Digital Signal Octave Codes (0B)

- Aliasing and Folding Frequencies

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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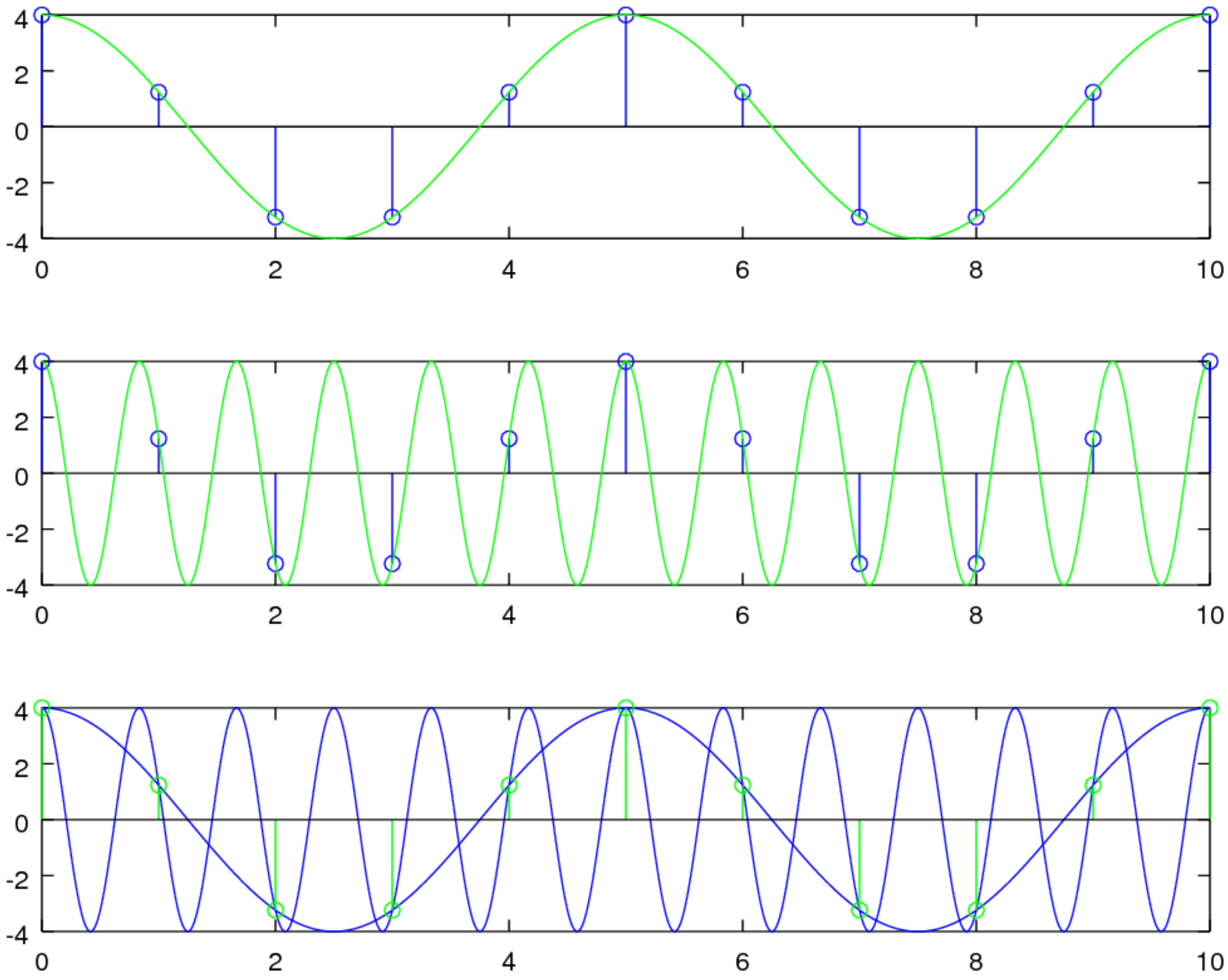
Based on
M.J. Roberts, Fundamentals of Signals and Systems
S.K. Mitra, Digital Signal Processing : a computer-based approach 2nd ed
S.D. Stearns, Digital Signal Processing with Examples in MATLAB

Aliasing Condition Examples

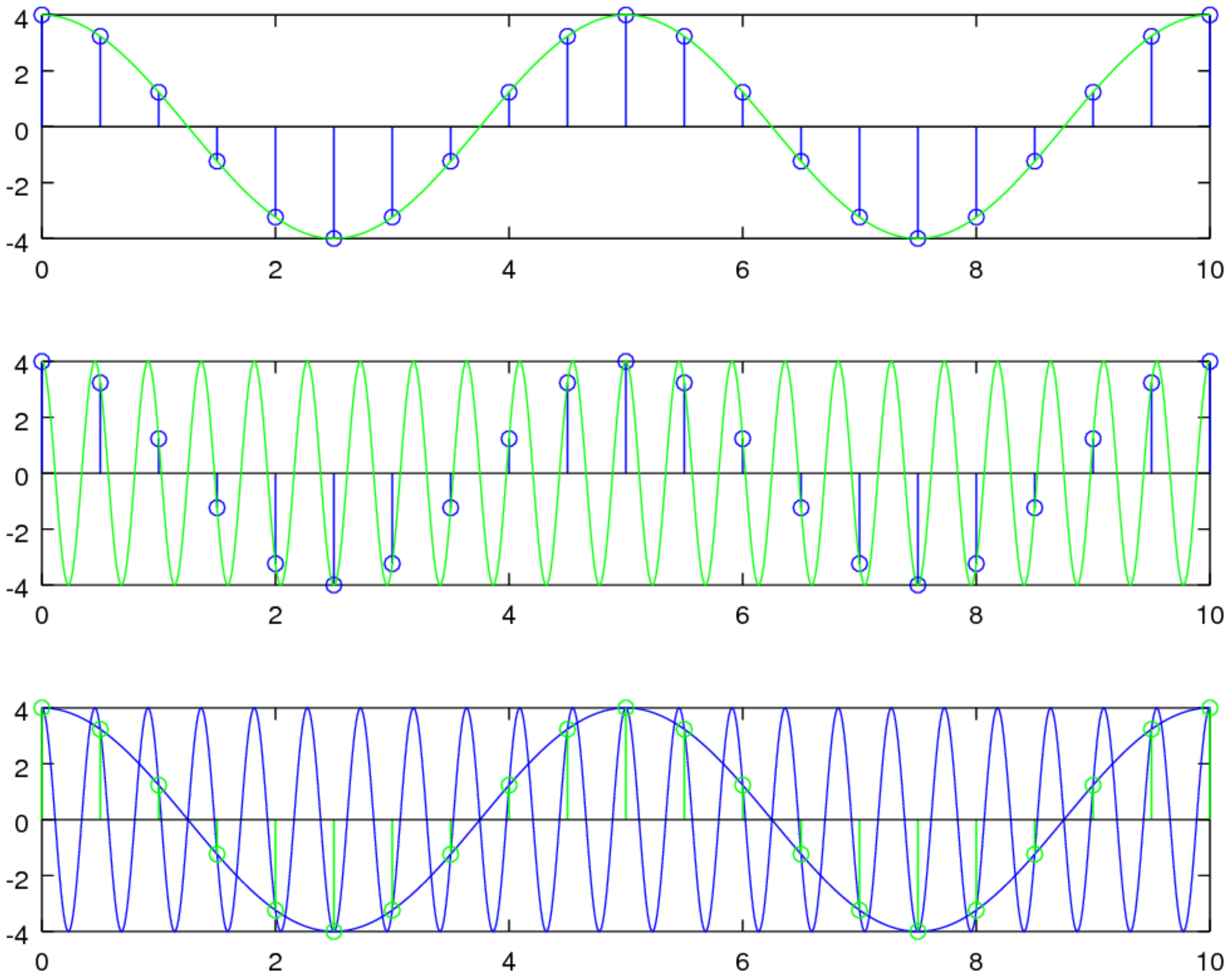



```
clf
n = [0:1:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(6/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);

subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

M.J. Roberts, Fundamentals of Signals and Systems

Aliasing Condition Examples

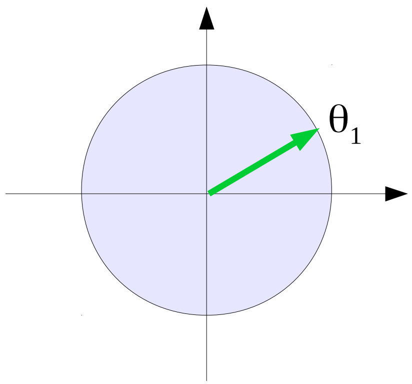


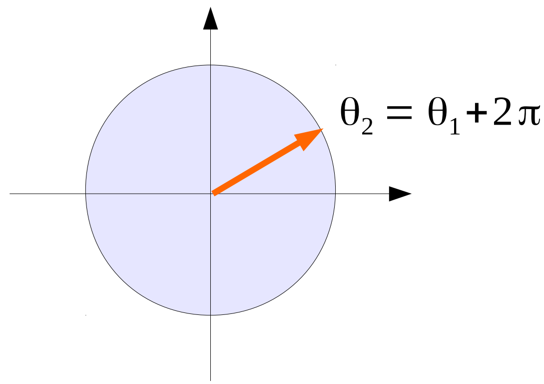

```
clf
n = [0:0.5:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(11/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(11/5)*t);

subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

M.J. Roberts, Fundamentals of Signals and Systems

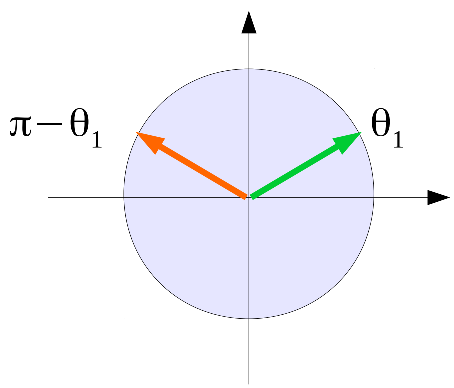
Identical Sine values and Cosine Values

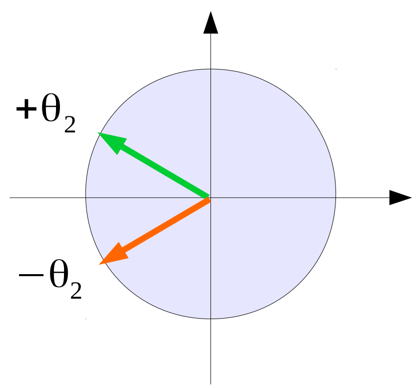


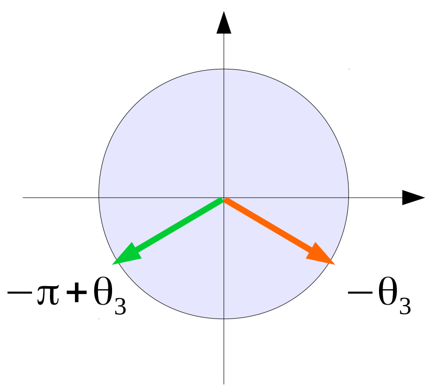


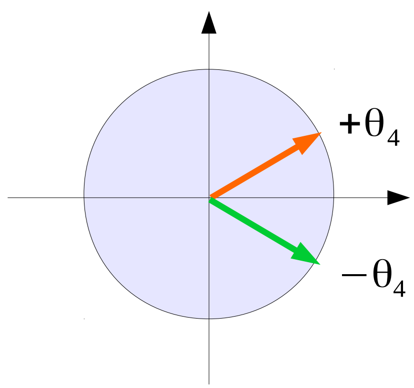
$$\omega_1 t - \omega_2 t = 2\pi$$

periodic condition









$$\omega_1 t + \omega_2 t = +\pi$$

$$\omega_1 t + \omega_2 t = 0$$

$$\omega_1 t + \omega_2 t = -\pi$$

$$\omega_1 t + \omega_2 t = 0$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$
$$-\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$
$$-\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

Identical Sine values and Cosine Values

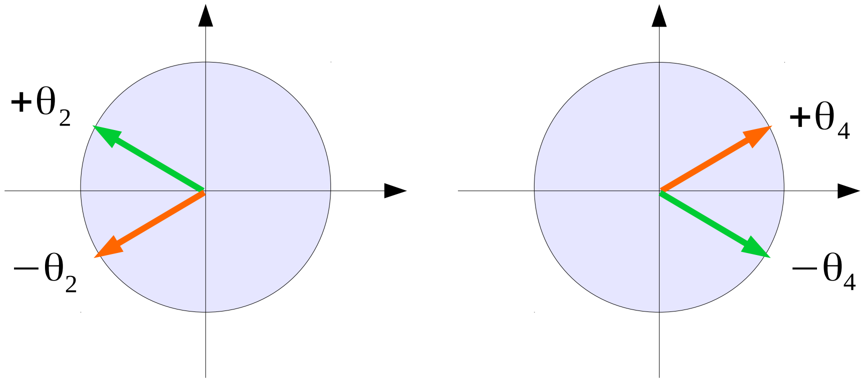
periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



Angles of identical trigonometric values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

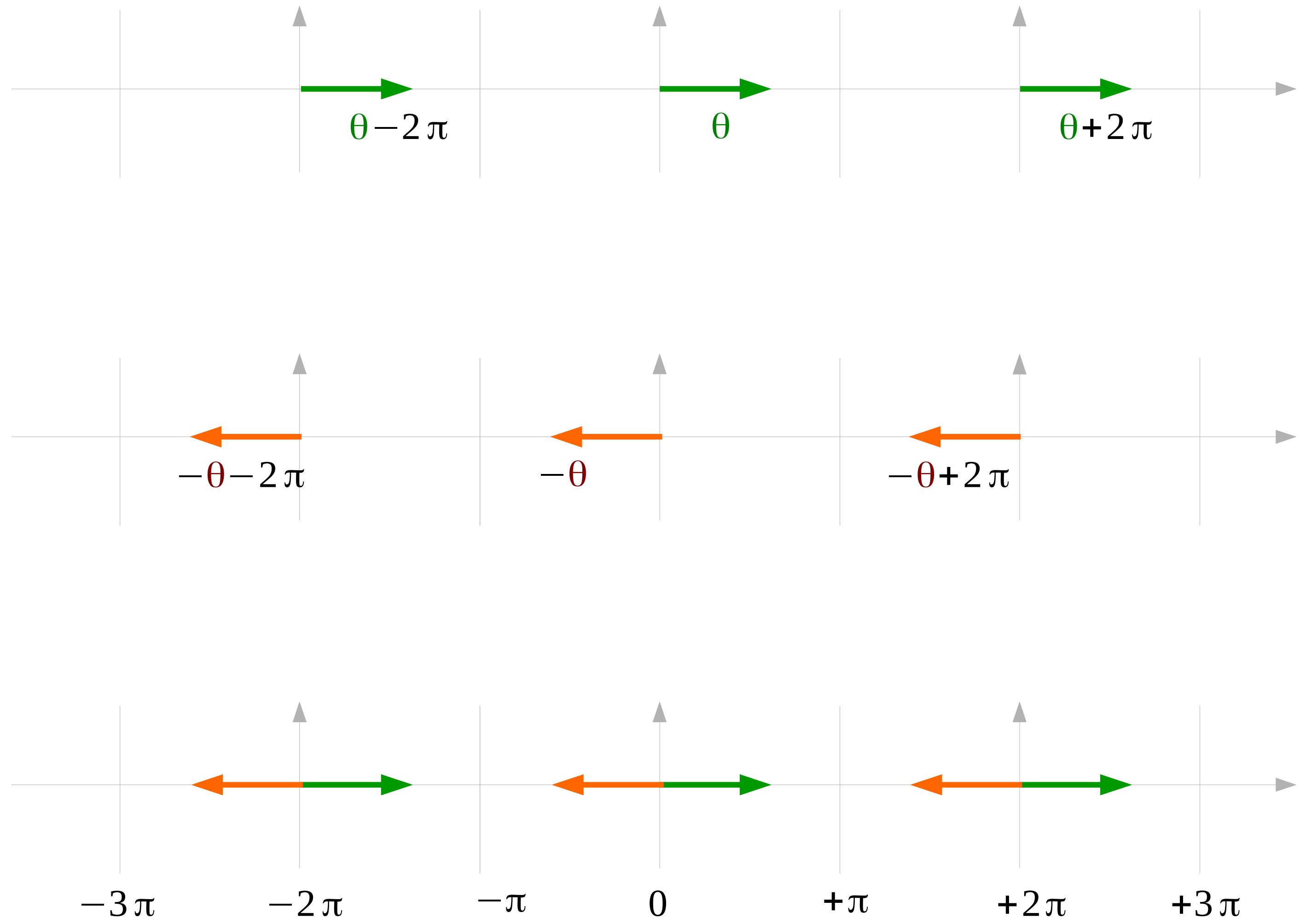
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



