

# Carry and Overflow

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- 1 "Self-service Linux: Mastering the Art of Problem Determination",

Mark Wilding

- 1 "Computer Architecture: A Programmer's Perspective", Bryant & O'Hallaron

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# Compiling 32-bit program on 64-bit gcc

- `gcc -v`
- `gcc -m32 t.c`
- `sudo apt-get install gcc-multilib`
- `sudo apt-get install g++-multilib`
- `gcc-multilib`
- `g++-multilib`
- `gcc -m32`
- `objdump -m i386`

- Carry flag and overflow flag
- Signed and unsigned computations
- Flags for an unsigned number
- Flags for a signed number
- Detecting errors in unsigned and signed arithmetic
- The verb to overflow v.s. the overflow flag

# Carry flag and overflow flag

- considering carry and overflow flags in **x86**
- do not confuse the **carry flag** with the **overflow flag** in integer arithmetic.
- the *ALU* always sets these flags appropriately when doing any integer math.
- these flags can occur on its *own*, or *both* together.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Signed and unsigned computations

- the CPU's ALU doesn't care or know whether **signed** or **unsigned** computations are performed;
- the ALU just performs integer arithmetic and sets the flags appropriately.
- It's up to the programmer to know which flag to check after the arithmetic is done.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Flags for an unsigned number

- if a word is treated as an **unsigned** number,
  - the **carry** flag must be used to check if the result is fit into  $n$ -bit or  $(n+1)$ -bit number
  - the **overflow** flag is *irrelevant* to an **unsigned** number arithmetic

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Flags for a signed number

- if a word is treated as an **signed** number,
  - the **carry** flag is *irrelevant* to an **signed** number arithmetic
  - the **overflow** flag must be used to check if the result is wrong or not

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Detecting errors in unsigned and signed arithmetic (1)

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**unsigned** integer  
arithmetic

---

**signed** integer  
arithmetic

---

**CF** Carry Flag

detects *overflows*  
extends an  $n$ -bit result  
into an  $(n+1)$ -bit result

---

**OF** Overflow Flag

detects *overflows*  
errors  
the result cannot be used

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Detecting errors in unsigned and signed arithmetic (2)

- **unsigned** integer arithmetic *overflow*  
is indicated by the **carry** flag
  - $P + P$  **CF=1** → carry out – the result is too large for an  $n$ -bit integer
  - $P - P$  **CF=1** → borrow in – the result is too small for an  $n$ -bit integer
- **signed** integer arithmetic *overflow*  
is indicated by the **overflow** flag
  - $P + P \rightarrow N$  **OF=1** → overflow – the result is not correct
  - $N + N \rightarrow P$  **OF=1** → overflow – the result is not correct
- $P$  (positive),  $N$  (negative)

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Detecting errors in unsigned and signed arithmetic (3)

- **unsigned** integer arithmetic *overflow* is indicated by the **carry** flag
  - the *overflowed*  $n$ -bit result can be extended into  $(n+1)$ -bit result by using the carry flag
- **signed** integer arithmetic *overflow* is indicated by the **overflow** flag
  - the *overflowed*  $n$ -bit result cannot be used

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# The verb to overflow v.s. the overflow flag (1)

- Do not confuse the English verb *to overflow* with the **overflow flag** in the ALU.
- The verb *to overflow* is used casually to indicate that some math result doesn't fit in the number of bits available;
- it could be integer math, or floating-point math, or whatever.
- The **overflow flag** is set specifically by the ALU it isn't the same as the casual English verb "to overflow"

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## The verb to overflow v.s. the overflow flag (2)

- In English, we may say "the binary/integer math overflowed the number of bits available for the result, causing the carry flag to come on".
- Note how this English usage of the verb "to overflow" is **not** the same as saying the **overflow flag** is on".
- A math result can overflow (the verb) the number of bits available without turning on the ALU **overflow flag**

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Computing Carry and Overflow Flags

## CF (carry flag) and OF (overflow flag) computation

ADD (addition)	SUB (subtraction)
$CF = C_n$	$CF = \overline{C_n}$
$OF = C_n \oplus C_{n-1}$	$OF = C_n \oplus C_{n-1}$
a 2's complement addition $A + B = A + B + 0$	a transformed addition $A - B = A + \overline{B} + 1$
$\{C_n, S_{n-1}\} = a_{n-1} + b_{n-1} + c_{n-1}$	$\{C_n, S_{n-1}\} = a_{n-1} + \overline{b_{n-1}} + c_{n-1}$
$\{C_{n-1}, S_{n-2}\} = a_{n-2} + b_{n-2} + c_{n-2}$	$\{C_{n-1}, S_{n-2}\} = a_{n-2} + \overline{b_{n-2}} + c_{n-2}$

