

The Random Variable

Young W Lim

April 27, 2020

Copyright (c) 2018 Young W. Lim. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 3.0 Unported" license.



Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

- 1 Definition
- 2 Conditions
- 3 Types

Random Variable Definition

Definition

a **real random variable**

a real **function** over a sample space $S = \{s_1, s_2, s_3, \dots, s_n\}$

a real random variable : capital letter X

a particular value : a lowercase letter x

a sample space $S = \{s_1, s_2, s_3, \dots, s_n\}$

an element of S : s

Random Variable Example

Example

$$X(s_1) = x_1$$

$$s_1 \longrightarrow x_1$$

$$X(s_2) = x_2$$

$$s_2 \longrightarrow x_2$$

...

...

$$X(s_n) = x_n$$

$$s_n \longrightarrow x_n$$

$$S = \{s_1, s_2, s_3, \dots, s_n\}$$

a sample space

 X

a random variable

Random Variable Conditions

- not multivalued
 - for every sample point in S ,
 - exactly one value of the random variable
- the set $\{X \leq x\}$:
 - an event for any real number x
 - sample points $s \in S$,
 - for which the random variable $X(s) < x$
- the probability $P\{X \leq x\}$
 - the sum of the probabilities of the events
 - corresponding to s
- $P\{X = -\infty\} = 0$, $P\{X = +\infty\} = 0$

Discrete Random Variables

Definition

Discrete Random Variables

a random variable X has only discrete values x_i ;
sample space S can

- a discrete set
- a continuous set
- a mixed set

Continuous Random Variables

Definition

Continuous Random Variables

a random variable X has a continuous range of values
sample space S must be

- a continuous set

Mixed Random Variables

Definition

Mixed Random Variables

a random variable X has
both a discrete values and
a continuous range of values

