Tree Traversal (1A)

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6/18/18

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This document was produced by using LibreOffice and Octave.

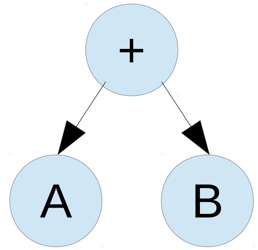
Infix, Prefix, Postfix Notations

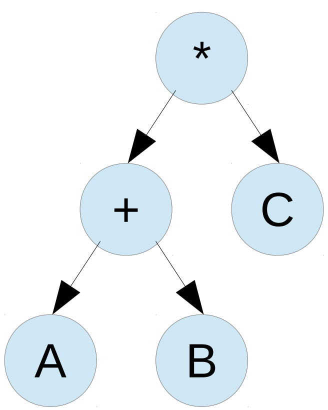
Infix Notation	Prefix Notation	Postfix Notation
A + B	+ A B	A B +
(A + B) * C	* + A B C	A B + C *
A * (B + C)	* A + B C	A B C + *
A / B + C / D	+ / A B / C D	A B / C D / +
((A + B) * C) – D	– * + A B C D	A B + C * D –

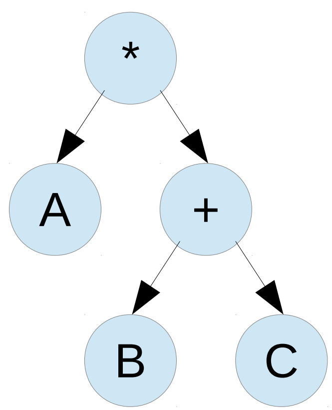
https://www.tutorialspoint.com/data_structures_algorithms/expression_parsing.html

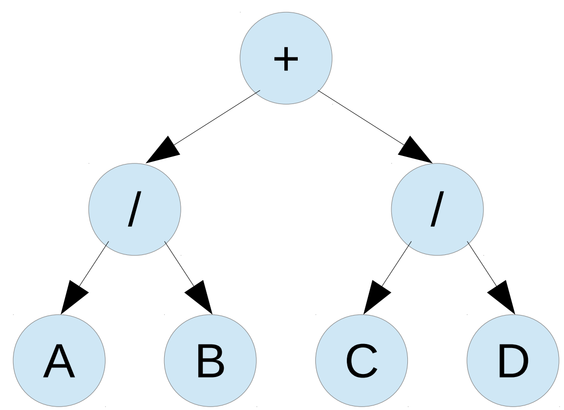
Infix, Prefix, Postfix Notations and Binary Trees

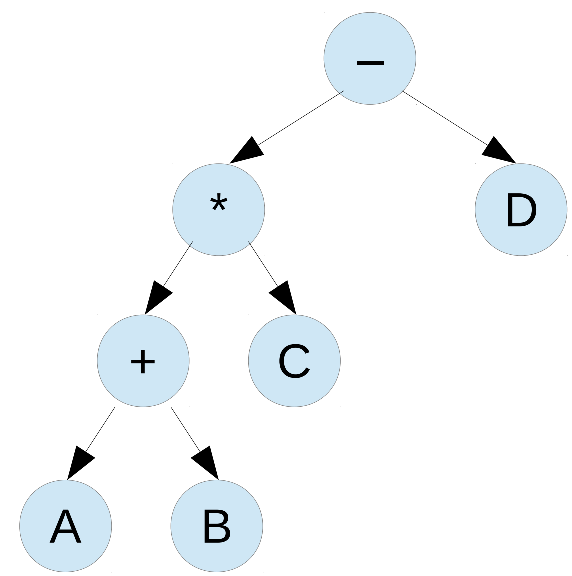
Infix Notation	Prefix Notation	Postfix Notation
A + B	+ A B	A B +
(A + B) * C	* + A B C	A B + C *
A * (B + C)	* A + B C	A B C + *
A / B + C / D	+ / A B / C D	A B / C D / +
((A + B) * C) – D	– * + A B C D	A B + C * D –











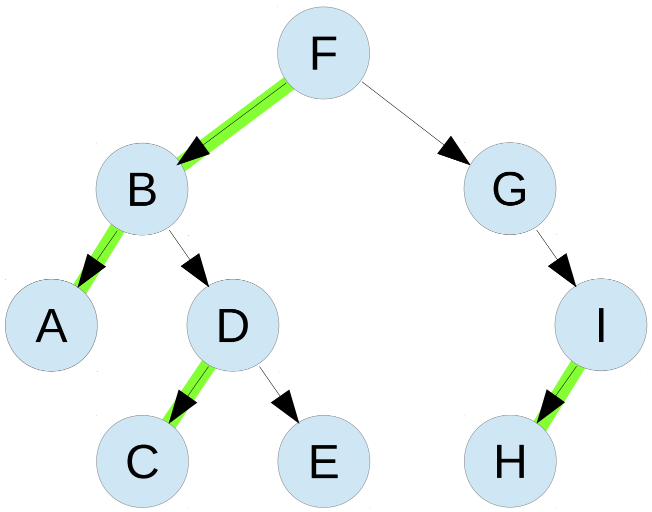
https://www.tutorialspoint.com/data_structures_algorithms/expression_parsing.htm