Identical Cosine Values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

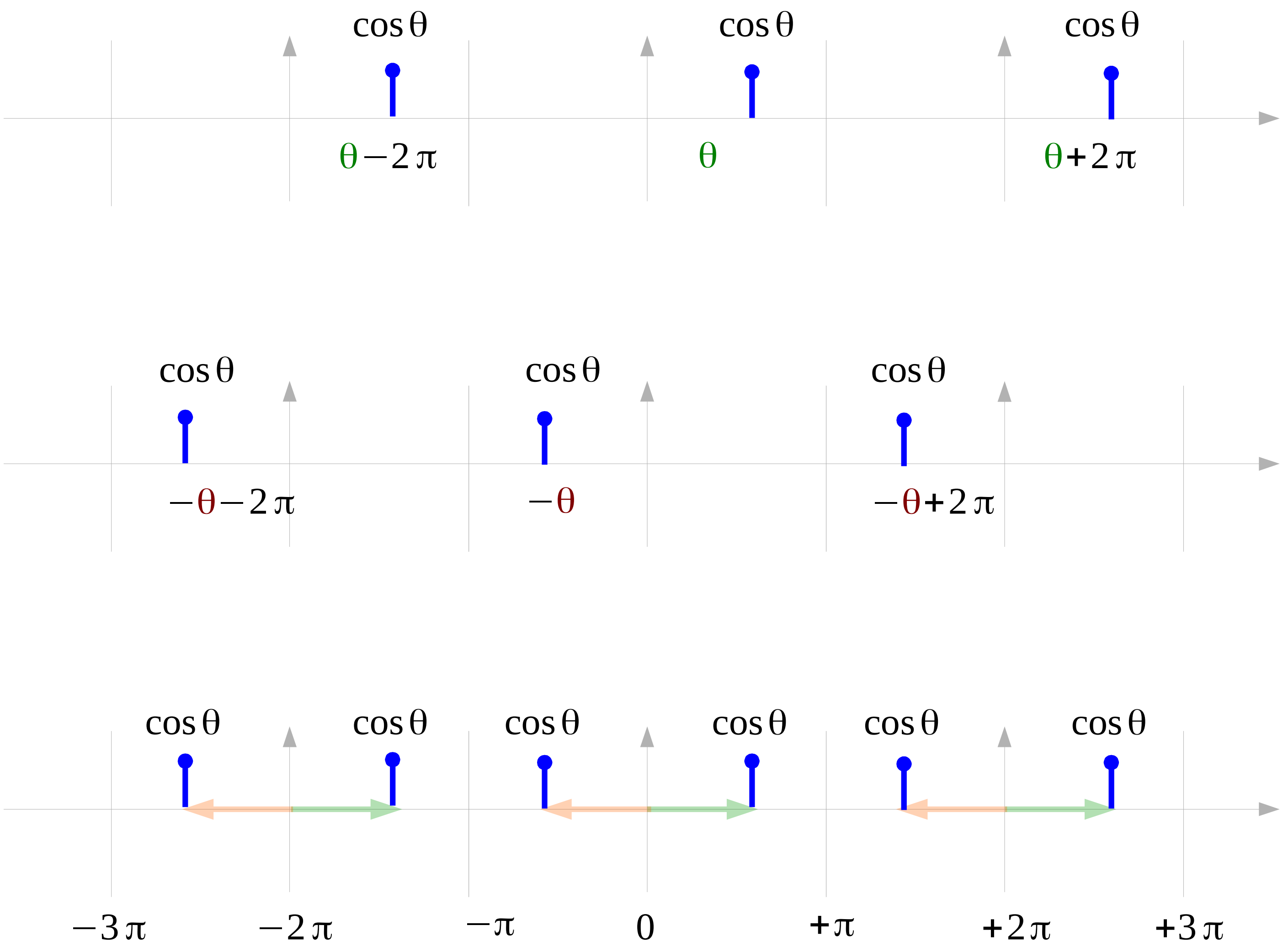
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



Identical Absolute Sine values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

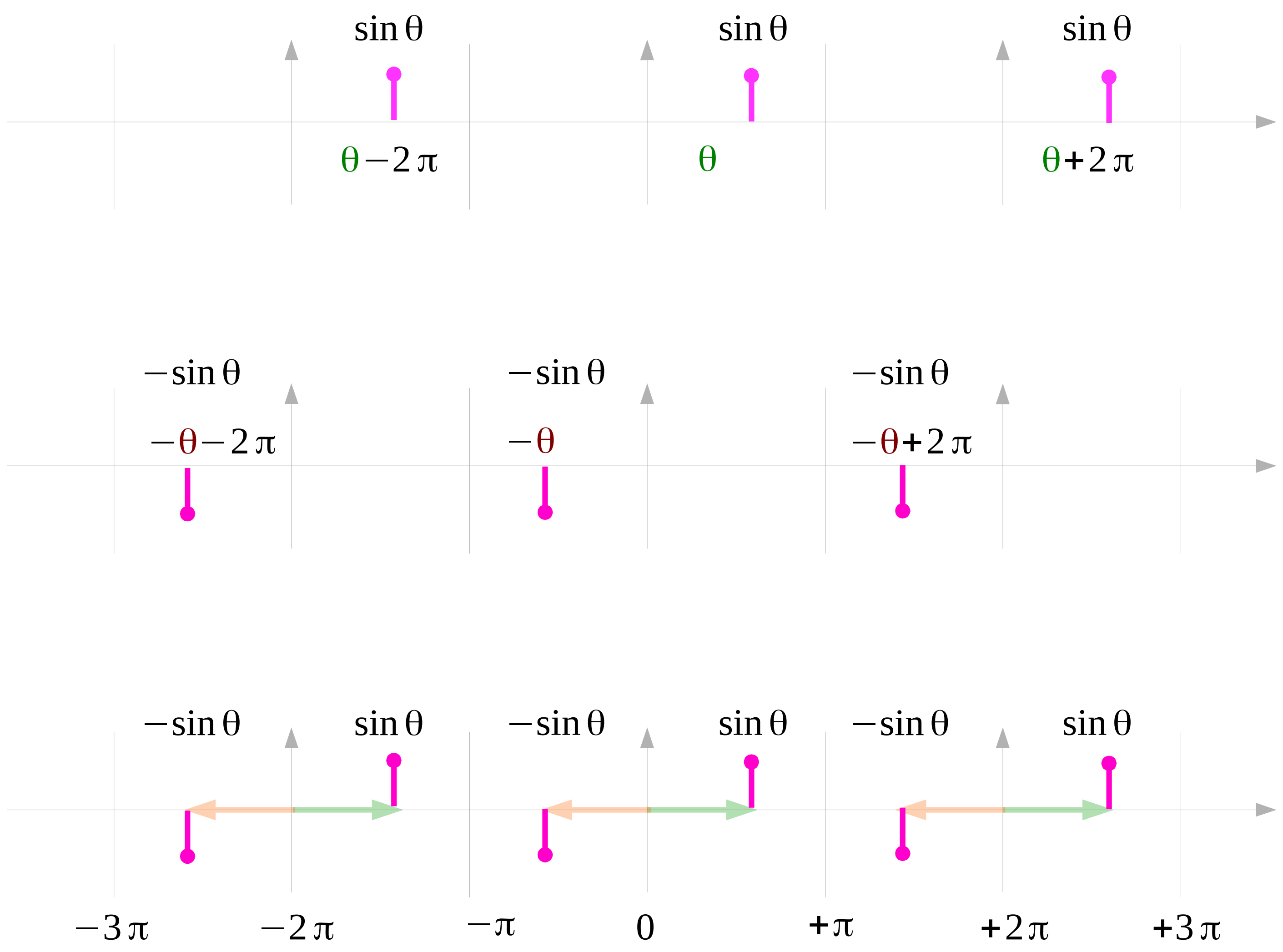
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



Spectrum Representation

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

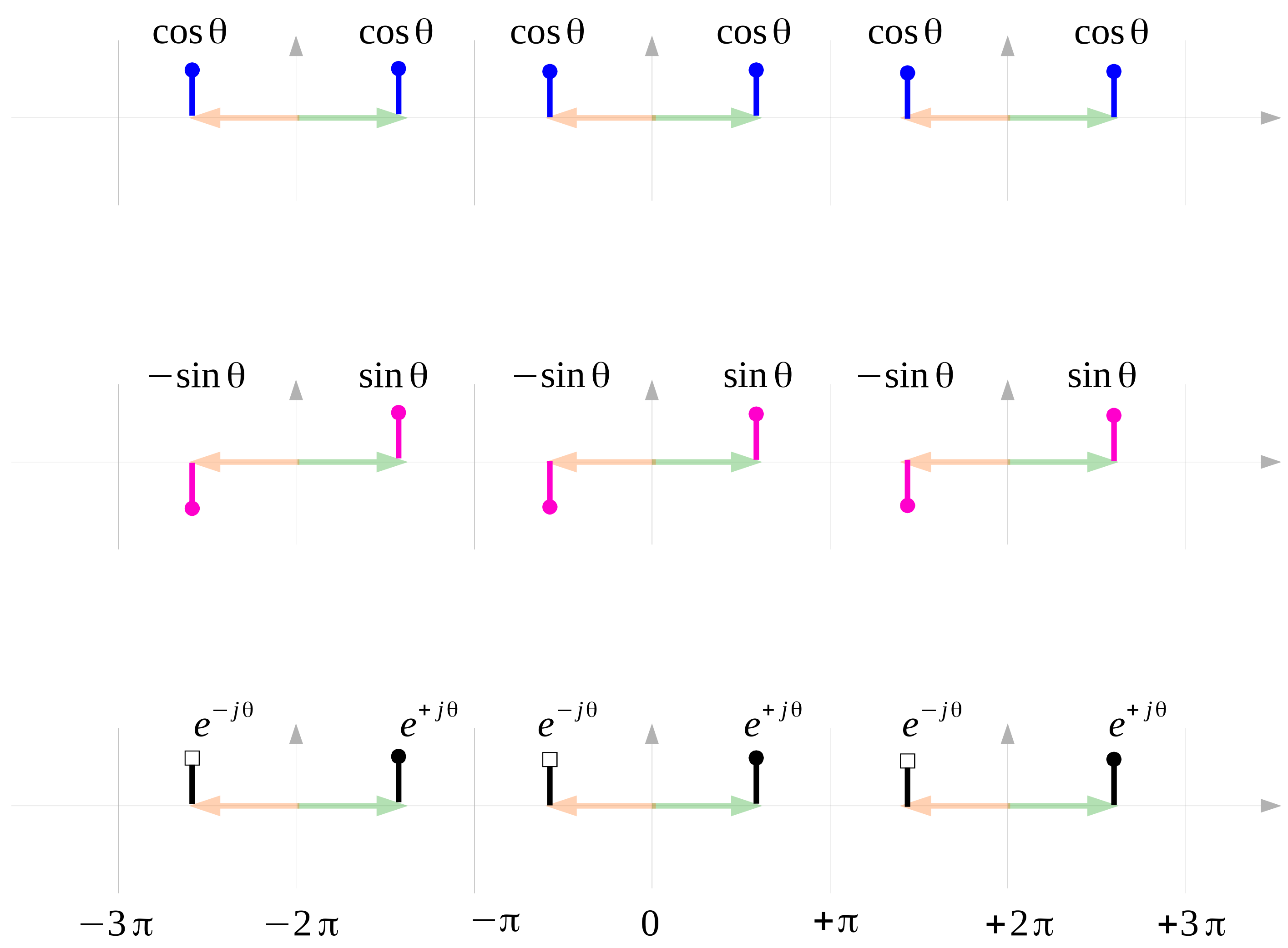
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



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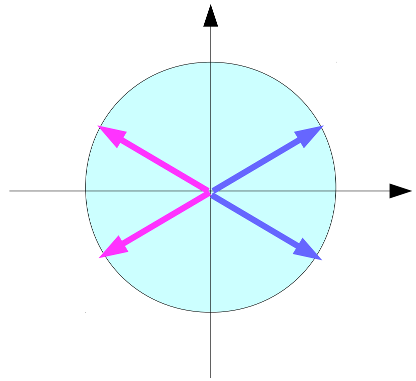
$\cos(2\pi f_1 t) = \cos(2\pi f_2 t)$

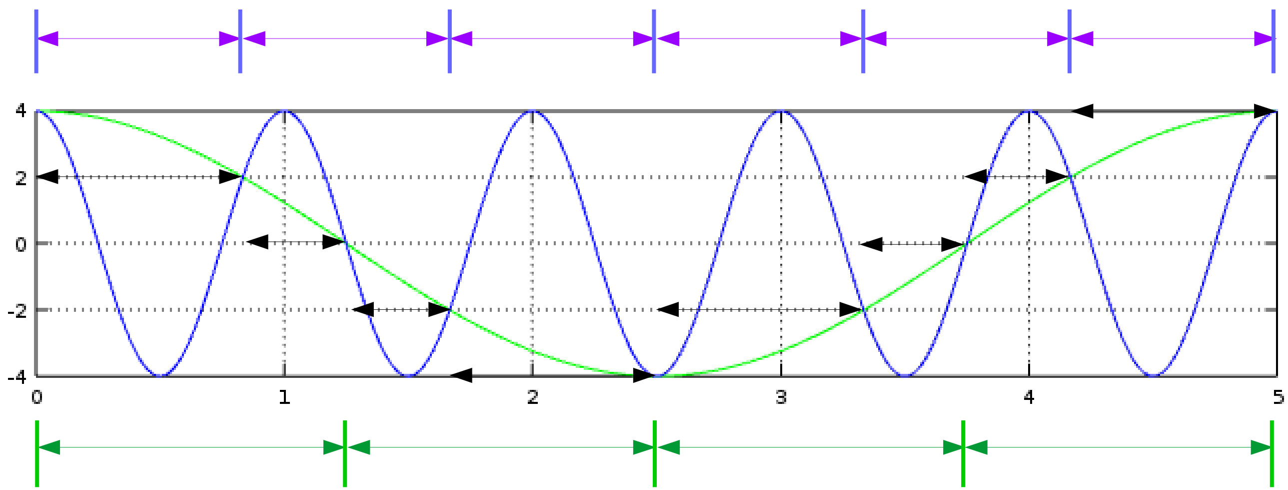
$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t - \omega_2 t = 2n\pi$$
$$\omega_1 t + \omega_2 t = 2n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = n \end{cases}$$

$$\begin{cases} t = \frac{5}{6}n \\ t = \frac{5}{4}n \end{cases}$$



$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \cdots$ $\omega_1 t + \omega_2 t = 2n\pi$

$\cos(\omega_1 t) = \cos(\omega_2 t)$

$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \cdots$ $\omega_1 t - \omega_2 t = 2n\pi$

$\cos(\omega_1 t) = \cos(\omega_2 t)$

Example Frequency Pairs

$f_1 = \frac{2}{5}$	$2\ cycles/5\,sec$	⟺	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{3}{5}$	$3\ cycles/5\,sec$	⟺	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{4}{5}$	$4\ cycles/5\,sec$	⟺	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{5}{5}$	$5\ cycles/5\,sec$	⟺	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{6}{5}$	$6\ cycles/5\,sec$	⟺	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{7}{5}$	$7\ cycles/5\,sec$	⟺	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{8}{5}$	$8\ cycles/5\,sec$	⟺	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$

Identical Cosine Value Conditions

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

$\omega_1 t + \omega_2 t = 2n\pi$ $\omega_1 t - \omega_2 t = 2n\pi$

Plotting the same valued **cosine** samples

```
clf
t = [0:500]/100;

n1 = 0: 5/2 : 5;
```

