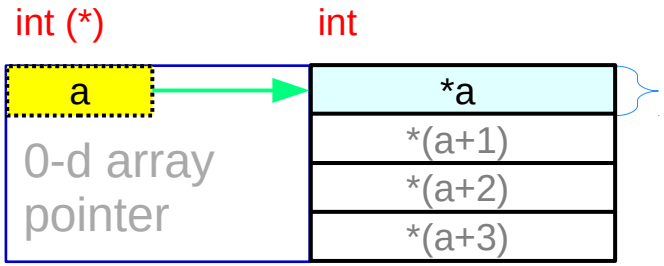
- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

Array **a** and pointer **a**



1-d array a specific array type

sizeof(a)



pointer a general pointer type

sizeof(a) = sizeof(*a) * 4

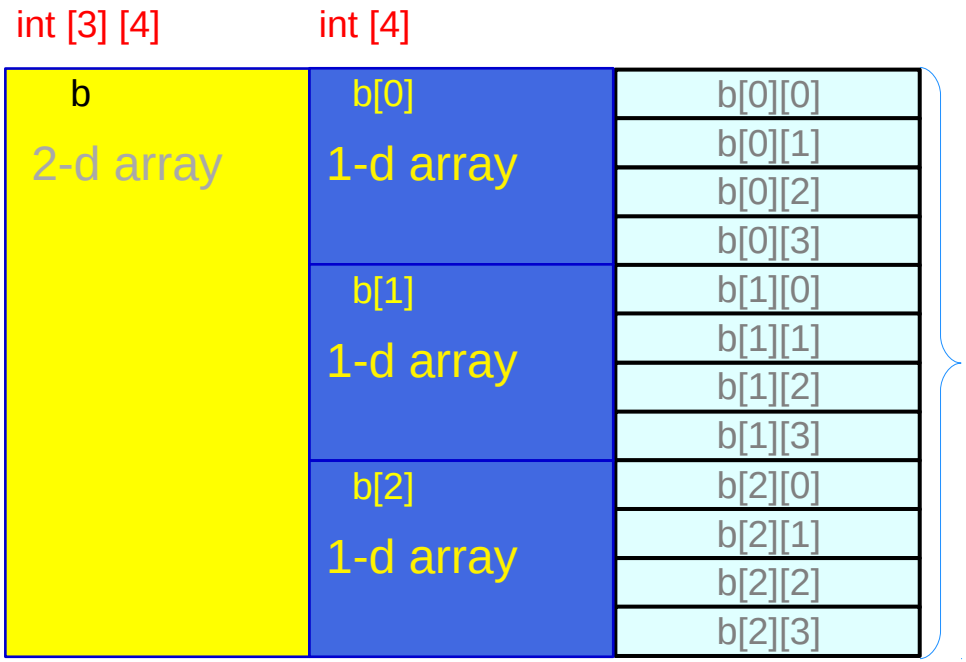
a is the name of a 1-d array
a also has a pointer type
a has the size of the array
a has the value of the starting address

a is a virtual array pointer

Array **b** and pointer **b**

2-d array b specific array type

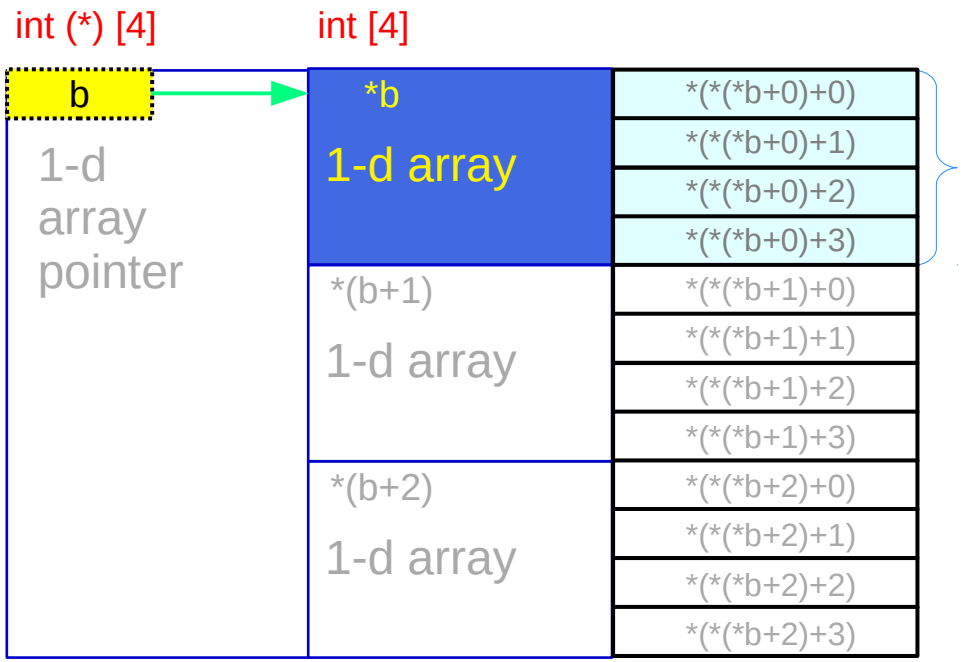
sizeof(b)



b is the name of a 2-d array
b has the size of the array

1-d array pointer b general pointer type

sizeof(b) = sizeof(*b) * 3



b also has a 1-d array pointer type
b has the value of the starting address

b is a virtual array pointer

Array **c**

3-d array c

specific array type

sizeof(c)

c is the name of a 3-d array
c has the size of the array

Pointer **c**

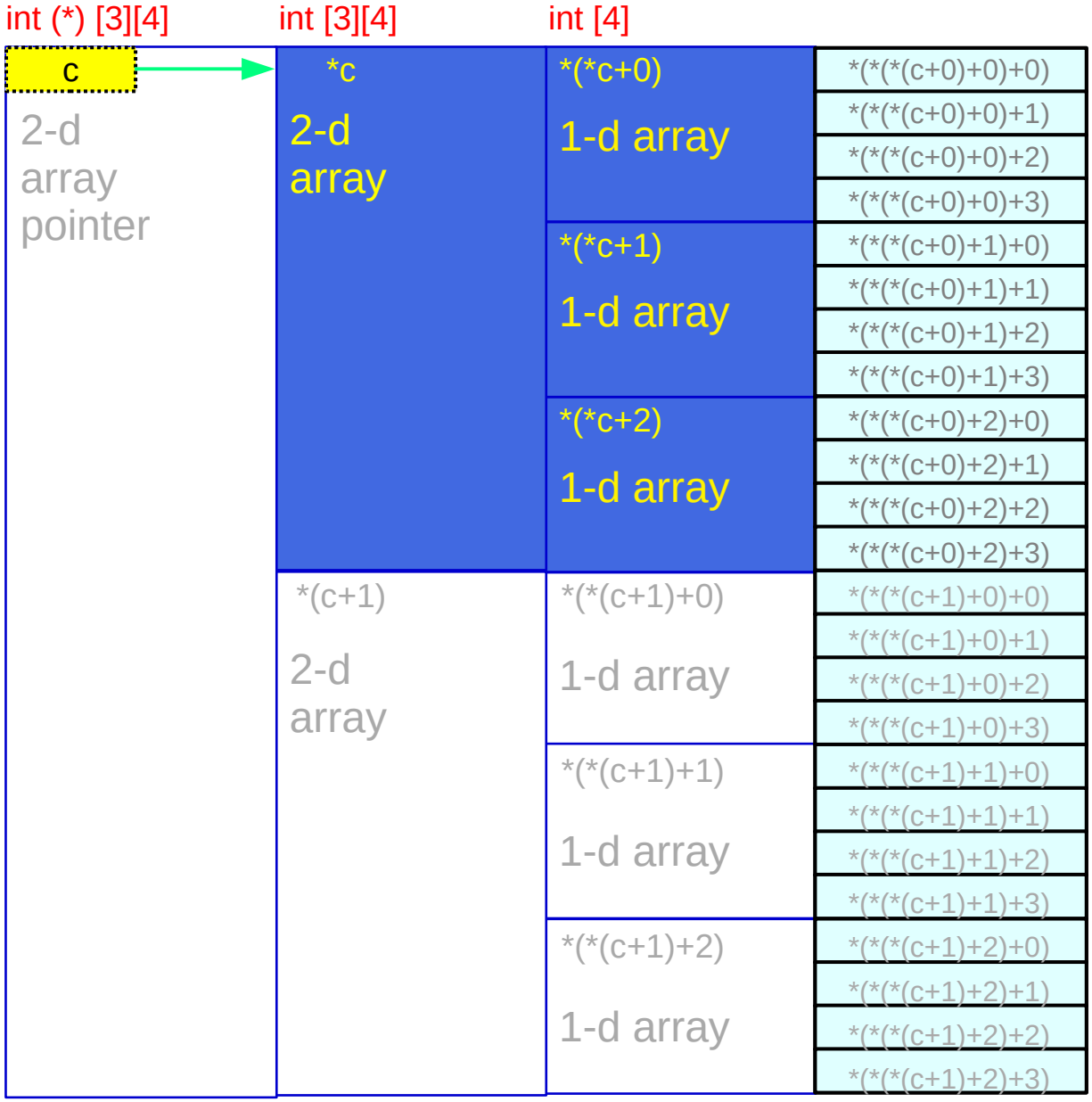
2-d array pointer c

general pointer type

sizeof(c) = sizeof(*c) * 2

c also has a 2-d array pointer type
c has the value of the starting address

c is a virtual array pointer



Types of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

c[i][j][k]	int	int
c[i][j] [k]	int [4] [k]	int (*) [k]
c[i] [j] [k]	int [3][4] [j] [k]	int (*)[4] [j] [k]
c [i] [j] [k]	int [2][3][4] [i] [j] [k]	int (*)[3][4] [i] [j] [k]
	array type (name)	**array pointer type**

Sizes of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

sizeof(**c[i][j][k]**) = sizeof(int)

sizeof(**c[i][j]**) = sizeof(int) * 4
[k] [k]

sizeof(**c[i]**) = sizeof(int) * 3 * 4
[j] [k] [j] [k]

sizeof(**c**) = sizeof(int) * 2 * 3 * 4
[i] [j] [k] [i] [j] [k]

Address values of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

c[i][j][k] = **&c[i][j][k]**

c[i][j]+k = **&c[i][j][0]** + **k** * sizeof(**c[i][j][k]**)

sizeof(***c[i][j]**)
= sizeof(**c[i][j][0]**) = sizeof(int)

c[i]+j = **&c[i][0][0]** + **j** * sizeof(**c[i][j]**)

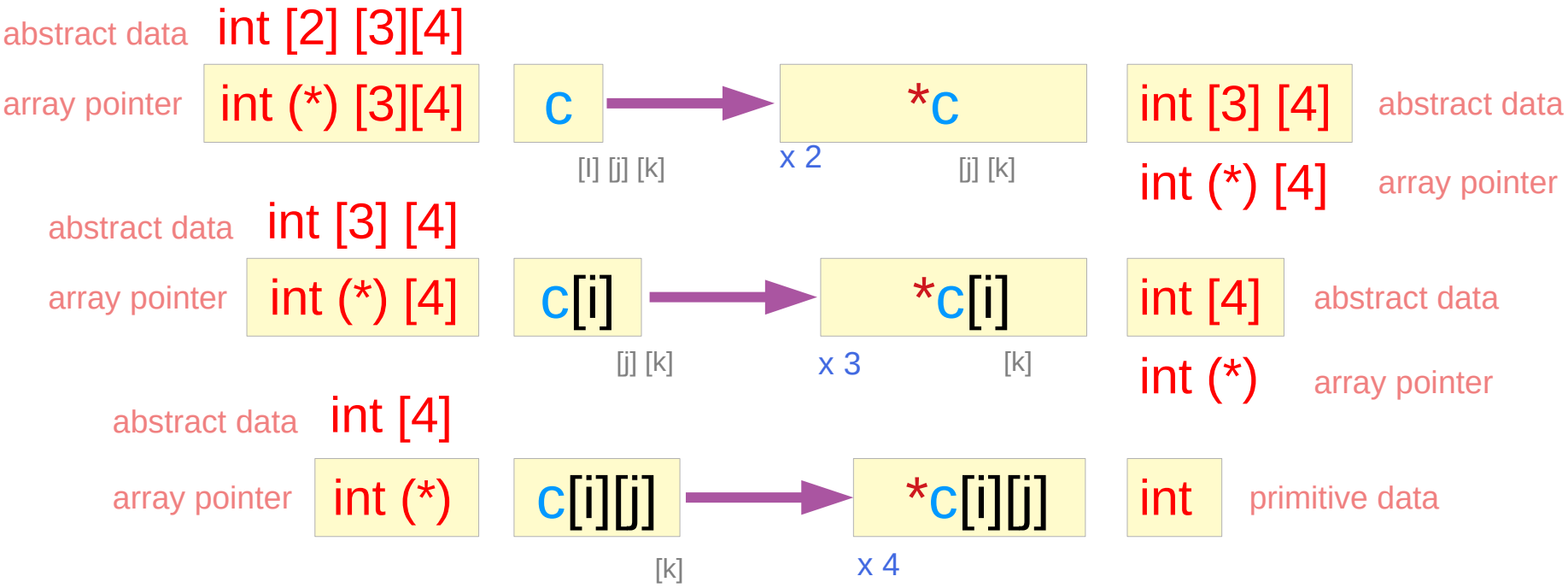
sizeof(***c[i]**)
= sizeof(**c[i][0]**) = sizeof(int) * 4
[k]

c+i = **&c[0][0][0]** + **i** * sizeof(**c[i]**)

sizeof(***c**)
= sizeof(**c[0]**) = sizeof(int) * 3 * 4
[j] [k]

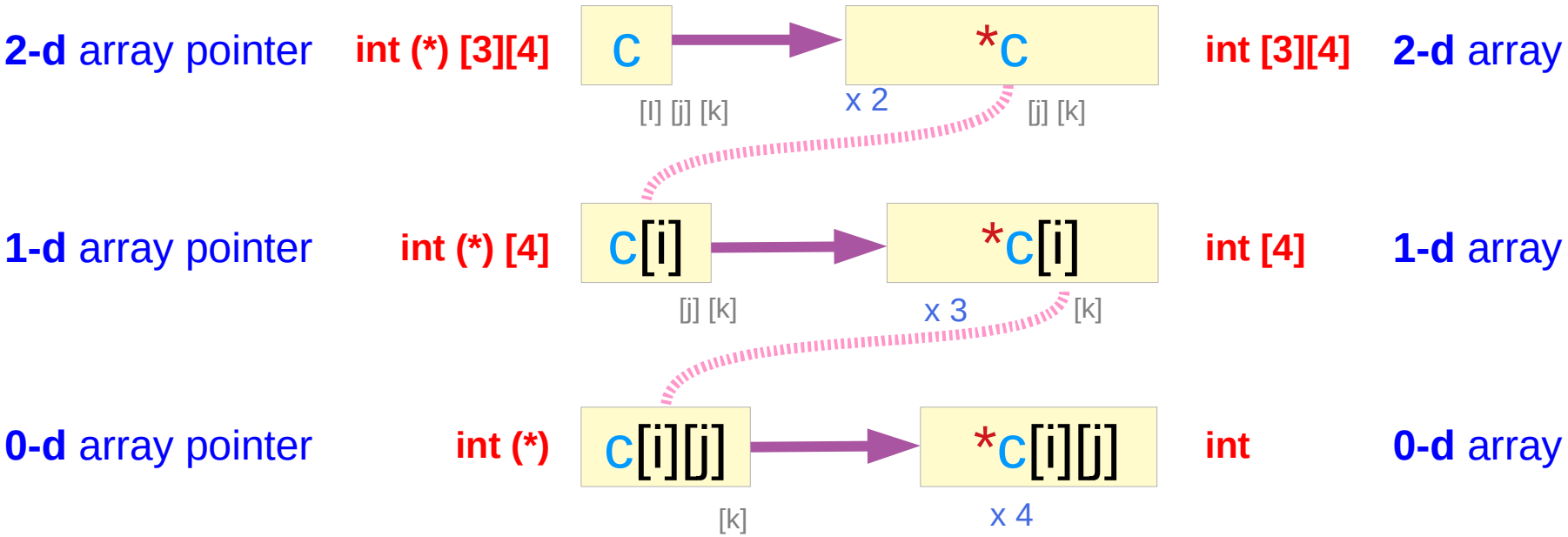
Types in a multi-dimensional **3-d** array

```
int     c [2][3][4];
```



Virtual array pointers and abstract data

```
int     c [2][3][4];
```

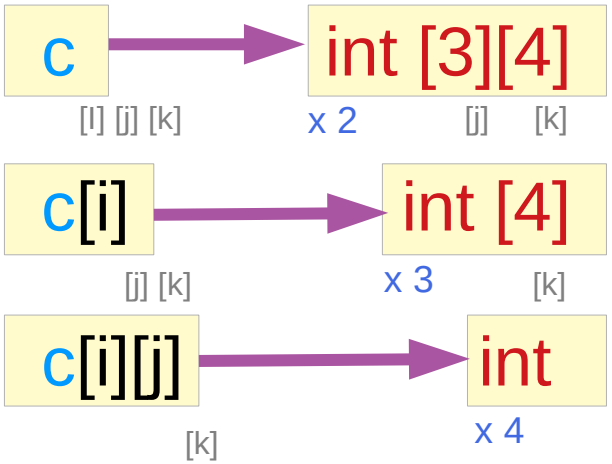


all these pointers are virtual, and take no actual memory locations

exploiting the **contiguity** of allocated memory locations

Abstract data sizes

```
int     c [2][3][4];
```



c, c[i], c[i][j] are virtual array pointers
and they are also abstract data (arrays)

when sizes are considered,
view them as abstract data (arrays)

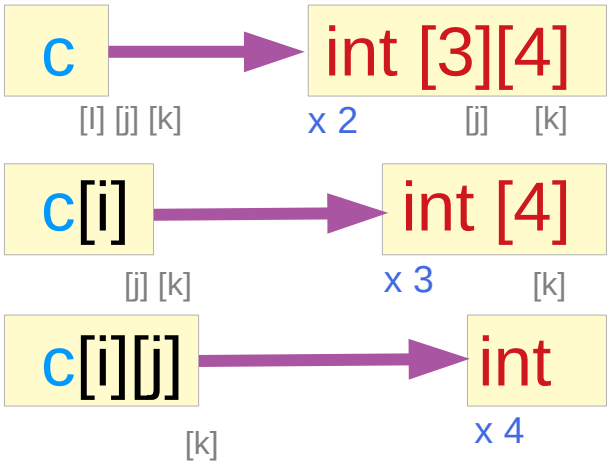
the size of a pointer type is fixed
Here, the sizes of virtual pointers are shown
i.e, the sizes of different abstract data types

sizeof(c)	=	sizeof(int [**2**][3][4])
sizeof(*c)	=	sizeof(int [3][4])
sizeof(c[i])	=	sizeof(int [**3**][4])
sizeof(*c[i])	=	sizeof(int [4])
sizeof(c[i][j])	=	sizeof(int [**4**])
sizeof(*c[i][j])	=	sizeof(int)

all are sizes of arrays

Virtual array pointer sizes and abstract data sizes

```
int     c [2][3][4];
```



size of a virtual array pointer	=	size of the pointed abstract data type	*	the number of such types

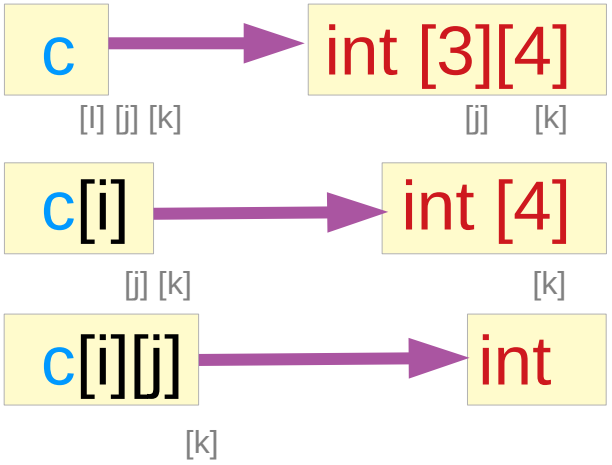
sizeof(c) = sizeof(*c) * **2**

sizeof(c[i]) = sizeof(*c[i]) * **3**

sizeof(c[i][j]) = sizeof(*c[i][j]) * **4**

Sizes of array pointer types

```
int     c [2][3][4];
```



not real array pointers
virtual array pointers

c int (*)[3][4] = sizeof(c)
sizeof(int (*) [3][4]) = pointer size ≠ sizeof(c)

c[i] int (*) [4] = sizeof(c[i])
sizeof(int (*) [4]) = pointer size ≠ sizeof(c[i])

c[i][j] int [4] = sizeof(c[i][j])
sizeof(int [4]) = pointer size ≠ sizeof(c[i][j])

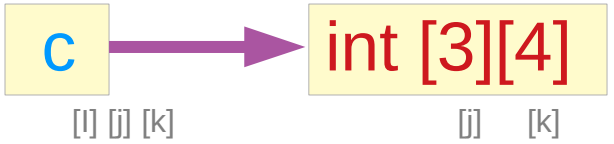
4 bytes for 32-bit machines
8 bytes for 64-bit machines

Virtual array pointer increment size

```
int     c [2][3][4];
```

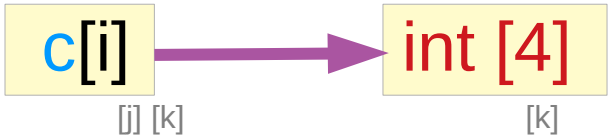
c points to a **2-d** array
increment size: sizeof(int[2][3][4])

int (*) [3][4]



c[i] points to an **1-d** array
increment size: sizeof(int[3][4])