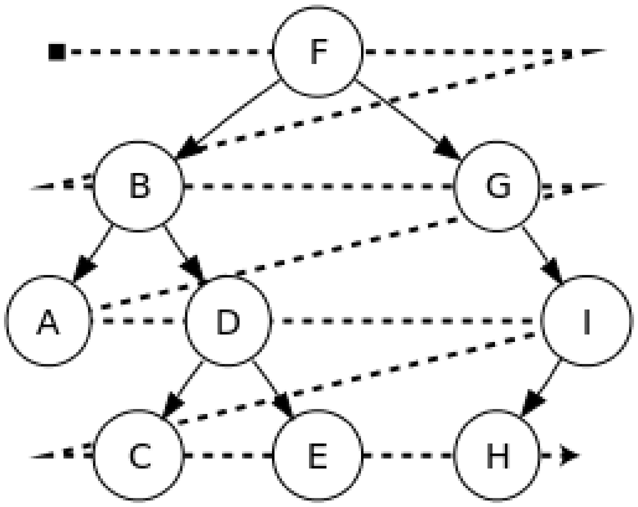
Tree Traversal

Depth First Search

- Pre-Order
- In-order
- Post-Order

Breadth First Search

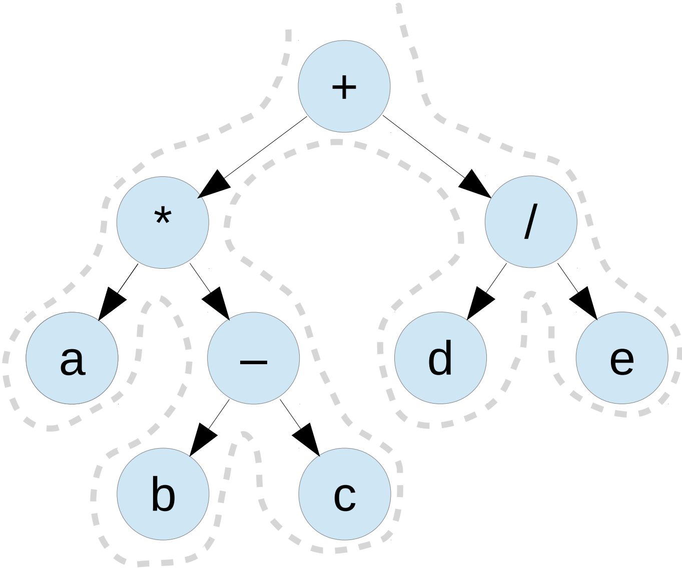




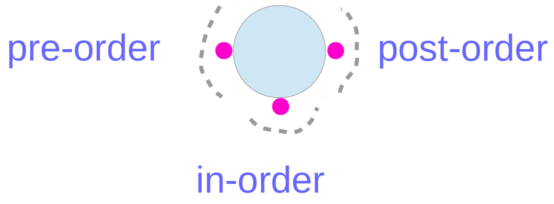
https://en.wikipedia.org/wiki/Morphism

Depth First Search on Binary Trees

Depth First Search

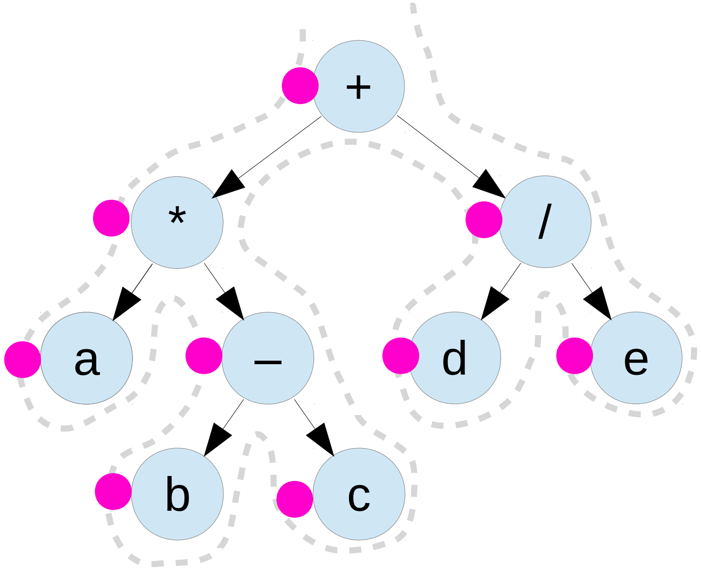


Three Variations
Pre-Order, In-Order, Post-Order



https://en.wikipedia.org/wiki/Tree_traversal

Pre-Order Binary Tree Traversals

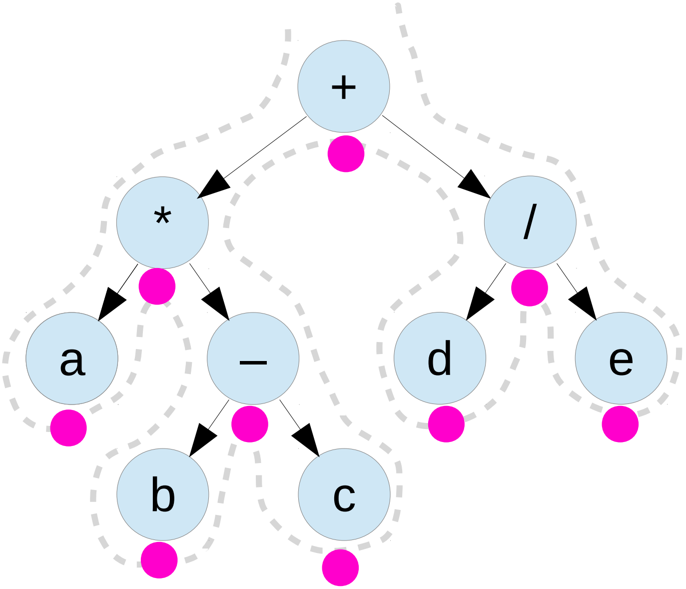


(a*(b-c))+(d/e)

a * b – c + d / e	Infix notation
+ * a – b c / d e	Prefix notation
a b c – * d e / +	Postfix notation

https://en.wikipedia.org/wiki/Tree_traversal

In-Order Binary Tree Traversals

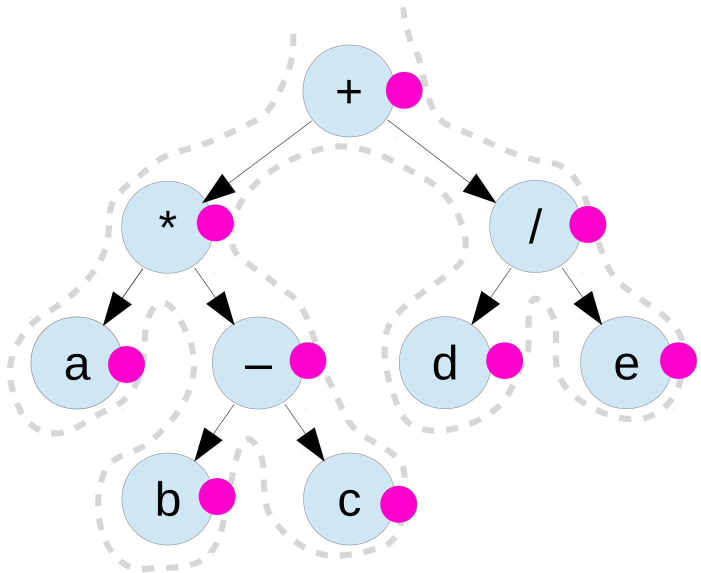


(a*(b-c))+(d/e)

a * b – c + d / e	Infix notation
+ * a – b c / d e	Prefix notation
a b c – * d e / +	Postfix notation

https://en.wikipedia.org/wiki/Tree_traversal

Post-Order Binary Tree Traversals



(a*(b-c))+(d/e)

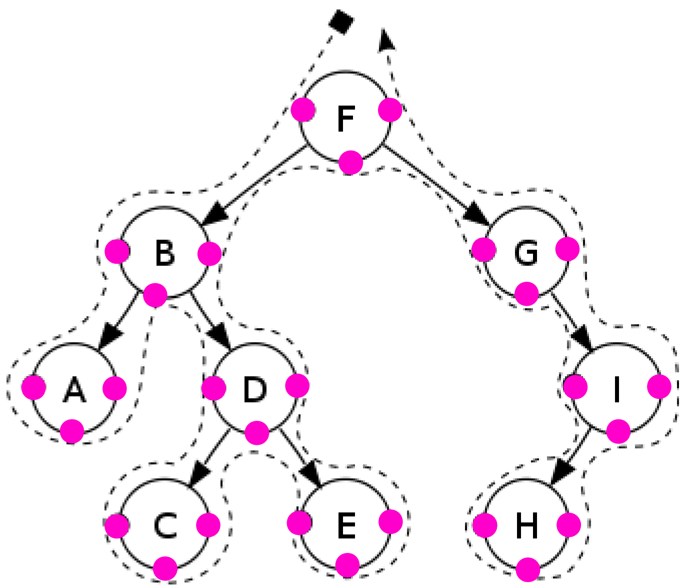
a * b – c + d / e	Infix notation
+ * a – b c / d e	Prefix notation
a b c – * d e / +	Postfix notation

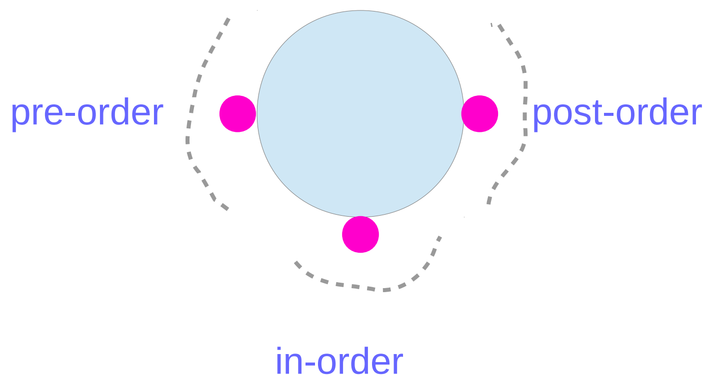
https://en.wikipedia.org/wiki/Tree_traversal

Binary Tree Traversal

Depth First Search
- Pre-Order
- In-order
- Post-Order

Breadth First Search





https://en.wikipedia.org/wiki/Tree_traversal

Pre-Order Traversal on Binary Trees

pre-order function

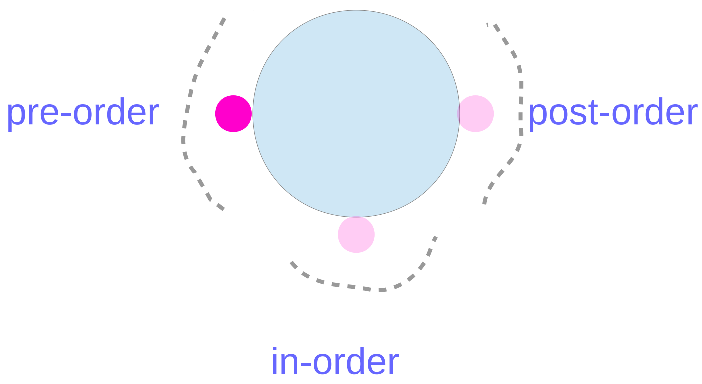
Check if the current node is empty / null.

<u>**Display**</u> the data part of the root (or current node).