[https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14\\_1](https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1)

- Examples of signed and unsigned integer arithmetic
- Carry flag in unsigned and signed computations
- Rules for the carry flag
- Method for computing the carry flag
- More examples of the carry flag



# TOC: Examples of signed and unsigned integer arithmetic

- Examples of interpreting **signed** and **unsigned** numbers
- Examples of **signed** and **unsigned** integer arithmetic
- 2's complements
- **Unsigned** subtraction
- **Signed** subtraction
- Interpreting the result as a **signed** or an **unsigned** integer
- Summary of **signed** and **unsigned** subtractions
- Examples of **unsigned** integer overflows
- Examples of **signed** integer overflows

# Examples of interpreting **signed** and **unsigned** numbers (1)

- interpreting 0xFFFFBDC3

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as an **unsigned** (positive) number    +0xFFFFBDC3    +4294950339<sub>10</sub>

---

as a **signed** (negative) number    -0x0000423D    -16957<sub>10</sub>

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of interpreting **signed** and **unsigned** numbers (2)

- interpreting 0xFFFFBDC3
  - as an **unsigned** (positive) number | +0xFFFFBDC3 | +4294950339<sub>10</sub> |

$$15 * 16^7 + 15 * 16^6 + 15 * 16^5 + 15 * 16^4 \\ + 11 * 16^3 + 13 * 16^2 + 12 * 16^1 + 3 * 16^0$$

- as a **signed** (negative) number | -0x0000423D | -16957<sub>10</sub> |

$$0 * 16^7 + 0 * 16^6 + 0 * 16^5 + 0 * 16^4 \\ + 4 * 16^3 + 2 * 16^2 + 3 * 16^1 + 13 * 16^0$$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of interpreting signed and unsigned numbers (3)

- the 2's complement of 0xFFFFBDC3 : 0x0000423D (= +16957<sub>10</sub>)

	F	F	F	F	B	D	C	3	
0xFFFFBDC3	0x1111	1111	1111	1111	1011	1101	1100	0011	
0x0000423D	0x0000	0000	0000	0000	0100	0010	0011	1100	(1's complement)
0x0000423D	0x0000	0000	0000	0000	0100	0010	0011	1101	(2's complement)
	0	0	0	0	4	2	3	D	

- the 2's complement of 0x0000423D : 0xFFFFBDC3 (= -16957<sub>10</sub>)

	0	0	0	0	4	2	3	D	
0x0000423D	0x0000	0000	0000	0000	0100	0010	0011	1101	
0x0000BDC2	0x1111	1111	1111	1111	1011	1101	1100	0010	(1's complement)
0xFFFFBDC3	0x1111	1111	1111	1111	1011	1101	1100	0011	(2's complement)
	F	F	F	F	B	D	C	3	

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of **signed** and **unsigned** integer arithmetic

- subtracting **0x0000618D** from **0x0000195D**

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**0x0000195D - 0x0000618D**      **unsigned** subtraction

subtraction by hand

---

**0x0000195D + (-0x0000618D)**      **signed** subtraction

the *transformed addition* using  
the 2's complement of subtrahend

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## 2's complements

- the 2's complement of **0x0000618D** : 0xFFFF8E73 (= -24973<sub>10</sub>)

		F	F	F	F	8	E	7	3	
0xFFFF9E73		0x1111_1111_1111_1111_1001_1110_0111_0011								
0x0000617C		0x0000_0000_0000_0000_0110_0001_1000_1100								(1's complement)
0x0000618D		0x0000_0000_0000_0000_0110_0001_1000_1101								(2's complement)
		0	0	0	0	6	1	8	D	

- the 2's complement of **0xFFFF8E73** : 0x0000618D (= +24973<sub>10</sub>)

		0	0	0	0	6	1	8	D	
0x0000618D		0x0000_0000_0000_0000_0110_0001_1000_1101								
0xFFFF9E72		0x1111_1111_1111_1111_1001_1110_0111_0010								(1's complement)
0xFFFF9E73		0x1111_1111_1111_1111_1001_1110_0111_0011								(2's complement)
		F	F	F	F	8	E	7	3	