$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

$\omega_1 t - \omega_2 t = 2n\pi$

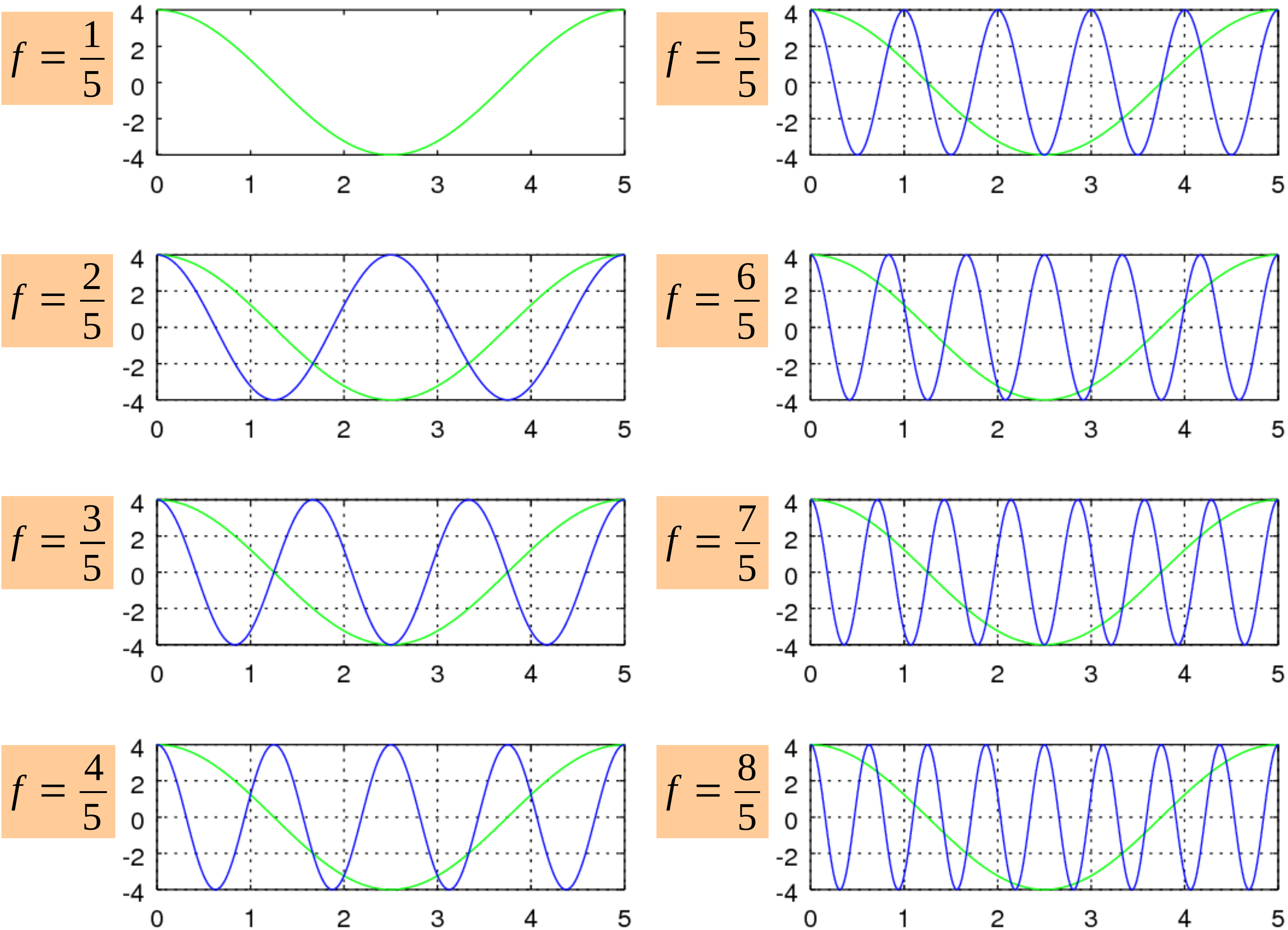
```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);

y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

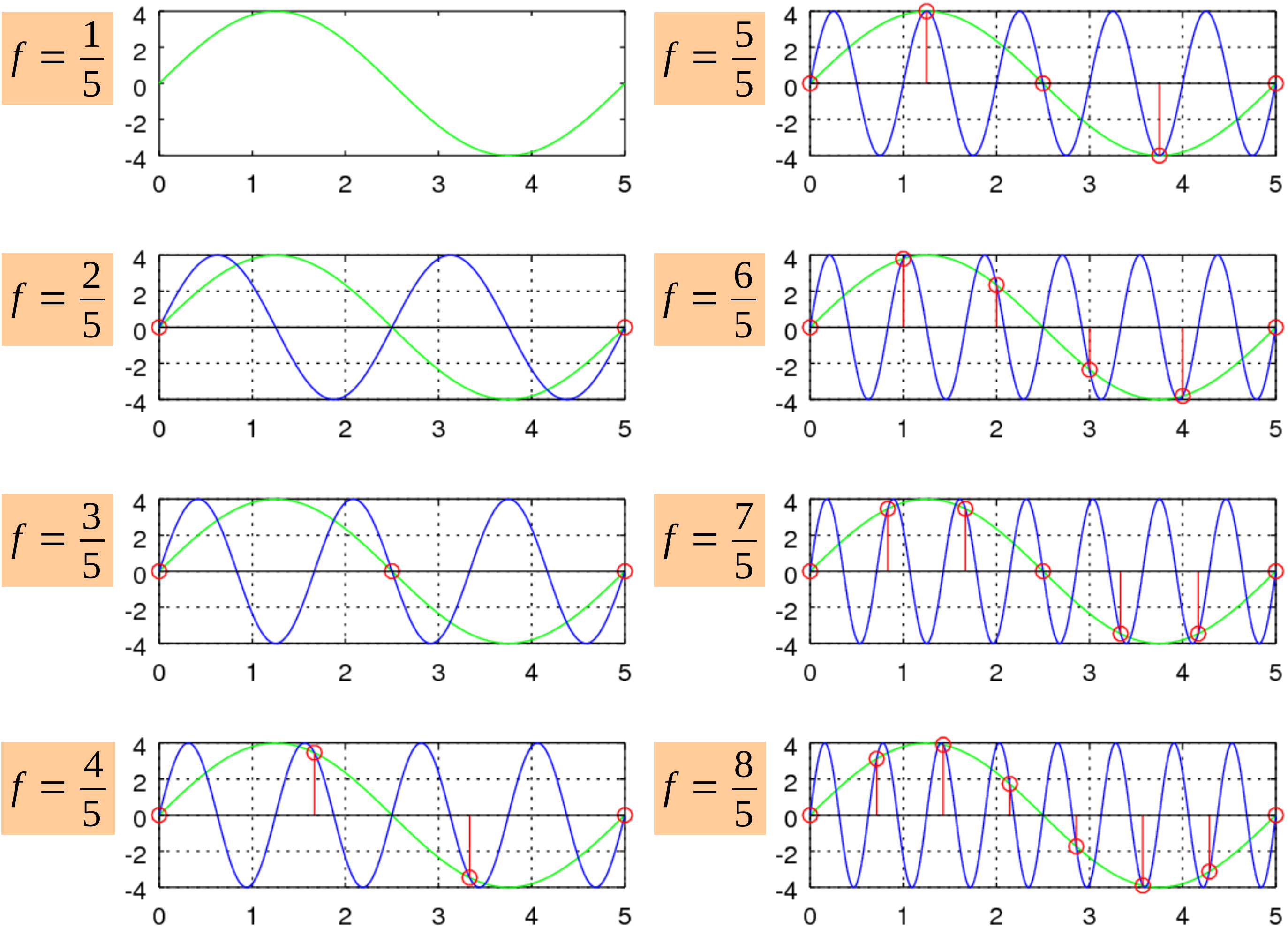
```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

Graphs of cos(2π(n/5)t) & cos(2π(1/5)t)




```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

Cosine values at $2\pi f_1 t + 2\pi f_2 t = 2n\pi$

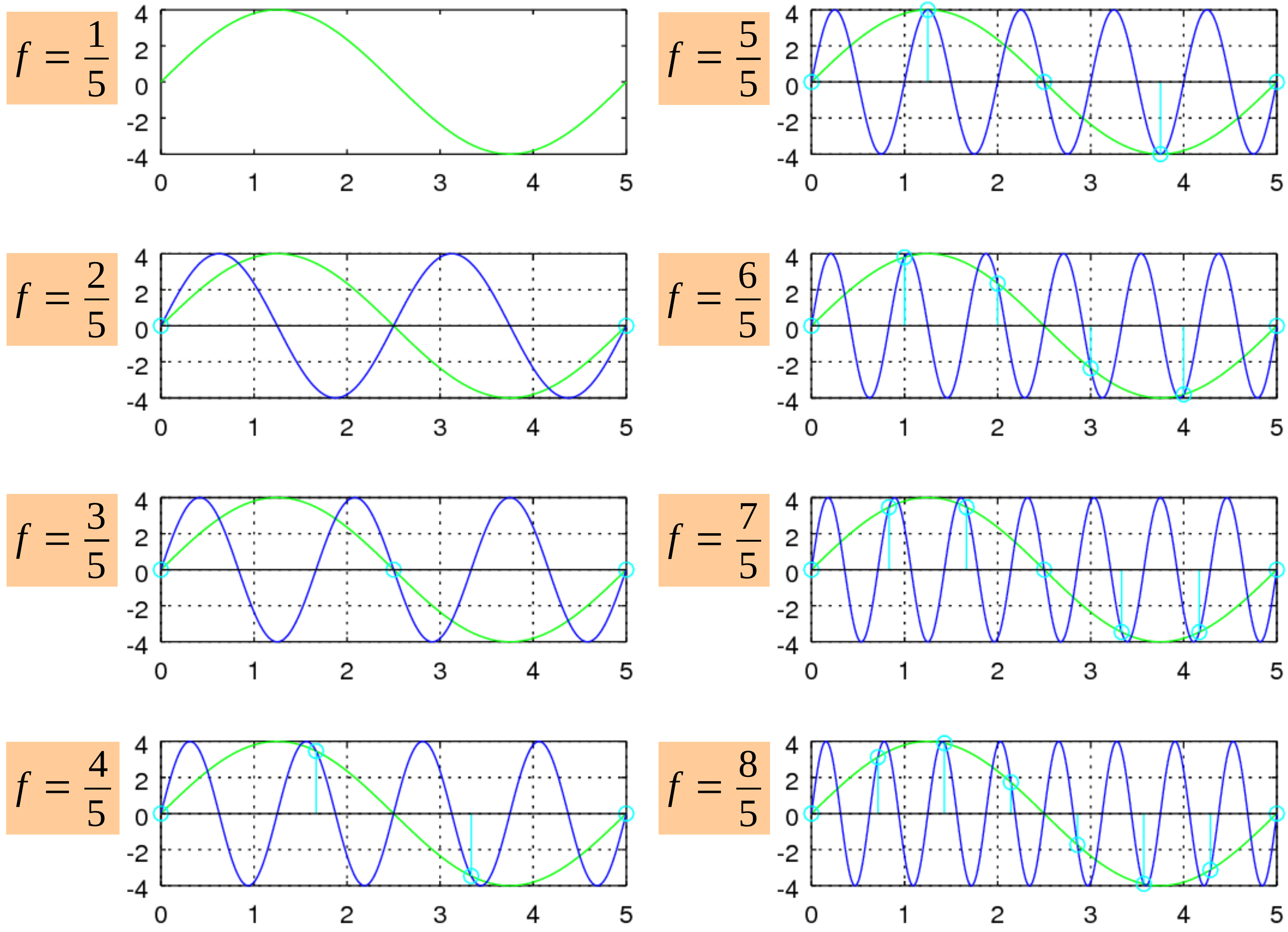


$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

Cosine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$



$\omega_1 t - \omega_2 t = 2n\pi$

```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

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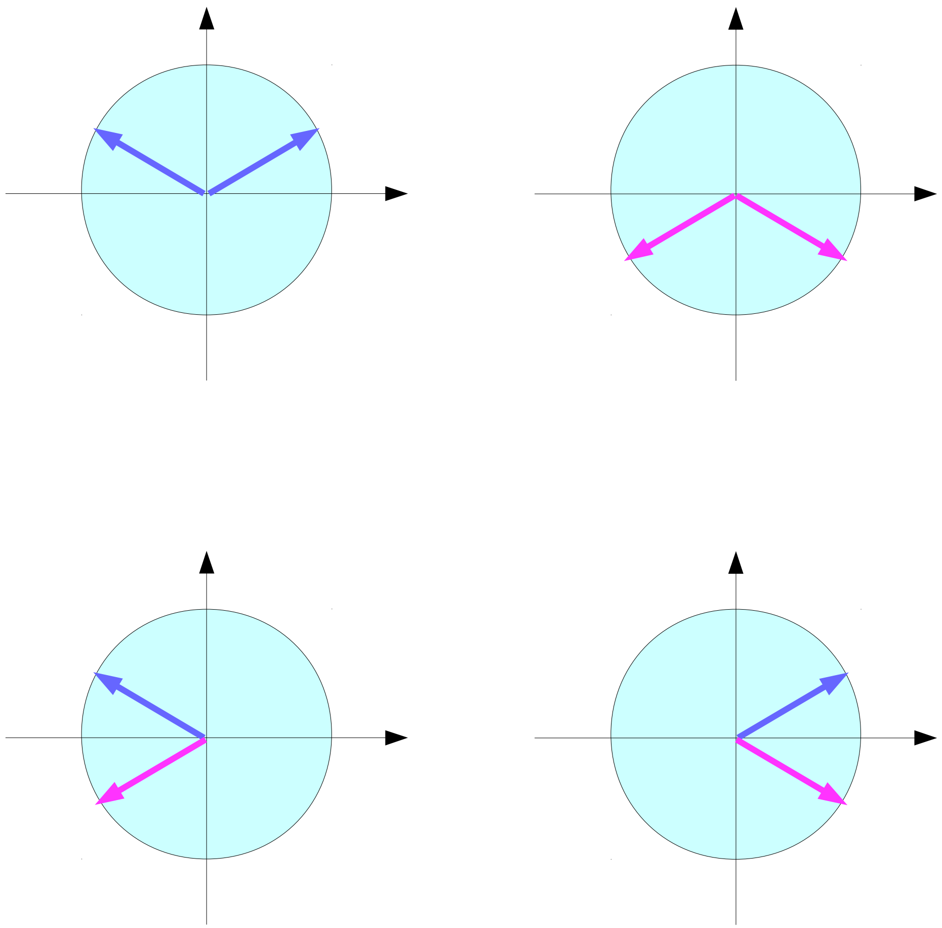
$2\pi f_1 t + 2\pi f_2 t = 2n\pi$, $(2n+1)\pi$ conditions



$$\omega_1 t + \omega_2 t = (2n+1)\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

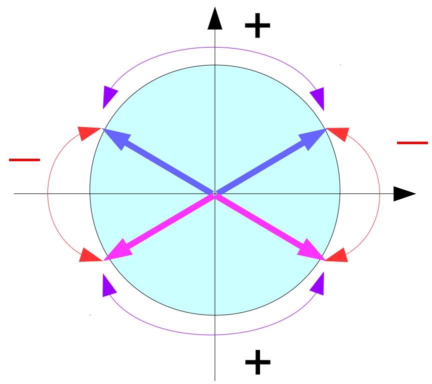
$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