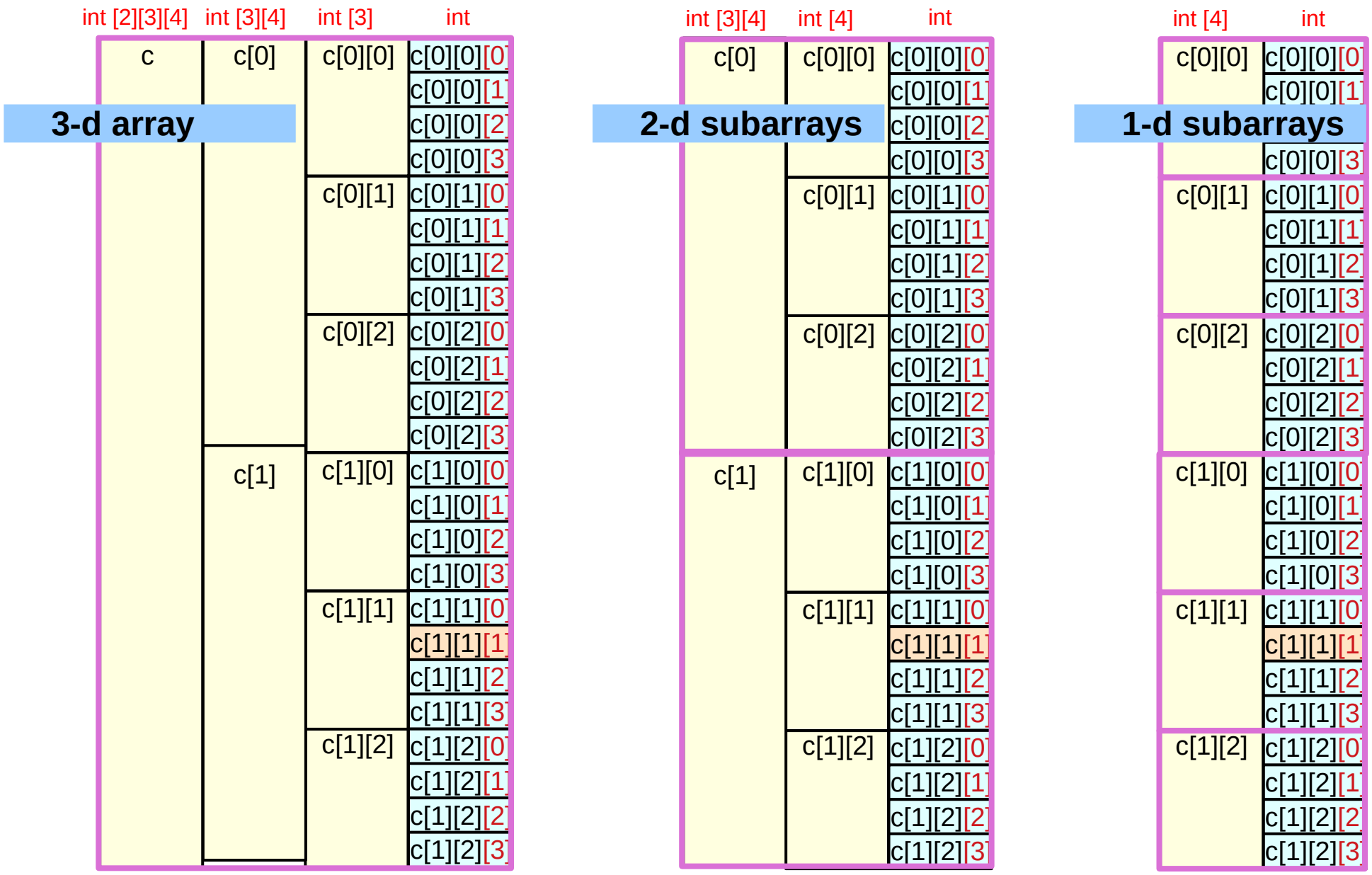
int (*) [4]



c[i][j] points to an integer
increment size: sizeof(int[4])

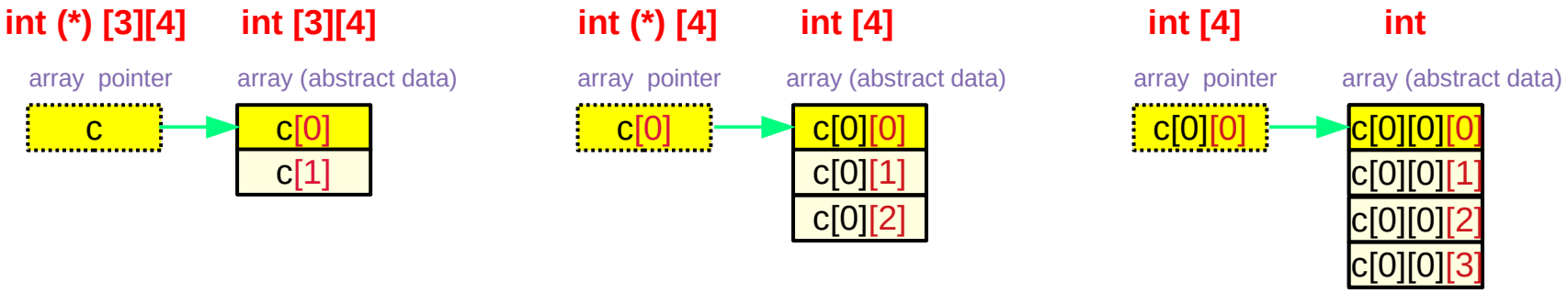
int (*)

Subarrays in a 3-d array



Virtual array pointer c, c[0], c[0][0] – types and sizes

Types – array pointers



Sizes – abstract data

sizeof(**c**)
sizeof(**int [2][3][4]**)
sizeof(**int**) * **2** * **3** * **4**

sizeof(**int [2][3][4]**) = 96
sizeof(**int (*)[3][4]**) = 4 / 8

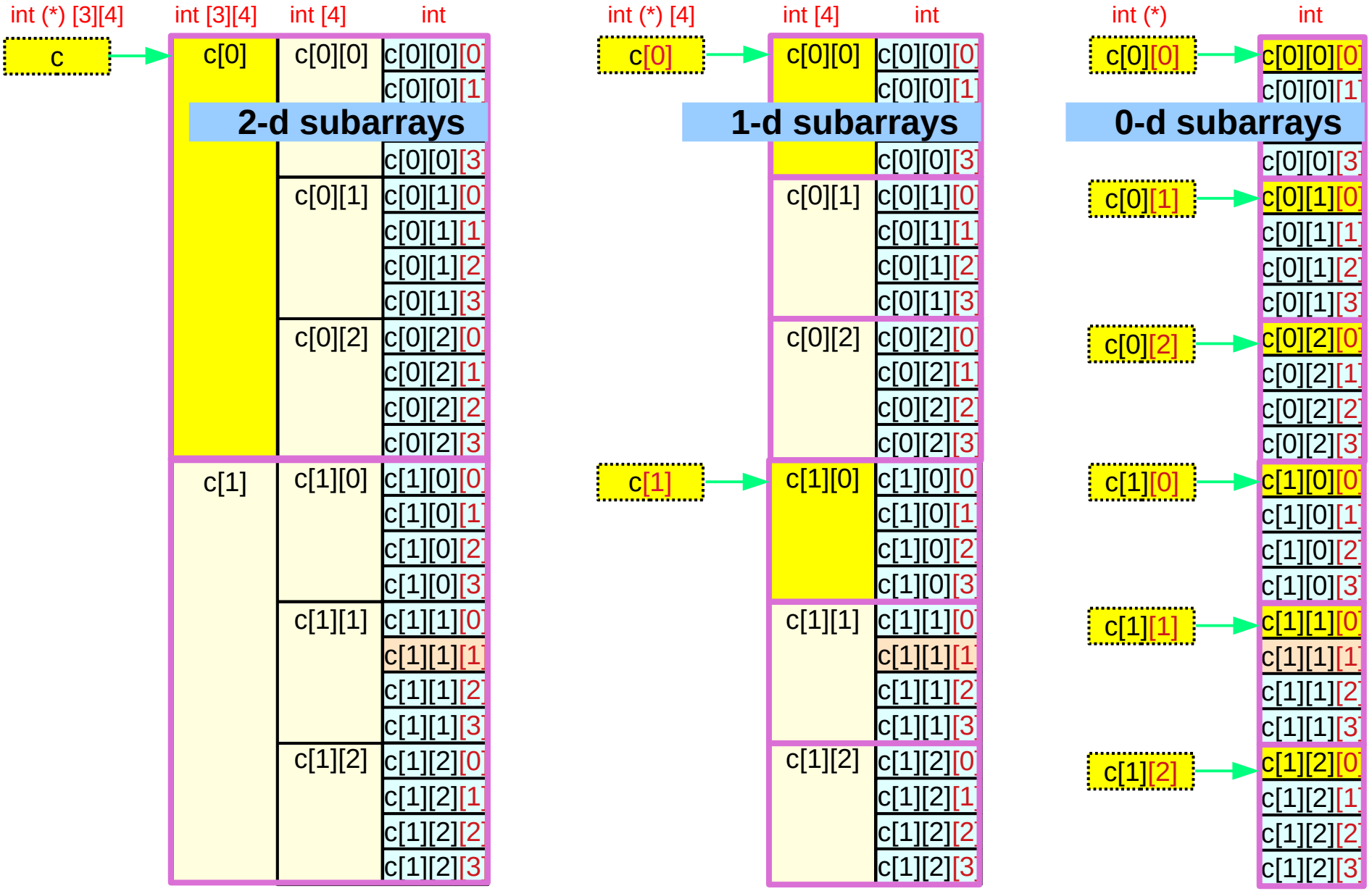
sizeof(**c[0]**)
sizeof(**int [3][4]**)
sizeof(**int**) * **3** * **4**

sizeof(**int [3][4]**) = 48
sizeof(**int (*)[4]**) = 4 / 8

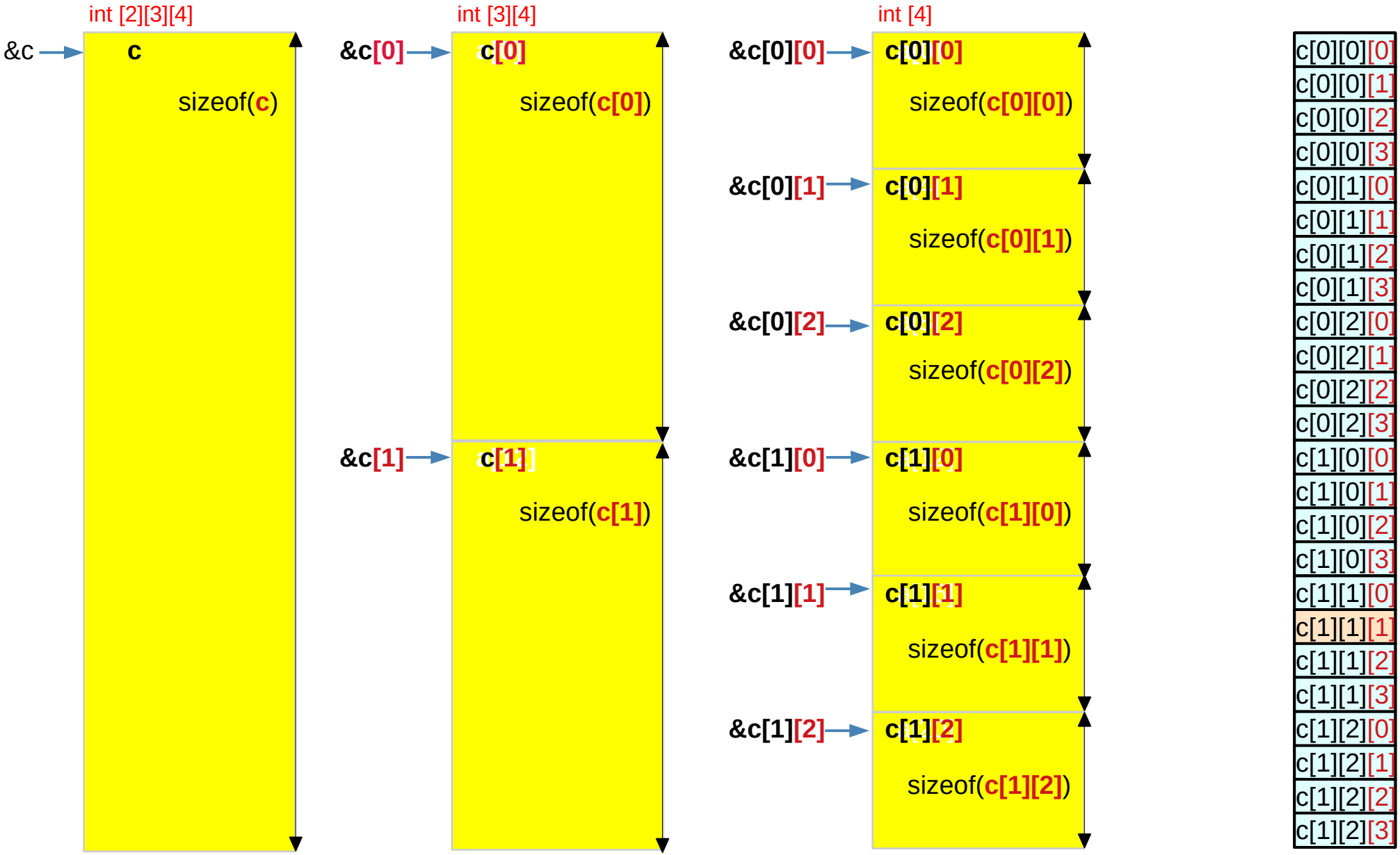
sizeof(**c[0][0]**)
sizeof(**int [4]**)
sizeof(**int**) * **4**

sizeof(**int [4]**) = 16
sizeof(**int (*)**) = 4 / 8

Pointers to subarrays in a 3-d array



Abstract Data c, c[i], c[i][j] – start addresses and sizes



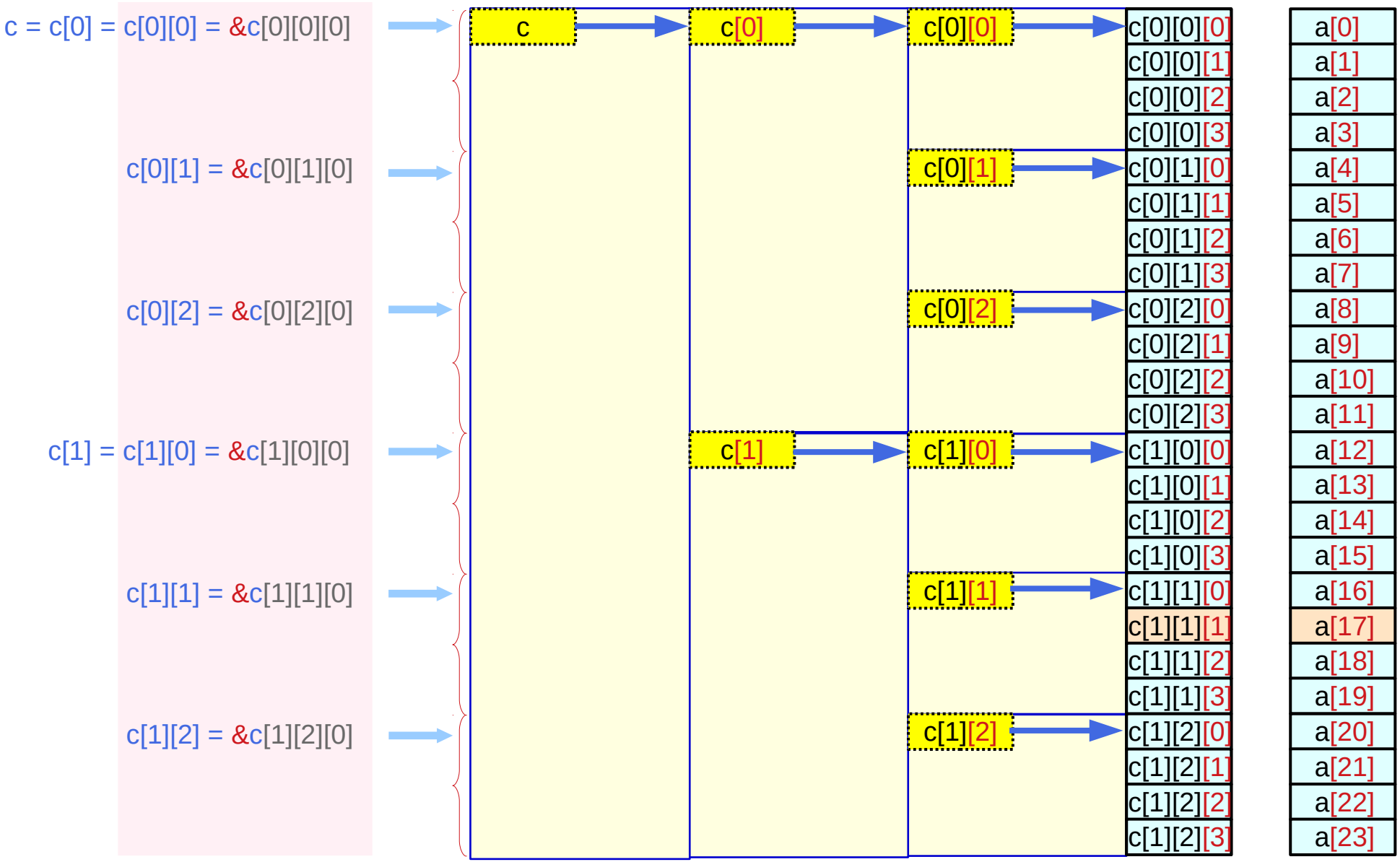
Virtual array pointers – types, sizes, and values

int c[2][3][4];	c[i][j]	c[i][j][0]	
type	int [4] int (*)	int int	• abstract data type • array pointer type
size	sizeof(c[i][j]) =	sizeof(c[i][j][0]) * 4	= sizeof(int) * 4
value (address)	c[i][j] =	&c[i][j][0]	

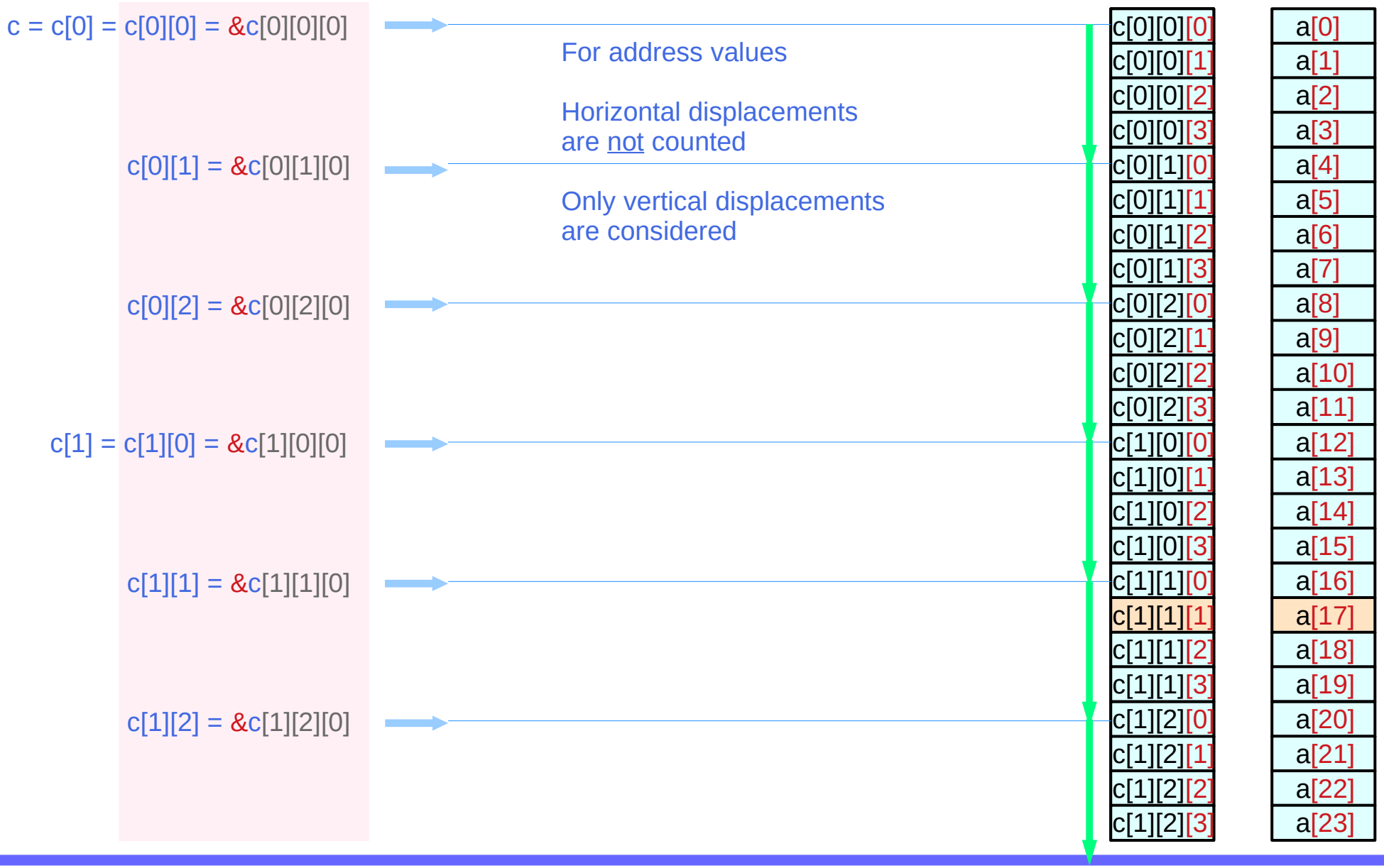
int c[2][3][4];	c[i]	c[i][0]	
type	int [3][4] int (*)[4]	int [4] int (*)	• abstract data type • array pointer type
size	sizeof(c[i]) =	sizeof(c[i][0]) * 3	= sizeof(int) * 4 * 3
value (address)	c[i] =	&c[i][0][0]	

int c[2][3][4];	c	c[0]	
type	int [2][3][4] int (*)[3][4]	int [3][4] int (*)[4]	• abstract data type • array pointer type
size	sizeof(c) =	sizeof(c[0]) * 2	= sizeof(int) * 4 * 3 * 2
value (address)	c =	&c[0][0][0]	