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Unsigned subtraction

- **0x0000195D - 0x0000618D** : **unsigned** subtraction  
subtraction by hand

		0	0	0	0	1	9	5	D	
0x0000195D		0x0000_0000_0000_0000_0001_1001_0101_1101								
- 0x0000618D		0x0000_0000_0000_0000_0110_0001_1000_1101								
		0	0	0	0	6	1	8	D	
-----										
0xFFFFB7D0	1	0x1111_1111_1111_1111_1011_0111_1101_0000								(hand subtraction)
	1	F	F	F	F	B	7	D	0	
	.									
	V	borrow (CF=1) : unsigned integer overflow								

- A **borrow** is indicated by the **carry** flag (CF=1)
  - whenever an **unsigned** integer overflow happened
  - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Signed subtraction

- $0x0000195D + (-0x0000618D)$  : signed subtraction  
the *transformed addition* using the 2's complement of subtrahend

```

          0  0  0  0  1  9  5  D
0x0000195D  0x0000_0000_0000_0000_0001_1001_0101_1101 (+0x0000195D)
+ 0xFFFF9E73  0x1111_1111_1111_1111_1001_1110_0111_0011 (-0x0000618D)
              F  F  F  F  9  E  7  3
-----
0xFFFFB7D0  0  0x1111_1111_1111_1111_1011_0111_1101_0000 (hand addition)
0           F  F  F  F  B  7  D  0
-0x00004830  .  0x0000_0000_0000_0000_0100_1000_0011_0000 (2's complement)
.           0  0  0  0  4  8  3  0
V no carry in the transformed addition (Cn=0) --> (CF=1)
```

- signed integer overflow is indicated by the **overflow** flag (OF)
  - the **carry** flag is set by the **inverted** carry of a transformed addition

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>



# Interpreting the result as a **signed** or an **unsigned** integer

- subtracting **0x0000618D** from **0x0000195D**  
the results of **unsigned** and **signed** subtractions have  
the same bit pattern **0xFFFFB7D0**

- the 2's complement of **0xFFFFB7D0** :  $0x00004830$  ( $= +18480_{10}$ )

	F	F	F	F	B	7	D	0
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							
0x0000482F	0x0000_0000_0000_0000_0100_1000_0010_1111							(1's complement)
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							(2's complement)
	0	0	0	0	4	8	3	0

- the 2's complement of **0x00004830** :  $0xFFFFB7D0$  ( $= -18480_{10}$ )

	0	0	0	0	4	8	3	0
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							
0xFFFFB7CF	0x1111_1111_1111_1111_1011_0111_1100_1111							(1's complement)
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							(2's complement)
	F	F	F	F	B	7	D	0

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Summary of signed and unsigned subtractions (1)

- subtracting  $0x0000618D$  from  $0x0000195D$ 
  - $0x0000195D - 0x0000618D$  : unsigned integer subtraction  
hand subtraction
  - $0x0000195D + (-0x0000618D)$  : signed integer subtraction  
the *transformed addition* using the 2's complement of the subtrahend
  - the same result :  $0xFFFFB7D0$  (the same bit pattern)
    - interpreting as a unsigned integer  $4294948816_{10}$   
 $0xFFFFB7D0$  with a borrow (CF=1)
    - interpreting as a signed integer  $-18480_{10}$   
 $-0x00004830$  (meaningless CF=1)

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Summary of **signed** and **unsigned** subtractions (2)

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0xFFFFB7D0    the result of **unsigned** subtraction    4294948816<sub>10</sub>  
with CF=1    with **unsigned** integer overflow

---

-0x00004830    the result of **signed** subtraction    -18480<sub>10</sub>

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of **unsigned** integer overflows

- $0x0000195D - 0x0000618D$  : **unsigned** subtraction
  - there is an **unsigned** integer overflow  
so the **carry** flag will be set ( $CF=1$ ) to indicate a **borrow**
  - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$   
(unsigned integers can't be negative),

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of signed integer overflows

- $0x0000195D + (-0x0000618D)$  : signed subtraction
  - there is no signed integer overflow  
the overflow flag won't be set (OF=0)
  - signed overflow occurs , in the transformed addition,
    - two *positive* numbers are added and  
the result is a *negative*, ( $P + P \rightarrow N$ ), or
    - two *negative* numbers are added and  
the result is a *positive*, ( $N + N \rightarrow P$ )