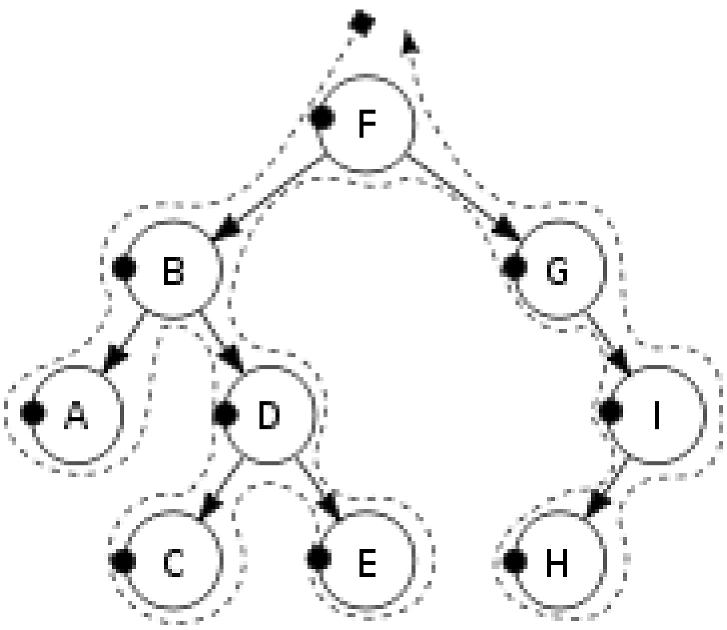
Traverse the **left** subtree by recursively calling the **pre-order** function.

Traverse the **right** subtree by recursively calling the **pre-order** function.

FBADCEGIH



https://en.wikipedia.org/wiki/Tree_traversal



In-Order Traversal on Binary Trees

in-order function

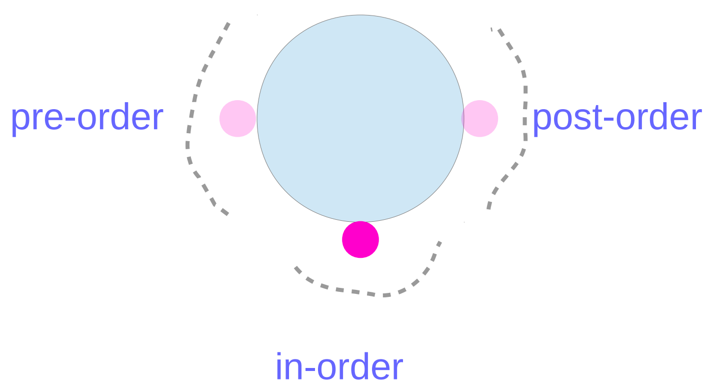
Check if the current node is empty / null.

Traverse the left subtree by recursively calling the **in-order** function.

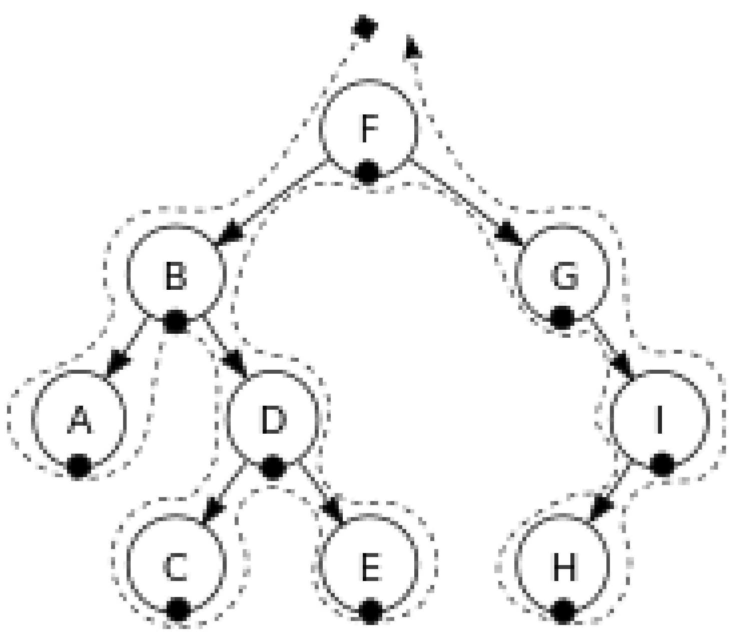
Display the data part of the root (or current node).

Traverse the right subtree by recursively calling the **in-order** function.

ABCDEFGHI



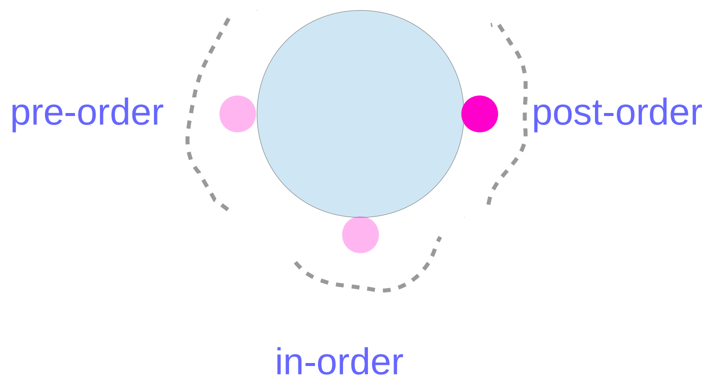
https://en.wikipedia.org/wiki/Tree_traversal

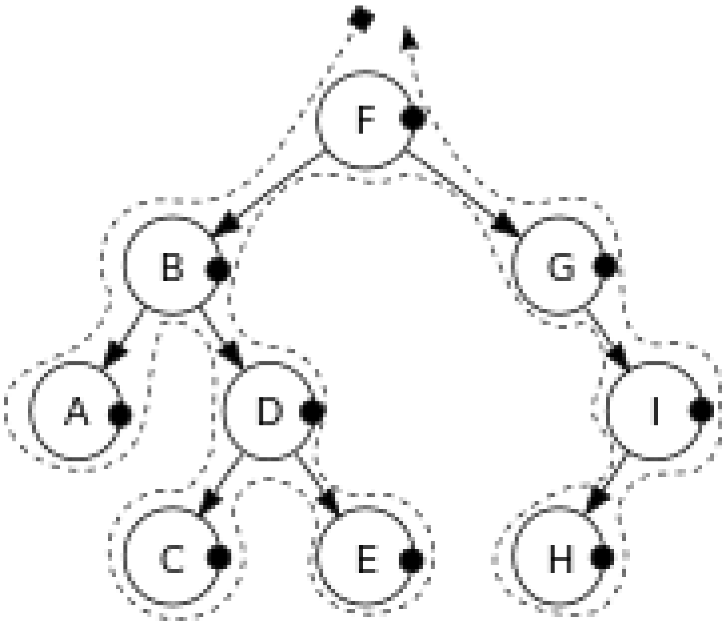


Post-Order Traversal on Binary Trees

post-order function
- Check if the current node is empty / null.
- **Traverse** the left subtree by recursively calling the **post-order** function.
- **Traverse** the right subtree by recursively calling the **post-order** function.
- **Display** the data part of the root (or current node).

ACEDBHIGH

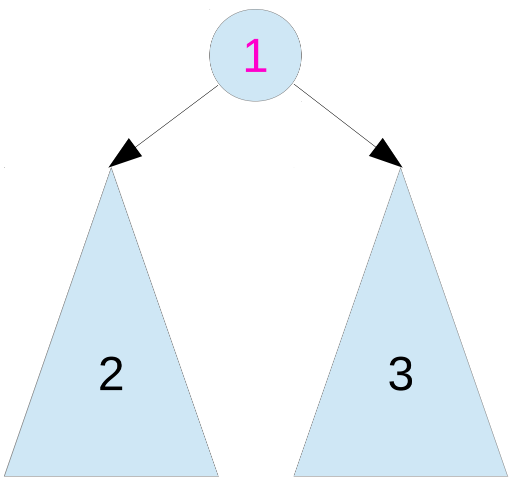




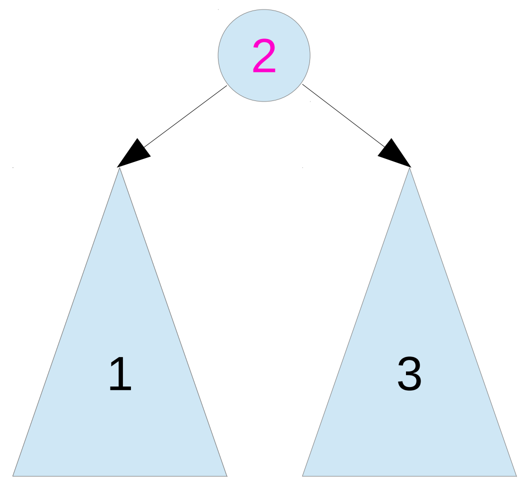
https://en.wikipedia.org/wiki/Tree_traversal