Virtual array pointer c, c[i], c[i][j] – values (addresses)



Virtual array pointer c, c[i], c[i][j] – vertical displacement



Virtual array pointer c, c[i], c[i][j] – values and types

c = c[0] = c[0][0] = &c[0][0][0]	means	value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0])	
		type(c) ≠ type(c[0]) ≠ type(c[0][0]) = type(&c[0][0][0])	
		int (*) [3][4] int (*) [4] int * int *	
c[0][1] = &c[0][1][0]	means	value(c[0][1]) = value(&c[0][1][0])	
		type(c[0][1]) = type(&c[0][1][0])	
		int * int *	
c[0][2] = &c[0][2][0]	means	value(c[0][2]) = value(&c[0][2][0])	
		type(c[0][2]) = type(&c[0][2][0])	
		int * int *	
c[1] = c[1][0] = &c[1][0][0]	means	value(c[1]) = value(c[1][0]) = value(&c[1][0][0])	
		type(c[1]) ≠ type(c[1][0]) = type(&c[1][0][0])	
		int (*) [4] int * int *	
c[1][1] = &c[1][1][0]	means	value(c[1][1]) = value(&c[1][1][0])	
		type(c[1][1]) = type(&c[1][1][0])	
		int * int *	
c[1][2] = &c[1][2][0]	means	value(c[1][2]) = value(&c[1][2][0])	
		type(c[1][2]) = type(&c[1][2][0])	
		int * int *	

Summary of virtual array pointers in a 3-d array

c[i] ≡ *(c +i)	c[i][j] ≡ *(c[i] +j)	c[i][j][k]≡ *(c[i][j] +k)
int (*) [3][4] 2-d array pointer c int [2] [3][4] 3-d array name c	int (*) [4] 1-d array pointers c[i] Int [3] [4] 2-d array names c[i]	int (*) 0-d array pointers c[i][j] int [4] 1-d array names c[i][j]
address value c + i &c[0][0][0] + i * sizeof(*c) &c[0][0][0] + i * sizeof(c[0]) &c[0][0][0] + i * 4 * 3 * 4	**address value** c[i] + j &c[i][0][0] + j * sizeof(*c[i]) &c[i][0][0] + j * sizeof(c[i][0]) &c[i][0][0] + j * 4 * 4	**address value** c[i][j] + k &c[i][j][0] + k * sizeof(*c[i][j]) &c[i][j][0] + k * sizeof(c[i][j][0]) &c[i][j][0] + k * 4
leading elements c[0][0][0]	**leading elements** c[0][0][0] c[1][0][0]	**leading elements** c[0][0][0] c[0][1][0] c[0][2][0] c[1][0][0] c[1][1][0] c[1][2][0]

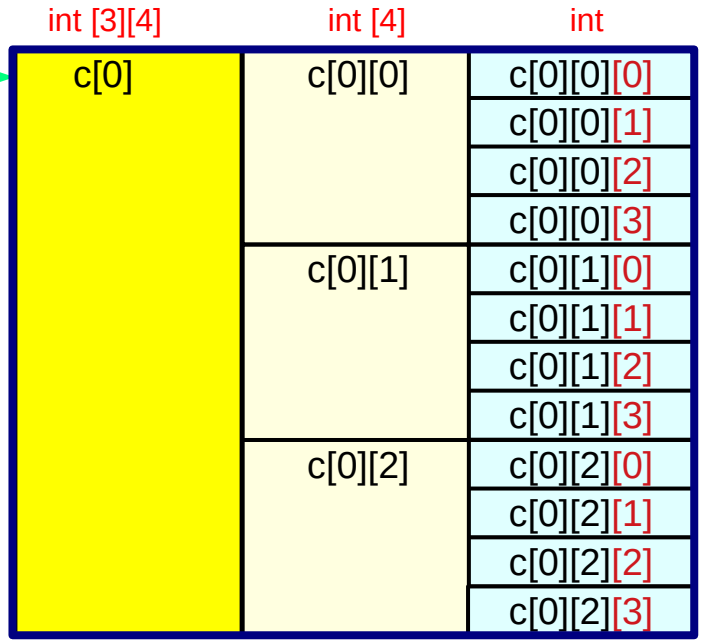
Dual type constraints in multi-dimensional arrays

Virtual pointers to subarrays in a 3-d array

virtual 2-d array pointer

sizeof(**c**) =
sizeof(**c[0]**) * **2**

```
int     c [2][3][4];
```

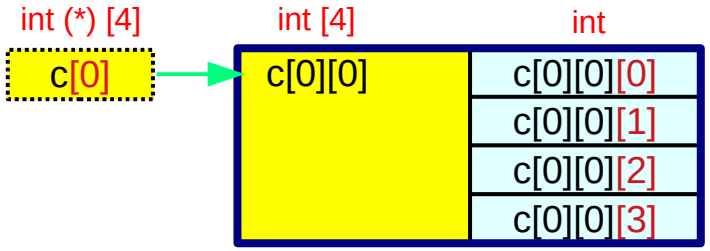


the first 2-d subarray

sizeof(**c[0]**) =
sizeof(**int [3][4]**)

virtual 1-d array pointer

sizeof(**c[0]**) =
sizeof(**c[0][0]**) * **3**

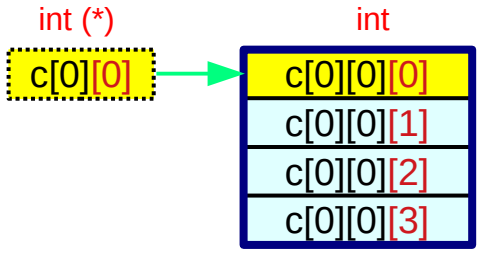


the first 1-d subarray

sizeof(**c[0][0]**) =
sizeof(**int [4]**)

virtual 0-d array pointer

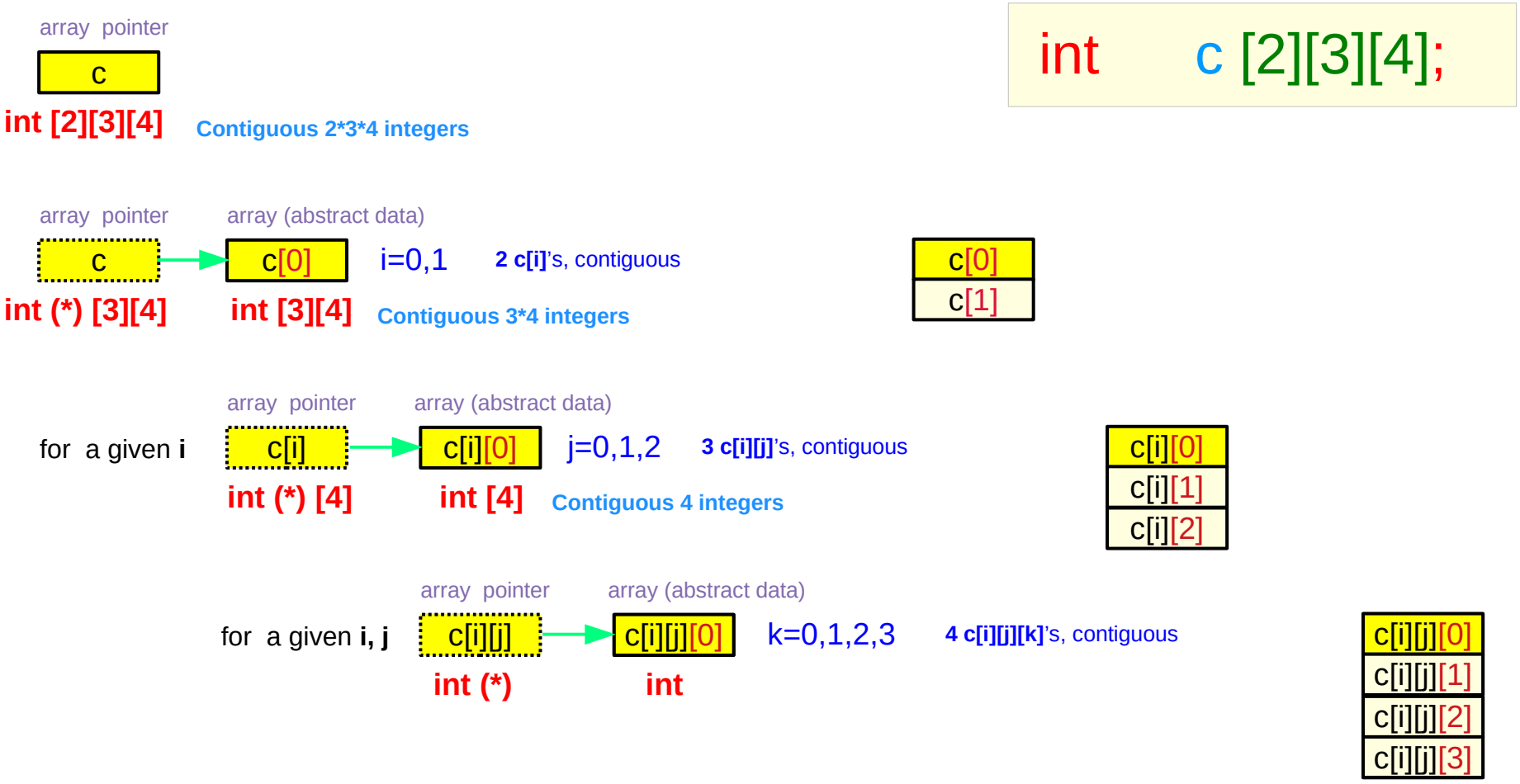
sizeof(**c[0][0]**) =
sizeof(**c[0][0][0]**) * **4**



the first 0-d subarray

sizeof(**c[0][0][0]**) =
sizeof(**int**)

Contiguous subarrays



Virtual assignments

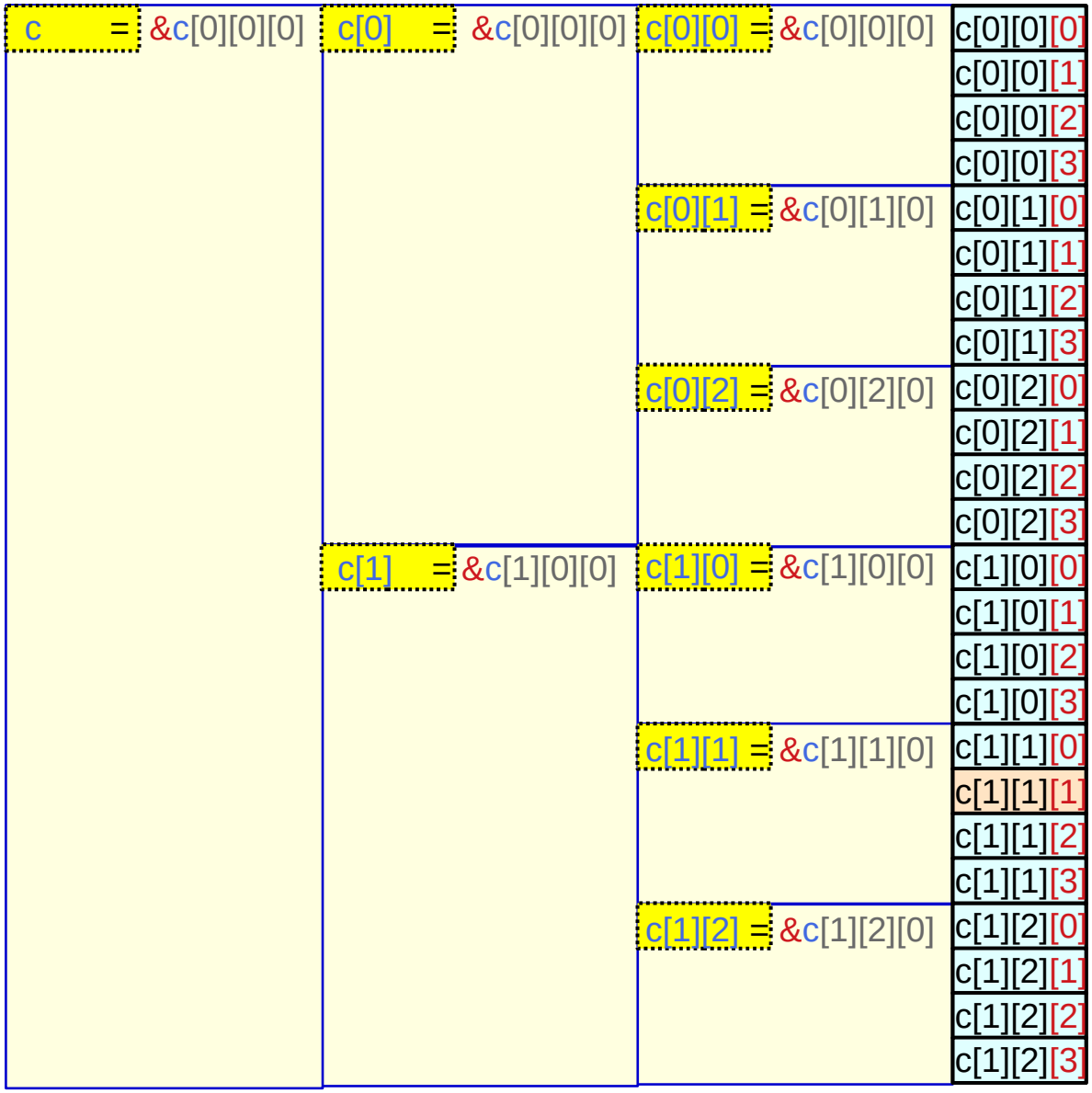
```
int     c [2][3][4];
```

```
        c [i][j][k];
```

virtual assignments

```
c       ⟵ &c[0][0][0]
c[i]    ⟵ &c[i][0][0]
c[i][j] ⟵ &c[i][j][0]
```

row major ordering
contiguous linear layout



Virtual assignments and type casts

```
int     c [2][3][4];
```

```
        c [i][j][k];
```

virtual assignments

```
c       <= &c[0][0][0]
c[i]    <= &c[i][0][0]
c[i][j] <= &c[i][j][0]
```

row major ordering
contiguous linear layout

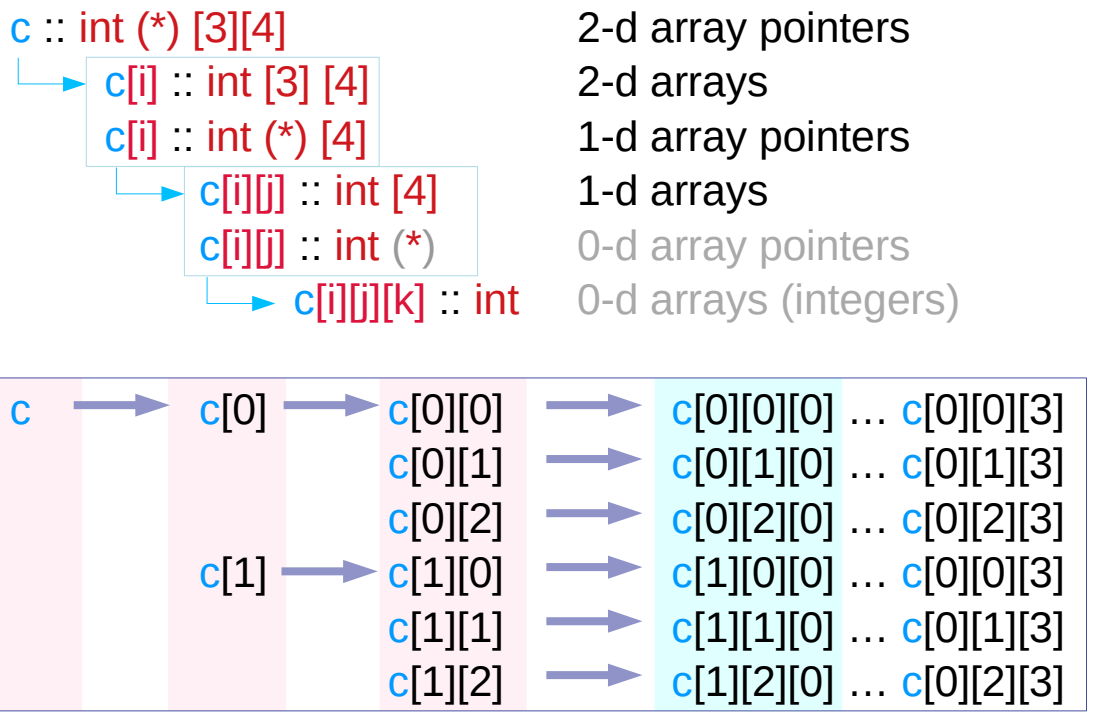
```
c       <= ( int (*)[3][4] )   &c[0][0][0]
c[i]    <= ( int (*)[4] )      &c[i][0][0]
c[i][j] <= ( int (*) )         &c[i][j][0]
```

type casts **address values**

if c, c[i], c[i][j] were real pointer variables,
type casts would be needed

Dual types of **c**, **c[i]**, **c[i][j]**

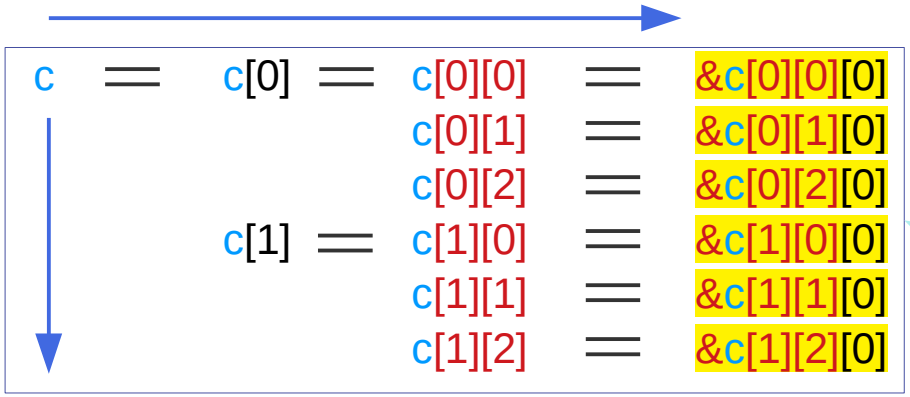
```
int     c [2][3][4];
```



Values of **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

virtual array pointers have address values
in each row in the following figure
have the same address value



Horizontal displacements are not counted
only **vertical displacements** are considered
for address values

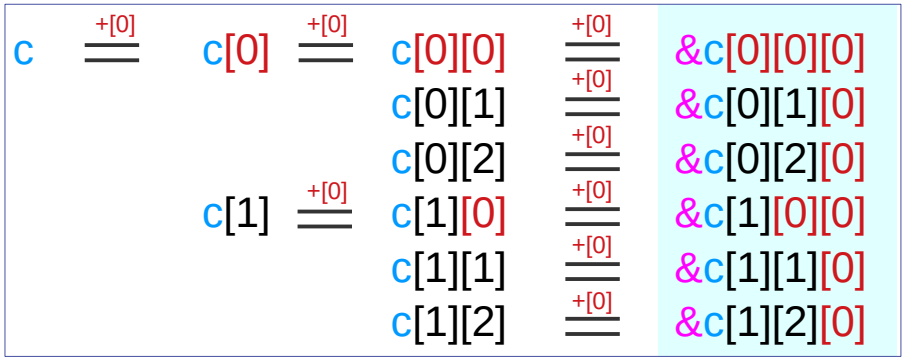
virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Finding address values of **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

append [0] to the right



int (*) [3][4] int (*) [4] int [4] int

c[0][0][0] : leading elements of **c**

&c[0][0][0]

c[i][0][0] : leading elements of **c[i]**

&c[0][0][0]

&c[1][0][0]

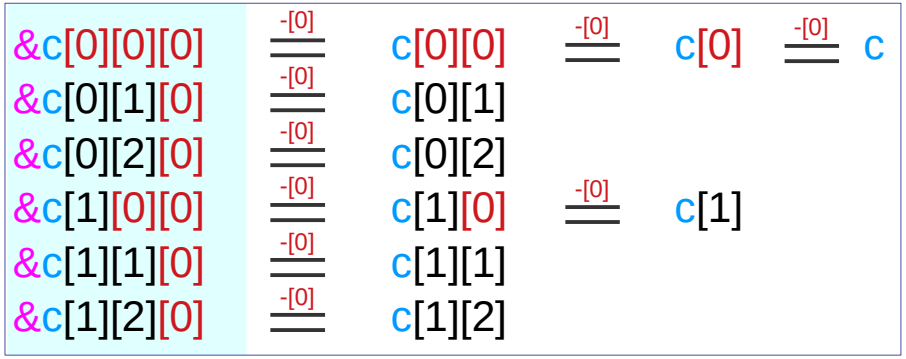
c[i][j][0] : leading elements of **c[i][j]**

&c[0][0][0]
&c[0][1][0]
&c[0][2][0]
&c[1][0][0]
&c[1][1][0]
&c[1][2][0]

Finding sub-array names with the address &c[i][j][0]

```
int     c [2][3][4];
```

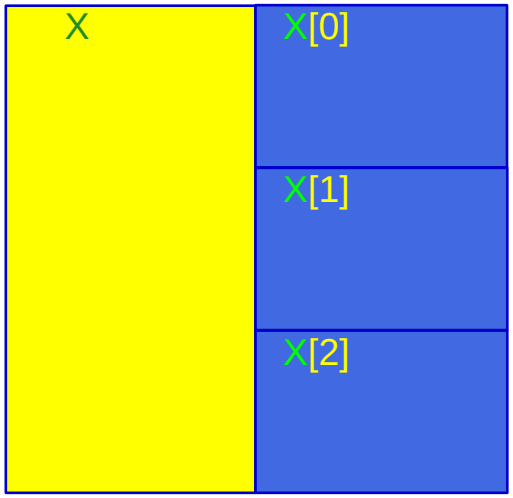
delete [0] from the right



c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

Dual type – virtual array pointer and abstract array

Abstract data (array) p[i]