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# TOC Carry flag in unsigned and signed computations

- 2's complement numbers : 4-bit
- Addend and augend in a  $n$ -bit addition
- Full adder operation in each bit position
- Internal and external carry bits
- Addition and Subtraction
- Using the Carry Flag as a borrow

## 2's complement numbers : 4-bit

---

0111	(+7)	1000	(-8)
0110	(+6)	1001	(-7)
0101	(+5)	1010	(-6)
0100	(+4)	1011	(-5)
0011	(+3)	1100	(-4)
0010	(+2)	1101	(-3)
0001	(+1)	1110	(-2)
0000	(0)	1111	(-1)

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Addend and augend in a $n$ -bit addition

$n$	bits	addened	$A$	$\{a_{n-1}, a_{n-2}, \dots, a_1, a_0\}$
$n$	bits	augend	$B$	$\{b_{n-1}, b_{n-2}, \dots, b_1, b_0\}$
$(n+1)$	bits	carry bits	$C$	$\{C_n, C_{n-1}, C_{n-2}, \dots, C_1, C_0\}$
$n$	bits	sum bits	$S$	$\{S_{n-1}, S_{n-2}, \dots, S_1, S_0\}$

external carry bits :  $C_n$  carry out,  $C_0$  carry in

$$\begin{array}{cccccc} a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 & \\ b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 & \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Full adder operation in each bit position

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full adder operation in the  $i^{\text{th}}$  bit position

$$\{C_{i+1}, S_i\} = a_i + b_i + C_i$$

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$$\begin{array}{r} a_i \\ b_i \\ C_i \\ \hline C_{i+1} \quad S_i \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Internal and external carry bits

external carries     $C_n$  output,  $C_0$  input  
 internal carries     $\{C_{n-1}, C_{n-2}, \dots, C_2, C_1\}$     output / input  
 sum bits             $\{S_{n-1}, S_{n-2}, \dots, S_1, S_0\}$     output

$$\begin{array}{rcccccc}
 & a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 \\
 & b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \\
 \hline
 C_n & C_{n-1} & C_{n-2} & \cdots & C_1 & C_0 \\
 \hline
 & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0
 \end{array}$$

$$\begin{array}{rcccccc}
 & a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 \\
 & b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \\
 & & & & & C_0 \\
 \hline
 C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0
 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Addition and Subtraction

- addition

$$\{C_n, S\} = A + B = A + B + 0$$

$$\begin{array}{rcccccc} & a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 \\ & b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \\ \hline & C_{n-1} & C_{n-2} & \cdots & C_1 & 0 \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

- subtraction - transformed addition

$$\{C_n, S\} = A - B = A + \overline{B} + 1$$

$$\begin{array}{rcccccc} & a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 \\ & b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \\ \hline & C_{n-1} & C_{n-2} & \cdots & C_1 & 1 \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Using the Carry Flag as a borrow (1)

- a **borrow** (CF=1) occurs in the **subtraction**  $A - B$  when  $b$  is larger than  $a$  ( $A < B$ ) as unsigned numbers
- Computer hardware can detect a **borrow** (CF=1) in **subtraction** by looking at whether a carry out (Cn) occurred in the transformed addition

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Using the Carry flag as a borrow (2)

- a **borrow** ( $CF=1$ ) occurs in the **subtraction**  $A - B$  ( $A < B$ ) as unsigned numbers
- a carry out ( $C_n$ ) in the transformed addition
  - If there is no **carry** ( $C_n=0$ ) then there is a **borrow** ( $CF=1$ )
  - If there is a **carry** ( $C_n=1$ ) then there is no **borrow** ( $CF=0$ )
  - **$CF = !C_n$**

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Using the Carry Flag as a borrow (3)

- the same *addition* and *subtraction* instructions are used for both **unsigned** and **signed** integer arithmetic.
  - no special *addition* and *subtraction* instructions for **unsigned** and **signed** integer arithmetic
- the only difference is
  - which flags you *test* afterwards and
  - how you *interpret* the result

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# TOC Rules for the carry flag

- 2's complement numbers : 4-bit
- The 1st rule for setting the carry flag
- The 2nd rule for setting the carry flag
- Cases for clearing the carry flag
- Computing CF in unsigned additions and subtractions