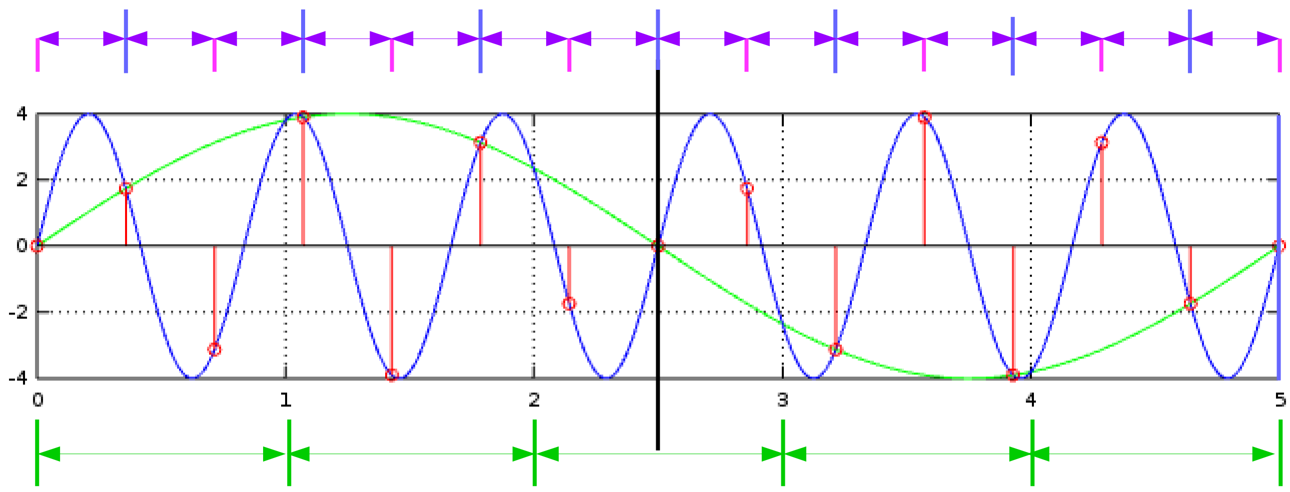
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\omega_1 t - \omega_2 t = 2n\pi$$
$$\omega_1 t + \omega_2 t = n\pi$$

$$\pm\sin(\omega_1 t) = \sin(\omega_2 t)$$



$$\begin{cases} \frac{6}{5}t + \frac{1}{5}t = \frac{n}{2} \\ \frac{6}{5}t - \frac{1}{5}t = n \end{cases} \qquad \begin{cases} t = \frac{5}{14}n \\ t = \frac{5}{5}n \end{cases}$$



$\frac{5}{14}, \frac{10}{14}, \frac{15}{14}, \cdots$ $\quad \omega_1 t + \omega_2 t = n\pi$

$\pm\sin(\omega_1 t) = \sin(\omega_2 t)$

$1, 2, 3, \cdots$ $\quad \omega_1 t - \omega_2 t = 2n\pi$

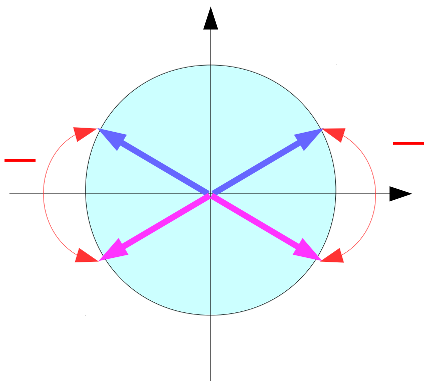
$\sin(\omega_1 t) = \sin(\omega_2 t)$

$\sin(2\pi f_1 t) = -\sin(2\pi f_2 t)$

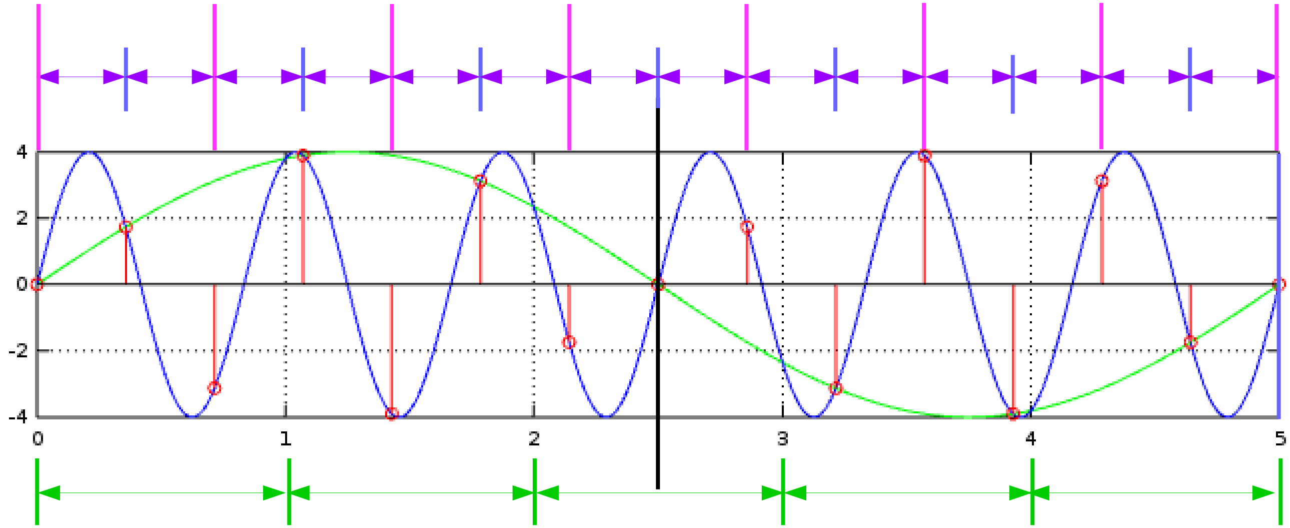
$\sin(\omega_1 t) = \sin(\omega_2 t)$

$\omega_1 t - \omega_2 t = 2n\pi$
$\omega_1 t + \omega_2 t = 2n\pi$

$-\sin(\omega_1 t) = \sin(\omega_2 t)$



$$\begin{cases} \frac{6}{5}t + \frac{1}{5}t = n \\ \frac{6}{5}t - \frac{1}{5}t = n \end{cases} \qquad \begin{cases} t = \frac{5}{7}n \\ t = \frac{5}{5}n \end{cases}$$



$\frac{5}{7}, \frac{10}{7}, \frac{15}{7}, \cdots$ $\quad \omega_1 t + \omega_2 t = 2n\pi$

$-\sin(\omega_1 t) = \sin(\omega_2 t)$

$1,\ 2,\ 3,\ \cdots$ $\quad \omega_1 t - \omega_2 t = 2n\pi$

$\sin(\omega_1 t) = \sin(\omega_2 t)$

Aliasing Condition Examples

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

Plotting the same valued **sine** samples

```
clf
t = [0:500]/100;

n1 = 0: 5/2 : 5;
```

$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

$\omega_1 t - \omega_2 t = 2n\pi$

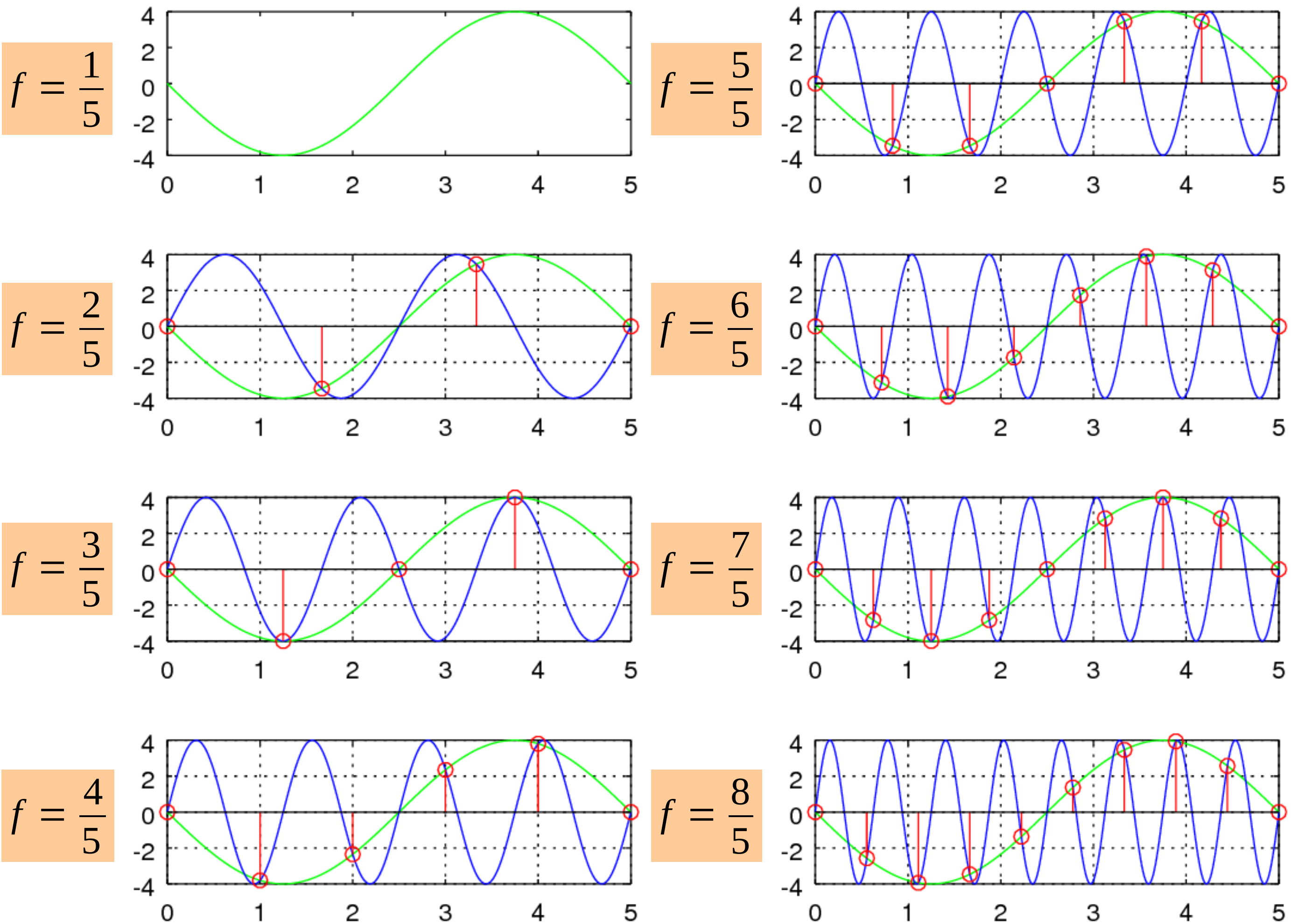
```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
yt1 =-4*sin(2*pi*(1/5)*t);
yt2 = 4*sin(2*pi*(2/5)*t);
yt3 = 4*sin(2*pi*(3/5)*t);
yt4 = 4*sin(2*pi*(4/5)*t);
yt5 = 4*sin(2*pi*(5/5)*t);
yt6 = 4*sin(2*pi*(6/5)*t);
yt7 = 4*sin(2*pi*(7/5)*t);
yt8 = 4*sin(2*pi*(8/5)*t);

y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

Sine values at $2\pi f_1 t + 2\pi f_2 t = 2n\pi$

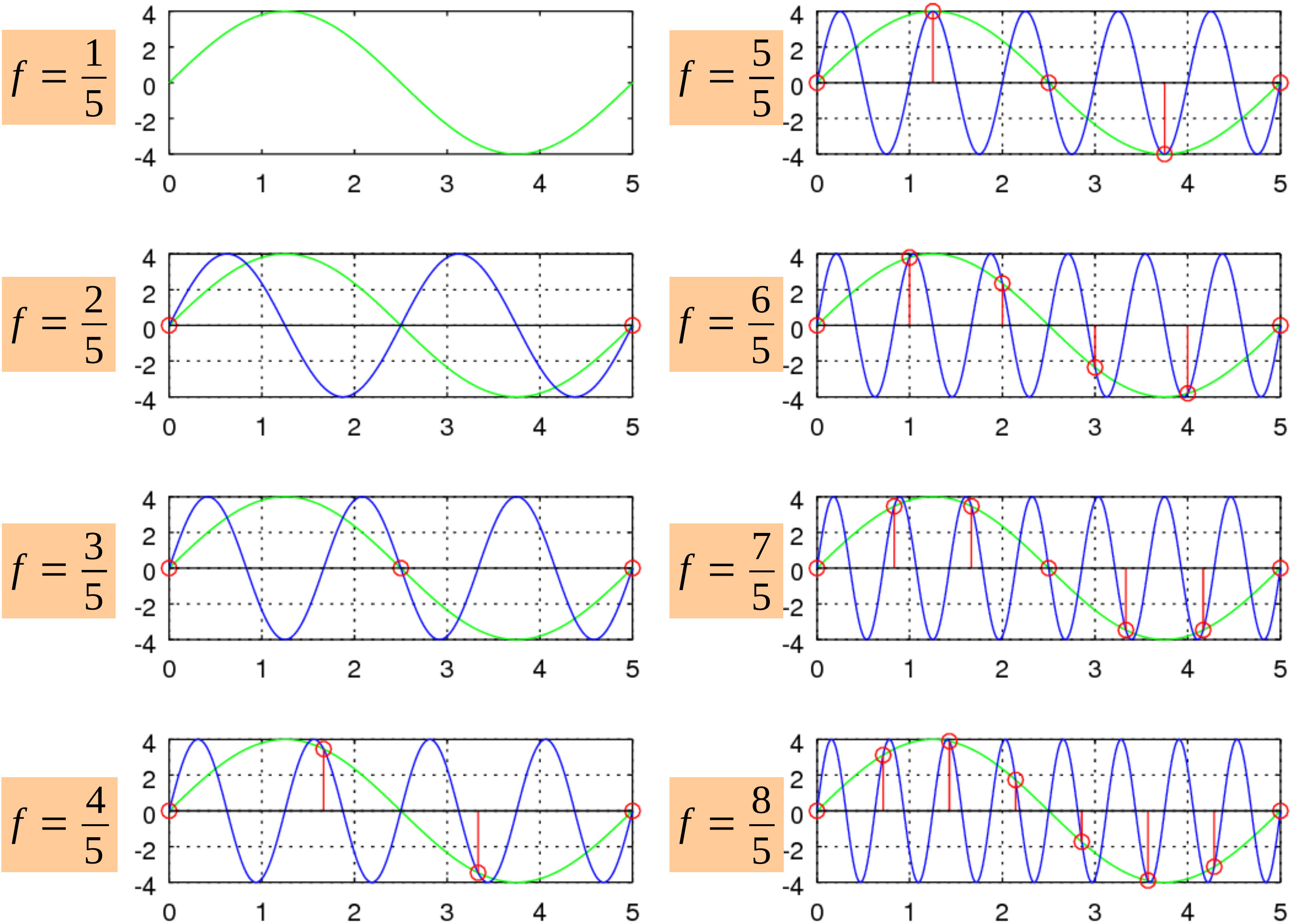


$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);

yt1 =–4*sin(2*pi*(1/5)*t);
```

Sine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$

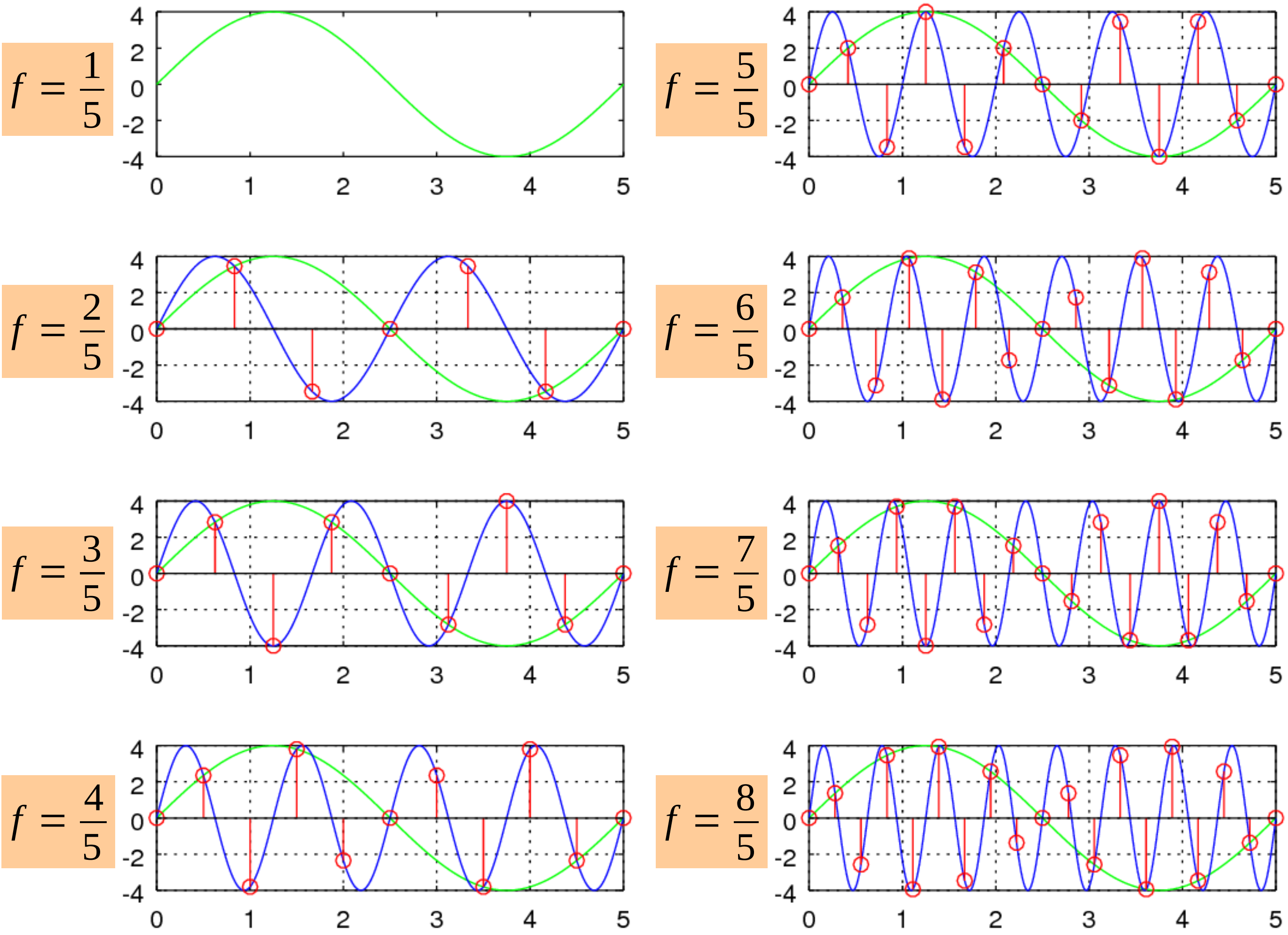


$$\omega_1 t - \omega_2 t = 2n\pi$$

```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);

yt1 =+4*sin(2*pi*(1/5)*t);
```