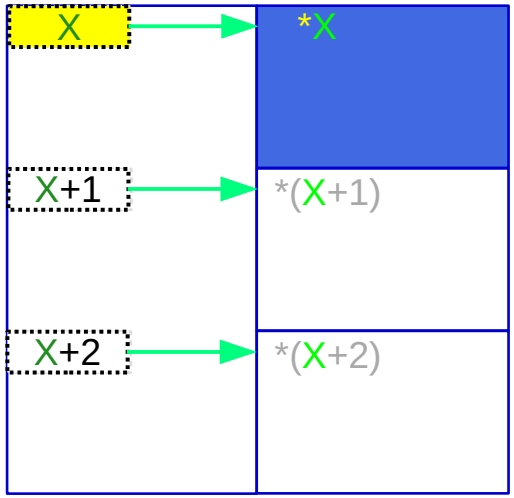


array (abstract data) array (abstract data)

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

&c[i][j]	=	&c[i][j][0]
&c[i]	=	&c[i][0]
&c	=	&c[0]

Virtual array pointer p[i]



array pointer array (abstract data)

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

c[i][j]	=	&c[i][j][0]
c[i]	=	&c[i][0]
c	=	&c[0]

Virtual array pointer values and addresses

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

Virtual array pointer

array pointer — array (abstract data)

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

```
&c[i][j][0]  = &c[i][j]
&c[i][0]     = &c[i]
&c[0]        = &c
```

Abstract data (array)

array (abstract data) — array (abstract data)

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

```
c[i][j]      = &c[i][j]
c[i]         = &c[i]
c            = &c
```

array (abstract data) — Address of an array pointer

c[i][j]	pointer value	=	pointer address	&c[i][j]
c[i]	pointer value	=	pointer address	&c[i]
c	pointer value	=	pointer address	&c

c[0] = c[0][0] relation

```
int     c [2][3][4];
```

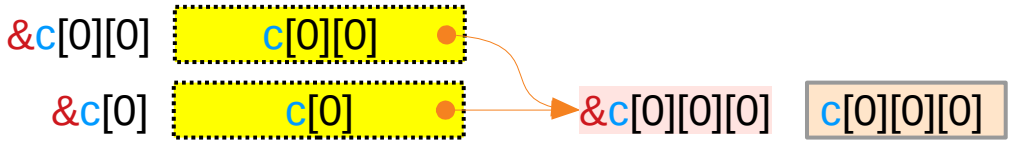
c == c[0] == c[0][0] == &c[0][0][0]

value(c[0]) = &c[0][0][0]
value(c[0][0]) = &c[0][0][0]

type(c[0]) = int (*)[4]
type(c[0][0]) = int [4]

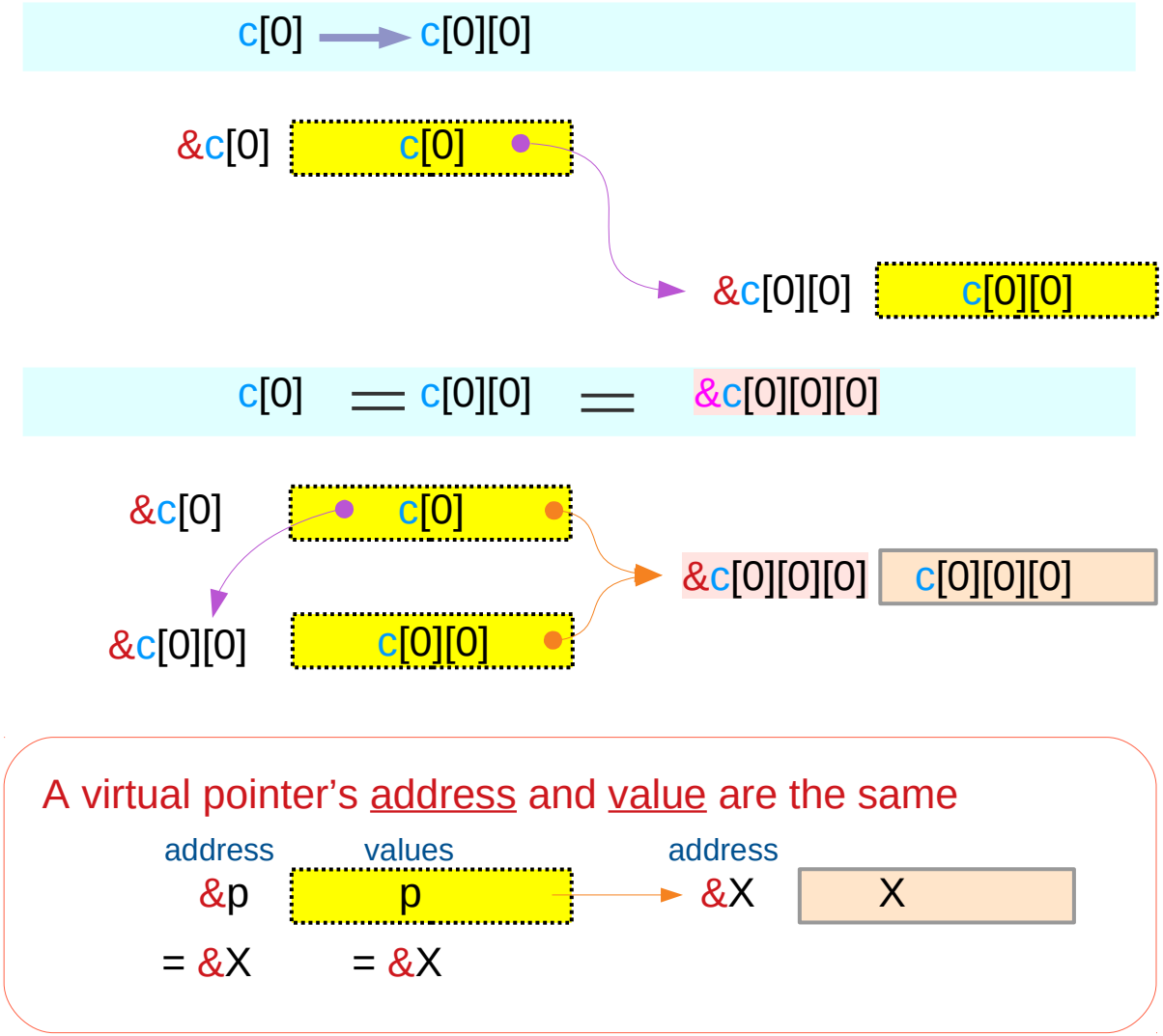
c[0] = c[0][0] *means*
value(c[0]) = value(c[0][0])

c[0] = c[0][0] *does **not** mean*
type(c[0]) = type(c[0][0])



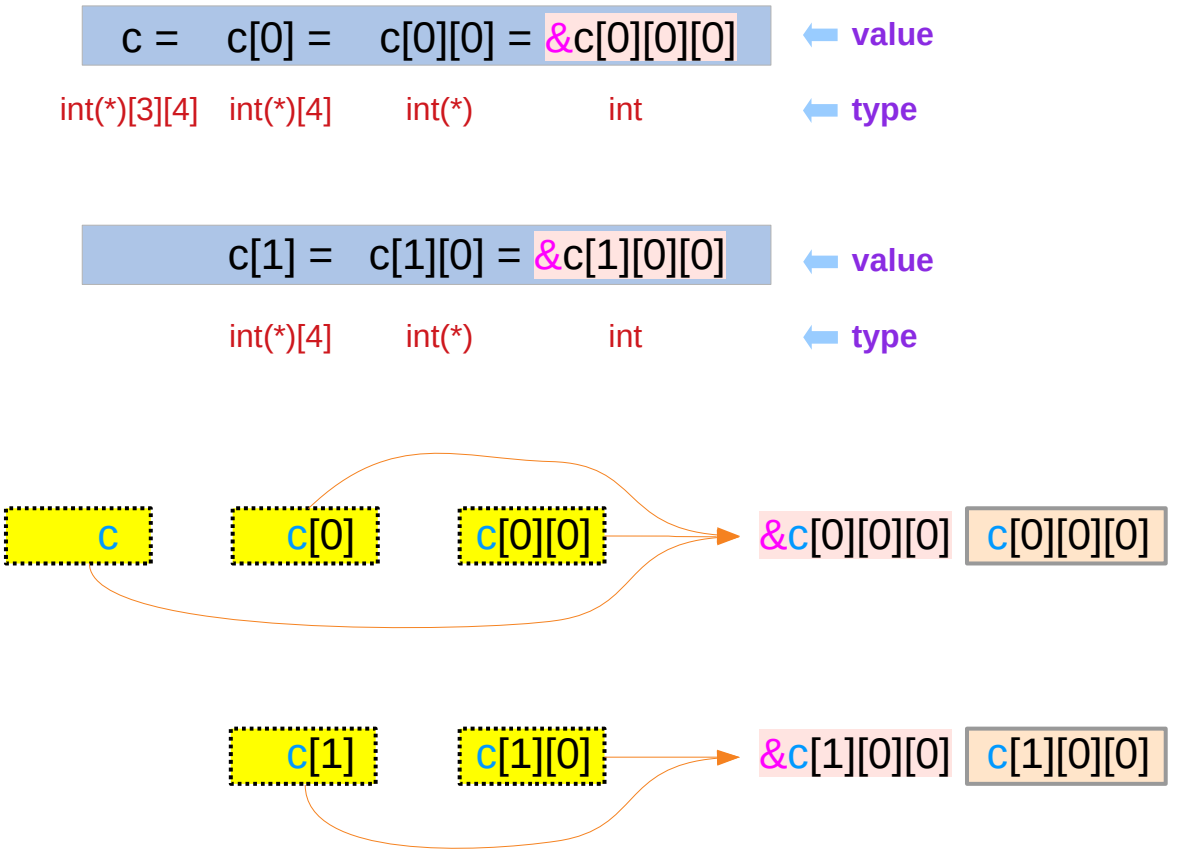
Addresses and Values of **c[0]** and **c[0][0]**

```
int     c [2][3][4];
```



c[i] and **c[i][0]** point to the same **c[i][0][0]**

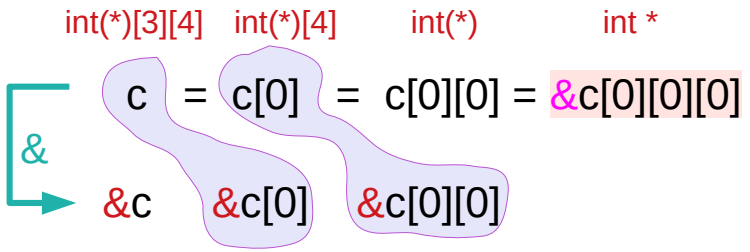
```
int     c [2][3][4];
```



These virtual pointers have different types
but the same value (address)

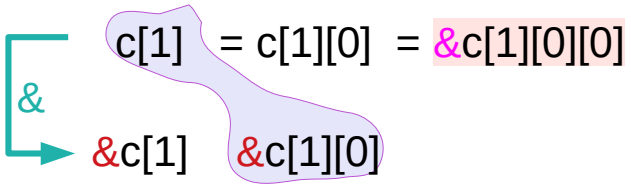
&c[i][0] and &c[i][0][0] – equivalence relations

```
int     c [2][3][4];
```



equivalences

c ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]



equivalences

c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]

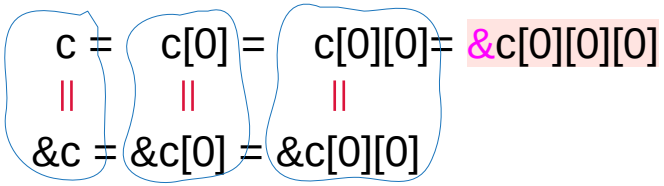
Horizontal displacements are not counted
only **vertical displacements** are considered
for **address values**

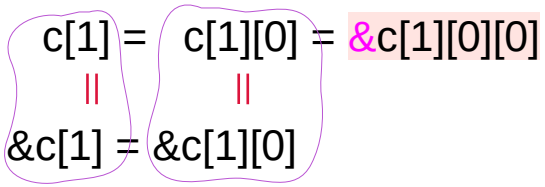
equivalences

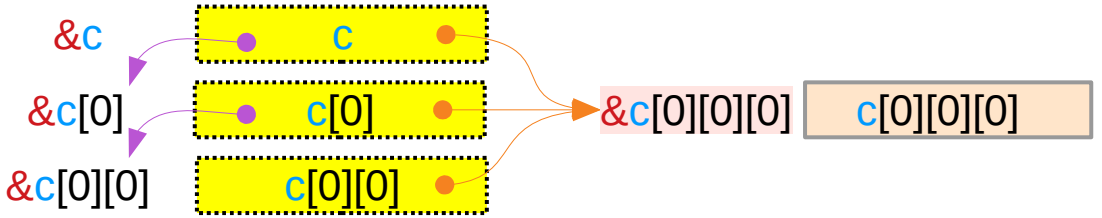
c ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]

c[i] = &c[i] and c[i][0] = &c[i][0]

```
int     c [2][3][4];
```



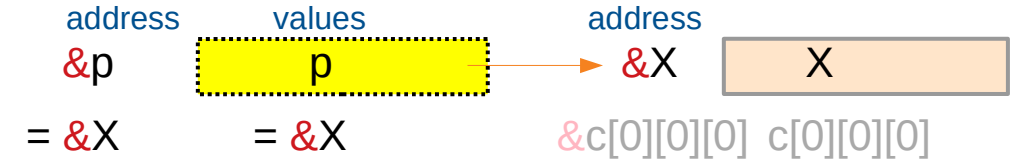


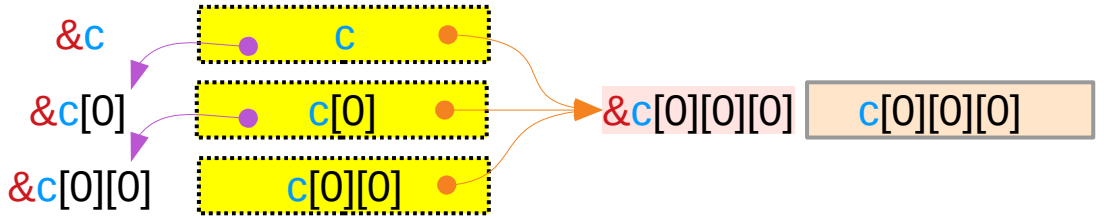


c[i] = &c[i] and c[i][0] = &c[i][0]

```
int     c [2][3][4];
```

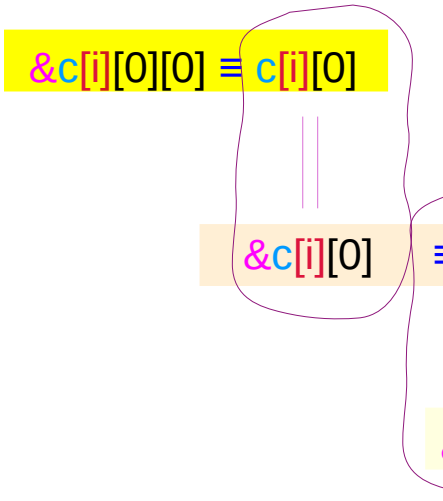
A virtual pointer's address and value are the same





c[i]

Array Pointers to **c[i][0][0]**

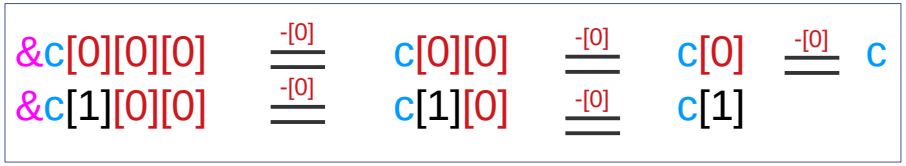


virtual pointers:
the address of a pointer is
the same as its value

= c + i*sizeof(*c)

= &c[0][0][0] + i*3*4

delete [0] from the right



Array Pointers to **c[i][j][0]**

&c[i][j][0] ≡ c[i][j]

||

&c[i][j] ≡ c[i] + j

= c[i] + j*sizeof(*c[i])

= c + i*sizeof(*c) + j*4

= &c[0][0][0] + i*3*4 + j*4

delete [0] from the right

&c[0][0][0]	-[0]	c[0][0]	-[0]	c[0]	-[0]	c
&c[0][1][0]	-[0]	c[0][1]				
&c[0][2][0]	-[0]	c[0][2]				
&c[1][0][0]	-[0]	c[1][0]	-[0]	c[1]		
&c[1][1][0]	-[0]	c[1][1]				
&c[1][2][0]	-[0]	c[1][2]				

Sub-array properties in multi-dimensional arrays

int c [2][3][4];  3-d access c **[i][j][k]**

2-d array pointer	c	int (*) [3][4]
1-d array pointers	c[i]	int (*) [4]
0-d array pointers	c[i][j]	int (*)

Hierarchical sub-arrays in a 3-d array

```
int     c [L][M][N];
```

```
        c [i][j][k]
```

left-to-right associativity

```
c    [i]  [j][k]
c[i] [j]  [k]
c[i][j]   [k]
```

Array Names and Types

Pointers to hierarchical sub-arrays

c	3-d array names	int (*) [M][N]	2-d array pointer
c[i]	2-d array names	int (*) [N]	1-d array pointer
c[i][j]	1-d array names	int (*)	0-d array pointer

Associativity and Equivalence Relations

left-to-right associativity

left-to-right associativity

((c [i])[j])[k] ≡ *(*(*(c +i) +j) +k)

X[n] ≡ *(X+n)

given c[i][j]	c[i][j][k]	≡	*(c[i][j]+k)	for all k
given c[i]	c[i][j]	≡	*(c[i]+j)	for all j
given c	c[i]	≡	*(c+i)	for all i

Requirements for the expression **c[i][j][k]**

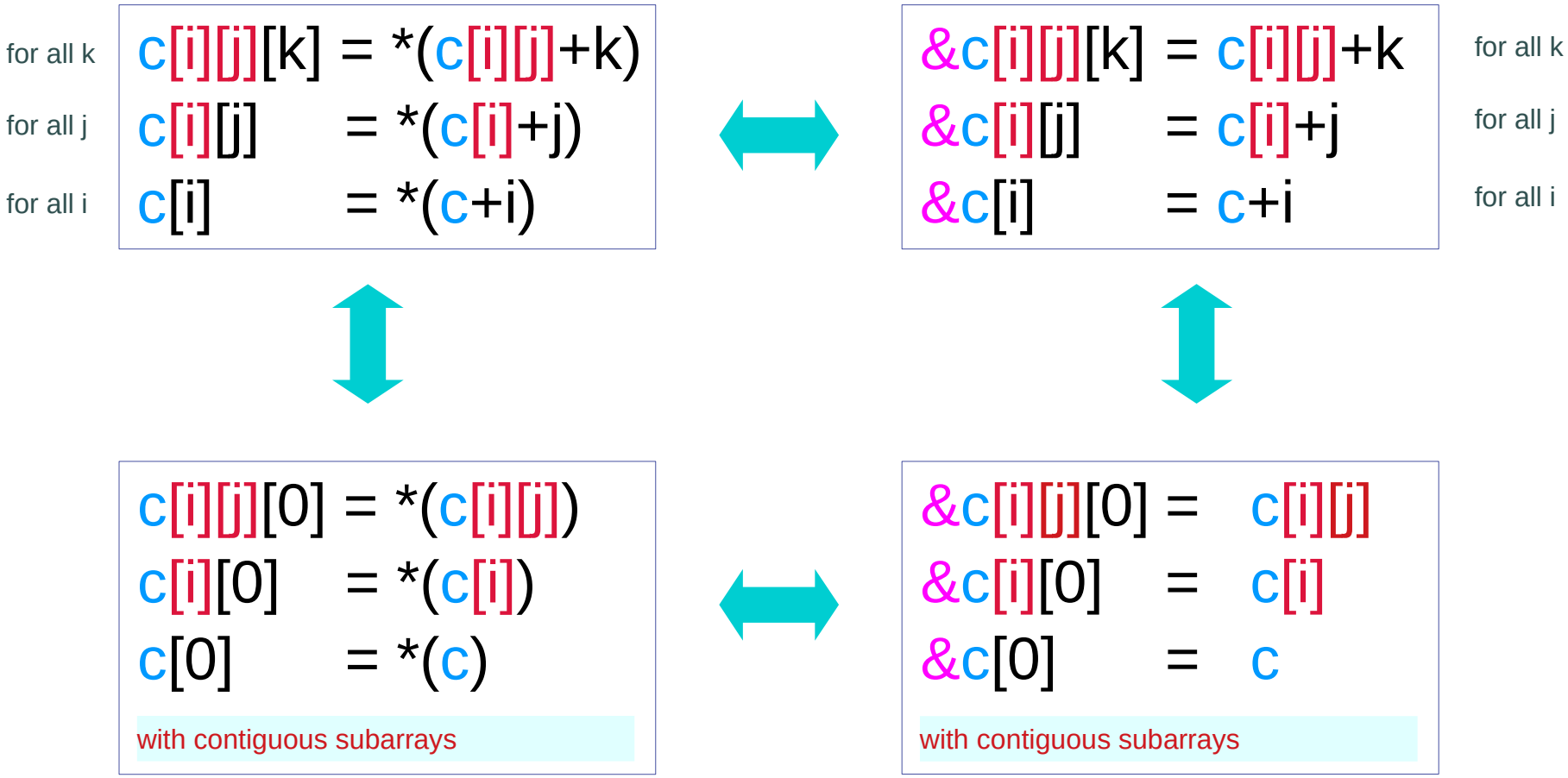
for a given c[i][j],	for all k	c[i][j][k] = *(c[i][j]+k)
for a given c[i],	for all j	c[i][j] = *(c[i]+j)
for a given c,	for all i	c[i] = *(c+i)