Recursive Algorithms

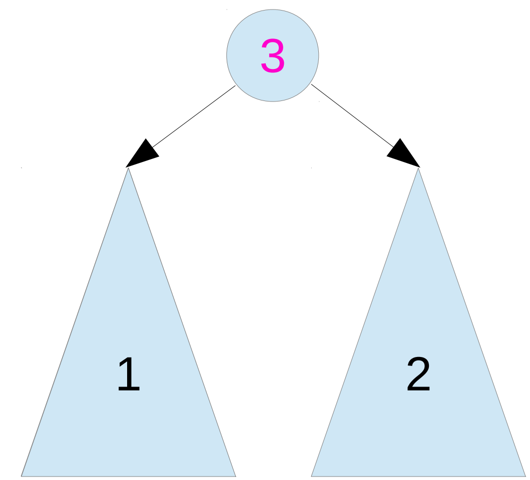
```
preorder(node)
  if (node = null)
    return
  visit(node)
  preorder(node.left)
  preorder(node.right)
```




```
inorder(node)
  if (node = null)
    return
  inorder(node.left)
  visit(node)
  inorder(node.right)
```



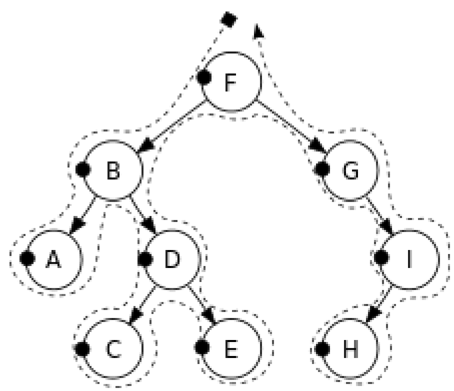

```
postorder(node)
  if (node = null)
    return
  postorder(node.left)
  postorder(node.right)
  visit(node)
```

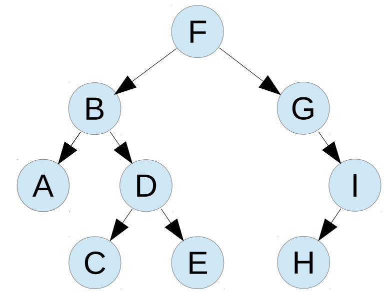


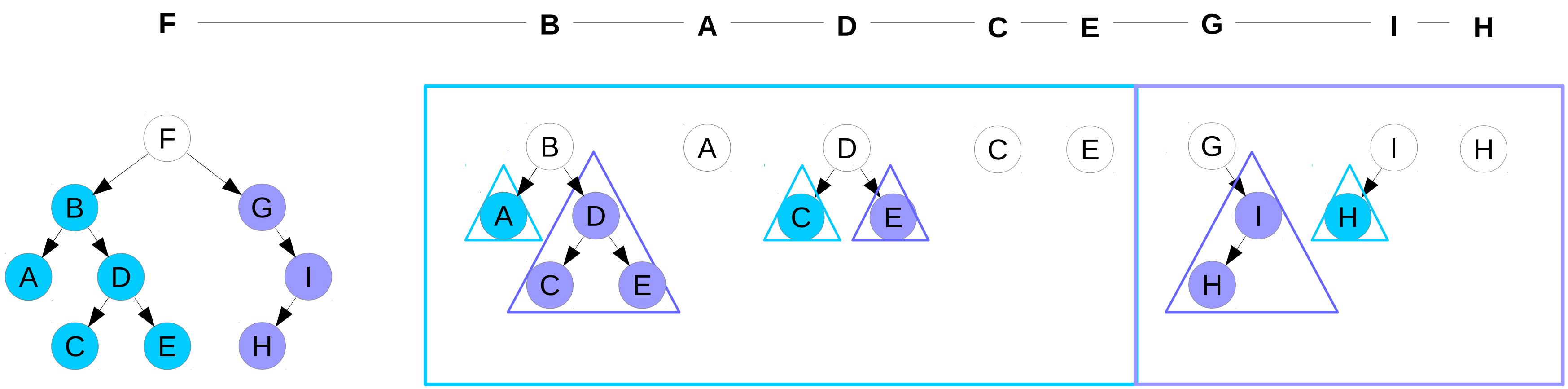
https://en.wikipedia.org/wiki/Tree_traversal

Pre-Order recursive algorithm

preorder(node)
  if (node = null)
    return
  visit(node)
  preorder(node.**left**)
  preorder(node.**right**)







https://en.wikipedia.org/wiki/Tree_traversal

Iterative Algorithms

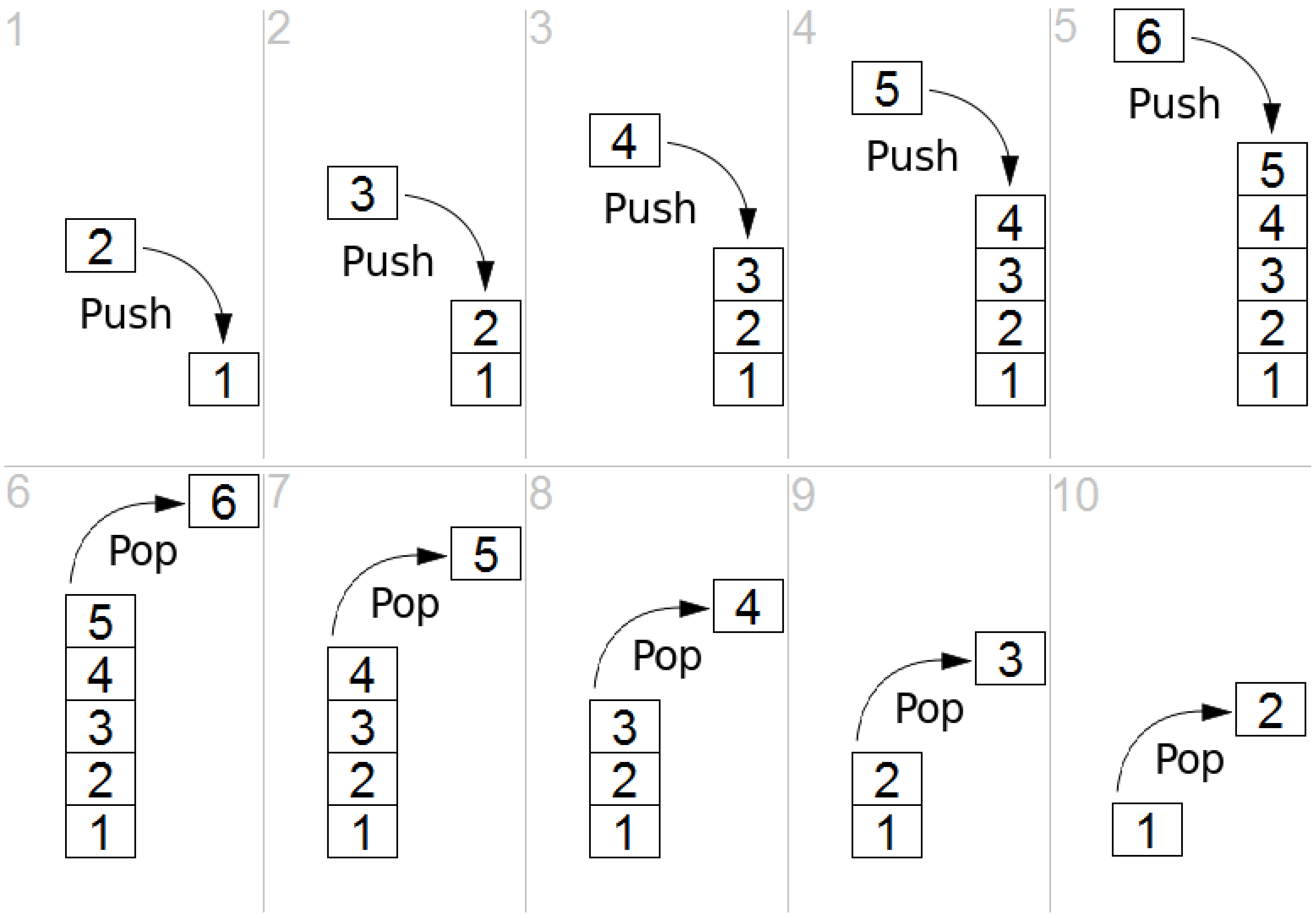
```
iterativePreorder(node)
  if (node = null)
    return
  s ← empty stack
  s.push(node)

  while (not s.isEmpty())
    node ← s.pop()
    visit(node)
    // right child is pushed first
    // so that left is processed first
    if (node.right ≠ null)
      s.push(node.right)
    if (node.left ≠ null)
      s.push(node.left)
```

https://en.wikipedia.org/wiki/Tree_traversal

```
iterativeInorder(node)
  s ← empty stack

  while (not s.isEmpty() or
           node ≠ null)
    if (node ≠ null)
      s.push(node)
      node ← node.left
    else
      node ← s.pop()
      visit(node)
      node ← node.right
```