Sine values at $2\pi f_1 t + 2\pi f_2 t = n\pi$

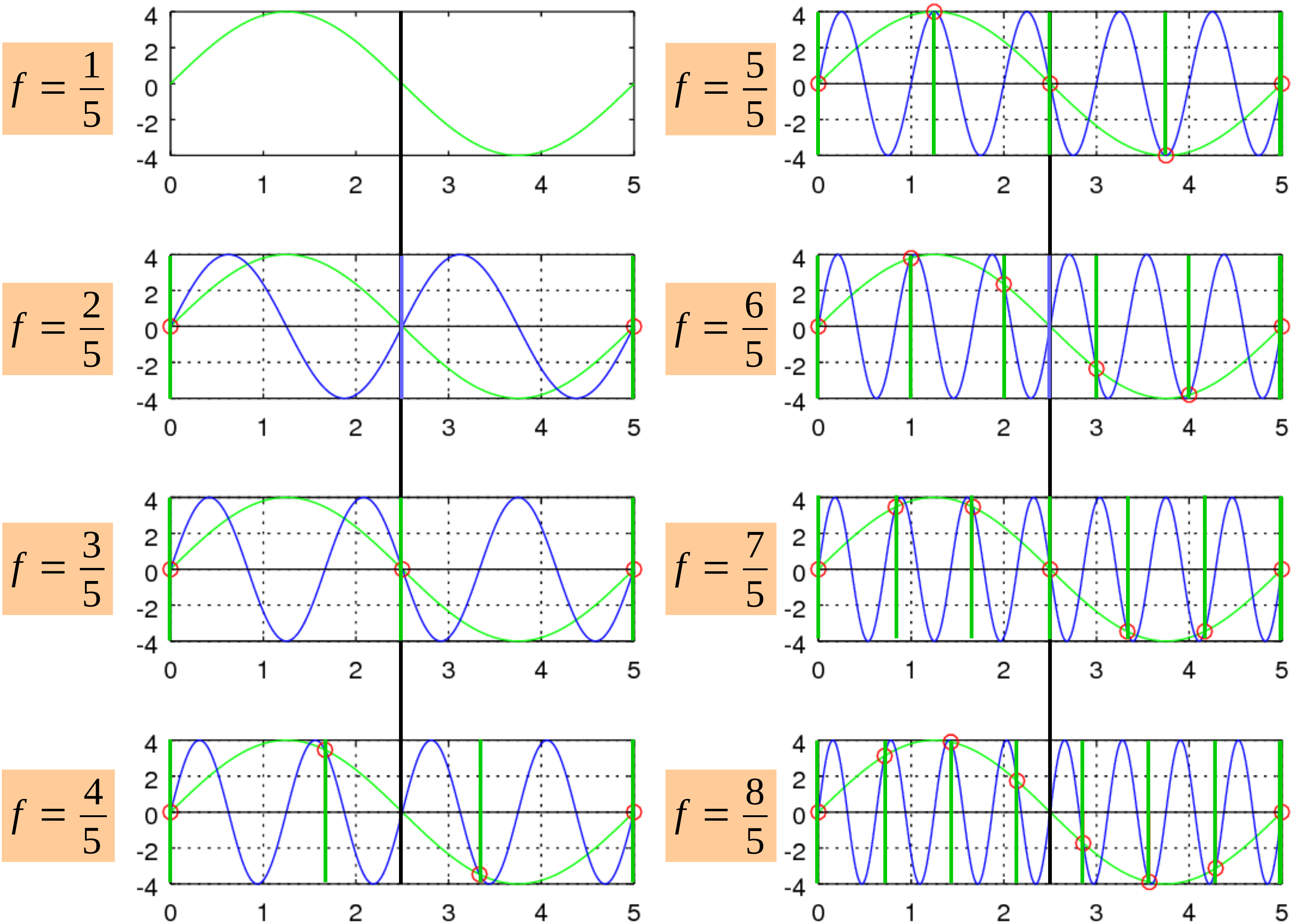


$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: (1/2)5/3 : 5;
n3 = 0: (1/2)5/4 : 5;
n4 = 0: (1/2)5/5 : 5;
n5 = 0: (1/2)5/6 : 5;
n6 = 0: (1/2)5/7 : 5;
n7 = 0: (1/2)5/8 : 5;
n8 = 0: (1/2)5/9 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

Sine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$

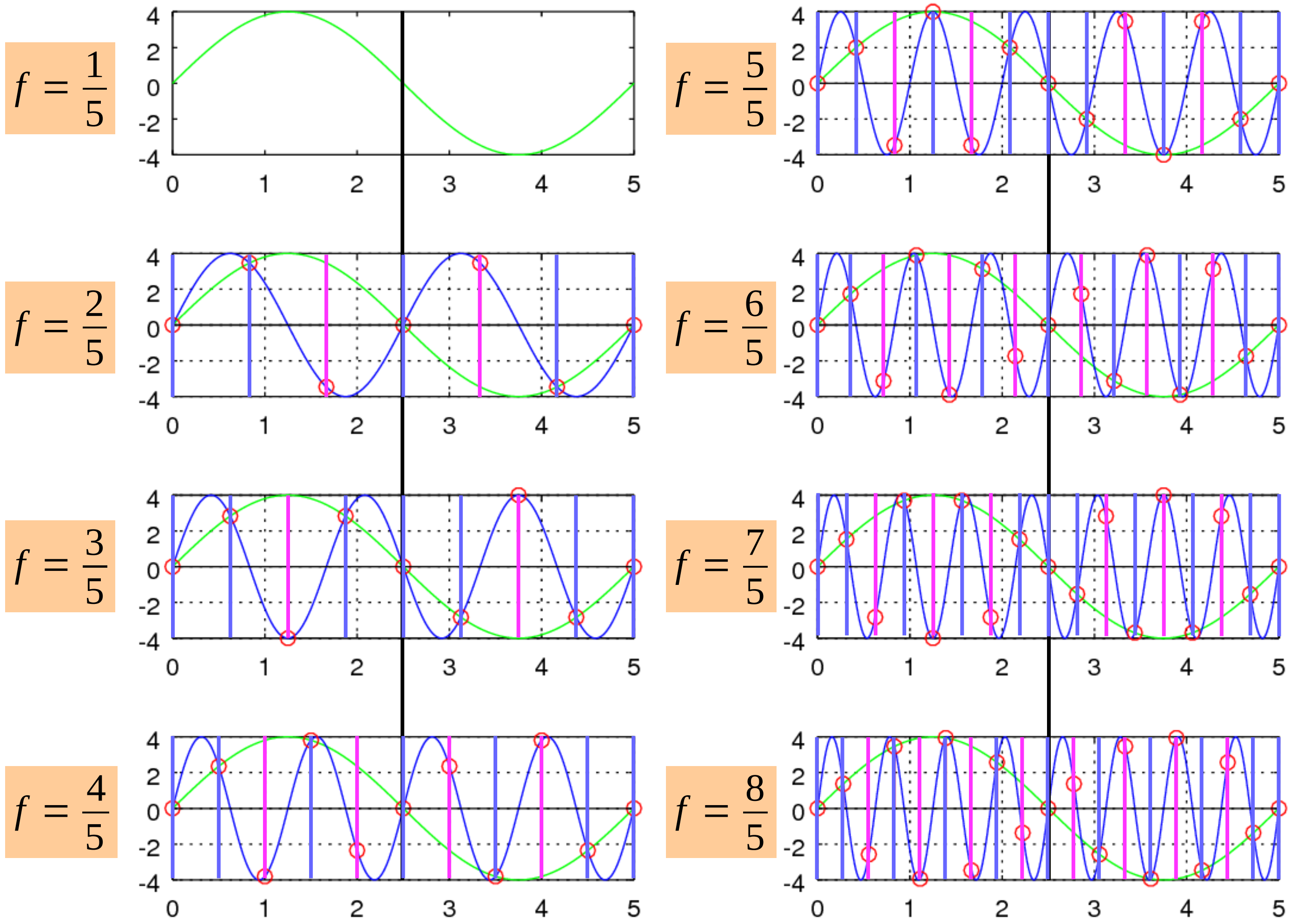


$\omega_1 t - \omega_2 t = 2n\pi$

```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

Aliasing Conditions for **Sine** waves (1)

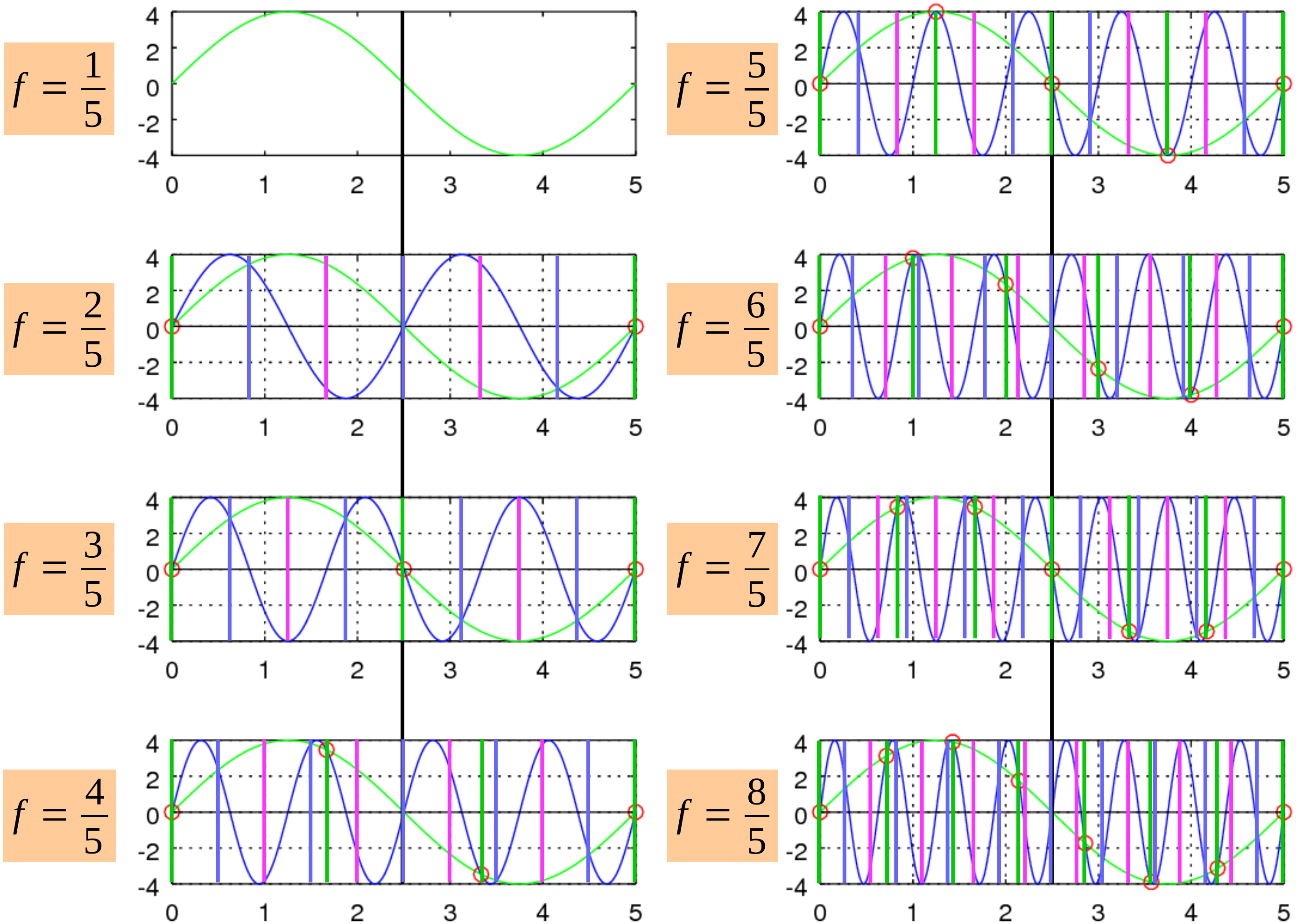


$\omega_1 t + \omega_2 t = 2 n \pi$

n2 = 0: (1/2)5/3 : 5;
n3 = 0: (1/2)5/4 : 5;
n4 = 0: (1/2)5/5 : 5;
n5 = 0: (1/2)5/6 : 5;
n6 = 0: (1/2)5/7 : 5;
n7 = 0: (1/2)5/8 : 5;
n8 = 0: (1/2)5/9 : 5;

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

Aliasing Conditions for **Sine** waves (2)

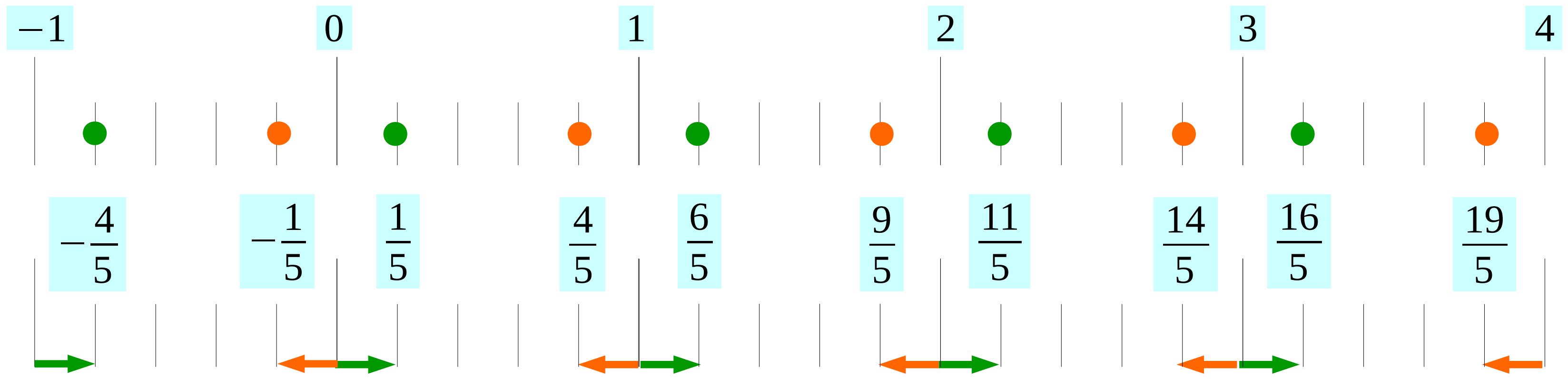


$\omega_1 t - \omega_2 t = 2 n \pi$

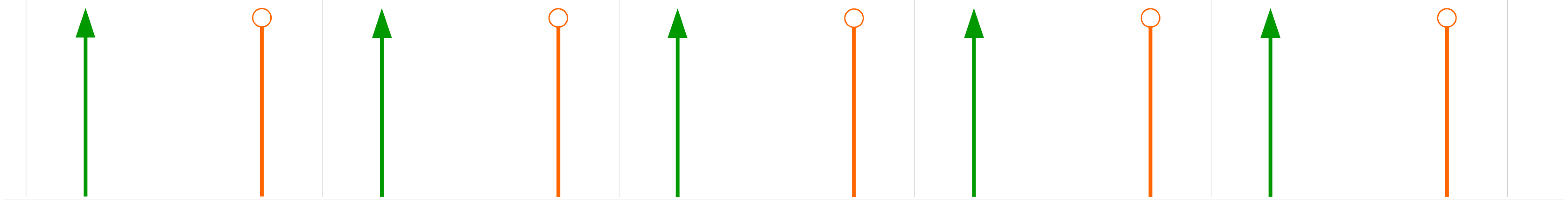
```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