3 contiguity requirements

for a given c[i][j],	contiguous c[i][j][k]'s
for a given c[i],	contiguous c[i][j]'s
for a given c,	contiguous c[i]'s

for a given subarray pointer	contiguous subarrays

Equivalent requirements for the expression **c[i][j][k]**



3-d access pattern **c[i][j][k]**

General requirements

```
c [i][j][k]
```

```
&c[i][j][k]  = c[i][j]+k   for all k
&c[i][j]     = c[i]+j      for all j
&c[i]        = c+i         for all i
```

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

```
c[i][j][k]   :: int
c[i][j]      :: int *
c[i]         :: int **
c            :: int ***
```

```
c[i]  ⟸  &b[i*3]
b[j]  ⟸  &a[j*4]
```

with contiguous a, b, c

Explicit
Arrays of pointers with
Multiple Indirection

N-dim Array approach

```
int c[2][3][4];
```

```
c[i][j][k] :: int
c[i][j]    :: int [4]
c[i]       :: int [3][4]
c          :: int [2][3][4]
```

```
c[i][j]  ⟸  &c[i][j][0]
c[i]     ⟸  &c[i][0][0]
c        ⟸  &c[0][0][0]
```

with contiguous c

Implicit
Nested
Virtual Array Pointers

3-d access pattern **c[i][j][k]** – array pointer approach

General requirements

```
c [i][j][k]
```

```
&c[i][j][k]  = c[i][j]+k   for all k
&c[i][j]     = c[i]+j      for all j
&c[i]        = c+i         for all i
```

N-dim array approach

```
int c[2][3][4];
```

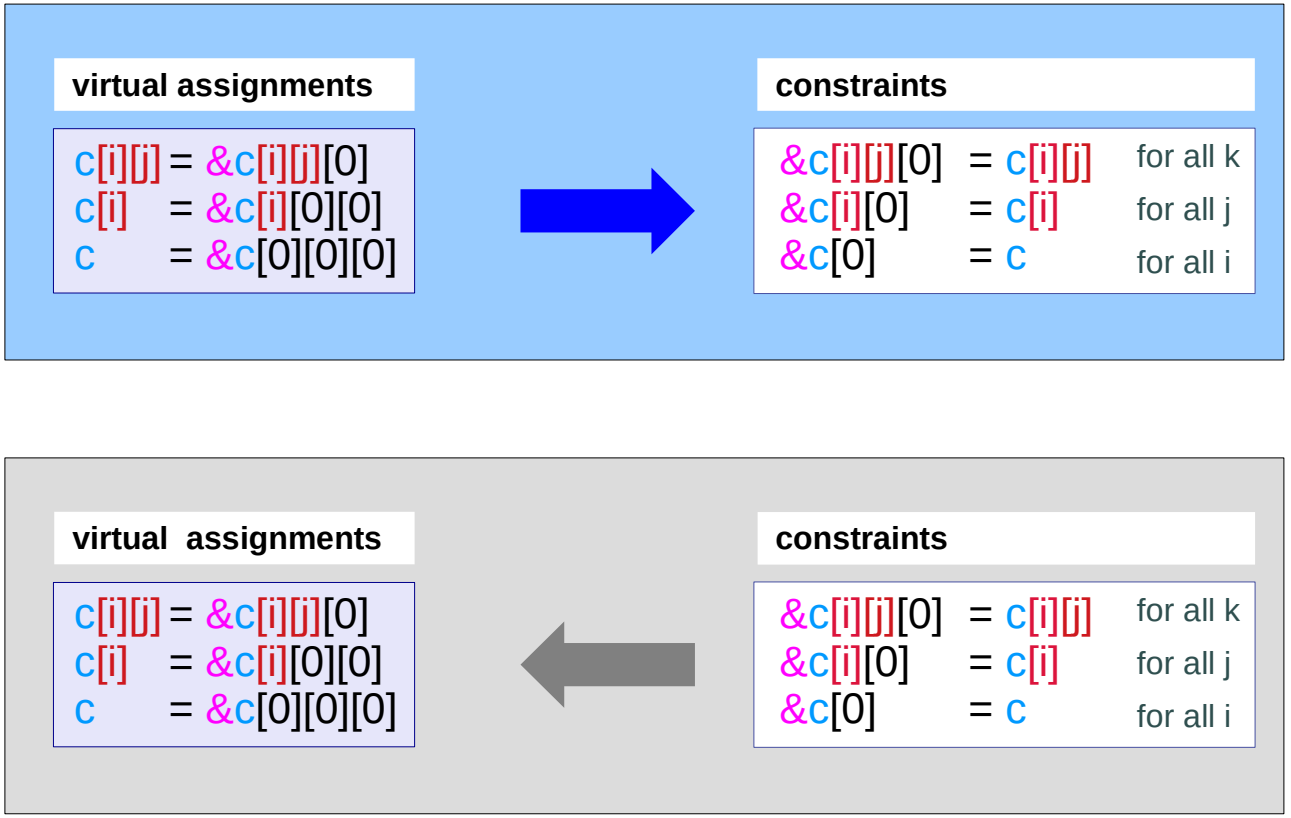
```
c[i][j][k]  :: int
c[i][j]     :: int [4]
c[i]        :: int [3][4]
c           :: int [2][3][4]
```

```
c[i][j]  ⟸ &c[i][j][0]
c[i]     ⟸ &c[i][0][0]
c        ⟸ &c[0][0][0]
```

with contiguous c

Implicit
Nested
Virtual Array Pointers

multi-dimensional arrays



General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

```
&c[i][j][k]  = c[i][j]+k     for all k
&c[i][j]     = c[i]+j        for all j
&c[i]        = c+i           for all i
```

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Virtual assignments

int (*)	c[i][j]	=	(int (*))	&c[i][j][0]
int (*) [4]	c[i]	=	(int (*) [4])	&c[i][0][0]
int (*) [3][4]	c	=	(int (*) [3][4])	&c[0][0][0]

Pointer Types

Sizes of abstract data types

int [4]	c[i][j]	size =4*4
int [3][4]	c[i]	size =3*4*4
int [2][3][4]	c	size =2*3*4*4

Abstract Data Types

Strides of array elements

c[i][j][0]	stride =4*4
c[i][0][0]	stride =3*4*4
c[0][0][0]	stride =2*3*4*4

k=[0:3]	c[i][j][k]	4 integers
j=[0:2], k=[0:3]	c[i][j][k]	3*4 integers
I=[0:1], j=[0:2], k=[0:3]	c[i][j][k]	2*3*4 integers

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

for all k
for all j
for all i

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

c[i][j]	has an address of	c[i][j][0]	
c[i]	has an address of	c[i][0][0]	
c	has an address of	c[0][0][0]	
Pointer Types			
c[i][j]+1	has an address of	c[i][j][1]	1 integer away
c[i]+1	has an address of	c[i][1][0]	4 integers away
c+1	has an address of	c[1][0][0]	3*4 integers away
Pointer Types			
c[i][j]+k	has an address of	c[i][j][k]	1*k integers away
c[i]+j	has an address of	c[i][j][0]	4*j integers away
c+i	has an address of	c[i][0][0]	3*4*i integers away
Pointer Types			

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

for all k
for all j
for all i

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0][0]
c	starts from	&c[0][0][0]

Abstract Data

c[i][j]	has a size of	c[i][j][k]	4	integers
c[i]	has a size of	c[i][j][k]	3*4	integers
c	has a size of	c[i][j][k]	2*3*4	integers

Abstract Data

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

```
&c[i][j][k]  = c[i][j]+k     for all k
&c[i][j]     = c[i]+j        for all j
&c[i]        = c+i           for all i
```

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c[i][j]   starts from   &c[i][j][0]
c[i]      starts from   &c[i][0][0]
c         starts from   &c[0][0][0]
```

Abstract Data

```
c[i]   and   c[i][0]   start from   &c[i][0][0]
c      and   c[0]      start from   &c[0][0][0]
```

Abstract Data — Abstract Data

```
c[i]   starts from   &c[i][0]
c      starts from   &c[0]
```

Abstract Data — Abstract Data

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

```
&c[i][j][k]  = c[i][j]+k     for all k
&c[i][j]     = c[i]+j        for all j
&c[i]        = c+i           for all i
```

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

`c[i]` and `c` start from `&c[i][0][0]` and `&c[0][0][0]`

Abstract Data

Abstract Data

`c[i]` has a size of 3*4 integers → `c[i][0]` has a size of 4 integers; `c[i][1]` has a size of 4 integers; `c[i][2]` has a size of 4 integers

`c[i]` is the name of an array which has 3 elements `c[i][0]` `c[i][1]` `c[i][2]`

points to `c[i][0]`

`&c[i][0]` = `c[i]`

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

for all k
for all j
for all i

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Each	c[i][j]	sub-array contains	4 integers
Each	c[i]	sub-array contains	3*4 integers
Each	c	sub-array contains	2*3*4 integers

Abstract Data Types

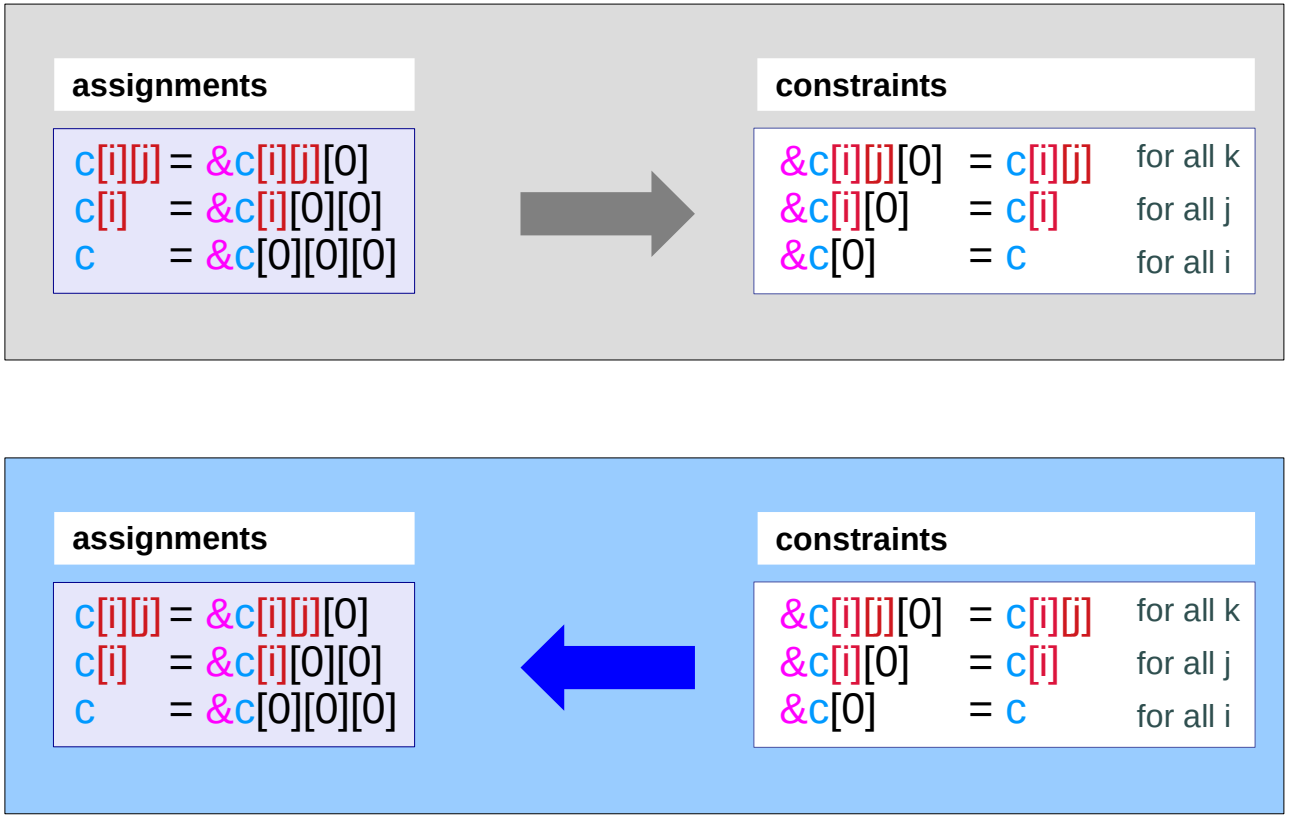
Each	c[i][j]+1	starts at	c[i][j][1]	1 integer away
Each	c[i]+1	starts at	c[i][1][0]	4 integers away
Each	c+1	starts at	c[1][0][0]	3*4 integers away

Pointer Types

Each	c[i][j]+k	starts at	c[i][j][k]	1*k integers away
Each	c[i]+j	starts at	c[i][j][0]	4*j integers away
Each	c+i	starts at	c[i][0][0]	3*4*i integers away

Pointer Types

multi-dimensional arrays



Subarray starting addresses

```
int     c [2][3][4];
```

Virtual array pointer

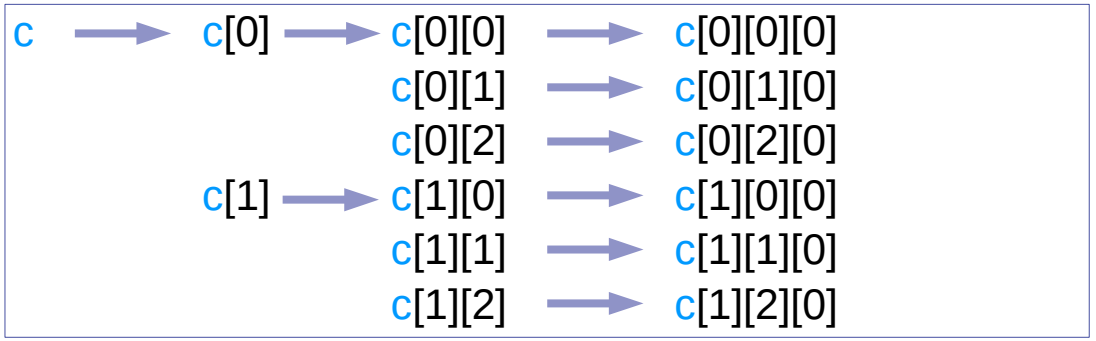
```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

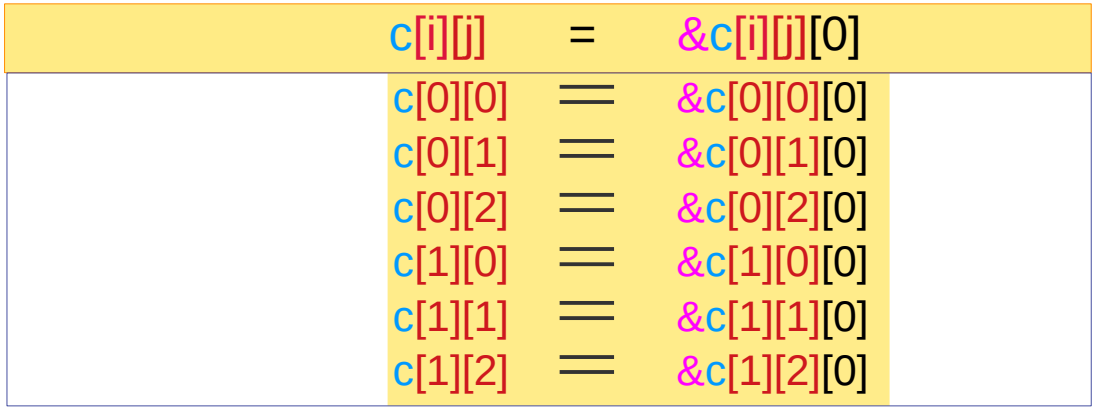
c[i][j] points to c[i][j][0]
c[i] points to c[i][0]
c points to c[0]

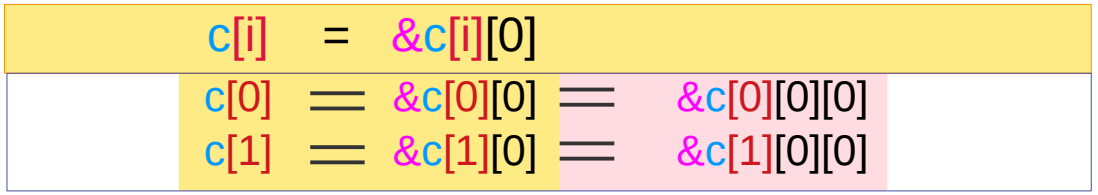
Abstract data (array)

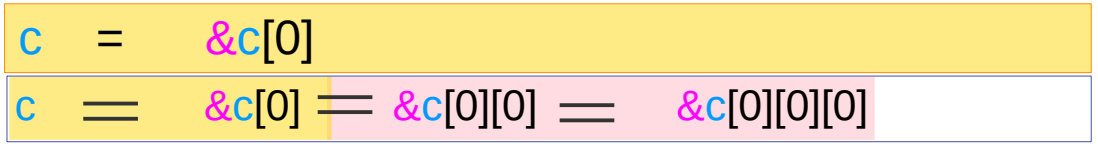
```
&c[i][j][0]  = &c[i][j]
&c[i][0]     = &c[i]
&c[0]        = &c
```

c[i][j] starts from &c[i][j][0]
c[i] starts from &c[i][0]
c starts from &c[0]