## 2's complement numbers : 4-bit

---

0111	(+7)	1000	(-8)
0110	(+6)	1001	(-7)
0101	(+5)	1010	(-6)
0100	(+4)	1011	(-5)
0011	(+3)	1100	(-4)
0010	(+2)	1101	(-3)
0001	(+1)	1110	(-2)
0000	(0)	1111	(-1)

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# The 1st rule for setting the carry flag

- 1 **CF = 1** : **carry** in **unsigned addition**
  - the **carry flag** is set if the **addition** of two **unsigned** numbers causes a **carry** out of the most significant bits added.
  - **unsigned integer overflow** in **unsigned addition**
  - *hand addition rule*

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# The 2nd rule for setting the carry flag

- ② **CF = 1 : borrow in unsigned subtraction**
  - the **carry flag** is also set if the **subtraction** of two **unsigned** numbers requires a **borrow** into the most significant bits subtracted.
  - **unsigned integer overflow in unsigned subtraction**
  - *hand subtraction rule*

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (1)

- Otherwise, the **carry flag** is turned off (zero).
  - all three interpretations have the same CF=1, the same S=0000

unsigned addition		signed addition		signed subtraction
0111 ( 7)		0111 (+7)		0111 (+7)
+1001 +( 9)		+1001 +(-7)		-0111 -(+7)
-----		-----		-----
10000 (16)		10000 ( 0)		10000 ( 0)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn		

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the carry flag (2)

- Otherwise, the **carry flag** is turned off (zero).
  - all three interpretations have the same CF=0, the same S=1111

unsigned addition		signed addition		signed subtraction
0111 ( 7)		0111 (+7)		0111 (+7)
+1001 +( 9)		+1001 +(-7)		-0111 -(+7)
-----		-----		-----
10000 (16)		10000 ( 0)		10000 ( 0)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn		

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Computing CF in unsigned additions and subtractions

- Computing CF in an **unsigned** addition
  - do the **signed** addition
  - $C_n$  is the carry out
  - $CF \leftarrow C_n$
- Computing CF in an **unsigned** subtraction
  - do the transformed **signed** addition
  - do the **signed** addition
  - $C_n$  is the carry out
  - $CF \leftarrow !C_n$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

- Carry flag computation

# Carry flag computation (1)

ADD (addition)	SUB (subtraction)
$CF = C_n$	$CF = \overline{C_n}$
normal carry of a 2's complement addition	<b>inverted</b> carry of a transformed addition
$A + B = A + B + 0$	$A - B = A + \overline{B} + 1$
$\{C_n, S_{n-1}\}$ $= a_{n-1} + b_{n-1} + c_{n-1}$	$\{C_n, S_{n-1}\}$ $= a_{n-1} + \overline{b_{n-1}} + c_{n-1}$

[https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14\\_1](https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1)

## Carry flag computation (2)

- In **unsigned** arithmetic,
  - the **carry flag** is used to detect *overflow*
  - the **carry flag** is used to extend *n-bit* result into *(n+1)-bit* result
  - for **addition**, the **carry flag** is a **carry out**
  - for **subtraction**, the **carry flag** is a **borrow in**
- In **signed** arithmetic,
  - the **carry flag** is useless
  - the **carry flag** neither detects overflow nor extends n-bit result

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Carry flag computation (3)

- In **unsigned** arithmetic,

---

Addition	<b>CF = 1</b> means <b>carry out</b>	when <b>C<sub>n</sub> = 1</b>
Subtraction	<b>CF = 1</b> means <b>borrow in</b>	when <b>C<sub>n</sub> = 0</b>

---

- **CF** - Carry Flag in x86
- **C<sub>n</sub>** - the normal carry out
  - the carry out of a 2's complement addition for **ADD**
  - the carry out of a *transformed* addition for **SUB**
- In **signed** arithmetic,
  - the **carry** flag is useless

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: More examples of the carry flag

- Summary I
- Summary II
- Cases for setting the carry flag
- Cases for clearing the carry flag

# Summary I

unsigned add/sub			signed addition			signed subtraction			CF	OF
1101	(13)		1101	(-3)		1101	(-3)			
+1110	+(14)	ADD	+1110	+(-2)	ADD	-0010	-(-2)			
-----	-----		-----	-----		-----	-----			
11011	(11)	(+16)	11011	(-5)		11011	(-5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
-1110	-(14)	SUB	+0010	+(+2)		-1110	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
10101	( 5)	(-16)	00101	(+5)		00101	(+5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
+0010	+( 2)	ADD	+0010	+(+2)	ADD	-1110	-(-2)			
-----	-----		-----	-----		-----	-----			
00101	( 5)	(+ 0)	00101	(+5)		00101	(+5)		0	0
1101	(13)		1101	(-3)		1101	(-3)			
-0010	-( 2)	SUB	+1110	+(-2)		-0010	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
11011	(11)	(-16)	11011	(-5)		11011	(-5)		0	0