Aliasing and Folding Frequencies (1/5 & 4/5)

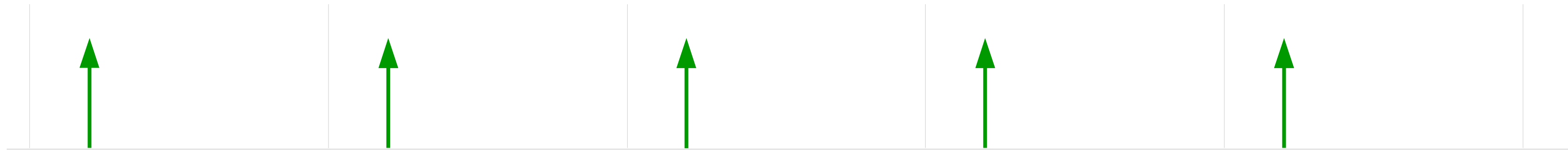


Aliased and Folded Sinusoidal Waves (1/5 & 4/5)

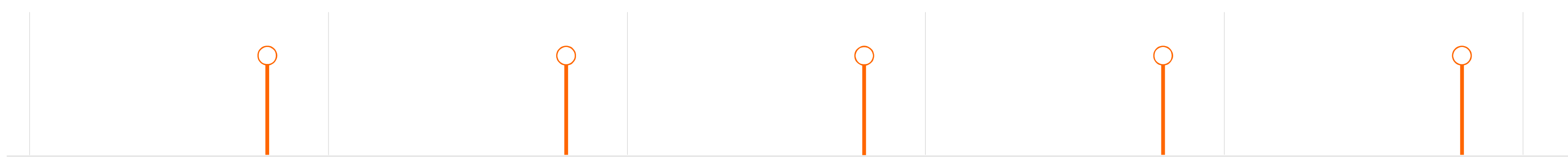


	$\cos\theta$ $\sin\theta$	$\cos\theta$ $-\sin\theta$	$\cos\theta$ $\sin\theta$	$\cos\theta$ $-\sin\theta$	$\cos\theta$ $\sin\theta$	$\cos\theta$ $-\sin\theta$	$\cos\theta$ $\sin\theta$	$\cos\theta$ $-\sin\theta$	$\cos\theta$ $\sin\theta$	$\cos\theta$ $-\sin\theta$
f	$-\frac{4}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	$\frac{9}{5}$	$\frac{11}{5}$	$\frac{14}{5}$	$\frac{16}{5}$	$\frac{19}{5}$
$\omega = 2\pi f$	$-2\pi\cdot\frac{4}{5}$	$-2\pi\cdot\frac{1}{5}$	$2\pi\cdot\frac{1}{5}$	$2\pi\cdot\frac{4}{5}$	$2\pi\cdot\frac{6}{5}$	$2\pi\cdot\frac{9}{5}$	$2\pi\cdot\frac{11}{5}$	$2\pi\cdot\frac{14}{5}$	$2\pi\cdot\frac{16}{5}$	$2\pi\cdot\frac{19}{5}$
$\theta = \omega t$	$-\frac{8\pi}{5}t$	$-\frac{2\pi}{5}t$	$\frac{2\pi}{5}t$	$\frac{8\pi}{5}t$	$\frac{12\pi}{5}t$	$\frac{18\pi}{5}t$	$\frac{22\pi}{5}t$	$\frac{28\pi}{5}t$	$\frac{32\pi}{5}t$	$\frac{38\pi}{5}t$

Aliased **cosine** & **sine** waves (1/5 & 4/5)

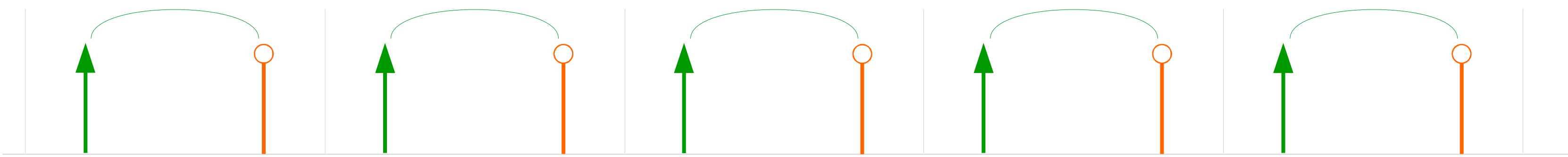


$-\frac{4}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	$\frac{11}{5}$	$\frac{16}{5}$	f
$-\frac{8\pi}{5}t$	$\frac{2\pi}{5}t$	$\frac{12\pi}{5}t$	$\frac{22\pi}{5}t$	$\frac{32\pi}{5}t$	$\theta = \omega t$
$\cos\left(-\frac{8\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{12\pi}{5}t\right)$	$\cos\left(\frac{22\pi}{5}t\right)$	$\cos\left(\frac{32\pi}{5}t\right)$	
$\sin\left(-\frac{8\pi}{5}t\right)$	$\sin\left(\frac{2\pi}{5}t\right)$	$\sin\left(\frac{12\pi}{5}t\right)$	$\sin\left(\frac{22\pi}{5}t\right)$	$\sin\left(\frac{32\pi}{5}t\right)$	

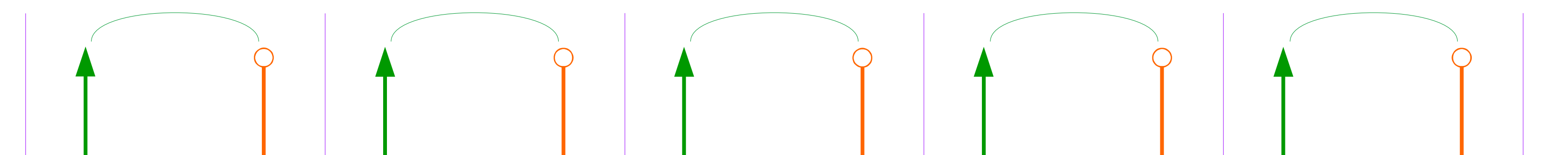


$-\frac{1}{5}$	$\frac{4}{5}$	$\frac{9}{5}$	$\frac{14}{5}$	$\frac{19}{5}$	f
$-\frac{2\pi}{5}t$	$\frac{8\pi}{5}t$	$\frac{18\pi}{5}t$	$\frac{28\pi}{5}t$	$\frac{38\pi}{5}t$	$\theta = \omega t$
$\cos\left(-\frac{2\pi}{5}t\right)$	$\cos\left(\frac{8\pi}{5}t\right)$	$\cos\left(\frac{18\pi}{5}t\right)$	$\cos\left(\frac{28\pi}{5}t\right)$	$\cos\left(\frac{38\pi}{5}t\right)$	
$\sin\left(-\frac{2\pi}{5}t\right)$	$\sin\left(\frac{8\pi}{5}t\right)$	$\sin\left(\frac{18\pi}{5}t\right)$	$\sin\left(\frac{28\pi}{5}t\right)$	$\sin\left(\frac{38\pi}{5}t\right)$	

Folded **cosine** & **sine** waves (1/5 & 4/5)

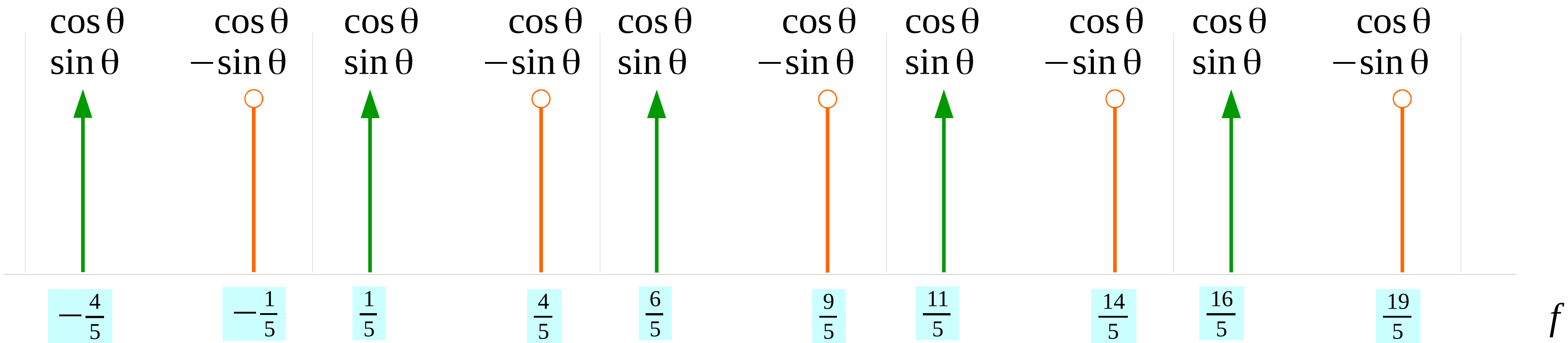


f	$-\frac{4}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	$\frac{9}{5}$	$\frac{11}{5}$	$\frac{14}{5}$	$\frac{16}{5}$	$\frac{19}{5}$
$\theta = \omega t$	$-\frac{8\pi}{5}t$	$-\frac{2\pi}{5}t$	$\frac{2\pi}{5}t$	$\frac{8\pi}{5}t$	$\frac{12\pi}{5}t$	$\frac{18\pi}{5}t$	$\frac{22\pi}{5}t$	$\frac{28\pi}{5}t$	$\frac{32\pi}{5}t$	$\frac{38\pi}{5}t$
	$\cos\left(-\frac{8\pi}{5}t\right)$	$\cos\left(-\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{8\pi}{5}t\right)$	$\cos\left(\frac{12\pi}{5}t\right)$	$\cos\left(\frac{18\pi}{5}t\right)$	$\cos\left(\frac{22\pi}{5}t\right)$	$\cos\left(\frac{28\pi}{5}t\right)$	$\cos\left(\frac{32\pi}{5}t\right)$	$\cos\left(\frac{38\pi}{5}t\right)$
	$\sin\left(-\frac{8\pi}{5}t\right)$	$\sin\left(-\frac{2\pi}{5}t\right)$	$\sin\left(\frac{2\pi}{5}t\right)$	$\sin\left(\frac{8\pi}{5}t\right)$	$\sin\left(\frac{12\pi}{5}t\right)$	$\sin\left(\frac{18\pi}{5}t\right)$	$\sin\left(\frac{22\pi}{5}t\right)$	$\sin\left(\frac{28\pi}{5}t\right)$	$\sin\left(\frac{32\pi}{5}t\right)$	$\sin\left(\frac{38\pi}{5}t\right)$



$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$	$\cos\left(\frac{2\pi}{5}t\right)$
$\sin\left(\frac{2\pi}{5}t\right)$	$-\sin\left(\frac{2\pi}{5}t\right)$	$\sin\left(\frac{2\pi}{5}t\right)$	$-\sin\left(\frac{2\pi}{5}t\right)$	$\sin\left(\frac{2\pi}{5}t\right)$	$-\sin\left(\frac{2\pi}{5}t\right)$	$\sin\left(\frac{2\pi}{5}t\right)$	$-\sin\left(\frac{2\pi}{5}t\right)$	$\sin\left(\frac{2\pi}{5}t\right)$	$-\sin\left(\frac{2\pi}{5}t\right)$

Sampled **cosine** and **sine** values (1/5 & 4/5)



Sampling Condition

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$\frac{2\pi}{5}t + \frac{8\pi}{5}t = 2n\pi$$

$$\frac{10\pi}{5}t = 2n\pi$$

$$t = n$$

$$\frac{8\pi}{5}t = \left(\frac{10}{5} - \frac{2}{5}\right)\pi t$$

$$= 2\pi\left(1 - \frac{1}{5}\right)t$$

$$\Rightarrow 2\pi\left(1 - \frac{1}{5}\right)n$$

when sampled at $t = n$

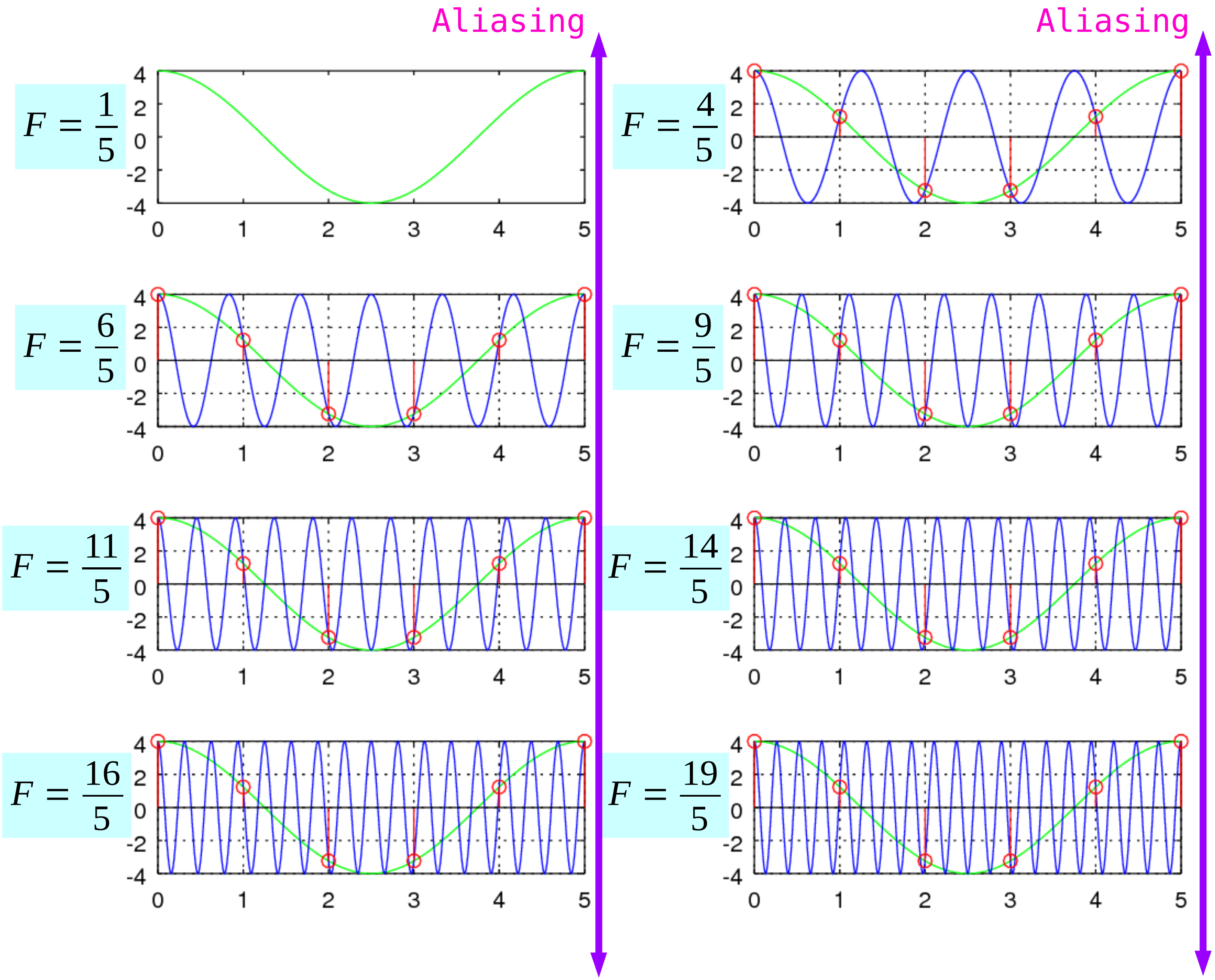
$$\cos\left(\frac{8\pi}{5}t\right) \Rightarrow \cos\left(2\pi\left(1 - \frac{1}{5}\right)n\right) = \cos\left(2\pi\frac{1}{5}n\right)$$

$$\sin\left(\frac{8\pi}{5}t\right) \Rightarrow \sin\left(2\pi\left(1 - \frac{1}{5}\right)n\right) = -\sin\left(2\pi\frac{1}{5}n\right)$$

$$\cos\left(\frac{2\pi}{5}t\right) \Rightarrow \cos\left(2\pi\frac{1}{5}n\right)$$

$$\sin\left(\frac{2\pi}{5}t\right) \Rightarrow \sin\left(2\pi\frac{1}{5}n\right)$$