General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

with contiguous subarrays

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

for all k
for all j
for all i

virtual assignments

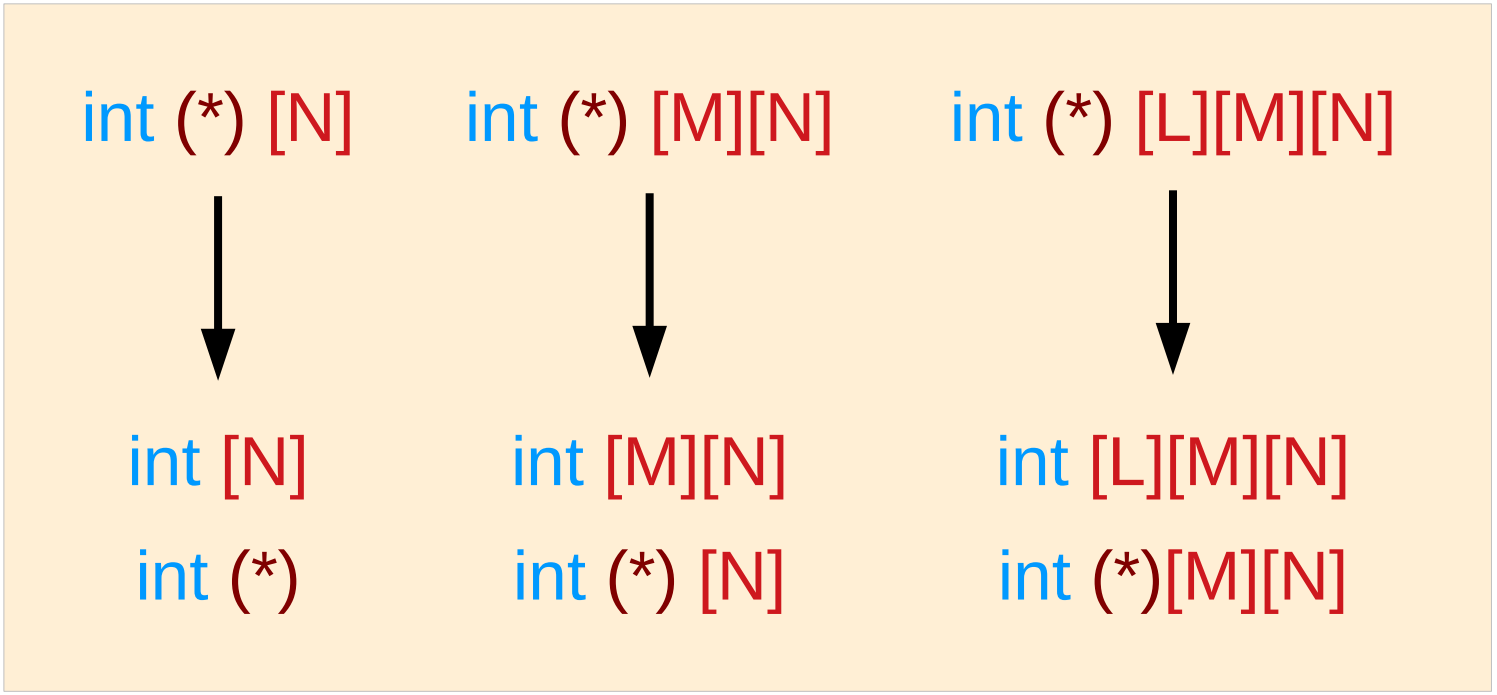
```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Contiguity Constraints

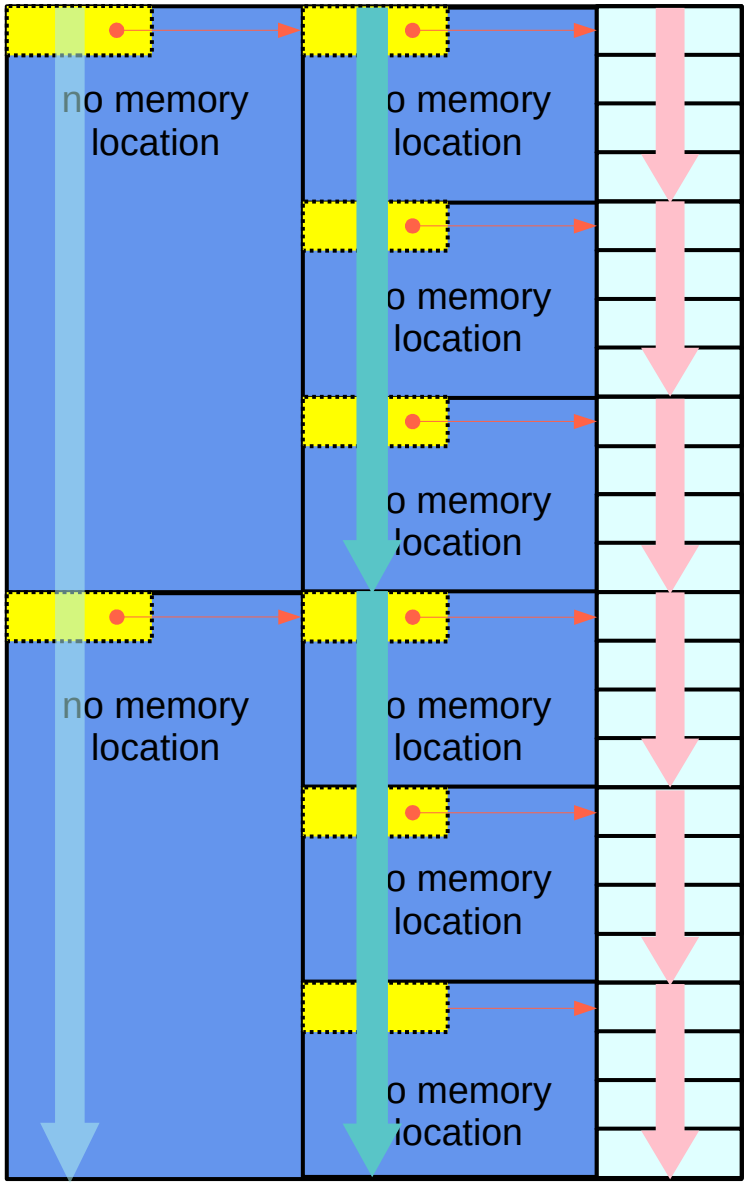
c [i][j][k];

Virtual Array Pointers and Contiguity

Using array pointers



Array pointer approach – contiguity constraints



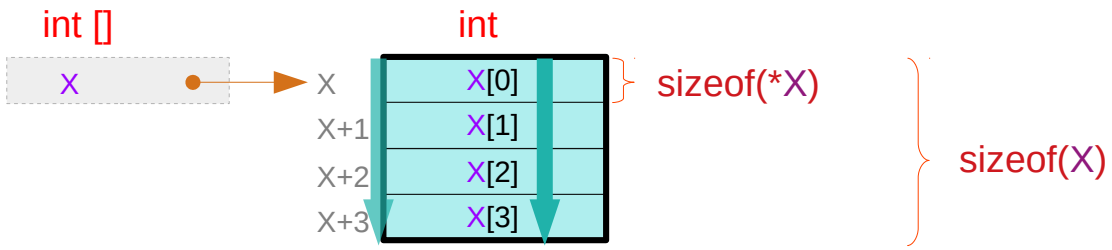
Array Pointer Approach
(pointer to arrays)

Equivalence and contiguity (1)

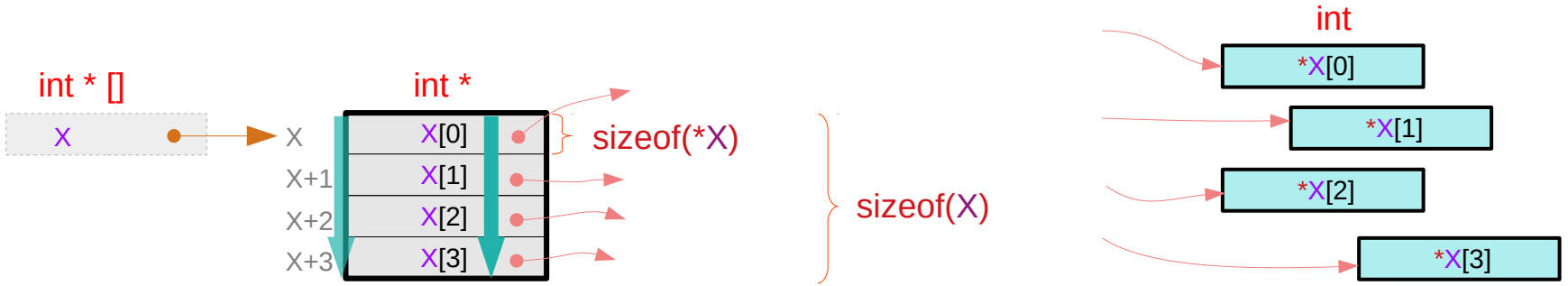
consecutive address　　consecutive data

*(X+n) ≡ X[n]

contiguous index : n



int X[4]; contiguous X[i] for a given X : **primitive types**



int * X[4]; contiguous X[i] for a given X : **pointer types**

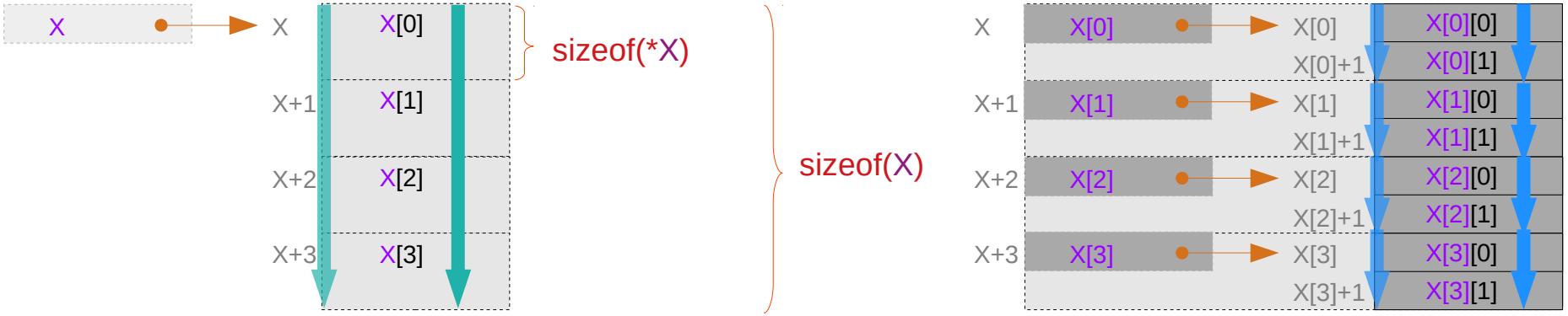
Equivalence and contiguity (2)

consecutive address　　consecutive data

*(X+n) ≡ X[n]

contiguous index : n

can be recursively applied



atype * X[4]; contiguous X[i] for a given X : **abstract data types**

Recursive applications of equivalences

By definition, contiguous memory locations are assumed

consecutive address　　consecutive data

*(X+n) ≡ X[n]

contiguous index : n

*(p[m]+n) ⟺ p[m][n]

X = p[m]　contiguous index : **n**

(*(p+m))[n]; ⟺ p[m][n];

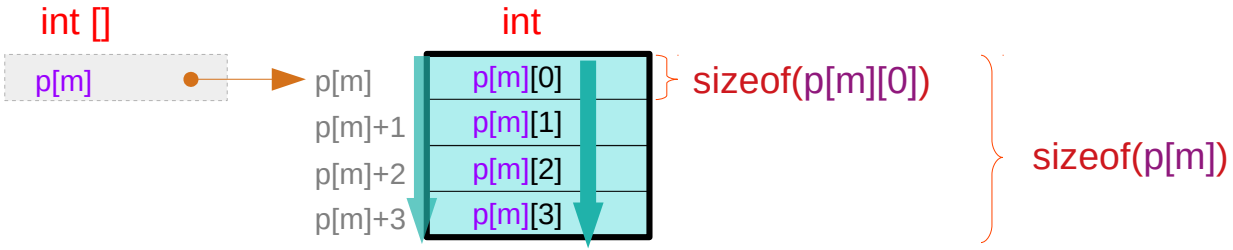
X = p　contiguous index : **m**

Equivalence for a given p[m] (1)

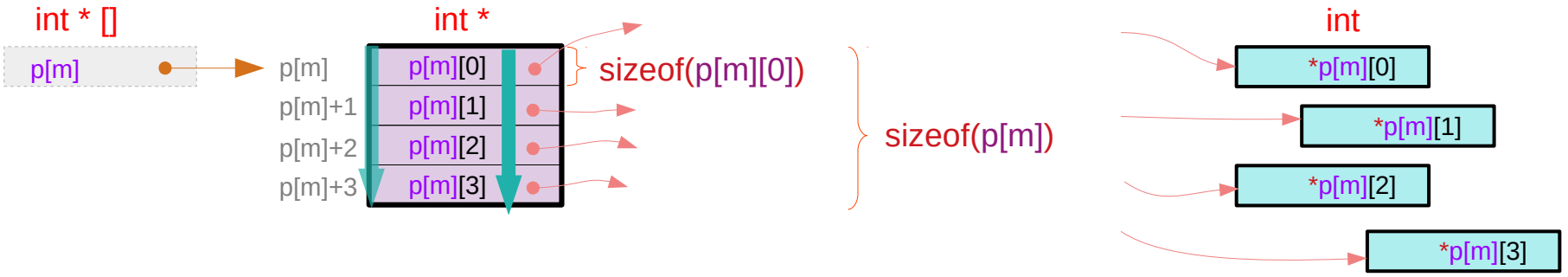
*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

int p[M][4]; contiguous p[m][n] for a given p[m] : **primitive types** m = 0, 1, …, M-1



int * p[M][4]; contiguous p[m][n] for a given p[m] : **pointer types** m = 0, 1, …, M-1

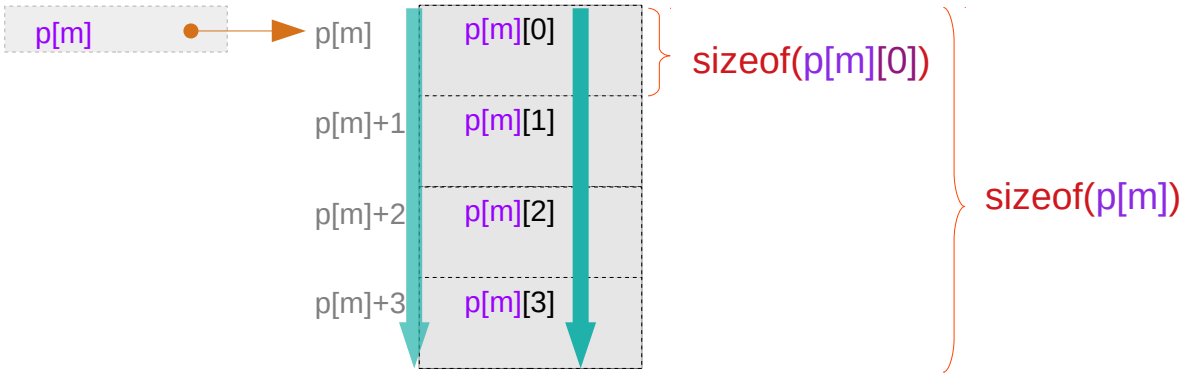


Equivalence for a given p[m] (2)

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

atype * p[M][4]; contiguous p[m][n] for a given p[m] : **abstract data types** m = 0, 1, …, M-1



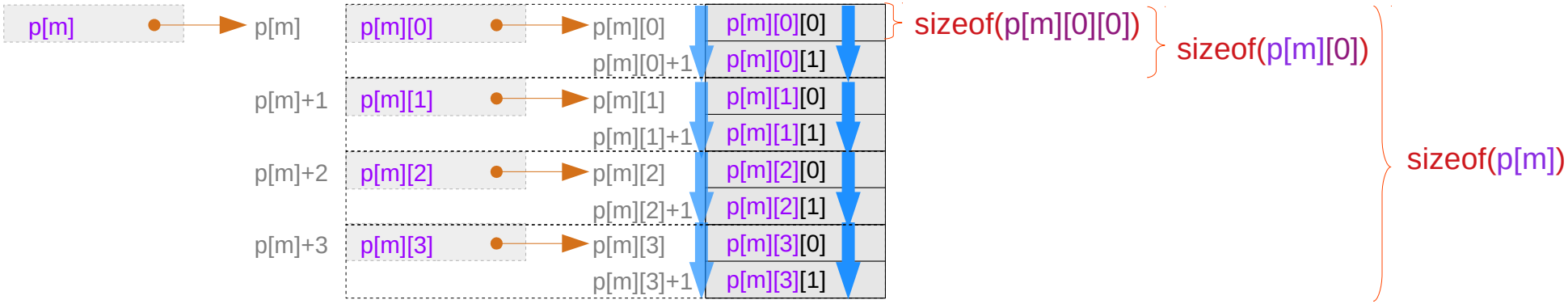
can be recursively applied

Equivalence for a given p[m][n]

*(p[m][n]+k) ⟺ p[m][n][k]

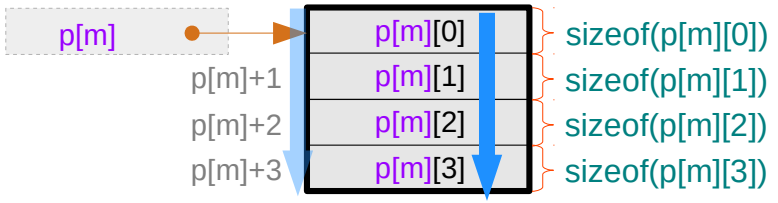
for a given p[m][n] contiguous index : **k**

atype * p[M][4][2]; contiguous p[m][n][k] for a given p[m][n] : **abstract data types** m = 0, 1, …, M-1



Contiguity constraints in multi-dimensional arrays

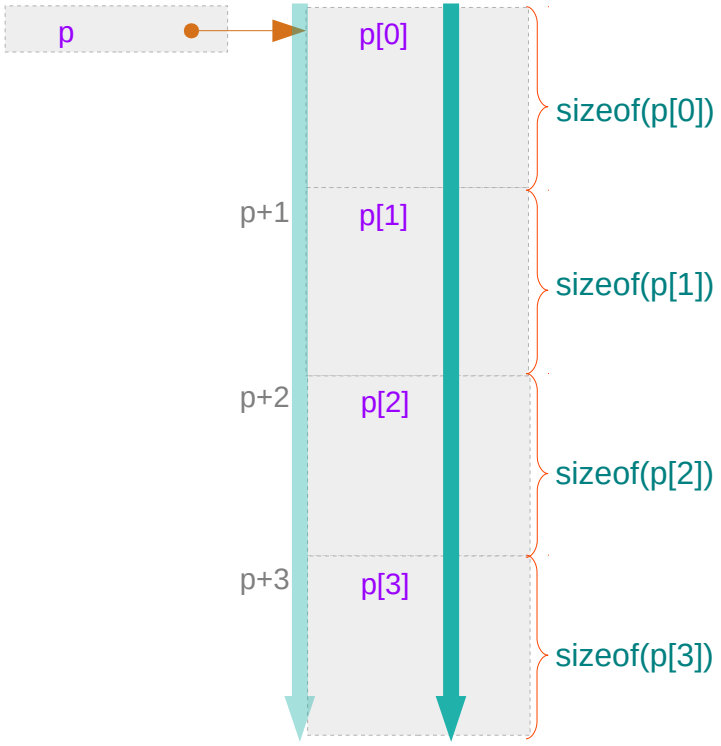
*(p[m]+n) ⟺ p[m][n]



for a given p[m], thus for a given p and m,
p[m][n]'s must be contiguous for all n.
p[m][0], p[m][1], … , p[m][N-1]

contiguous index : n

*(p+m) ⟺ p[m]



for a given p,
p[m]'s must be contiguous for all m.
p[0], p[1], … , p[M-1]

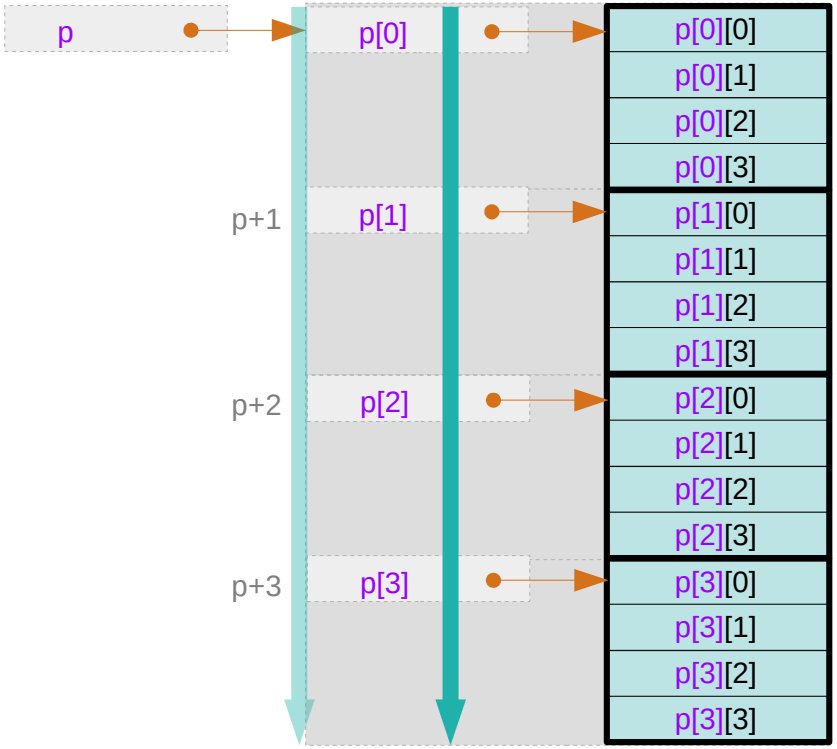
contiguous index : m

Contiguity constraints for p

*(p+m) ⟺ p[m]

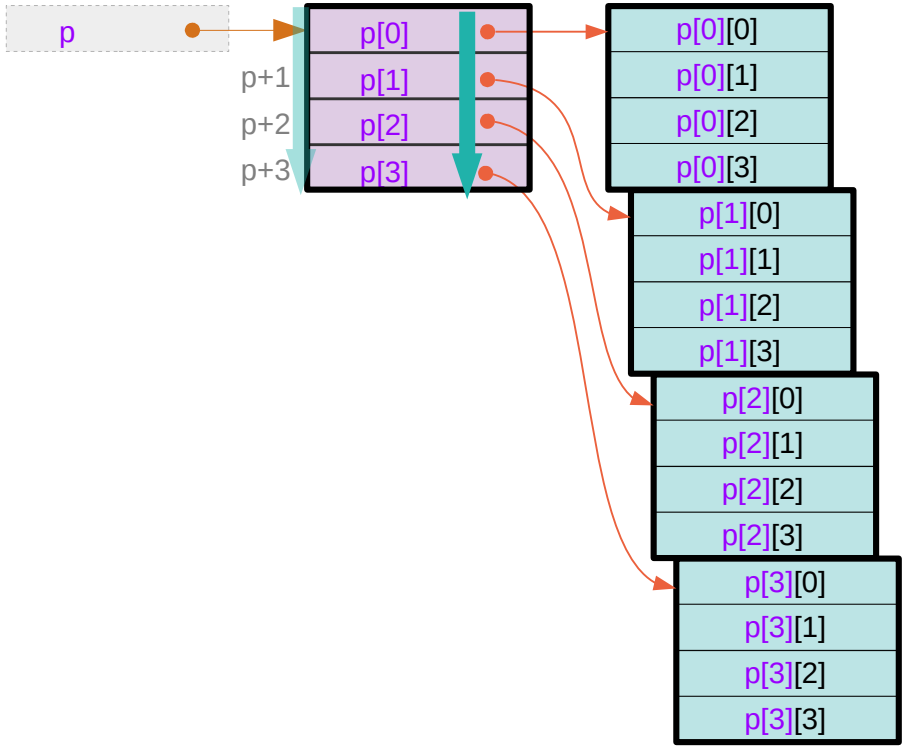
for a given p contiguous index : **m**

2-d array name **1-d** array names



contiguous p[m] ➡ contiguous p[m][n]

1-d array of pointers



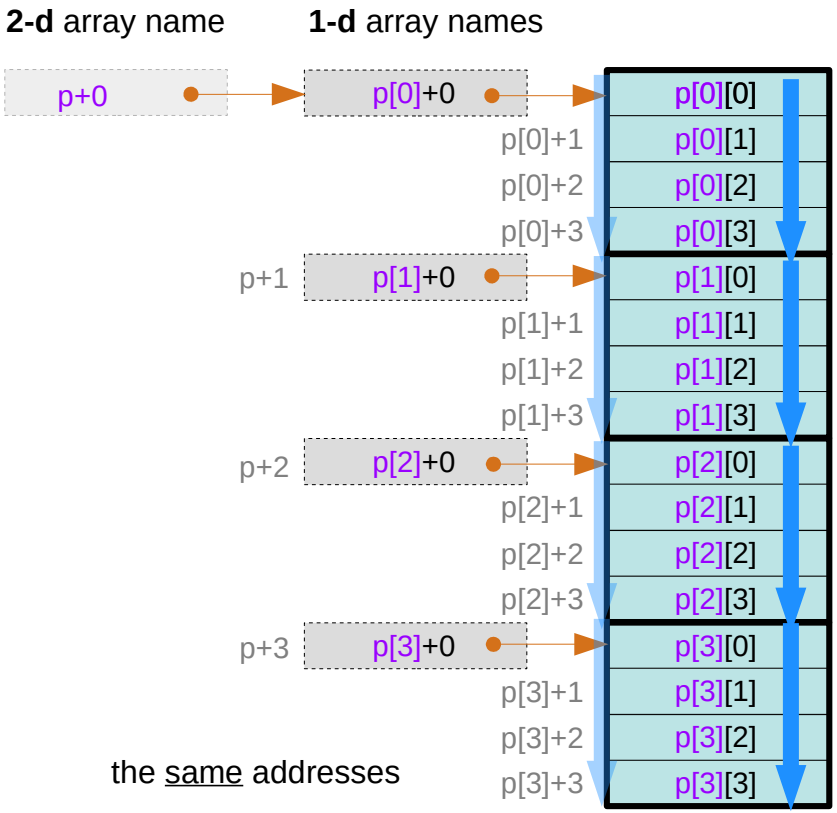
contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints for p[m] – using array pointers

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**



contiguous p[m] ➡ contiguous p[m][n]

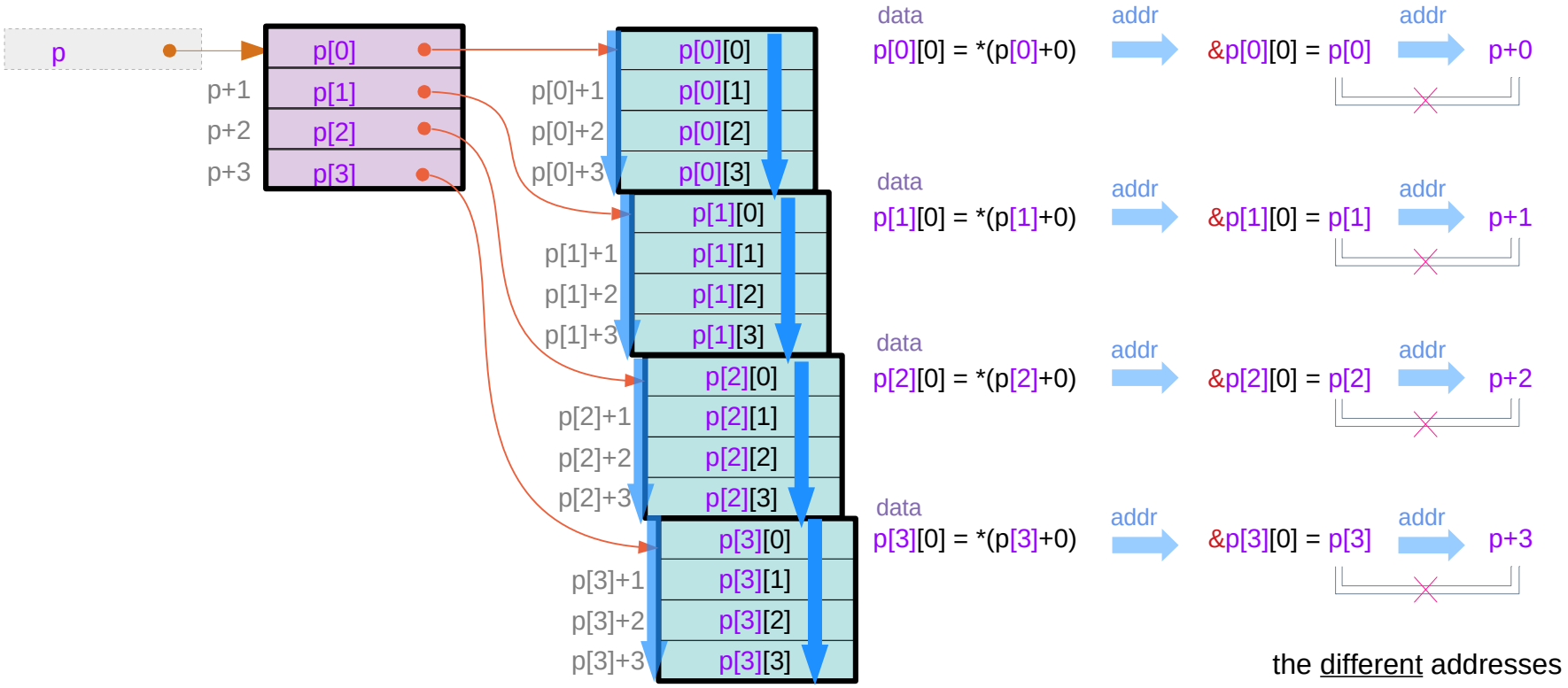
virtual array pointer ⟺ no real memory locations

Contiguity constraints for p[m] – using pointer arrays

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

1-d array of pointers



contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints for 2-d arrays

int a[M][N] ;

*(a+m) ⟺ a[m]

a[0], a[1], … , a[M-1]
are contiguous

*(a[m]+n) ⟺ a[m][n]

a[m][0], a[m][1], … , a[m][N-1]
are contiguous