# Summary II

unsigned add/sub			signed addition			signed subtraction			CF	OF
1011	(11)		1011	(-5)		1011	(-5)			
+1100	+(12)	ADD	+1100	+(-4)	ADD	-0100	-(+4)			
-----	-----		-----	-----		-----	-----			
10111	( 7) (+16)		10111	(+7)		10111	(+7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
-1100	-(12)	SUB	+0100	+(+4)		-1100	-(-4)	SUB		
-----	-----		-----	-----		-----	-----			
11001	( 9) (-16)		01001	(-7)		01001	(-7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
+0100	+( 4)	ADD	+0100	+(+4)	ADD	-1100	-(-4)			
-----	-----		-----	-----		-----	-----			
01001	( 9) (+ 0)		01001	(-7)		01001	(-7)		0	1
1011	(11)		1011	(-5)		1011	(-5)			
-0100	-( 4)	SUB	+1100	+(-4)		-0100	-(+4)	SUB		
-----	-----		-----	-----		-----	-----			
00111	( 7) ( 0)		10111	(+7)		10111	(+7)		0	1

# Cases for setting the carry flag (1) CF=1, OF=0

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1101 (-3)		1101 (-3)
+1110 +(14) ADD		+1110 +(-2) ADD		-0010 -(+2)
-----		-----		-----
11011 (11) (+16)		11011 (-5)		11011 (-5)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

\* CF=1, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (2) CF=1, OF=0

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0011 (3)		0011 (+3)		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 0101		S = 0101		S = 0101
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		CF <- !Cn (for unsigned subtraction)
-----		-----		-----

\* CF=1, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (3) CF=1, OF=1

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition		signed subtraction
1011 (11)		1011 (-5)		1011 (-5)
+1100 +(12) ADD		+1100 +(-4) ADD		-0100 -(+4)
-----		-----		-----
10111 ( 7) (+16)		10111 (+7)		10111 (+7)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means +16		CF meaningless		CF meaningless
S = 0111		S = 0111		S = 0111
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

\* CF=1, S=0111, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (4) CF=1, OF=1

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0101 (5)		0101 (+5)		0101 (+5)
-1100 -(12) SUB		+0100 +(4)		-1100 -(-4) SUB
-----		-----		-----
11001 (9) (-16)		01001 (-7)		01001 (-7)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 1001		S = 1001		S = 1001
* think hand subtraction		* think Cn of the transformed addition		CF <- !Cn (for unsigned subtraction)

\* CF=1, S=1001, OF=1 for all three interpretations



# Cases for clearing the carry flag (1) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition	signed subtraction
0011 ( 3)		0011 (+3)	0011 (+3)
+0010 +( 2) ADD		+0010 +( +2) ADD	-1110 -(-2)
-----		-----	-----
00101 ( 5) (+ 0)		00101 (+5)	00101 (+5)
CF=0		Cn=0 -> CF=0	Cn=0 -> CF=0
CF means 0		CF meaningless	CF meaningless
S = 0101		S = 0101	S = 0101
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn (for unsigned addition)	

\* CF=0, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (2) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition	* signed addition	signed subtraction
1101 (13)	1101 (-3)	1101 (-3)
-0010 -( 2) SUB	+1110 +(-2)	-0010 -(+2) SUB
-----	-----	-----
11011 (11) (-16)	11011 (-5)	11011 (-5)
CF=0	Cn=0 -> CF=0	Cn=0 -> CF=0
CF means 0	CF meaningless	CF meaningless
S = 1011	S = 1011	S = 1011
-----	-----	-----
* think hand subtraction	* think Cn of the corresponding addition	
	CF <- Cn (for unsigned addition)	

\* CF=0, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (3) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition	signed subtraction
0101 ( 5)		0101 (+5)	0101 (+5)
+0100 +( 4) ADD		+0100 +( +4) ADD	-1100 -(-4)
-----		-----	-----
01001 ( 9) (+ 0)		01001 (-7)	01001 (-7)
CF=0		Cn=0 -> CF=0	Cn=0 -> CF=0
CF means +0		CF meaningless	CF meaningless
S = 1001		S = 1001	S = 1001
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn (for unsigned addition)	