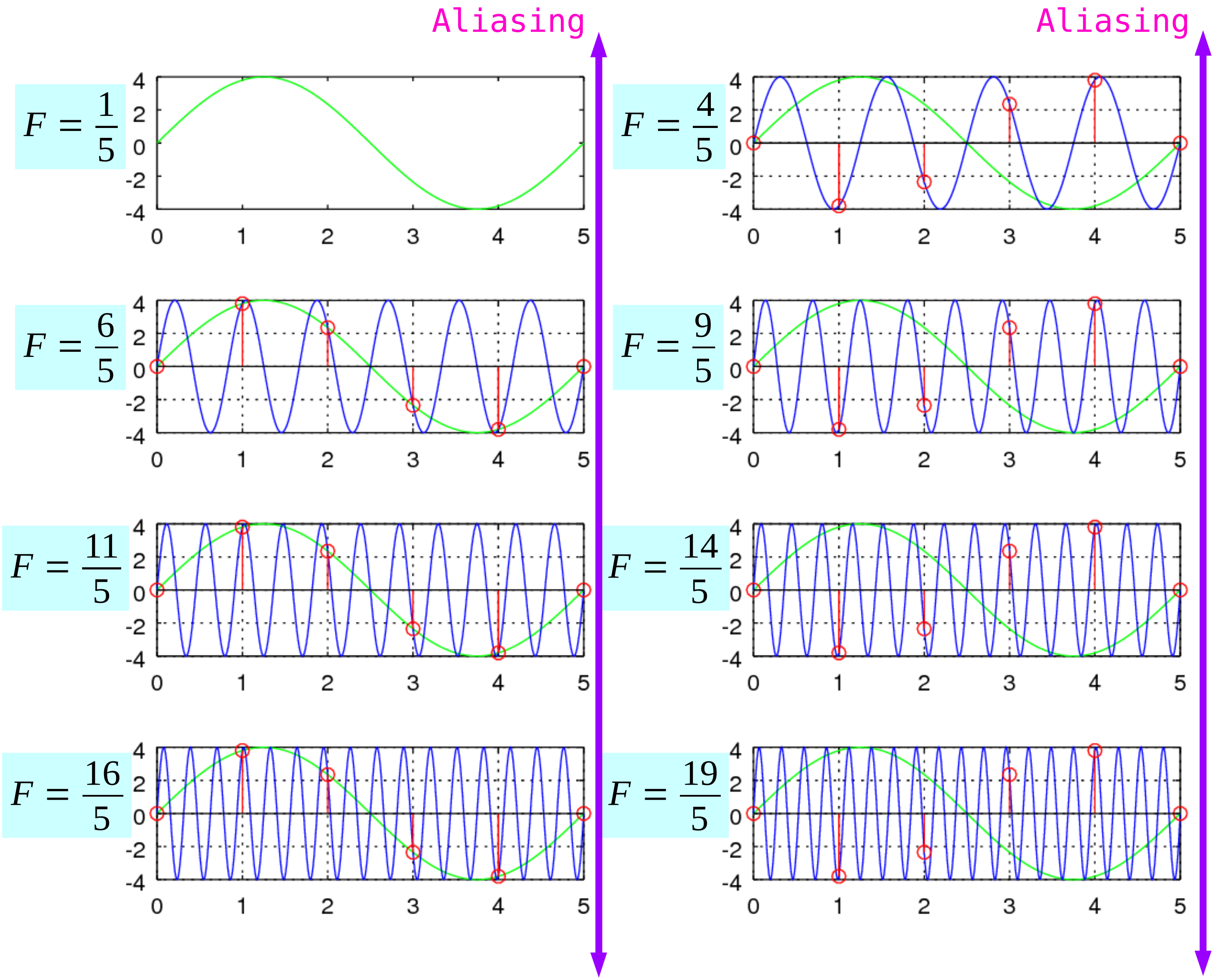
Sampled values of <u>aliased</u> **cosine** waves (1/5 & 4/5)



$\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = \cos\theta_4$

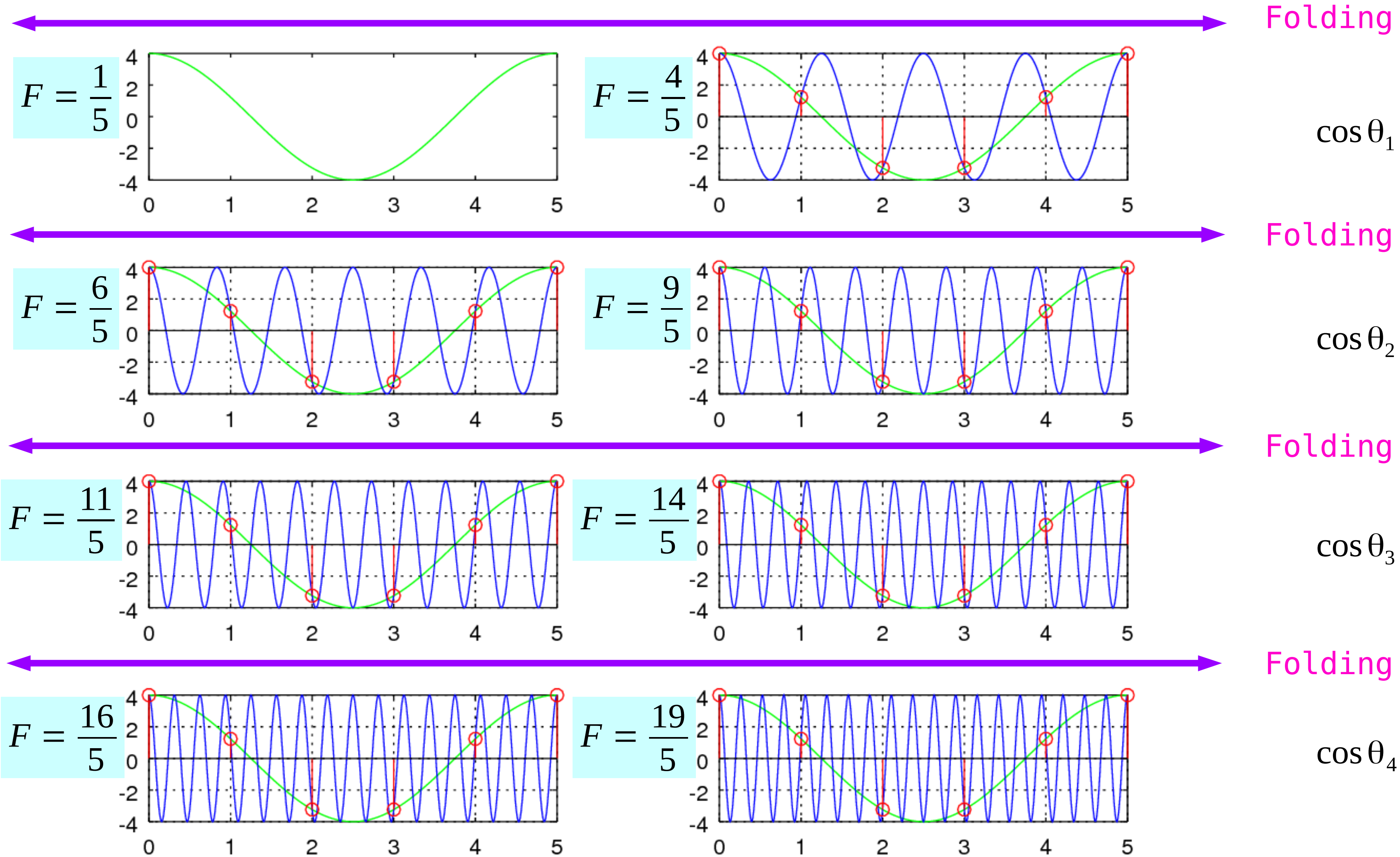
$\cos\theta_5 = \cos\theta_6 = \cos\theta_7 = \cos\theta_8$

Sampled values of aliased **sine** waves (1/5 & 4/5)



$\sin\theta_1$	$-\sin\theta_5$
$\Vert$	$\Vert$
$\sin\theta_2$	$-\sin\theta_6$
$\Vert$	$\Vert$
$\sin\theta_3$	$-\sin\theta_7$
$\Vert$	$\Vert$
$\sin\theta_4$	$-\sin\theta_8$

Sampled values of <u>folded</u> **cosine** waves (1/5 & 4/5)



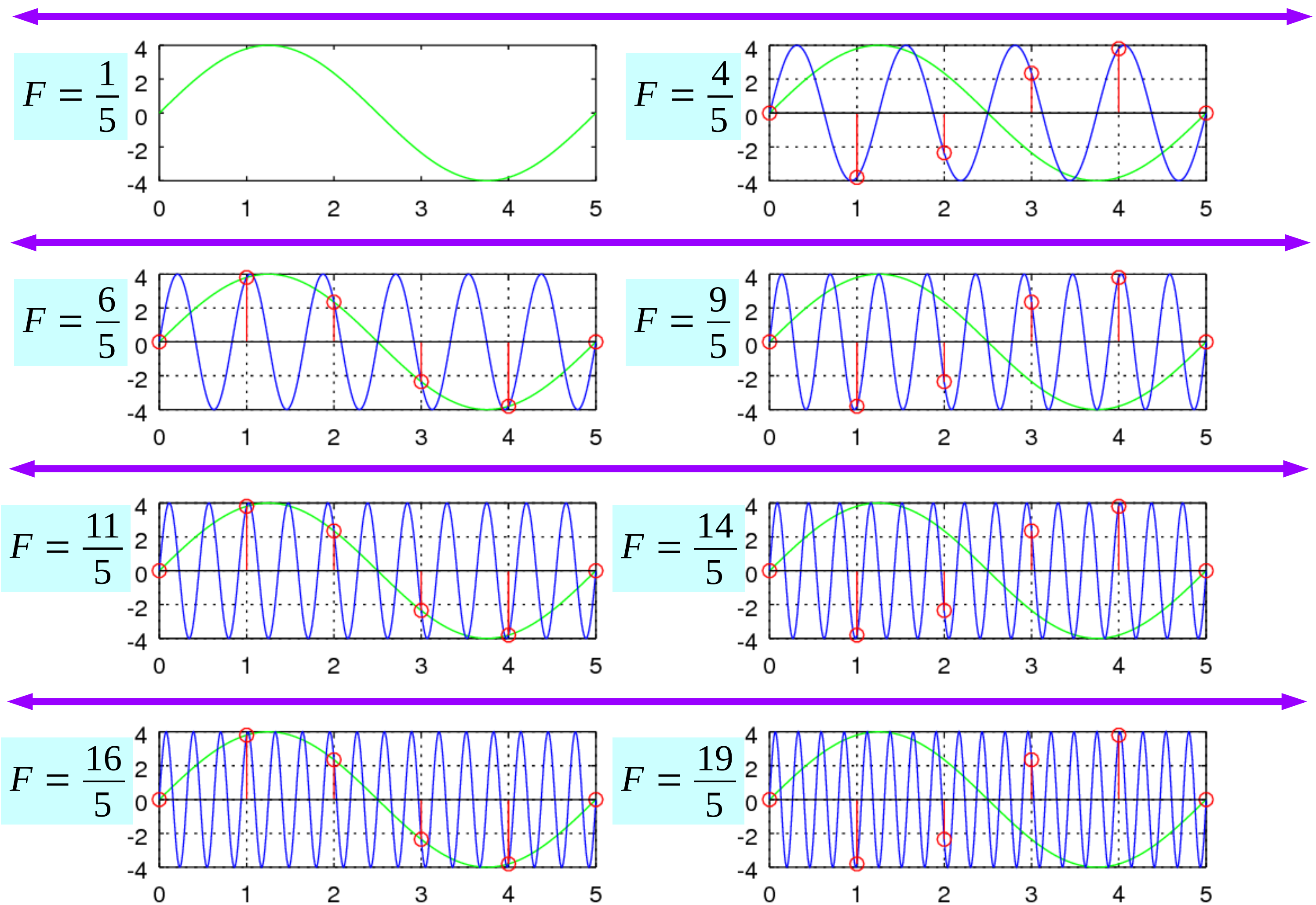
$\cos\theta_1 = \cos\theta_5$

$\cos\theta_2 = \cos\theta_6$

$\cos\theta_3 = \cos\theta_7$

$\cos\theta_4 = \cos\theta_8$

Sampled values of folded **sine** waves (1/5 & 4/5)



Folding

$\sin\theta_1 \equiv -\sin\theta_5$

Folding

$\sin\theta_2 \equiv -\sin\theta_6$

Folding

$\sin\theta_3 \equiv -\sin\theta_7$

Folding

$\sin\theta_4 \equiv -\sin\theta_8$

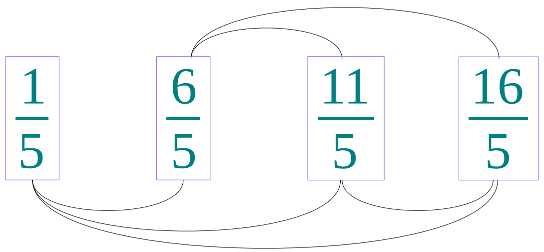
Plotting Aliased & Folded Waves (1/5 & 4/5)

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
yt3 = 4*cos(2*pi*(11/5)*t);
yt4 = 4*cos(2*pi*(16/5)*t);
yt5 = 4*cos(2*pi*(4/5)*t);
yt6 = 4*cos(2*pi*(9/5)*t);
yt7 = 4*cos(2*pi*(14/5)*t);
yt8 = 4*cos(2*pi*(19/5)*t);

n1 = 0: 5/5 : 5;
n2 = 0: 5/5 : 5;
n3 = 0: 5/5 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/5 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/5 : 5;
n8 = 0: 5/5 : 5;
```

```
y2 = 4*cos(2*pi*(6/5)*n2);
y3 = 4*cos(2*pi*(11/5)*n2);
y4 = 4*cos(2*pi*(16/5)*n2);
y5 = 4*cos(2*pi*(4/5)*n5);
y6 = 4*cos(2*pi*(9/5)*n5);
y7 = 4*cos(2*pi*(14/5)*n5);
y8 = 4*cos(2*pi*(19/5)*n5);


subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n2, y3, 'r');
```

```
subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n2, y4, 'r');

subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n5, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n5, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n5, y8, 'r');
```

Sampling period for aliasing and folding frequencies (1)



A+B

$\frac{7}{5}$ $\frac{12}{5}$ $\frac{17}{5}$ $\frac{22}{5}$ $\frac{27}{5}$ $\Rightarrow$ $\frac{5}{7}$ $\frac{5}{12}$ $\frac{5}{17}$ $\frac{5}{22}$ $\frac{5}{27}$

A-B

$\frac{5}{5}$ $\frac{10}{5}$ $\frac{15}{5}$ $\Rightarrow$ $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\Rightarrow$ 1

Sampling period for aliasing and folding frequencies (2)

$\frac{4}{5}$ $\frac{9}{5}$ $\frac{14}{5}$ $\frac{19}{5}$

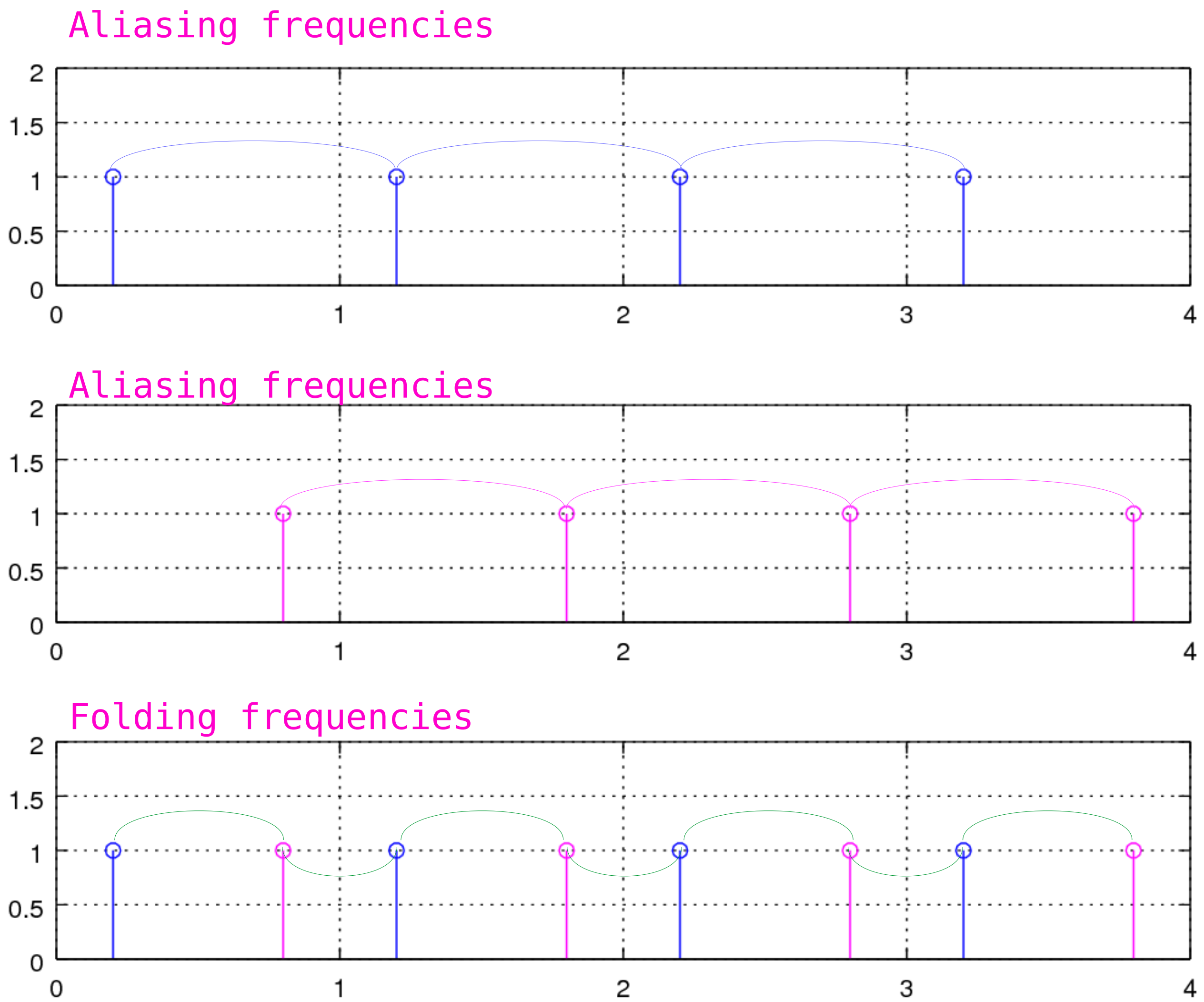
A+B

$\frac{13}{5}$ $\frac{18}{5}$ $\frac{23}{5}$ $\frac{28}{5}$ $\frac{33}{5}$ → $\frac{5}{13}$ $\frac{5}{18}$ $\frac{5}{23}$ $\frac{5}{28}$ $\frac{5}{33}$

A-B

$\frac{5}{5}$ $\frac{10}{5}$ $\frac{15}{5}$ → $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ → 1

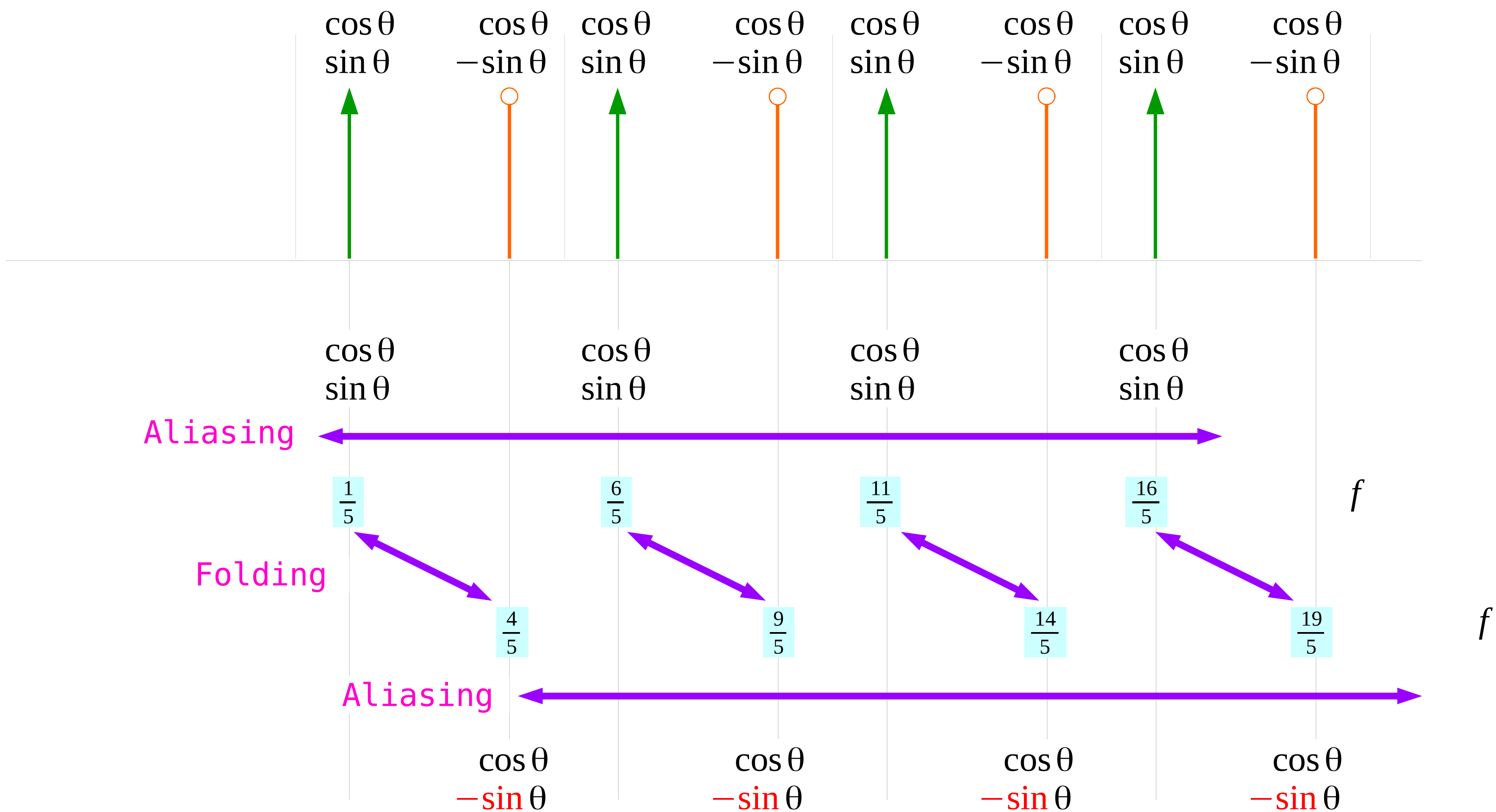
Aliasing and Folding Frequencies (1/5 & 4/5)



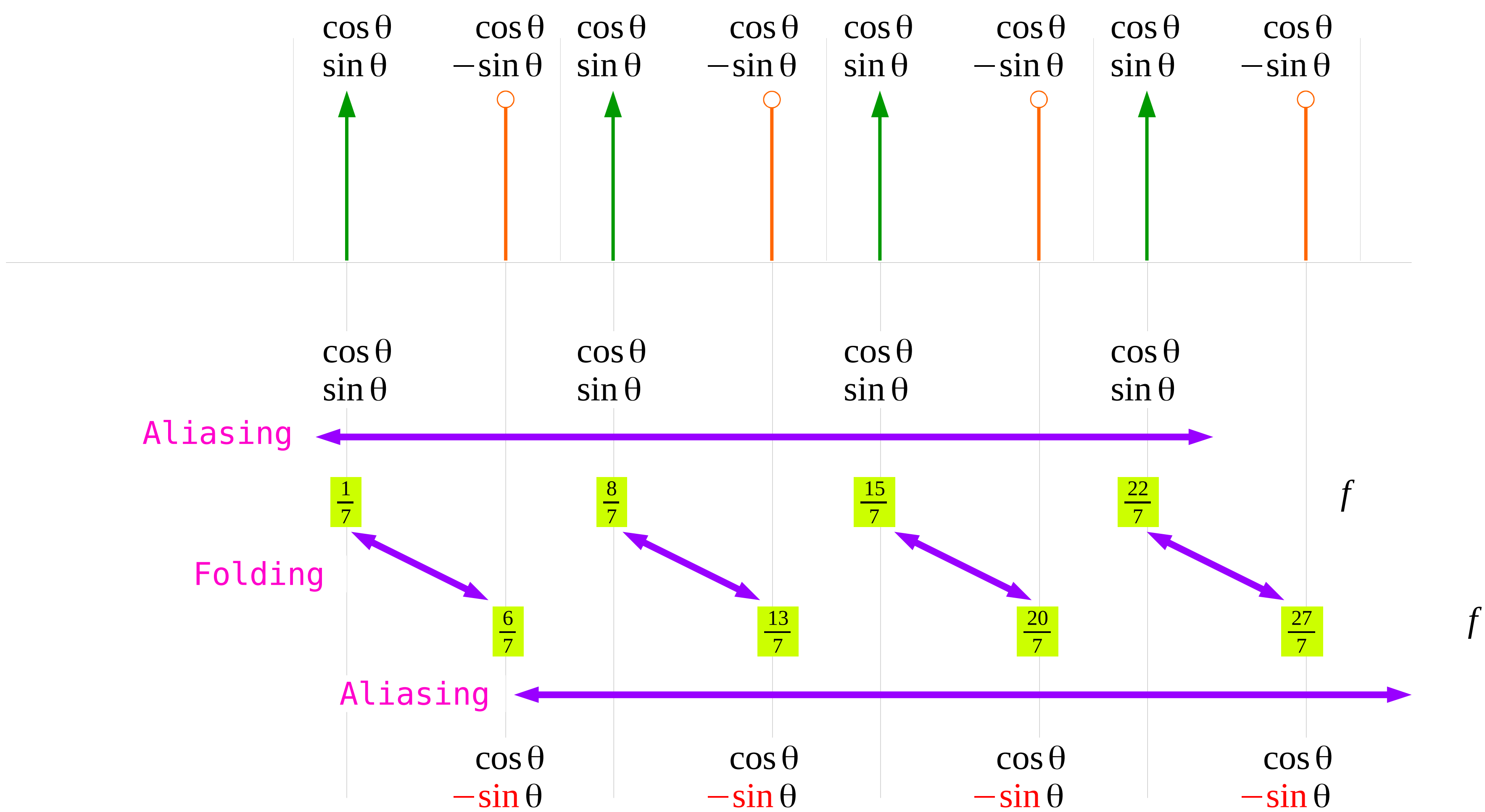

```
n1 = [1/5, 6/5, 11/5, 16/5];
n2 = [4/5, 9/5, 14/5, 19/5];

y1 = [1, 1, 1, 1];
y2 = [1, 1, 1, 1];

subplot(3, 1, 1)
stem(n1, y1, 'b'); grid on;
axis([0, 4, 0, 2]);

subplot(3, 1, 2)
stem(n2, y2, 'm'); grid on;
axis([0, 4, 0, 2]);

subplot(3, 1, 3)
stem(n1, y1, 'b'); hold on;
stem(n2, y2, 'm'); grid on;
axis([0, 4, 0, 2]);
```

J.H. McClellan, et al., Signal Processing First

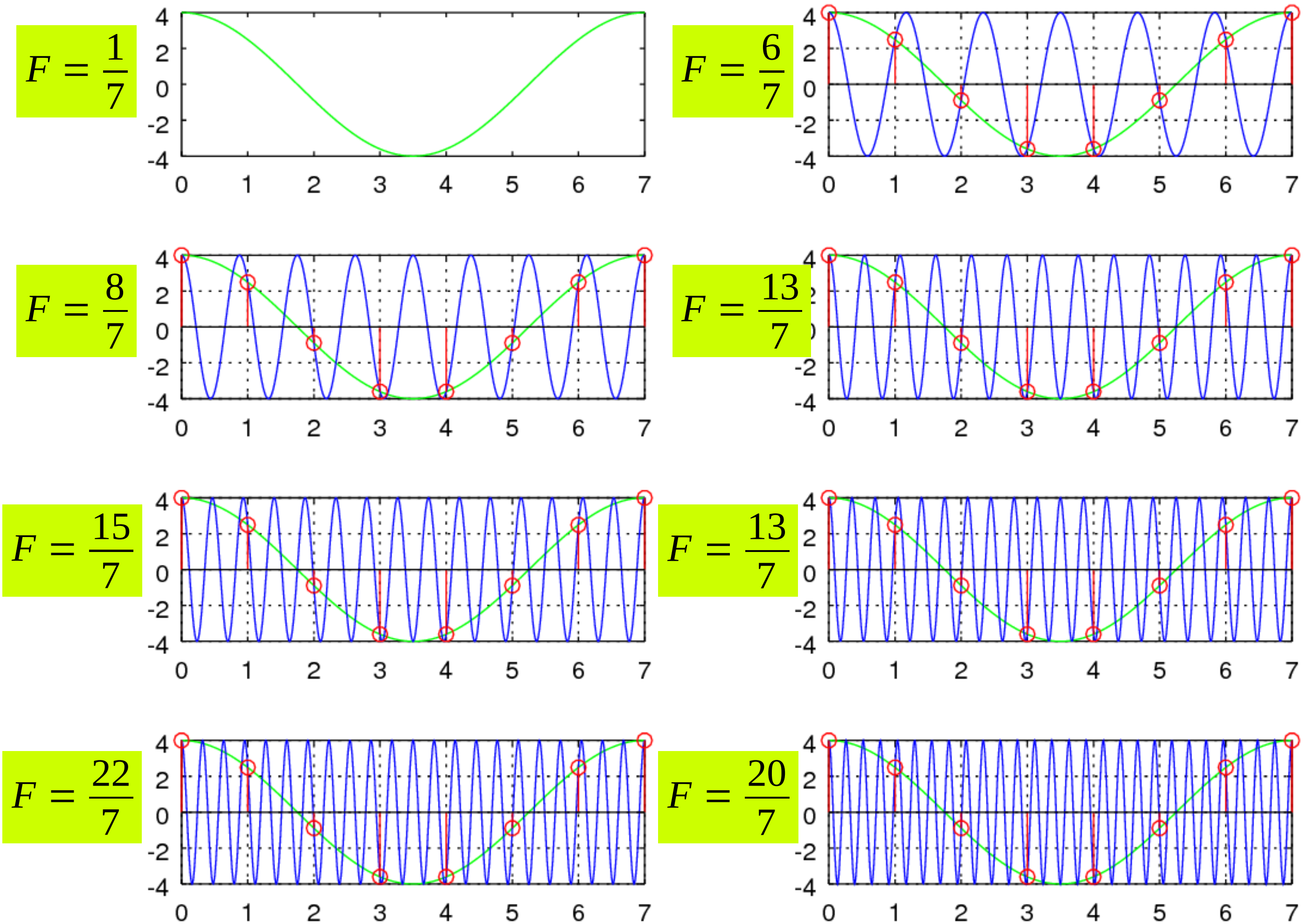
Aliased and Folded Sinusoidal Waves (1/5 & 4/5)



Aliased and Folded Sinusoidal Waves (1/7 & 6/7)



Graphs of cos(2π(n/7)t) & cos(2π(1/7)t)



References

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[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M.J. Roberts, Fundamentals of Signals and Systems
[4] S.J. Orfanidis, Introduction to Signal Processing
[5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

[6] A “graphical interpretation” of the DFT and FFT, by Steve Mann