int (*b)[N] ;

*(b+m) ⟺ b[m]

b[0], b[1], … , b[M-1]
are contiguous

*(b[m]+n) ⟺ b[m][n]

b[m][0], b[m][1], … , b[m][N-1]
are contiguous

int * c[M] ;

*(c+m) ⟺ c[m]

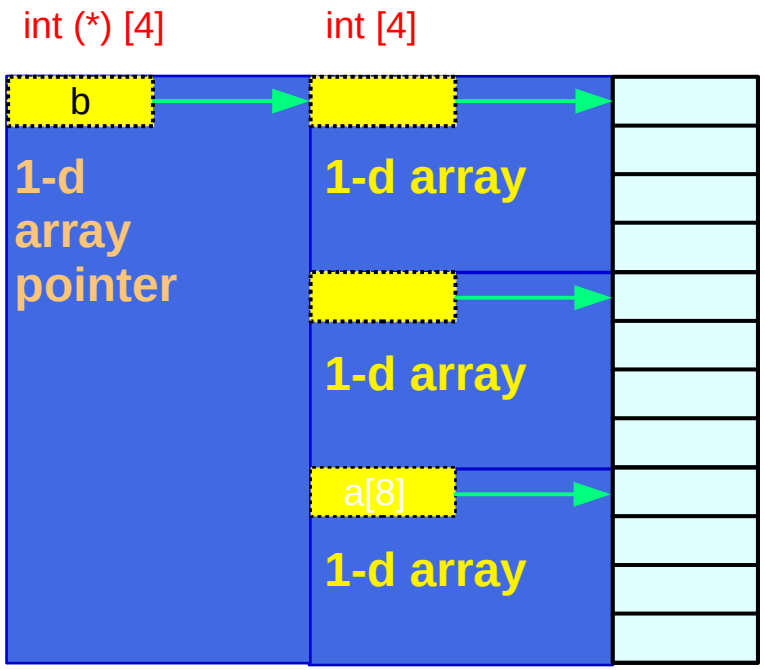
c[0], c[1], … , c[M-1]
are contiguous

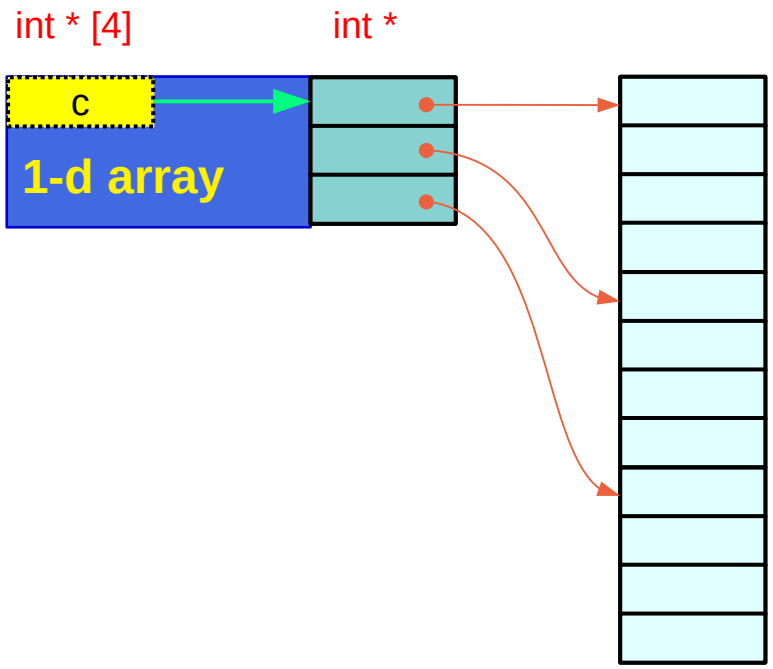
*(c[m]+n) ⟺ c[m][n]

c[m][0], c[m][1], … , c[m][N-1]
are contiguous

a set of assignments of pointers
are necessary for this contiguity

Pointer Arrays vs Array Pointers





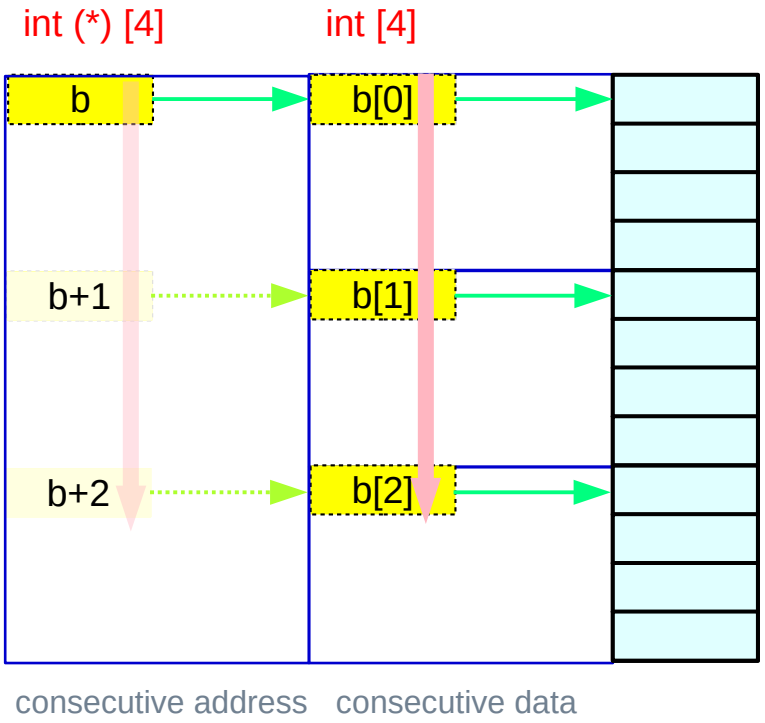
int (*b)[N] ;

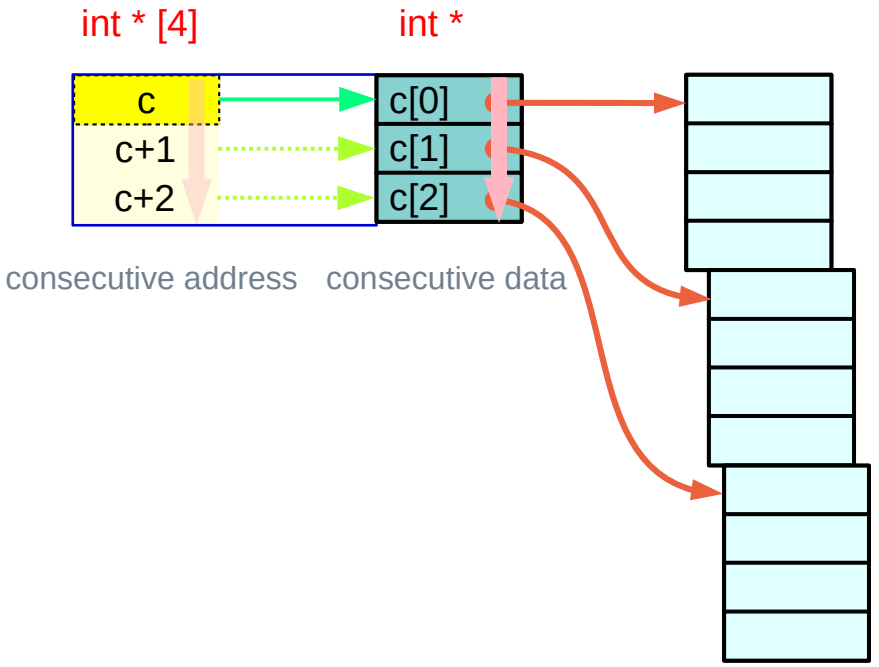
int * c[M] ; with proper assignments

*(b+m) ⟺ b[m]
*(b[m]+n) ⟺ b[m][n]

*(c+m) ⟺ c[m] or
*(c[m]+n) ⟺ c[m][n]

Pointer Arrays vs Array Pointers





int (*b)[N] ;

*(b+m) ⟺ b[m]

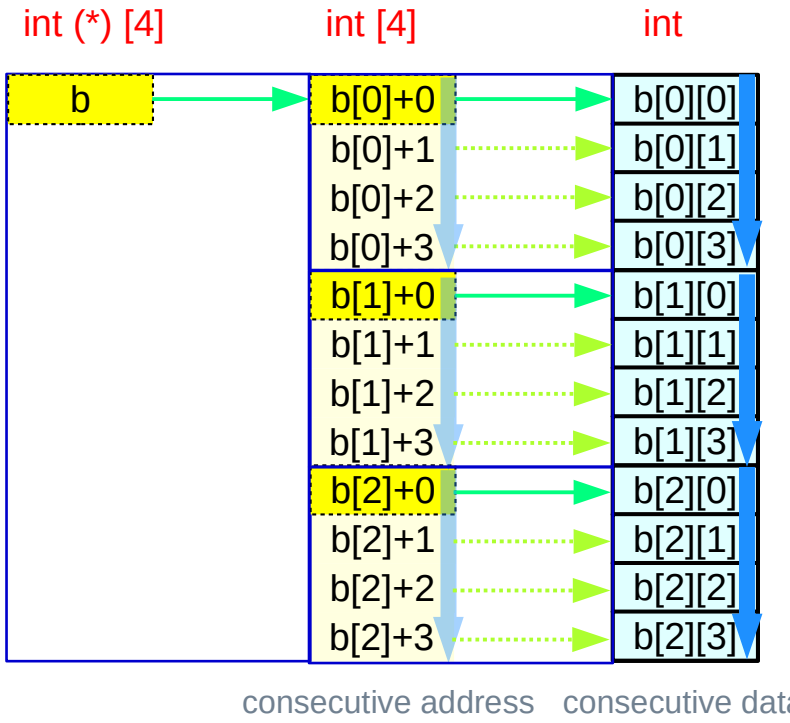
*(b[m]+n) ⟺ b[m][n]

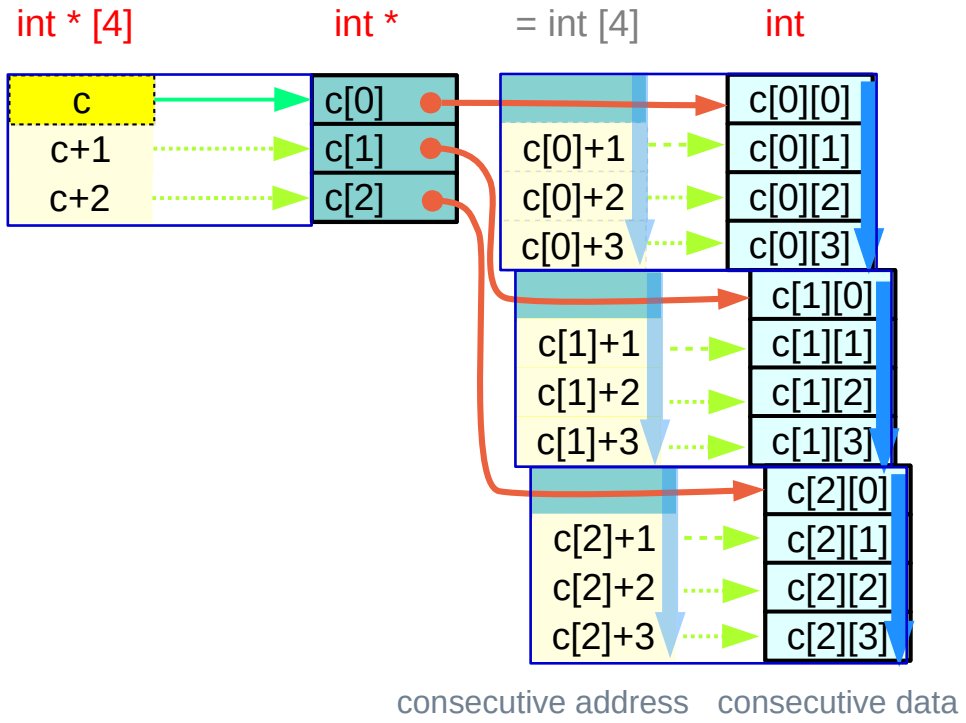
int * c[M] ; with proper assignments

*(c+m) ⟺ c[m] or

*(c[m]+n) ⟺ c[m][n]

Pointer Arrays vs Array Pointers





int (*b)[N] ;

*(b+m) ⟺ b[m]

*(b[m]+n) ⟺ b[m][n]

int * c[M] ; with proper assignments

*(c+m) ⟺ c[m]

*(c[m]+n) ⟺ c[m][n]

Three contiguity constraints for 3-d arrays

Pointer Array Approach (array of pointers)

c[i][j][k] ➡ *(c[i][j] +k)
*(c[i][j] +k) ➡ *(*(c[i] +j) +k)
((c[i] +j) +k) ➡ *(*(*(c +i) +j) +k)

contiguous **int** int
contiguous pointers to **int** int *
contiguous double pointers to **int** int **

the contiguity constraints are satisfied by
allocating arrays of pointers

Array Pointer Approach (pointer to arrays)

c[i][j][k] ➡ *(c[i][j] +k)
*(c[i][j] +k) ➡ *(*(c[i] +j) +k)
((c[i] +j) +k) ➡ *(*(*(c +i) +j) +k)

contiguous **0-d** arrays int int
contiguous **1-d** arrays int [4] int *
contiguous **2-d** arrays int [3][4] int (*) [4]

The contiguity constraints are satisfied by
row major ordered linear data layout

Contiguous array pointers c[i][j][k] ≡ *(c[i][j] +k)

c[0][0][0] = *(c[0][0] + 0)
c[0][0][1] = *(c[0][0] + 1)
c[0][0][2] = *(c[0][0] + 2)
c[0][0][3] = *(c[0][0] + 3)
c[0][1][0] = *(c[0][1] + 0)
c[0][1][1] = *(c[0][1] + 1)
c[0][1][2] = *(c[0][1] + 2)
c[0][1][3] = *(c[0][1] + 3)

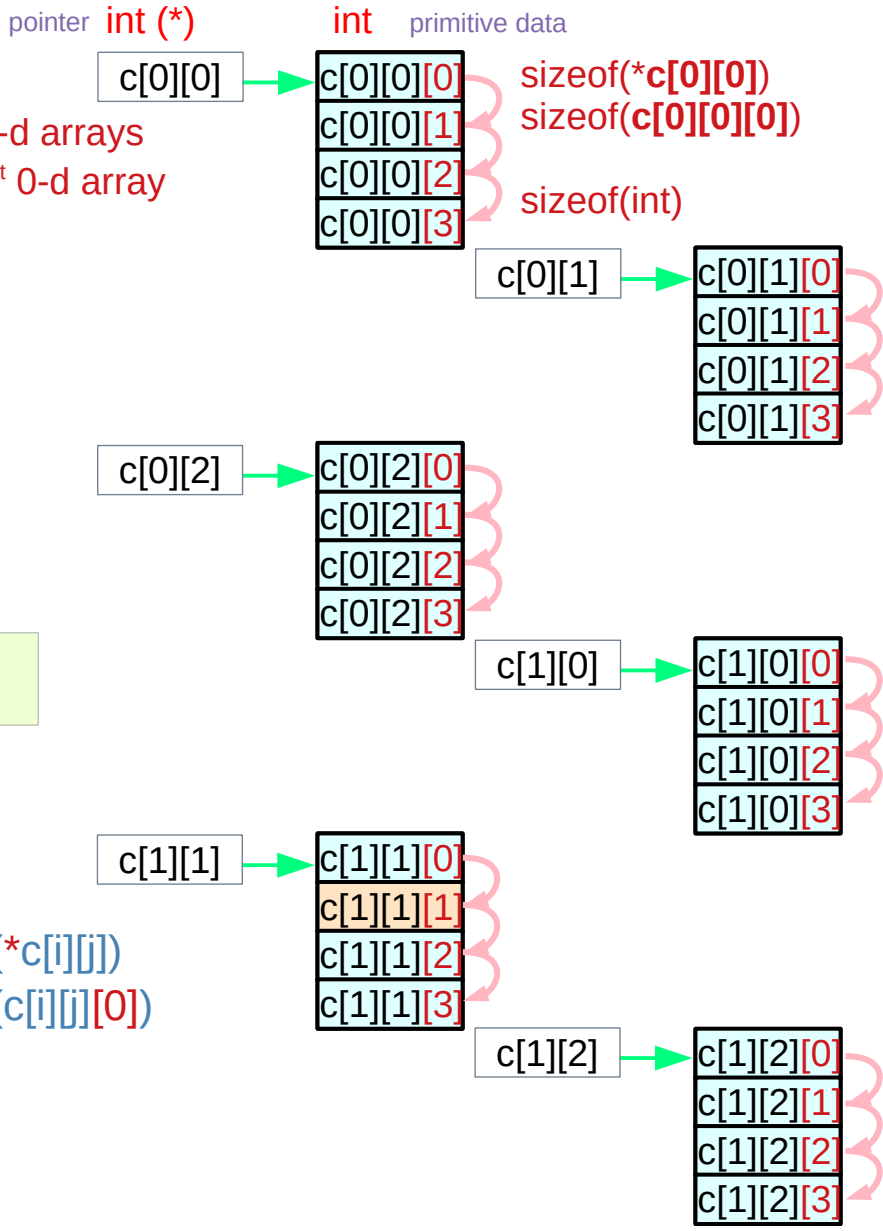
contiguous 1-d
array elements

c[i][j]
int [4] **4** contiguous 0-d arrays
int * points to the 1st 0-d array
int 0-d array

sizeof(**c[i][j]**) [k]
sizeof(**c[i][j][k]**) * 4
sizeof(**int**) * 4

int c[2][3][4];

Address Value
c[i][j] + k
&c[i][j][0] + k * sizeof(*c[i][j])
&c[i][j][0] + k * sizeof(c[i][j][0])
&c[i][j][0] + k * 4

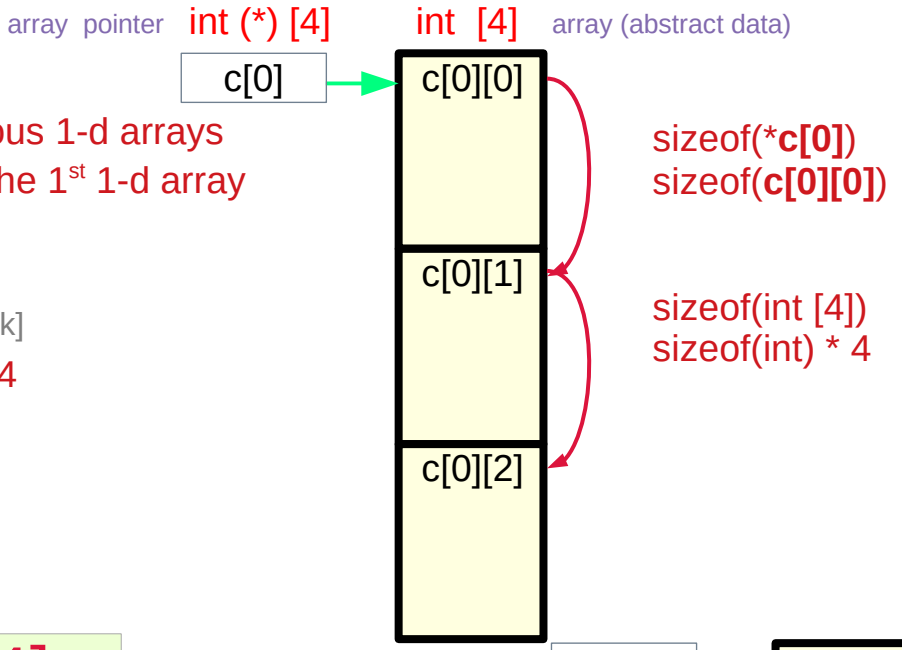


Contiguous array pointers c[i][j] ≡ *(c[i] +j)

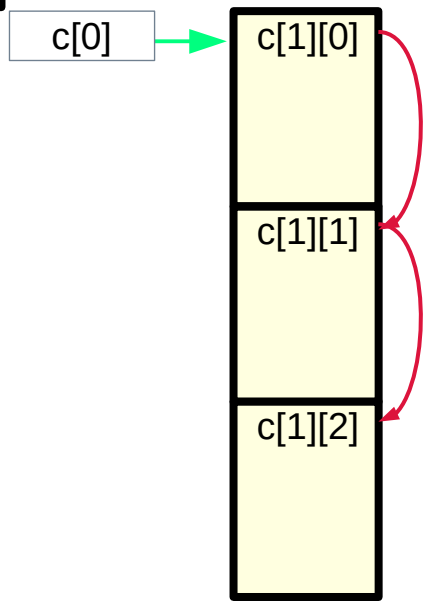
c[0][0] = *(c[0] + 0)
c[0][1] = *(c[0] + 1)
c[0][2] = *(c[0] + 2)
c[1][0] = *(c[1] + 0)
c[1][1] = *(c[2] + 1)
c[1][2] = *(c[3] + 2)

c[i]
int [3][4] **3** contiguous 1-d arrays
int (*) [4] points to the 1st 1-d array
int [4] 1-d array

sizeof(**c[i]**) [j] [k]
sizeof(**c[i][j][k]**) * 3 * 4
sizeof(**int**) * 3 * 4



int c[2][3][4];



Address Value
c[i] + j
&c[i][0][0] + j * sizeof(*c[i])
&c[i][0][0] + j * sizeof(c[i][0])
&c[i][0][0] + j * 4 * 4

Contiguous array pointers c[i] ≡ *(c +i)

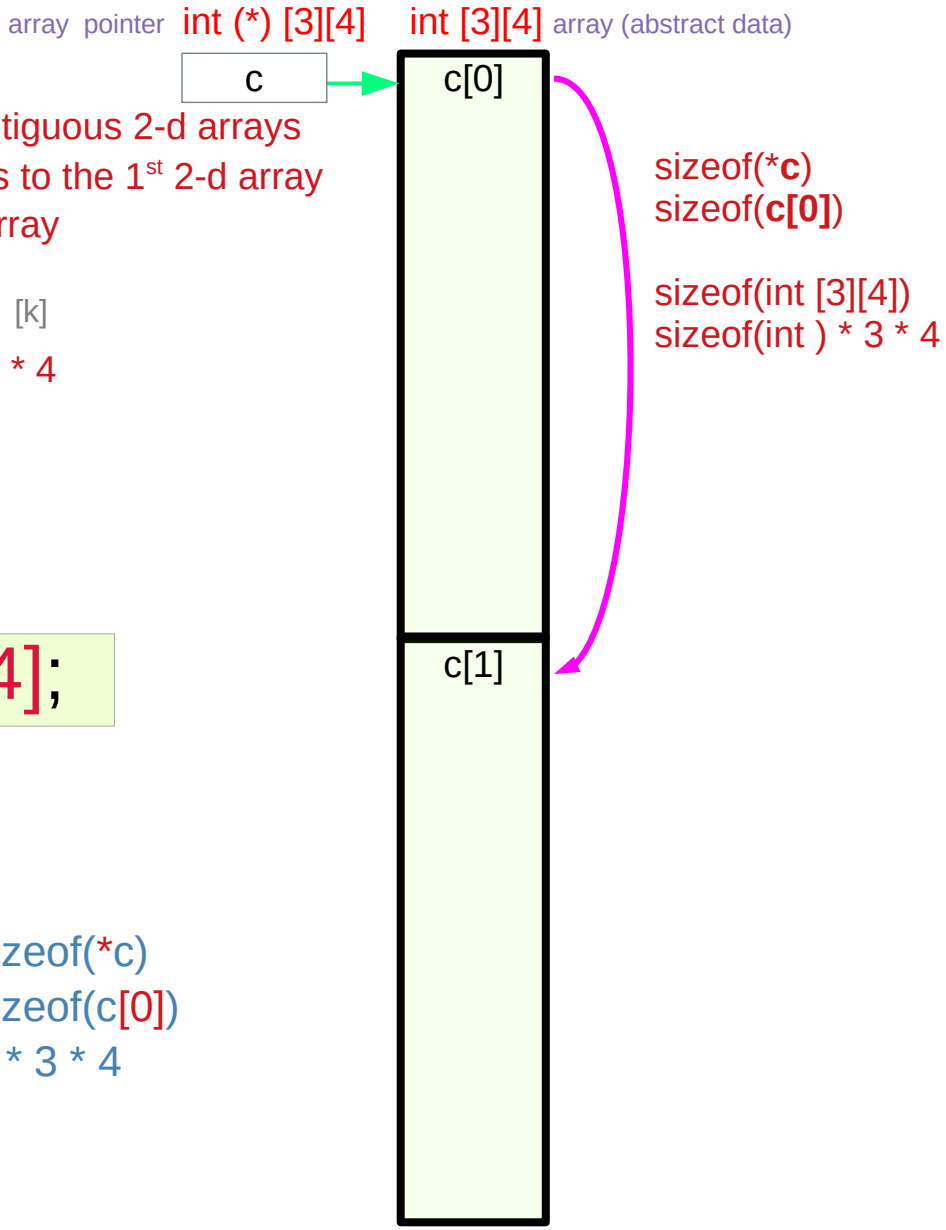
c[0] = *(c + 0)
c[1] = *(c + 1)

c
int [2][3][4] **2** contiguous 2-d arrays
Int (*) [3][4] points to the 1st 2-d array
int [3][4] 2-d array

sizeof(**c**) [i] [j] [k]
sizeof(**c[i][j][k]**) * 2 * 3 * 4
sizeof(**int**) * 2 * 3 * 4

int c[2][3][4];

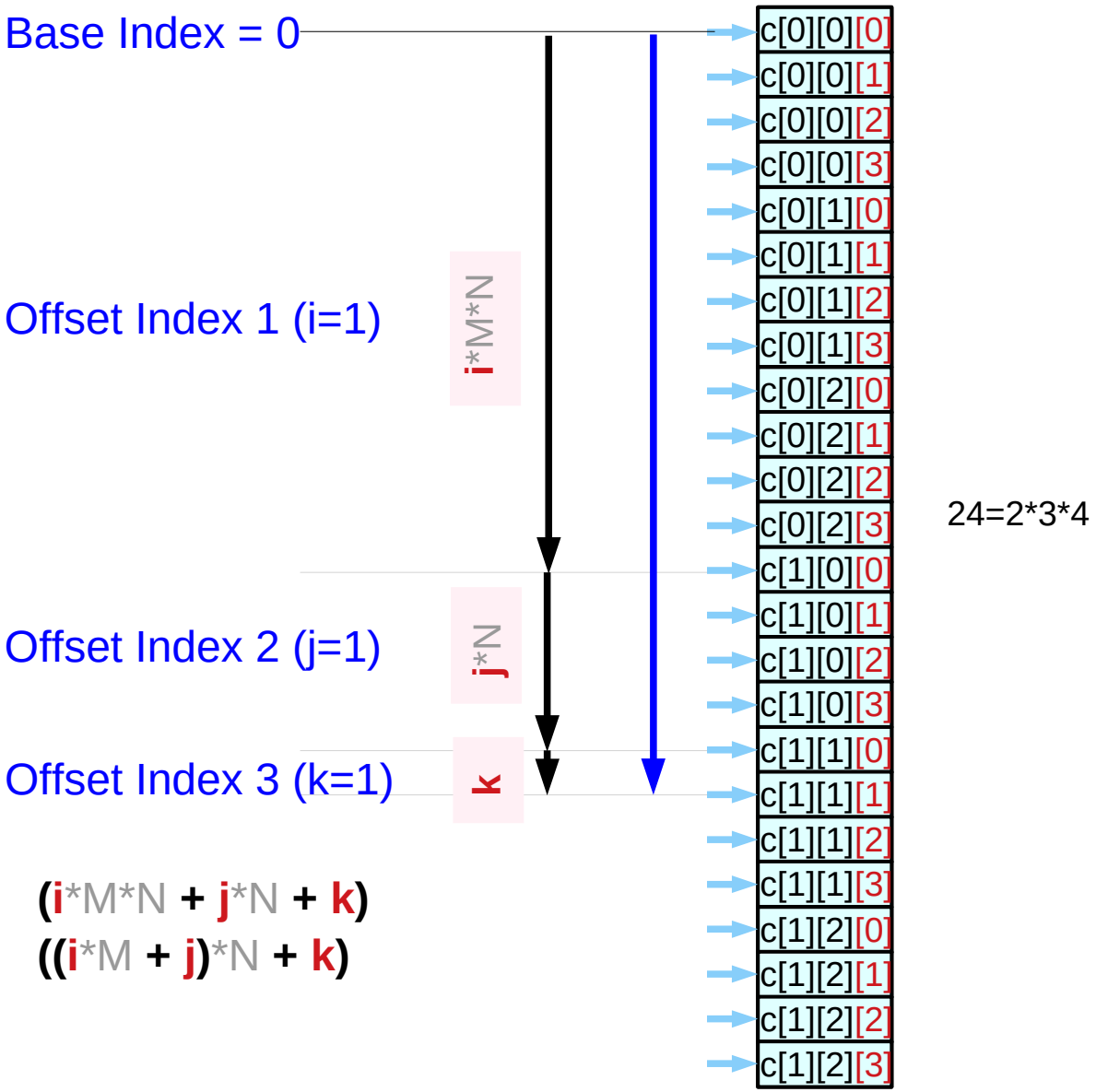
Address Value
c + i
&c[0][0][0] + i * sizeof(*c)
&c[0][0][0] + i * sizeof(c[0])
&c[0][0][0] + i * 4 * 3 * 4



Contiguous linear layout

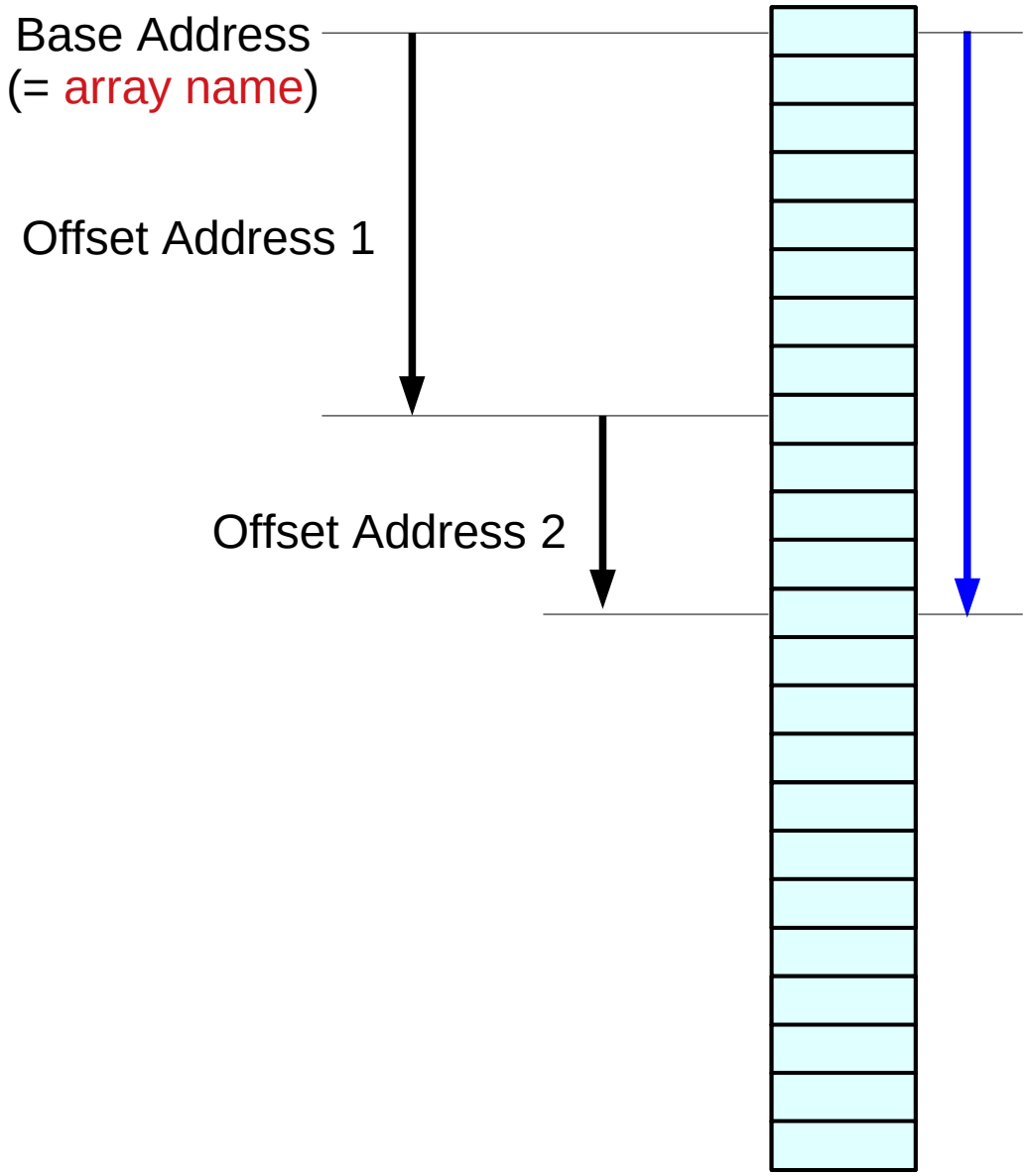
```
int     c [L][M][N];