\* CF=0, S=1001, OF=1 for all three interpretations

# Cases for clearing the carry flag (4) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned subtraction		signed addition		* signed subtraction
1011 (11)		1011 (-5)		1011 (-5)
-0100 -( 4) SUB		+1100 +(-4)		-0100 -(+4) SUB
-----		-----		-----
00111 ( 7) ( 0)		10111 (+7)		10111 (+7)
CF=0		Cn=1 -> CF=0		Cn=1 -> CF=0
CF means 0		CF meaningless		CF meaningless
S = 0111		S = 0111		S = 0111
* think hand subtraction		* think Cn of the transformed addition		
		CF <- !Cn (for unsigned subtraction)		

\* CF=0, S=0111, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: Overflow flag

- Overflow flag in unsigned and signed computations
- Rules for the overflow flag
- Method 1 for computing the overflow flag
- Method 2 for computing the overflow flag
- More examples of the overflow flag

- Overflow flag

# Overflow flag (1)

- only need to look at the **sign bits** (leftmost) of the three numbers

$$\begin{array}{rclcl} \text{augend} & + & \text{addend} & = & \text{sum} \\ \text{minuend} & - & \text{subrahend} & = & \text{difference} \end{array}$$

to decide if the **overflow** flag is turned on or off.

- overflow** flag is based on **signed** arithmetic

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Overflow flag (2)

- in **signed** arithmetic,
  - watch the **overflow** flag to detect errors
  - **overflow** flag on means the result is wrong
  - errors can be detected by examining the sign of the result, in the 2's complement arithmetic
- in **unsigned** arithmetic,
  - the **overflow** flag tells you nothing interesting

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Overflow flag (3)

- when two positive numbers are added
  - if the result is a negative, ( $P + P \rightarrow N$ ), then overflow
  - if the result is a positive, ( $P + P \rightarrow P$ ), then no overflow
- when two negative numbers are added
  - the result is a positive, ( $N + N \rightarrow P$ ), then overflow
  - the result is a negative, ( $N + N \rightarrow N$ ), then no overflow

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow flag (4)

- adding negative and positive numbers cannot be wrong, because the sum is between the addends.
  - mixed-sign addition never turns on the **overflow** flag.
  - opposite signed numbers are added, then no **overflow**
  - both of the addends lies in the allowable range of numbers, and their sum is between the addends, therefore the sum lies also in the allowable range
- $(P + N \rightarrow P)$  no overflow
- $(P + N \rightarrow N)$  no overflow
- $(N + P \rightarrow P)$  no overflow
- $(N + P \rightarrow N)$  no overflow

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Rules for the overflow flag

- the 1st rule for setting OF
- the 2nd rule for setting OF
- cases for clearing OF (1 ~ 6)

# Overflow flag setting and clearing conditions

ADD conditions      SUB conditions

OF=1	$P + P \rightarrow N$	$P - N \rightarrow N$	$C_n \oplus C_{n-1} = 1$
OF=1	$N + N \rightarrow P$	$N - P \rightarrow P$	$C_n \oplus C_{n-1} = 1$
OF=0	$P + P \rightarrow P$	$P - N \rightarrow P$	$C_n \oplus C_{n-1} = 0$
OF=0	$N + N \rightarrow N$	$N - P \rightarrow N$	$C_n \oplus C_{n-1} = 0$
OF=0	$P + N \rightarrow P$	$P - N \rightarrow P$	$C_n \oplus C_{n-1} = 0$
OF=0	$P + N \rightarrow N$	$P - P \rightarrow N$	$C_n \oplus C_{n-1} = 0$
OF=0	$N + P \rightarrow P$	$N - N \rightarrow P$	$C_n \oplus C_{n-1} = 0$
OF=0	$N + P \rightarrow N$	$N - P \rightarrow N$	$C_n \oplus C_{n-1} = 0$

$$+P = -(-P) = -N$$

$$+N = -(-N) = -P$$

# The 1st rule for setting the overflow flag

- 1 If the **sum** of two **signed** numbers with the sign bits off (0, 0) yields a result number with the sign bit on (1) the **overflow flag** is turned on ( $OF = 1 : P + P \rightarrow N$ )

signed addition

```
0100 carries
 0100 (+4)
+0100 +(4)
-----
01000 (-8)
```

signed subtraction

```
0100 (+4)
-1100 -(-4)
-----
01000 (-8)
```

unsigned addition

```
0100 ( 4)
+0100 +( 4)
-----
01000 ( 8)
```

$$\bullet OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 1 = 1$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# The 2nd rule for setting the overflow flag