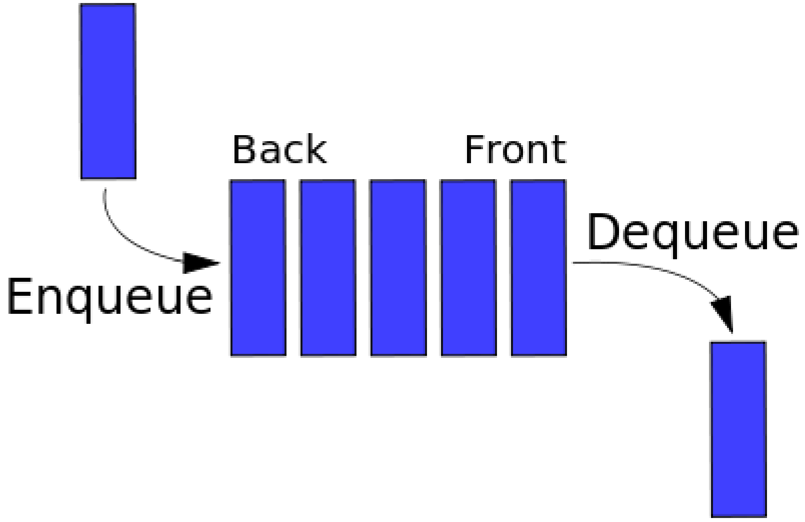
```
iterativePostorder(node)
  s ← empty stack
  lastNodeVisited ← null

  while (not s.isEmpty() or node ≠ null)
    if (node ≠ null)
      s.push(node)
      node ← node.left
    else
      peekNode ← s.peek()
      // if right child exists and traversing
      // node from left child, then move right
      if (peekNode.right ≠ null and
         lastNodeVisited ≠ peekNode.right)
        node ← peekNode.right
      else
        visit(peekNode)
        lastNodeVisited ← s.pop()
```

Stack



https://en.wikipedia.org/wiki/Stack_(abstract_data_type)

Queue



https://en.wikipedia.org/wiki/Queue_(abstract_data_type)#/media/File:Data_Queue.sv
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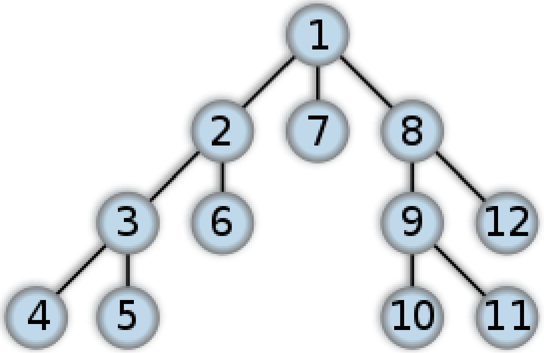
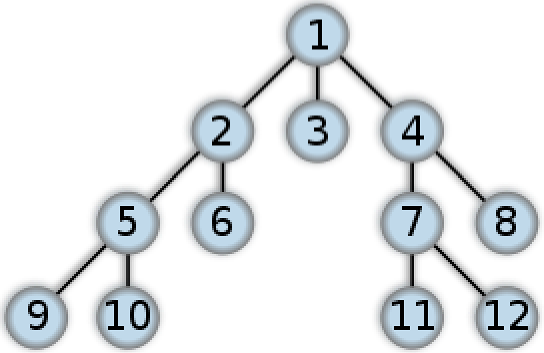
Search Algorithms

DFS (Depth First Search)



BFS (Breadth First Search)



https://en.wikipedia.org/wiki/Breadth-first_search, /Depth-first_search

DFS Algorithm

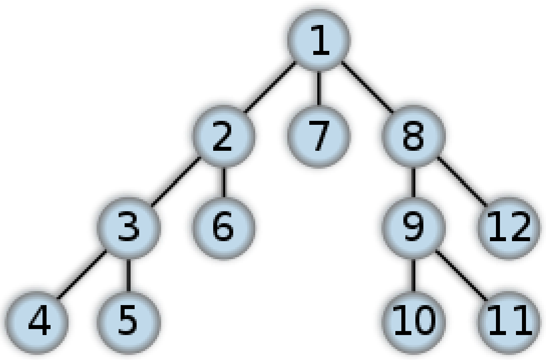
A recursive implementation of DFS:

```
procedure DFS(G,v):
    label v as discovered
    for all edges from v to w in G.adjacentEdges(v) do
        if vertex w is not labeled as discovered then
            recursively call DFS(G,w)
```

A non-recuUrsive implementation of DFS:

```
procedure DFS-iterative(G,v):
    let S be a stack
    S.push(v)
    while S is not empty
        v = S.pop()
        if v is not labeled as discovered:
            label v as discovered
            for all edges from v to w in G.adjacentEdges(v) do
                S.push(w)
```

DFS (Depth First Search)

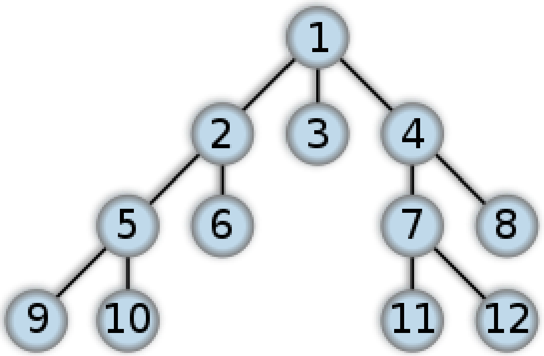


https://en.wikipedia.org/wiki/Breadth-first_search, /Depth-first_search

BFS Algorithm

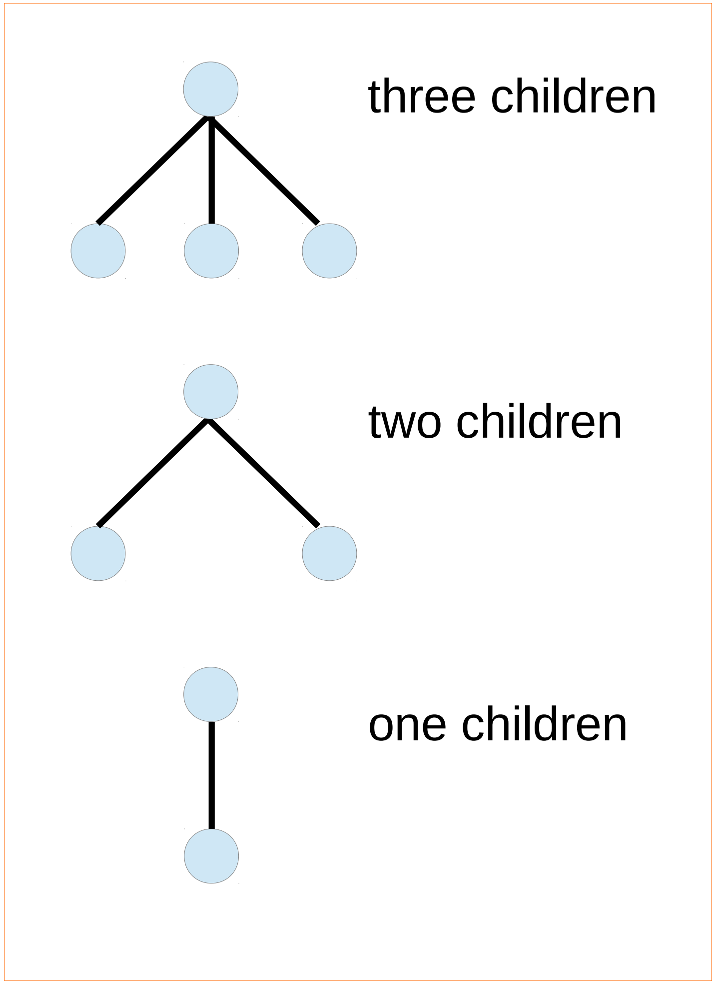
```
Breadth-First-Search(Graph, root):

    create empty set S     // admissiable node set
    create empty queue Q

    add root to S
    Q.enqueue(root)

    while Q is not empty:
        current = Q.dequeue()
        if current is the goal:
            return current
        for each node n that is adjacent to current:
            if n is not in S:
                add n to S
                n.parent = current
                Q.enqueue(n)
```

