Distribution Functions

Young W Lim

April 28, 2020

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Cumulative Distribution Function

Definition

$F_X(x) = P\{X \le x\}$

- $F_X(x)$: a cumulative distribution function of x
- x : any real number $(-\infty < x < +\infty)$
- $P\{X \le x\}$ is the probability of the event $\{X \le x\}$

CDF of a continuous R.V.

Definition

$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$

- $F_x(x)$: the cdf of a continuous random variable X
- the integral of its probability density function (pdf) $f_X(x)$

The properties of a distribution function

- $F_X(x = -\infty) = 0$
- $F_X(x = +\infty) = 1$
- $0 \leq F_X(x) \leq 1$
- $x_1 < x_2 \implies F_X(x_1) \leq F_X(x_2)$
- $F_X(x_2) - F_X(x_1) = P\{x_1 < X \leq x_2\}$
- $F_X(x^+) = F_X(x)$

$P\{x_1 < X \leq x_2\}$

Theorem

$$P\{x_1 < X \leq x_2\} = F_X(x_2) - F_X(x_1) \qquad (x_1 < x_2)$$

- $\{X \leq x_1\} \cup \{x_1 < X \leq x_2\} = \{X \leq x_2\}$
- $\{x_1 < X \leq x_2\} = \{X \leq x_2\} - \{X \leq x_1\}$
- $P\{x_1 < X \leq x_2\} = P\{X \leq x_2\} - P\{X \leq x_1\}$

CDF of a discrete R.V.

Definition

$$F_X(x) = \sum_{i=1}^{N} P\{X = x_i\} u(x - x_i) = \sum_{i=1}^{N} P(x_i) u(x - x_i)$$

- $P\{X = x_i\} = P(x_i)$
- $u(x - x_i) = \begin{cases} 1 & x \geq x_i \\ 0 & x < x_i \end{cases}$
- $F_X(x) = \sum_{i=1}^{N} P(x_i) u(x - x_i)$