

# DM (Delta Modulation)

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# Delta Modulation

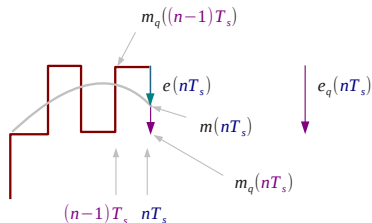
- oversampled (much higher than the Nyquist rate)
- increased correlation
- a simple quantization scheme possible
- a staircase approximation
- only two quantization levels  $+\Delta, -\Delta$
- assume that the input signal does not change rapidly

# The Principle of DM

$m(t)$  : the input signal

$m_q(t)$  : its staircase approximation

$T_s$ : the sampling period



$e(nT_s)$  : an error signal

$m(nT_s)$  : the current sample value

$m_q((n-1)T_s)$  : the latest approximation

$e_q(nT_s)$  : an quantized error signal

$sgn()$  : the signum function

# Delta Modulation Equations

## Delta Modulation Principle

- $e(nT_s) = m(nT_s) - m_q((n-1)T_s)$
- $e_q(nT_s) = \Delta \operatorname{sgn}[e(nT_s)]$
- $m_q(nT_s) = m_q((n-1)T_s) + e_q(nT_s)$

# Sample-and-Hold Filter

## Flat-top PAM Pulses

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$s(t) = m_{\delta}(t) \star h(t)$$

## The rectangular Signal

$$h(t) = \text{rect}\left(\frac{t-T/2}{T}\right) \begin{cases} 1 & (0 < t < T) \\ 0.5 & (t = 0, T) \\ 0 & (\text{otherwise}) \end{cases}$$

## Idealy Sampled Signal

$$m_{\delta}(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

# Flat-top PAM Pulses

## Flat-top PAM Pulses

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$s(t) = m_{\delta}(t) \star h(t)$$

$$\begin{aligned} s(t) &= m_{\delta}(t) \star h(t) = \int_{-\infty}^{+\infty} m_{\delta}(\tau)h(t - \tau)d\tau \\ &= \int_{-\infty}^{+\infty} \left[ \sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s) \right] h(t - \tau)d\tau \\ &= \sum_{n=-\infty}^{\infty} m(nT_s) \left\{ \int_{-\infty}^{+\infty} [\delta(\tau - nT_s)] h(t - \tau)d\tau \right\} \\ &= \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s) \end{aligned}$$

# Fourier Transform of PAM Pulses

## Fourier Transform of a Sampled Signal

$$g_{\delta}(t) \Leftrightarrow G_{\delta}(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$

$$\sum_{n=-\infty}^{\infty} g(nT_s) \delta(t - nT_s) \Leftrightarrow \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi nT_s f)$$

## Fourier Transform of Flat-top PAM Pulses

$$s(t) = m_{\delta}(t) \star h(t)$$

$$S(f) = M_{\delta}(f)H(f) = \left[ f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \right] H(f)$$



# Reference

[1] S. Haykin, M Moher, “Introduction to Analog and Digital Communications”, 2ed