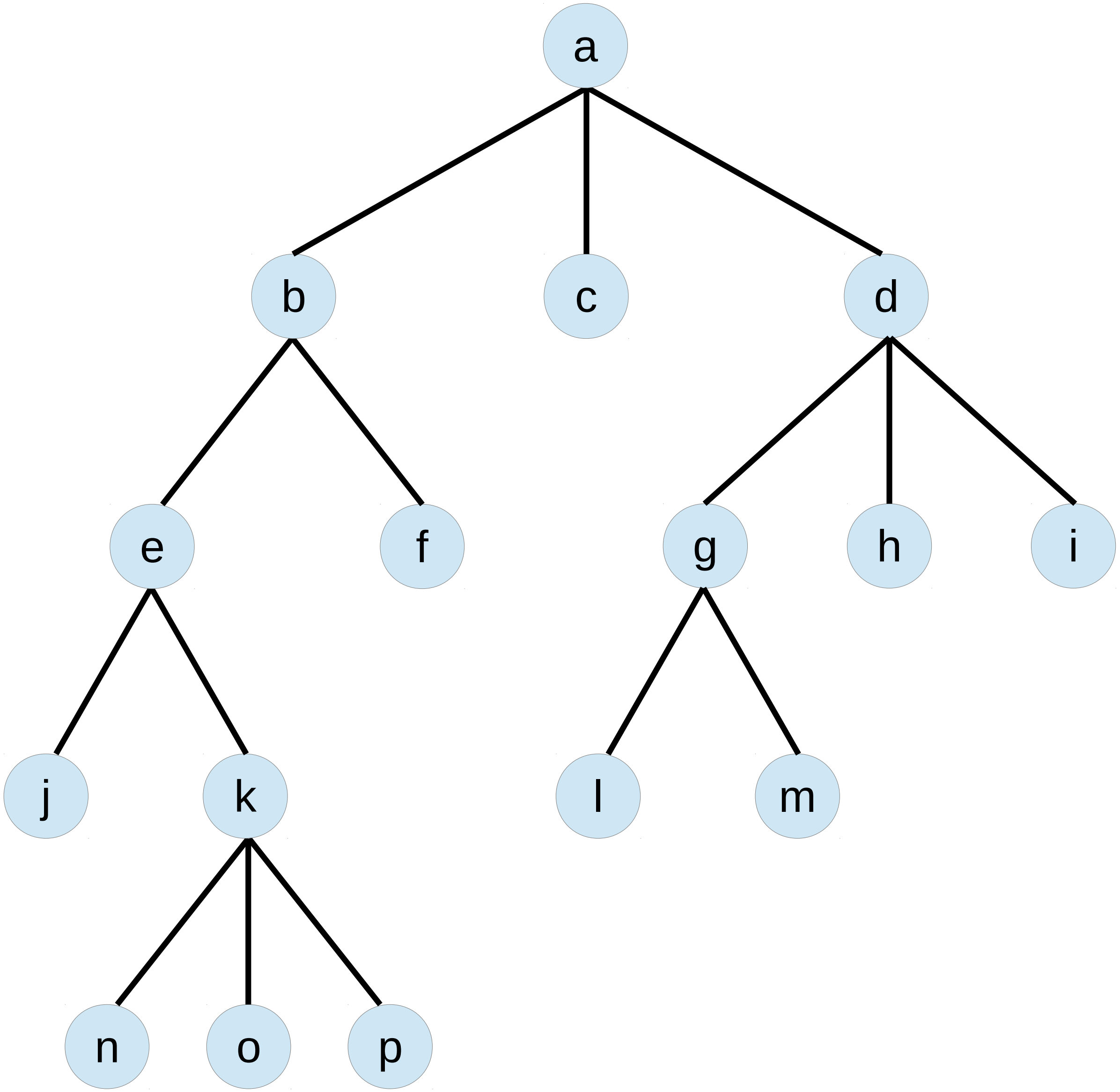
BFS (Breadth First Search)



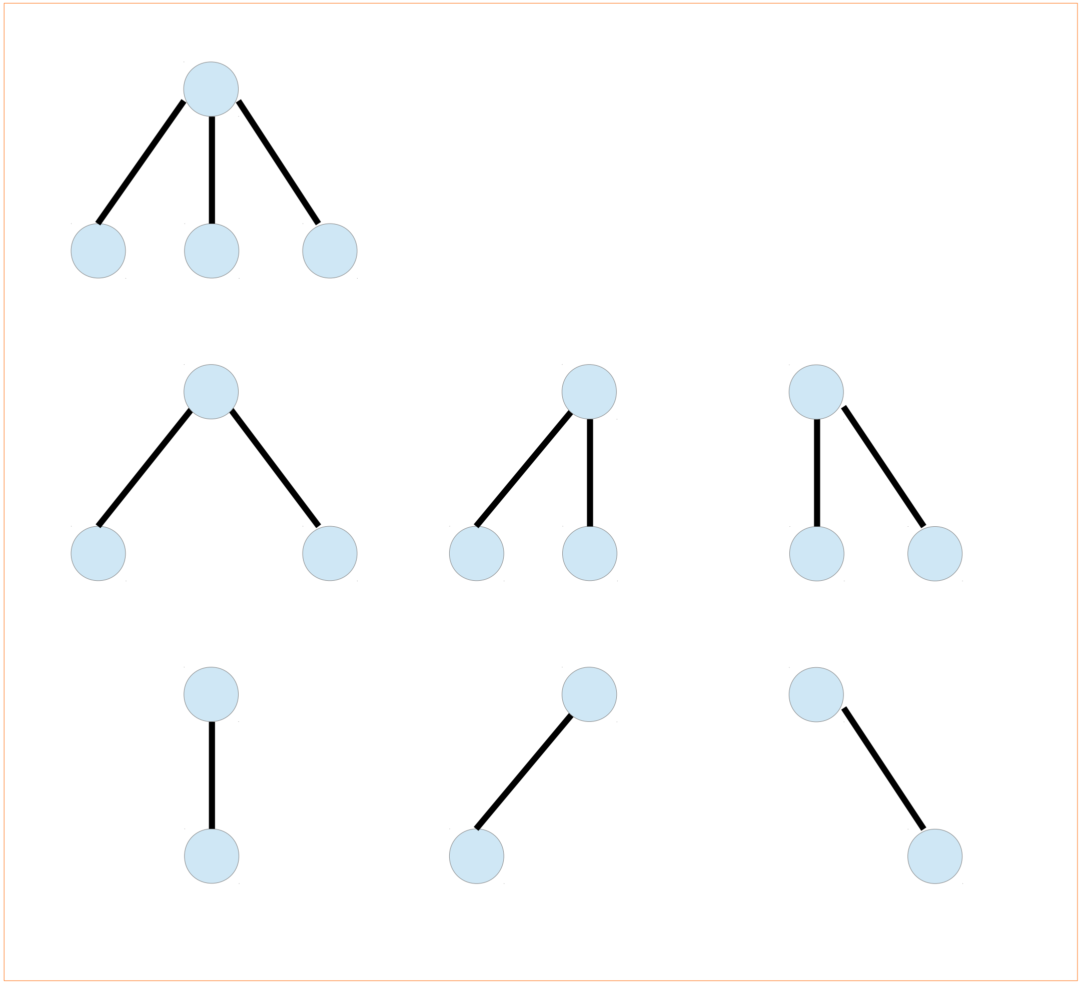
https://en.wikipedia.org/wiki/Breadth-first_search, /Depth-first_search