        c [i][j][k];
```

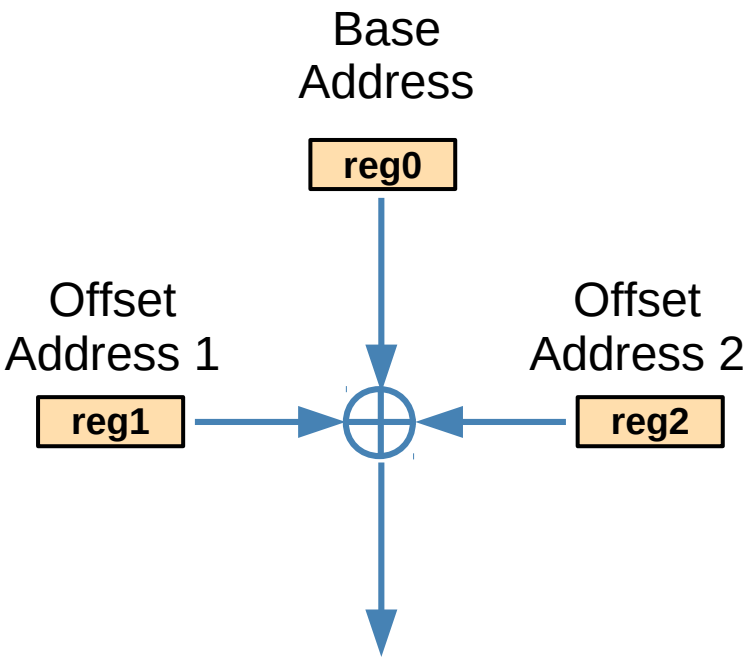
L	M	N
i	**j**	**k**
i*M*N	**j*N**	**k**



(i*M*N + j*N + k)
((i*M + j)*N + k)

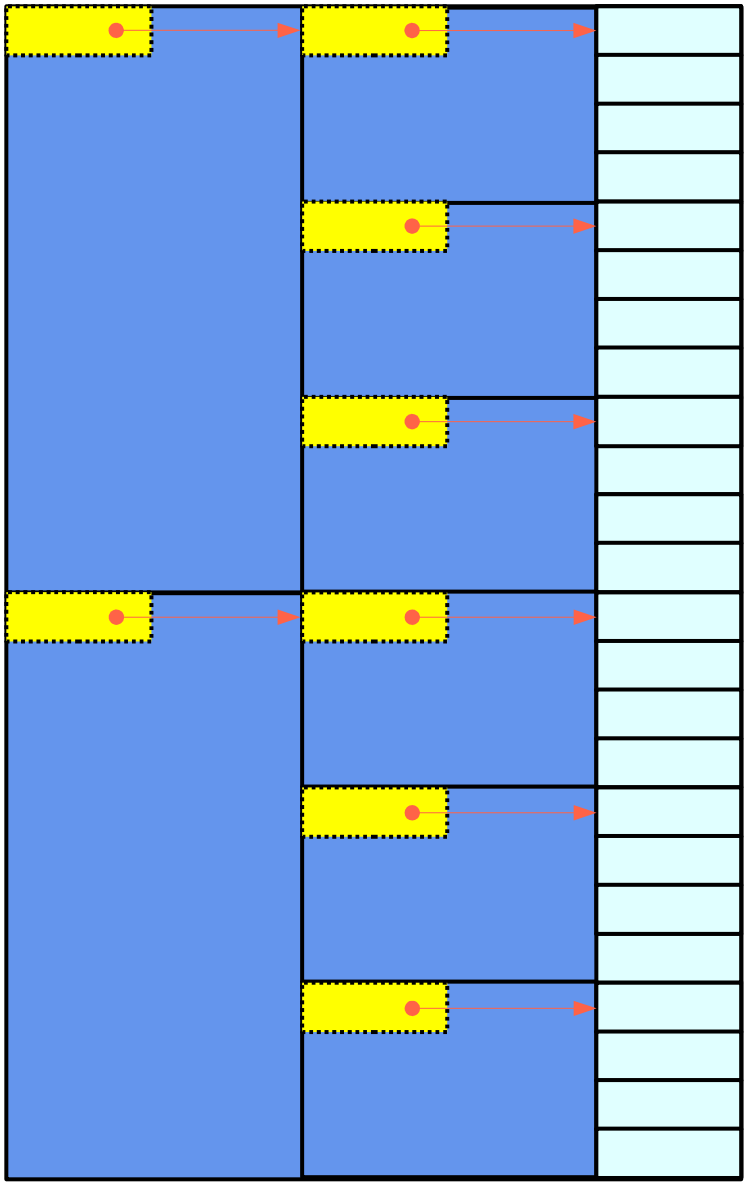
Base and Offset Addressing

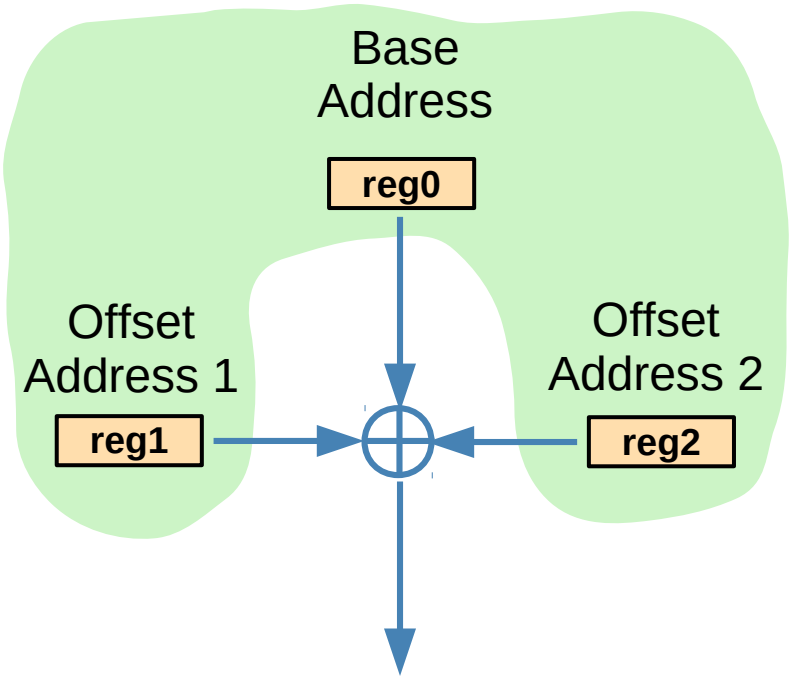




compiler
assembly instruction
registers in the CPU

Array Pointer Approach





register based address **computations**
eliminate the pointer arrays – by a compiler

Array Pointer Approach
(pointer to arrays)

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun