DM (Delta Modulation)

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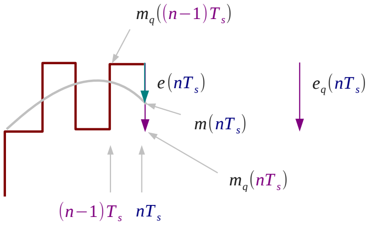
October 11, 2013

Delta Modulation

- oversampled (much higher than the Nyquist rate)
- increased correlation
- a simple quantization scheme possible
- a staircase approximation
- only two quantization levels $+\Delta, -\Delta$
- assume that the input signal does not change rapidly

The Principle of DM

$m(t)$: the input signal
$m_q(t)$: its staircase approximation
T_s: the sampling period



$e(nT_s)$: an error signal
$m(nT_s)$: the current sample value
$m_q((n-1)T_s)$: the latest approximation
$e_q(nT_s)$: an qunantized error signal
$sgn()$: the signum function

Delta Modulation Equations

Delta Modulation Principle

- $e(nT_s) = m(nT_s) - m_q((n-1)T_s)$
- $e_q(nT_s) = \Delta sgn[e(nT_s)]$
- $m_q(nT_s) = m_q((n-1)T_s) + e_q(nT_s)$

Sample-and-Hold Filter

Flat-top PAM Pulses

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$s(t) = m_\delta(t) \star h(t)$$

The rectangular Signal

$$h(t) = rect(\frac{t-T/2}{T}) \qquad \begin{cases} 1 & (0 < t < T) \\ 0.5 & (t = 0,\ T) \\ 0 & (otherwise) \end{cases}$$

Idealy Sampled Signal

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

Flat-top PAM Pulses

Flat-top PAM Pulses

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$s(t) = m_\delta(t) \star h(t)$$

$$\begin{aligned}
s(t) = m_\delta(t) \star h(t) &= \int_{-\infty}^{+\infty} m_\delta(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{+\infty} \left[\sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s) \right] h(t-\tau)d\tau \\
&= \sum_{n=-\infty}^{\infty} m(nT_s) \left\{ \int_{-\infty}^{+\infty} [\delta(\tau - nT_s)]\, h(t-\tau)d\tau \right\} \\
&= \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)
\end{aligned}$$

Fourier Transform of PAM Pulses

Fourier Transform of a Sampled Signal

$$g_\delta(t) \leftrightarrows G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$

$$\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \leftrightarrows \sum_{n=-\infty}^{\infty} g(nT_s) exp(-j2\pi nT_s f)$$

Fourier Transform of Flat-top PAM Pulses

$$s(t) = m_\delta(t) \star h(t)$$

$$S(f) = M_\delta(f)H(f) = \left[f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \right] H(f)$$

Reference

[1] S. Haykin, M Moher, “Introduction to Analog and Digital Communications”, 2ed