Ternary Tree Example





Children of a Ternary Tree

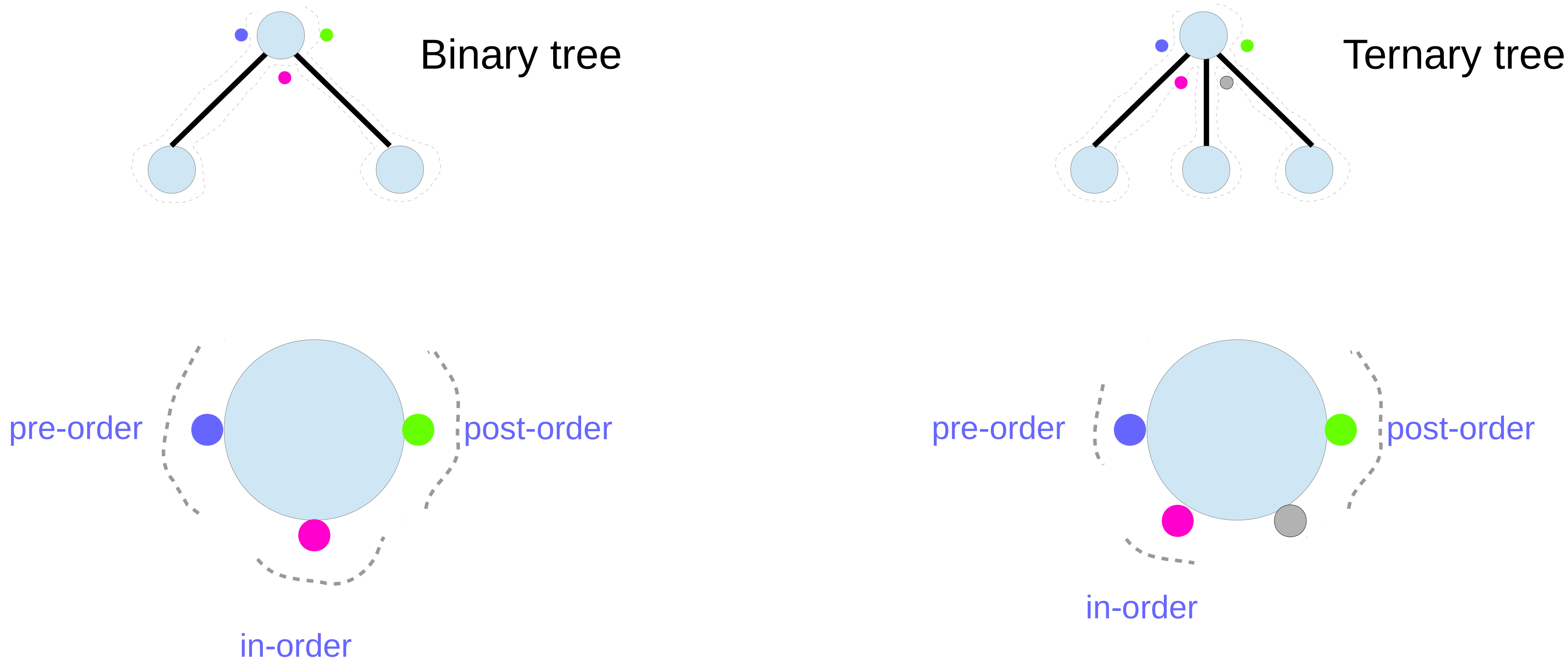


three children

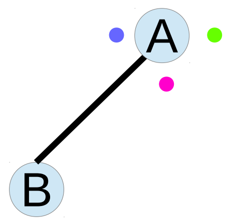
two children

one children

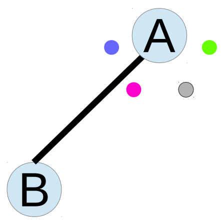
Ternary Tree Traversal



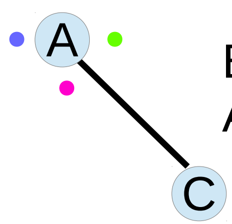
Ternary Tree In-Order Traversal



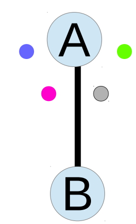
Binary tree
B-A



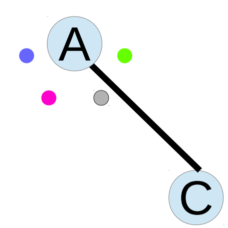
Ternary tree
B-A



Binary tree
A-B



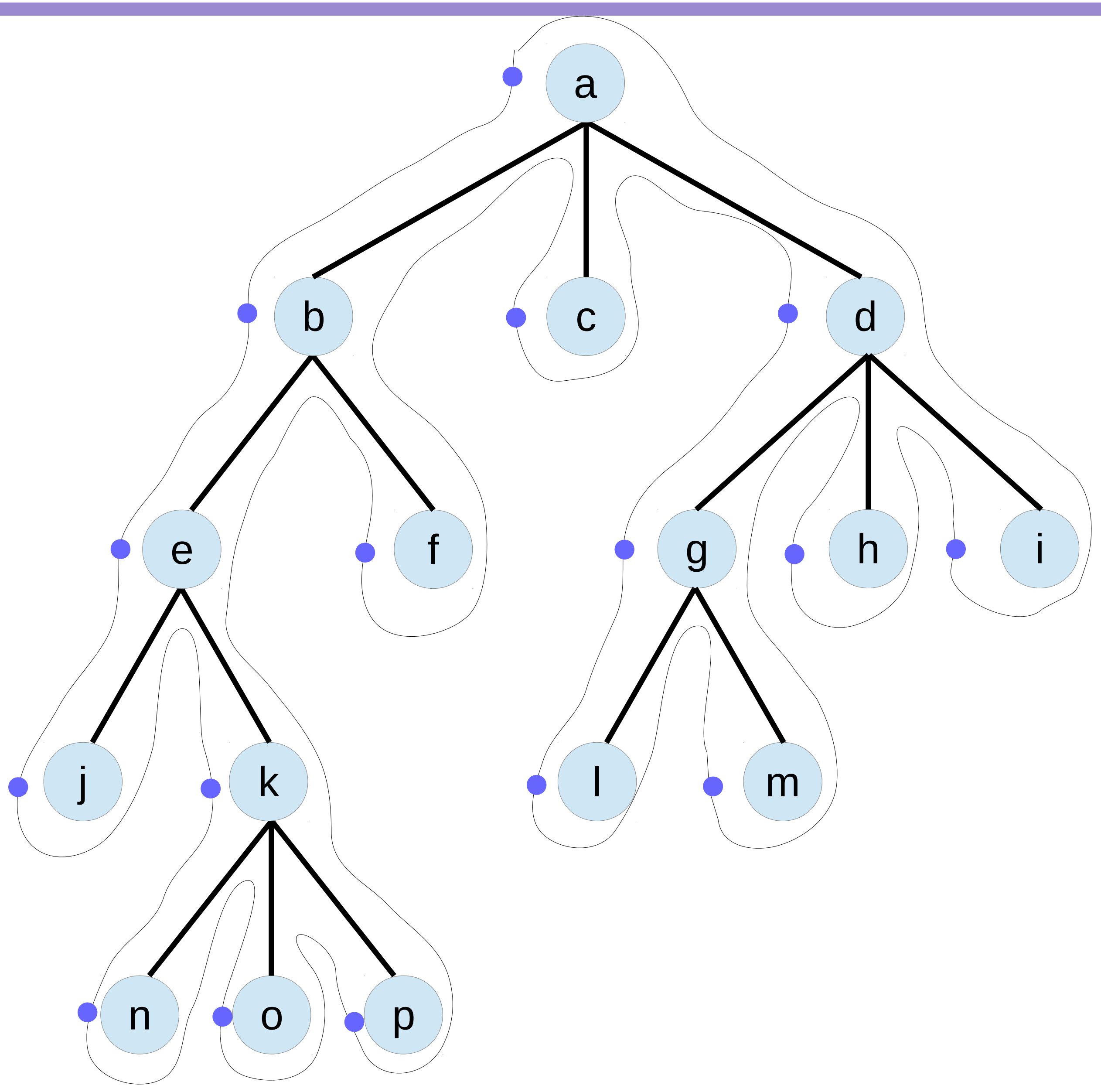
Ternary tree
A-B



Ternary tree
A-C

Pre-Order Traversal on Ternary Trees

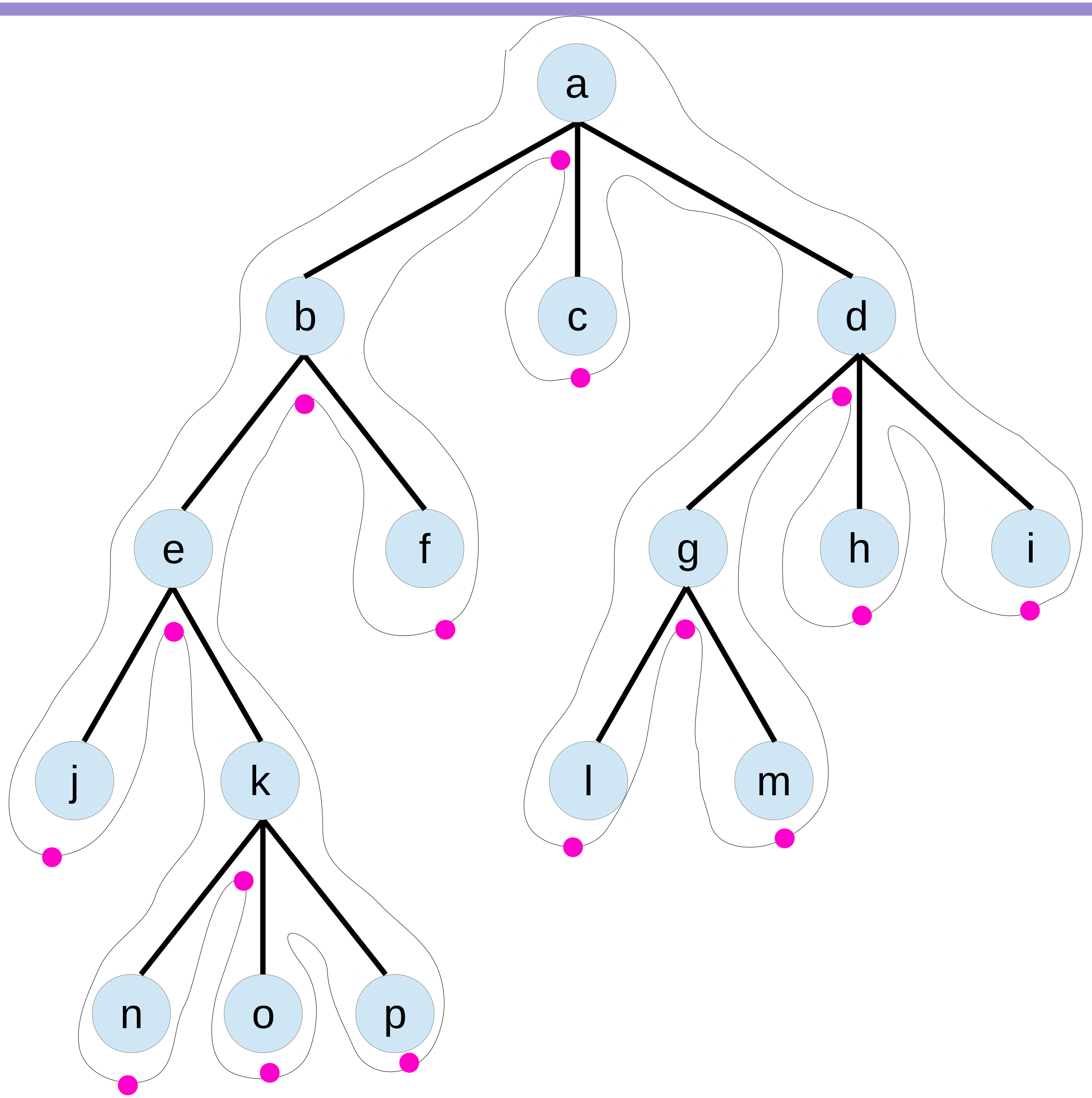
a-b-e-j-k-n-o-p-f-c-d-g-l-m-h-i



Rosen

In-Order Traversal on Ternary Trees

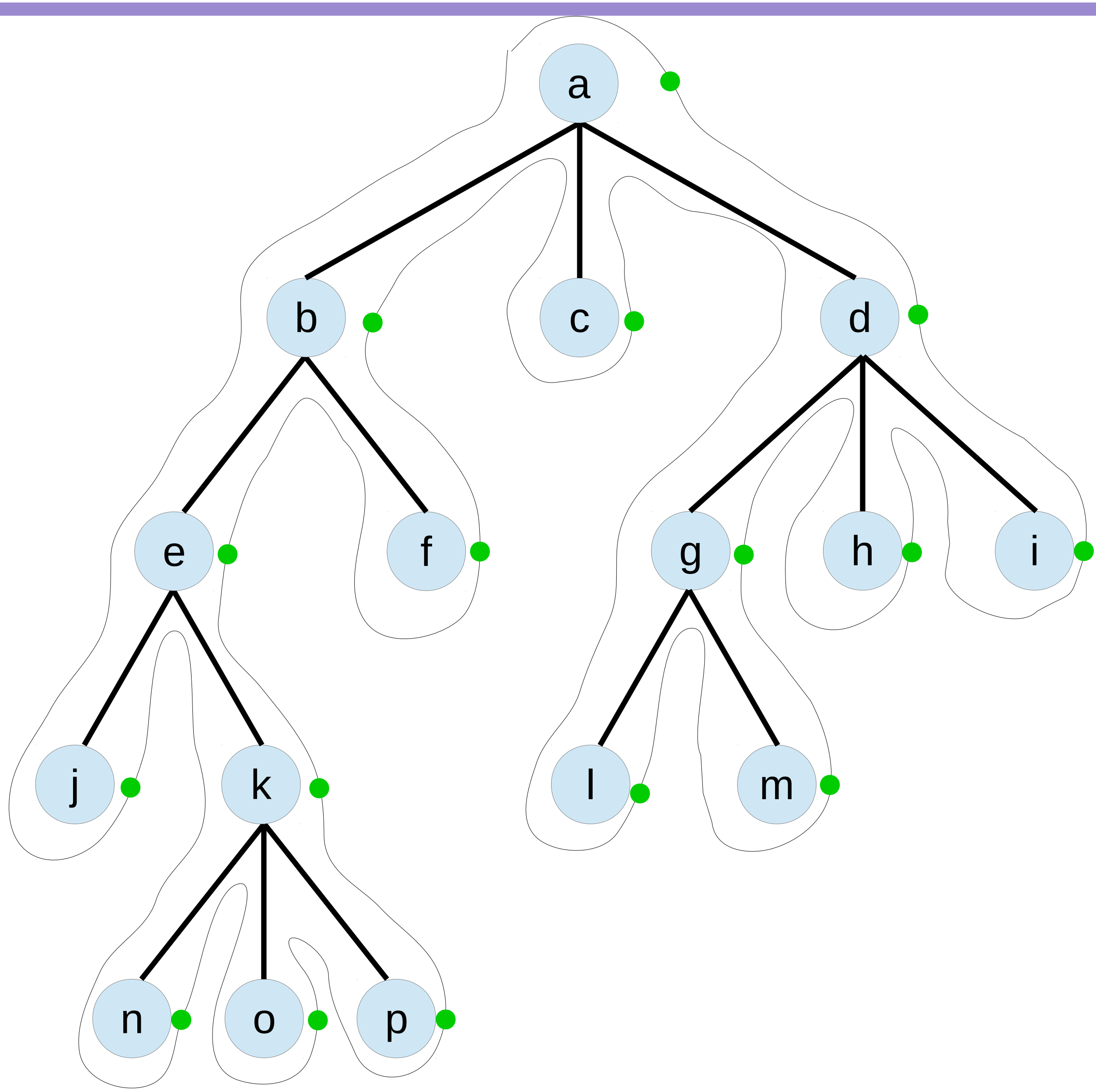
j-e-n-k-o-p-b-f-a-c-l-g-m-d-h-i



Rosen

Post-Order Traversal on Ternary Trees

j-n-o-p-k-e-f-b-c-l-m-g-h-i-d-a



Rosen

Ternary

Ternary

Etymology
Late Latin ternarius (“consisting of three things”), from terni (“three each”).
Adjective

ternary (not comparable)
- Made up of three things; treble, triadic, triple, triplex
- Arranged in groups of three
- (mathematics) To the base three [quotations ▼]
- (mathematics) Having three variables

https://en.wiktionary.org/wiki/ternary

The sequence continues with **quaternary**, **quinary**, **senary**, **septenary**, **octonary**, **nonary**, and **denary**, although most of these terms are rarely used. There's no word relating to the number eleven but there is one that relates to the number twelve: **duodenary**.

https://en.oxforddictionaries.com/explore/what-comes-after-primary-secondary-tertiary

References

[1] http://en.wikipedia.org/
[2]