- 2 If the **sum** of two numbers with the sign bits on (1, 1) yields a result number with the sign bit off (0) the **overflow flag** is turned on. ( $OF = 1 : N + N \rightarrow P$ )

signed addition

```
1001 carries
 1001 (-7)
+1001 +(-7)
-----
10010 ( 2)
```

signed subtraction

```
1001 (-7)
-0111 -(+7)
-----
10010 ( 2)
```

unsigned addition

```
1001 ( 9)
+1001 +( 9)
-----
10010 (18)
```

$$\bullet OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 0 = 1$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (1)

- **overflow flag** is turned off. ( $OF = 0 : P + P \rightarrow P$ )

signed addition

```
0011  carries
0011  (+3)
+0011  +(+3)
-----
00110  (+6)
```

signed subtraction

```
0011  (+3)
-1101  -(-3)
-----
00110  (+6)
```

unsigned addition

```
0011  ( 3)
+0011  +( 3)
-----
00110  ( 6)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the overflow flag (2)

- **overflow flag** is turned off. ( $OF = 0 : N + N \rightarrow N$ )

signed addition

```
1101 carries
 1101 (-3)
+1101 +(-3)
-----
11010 (-6)
```

signed subtraction

```
1101 (-3)
-0011 -(+3)
-----
11010 (-6)
```

unsigned addition

```
1101 (13)
+1101 +(13)
-----
11010 (26)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Cases for clearing the overflow flag (3)

- **overflow flag** is turned off. ( $OF = 0 : P + N \rightarrow P$ )

signed addition

```
1100 carries
 0100 (+4)
+1101 +(-3)
-----
10001 (+1)
```

signed subtraction

```
0100 (+4)
-0011 -(+3)
-----
10001 (+1)
```

unsigned addition

```
0100 ( 4)
+1101 +(13)
-----
10001 (17)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (4)

- **overflow flag** is turned off. ( $OF = 0 : P + N \rightarrow N$ )

signed addition

```
0000 carries
0011 (+3)
+1100 +(-4)
-----
01111 (-1)
```

signed subtraction

```
0011 (+3)
-0100 -(+4)
-----
01111 (-1)
```

unsigned addition

```
0011 ( 3)
+1100 +(12)
-----
01111 (15)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (5)

- **overflow flag** is turned off. ( $OF = 0 : N + P \rightarrow P$ )

signed addition

```
1100 carries
 1101 (-3)
+0100 (+4)
-----
10001 (+1)
```

signed subtraction

```
0011 (-3)
-1100 -(-4)
-----
10001 (+1)
```

unsigned addition

```
1101 (13)
+0100 +( 4)
-----
10001 (17)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (6)

- **overflow flag** is turned off. ( $OF = 0 : N + P \rightarrow N$ )

signed addition

```
0000 carries
1100 (-4)
+0011 +(+3)
-----
01111 (-1)
```

signed subtraction

```
0100 (-4)
-1101 -(-3)
-----
01111 (-1)
```

unsigned addition

```
1100 (12)
+0011 +( 3)
-----
01111 (15)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Method 1 for computing the overflow flag

- Adding two numbers with the same sign
- Overflow conditions for additions and subtractions
- Overflow condition for an addition
- Overflow conditions for a subtraction
- Overflow in signed computations

# Adding two numbers with the same sign

- **overflow** can only happen when adding two numbers of the same sign results in a different sign ( $P + P \rightarrow N$ ,  $N + N \rightarrow P$ )

- $n$ -bit **signed** binary arithmetic  $A + B = C$

$$A = (a_{n-1}, \dots, a_1, a_0)$$

$$B = (b_{n-1}, \dots, b_1, b_0)$$

$$C = (c_{n-1}, \dots, c_1, c_0)$$

- to detect overflow
  - only the **sign** bits are considered
  - **msb** (most significant bit)  $a_{n-1}, b_{n-1}, c_{n-1}$
  - the other bits are ignored

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow conditions for additions and subtractions

- with two operands ( $A$  and  $B$ ) and one result ( $C$ ), three sign bits ( $a_{n-1}, b_{n-1}, c_{n-1}$ ) are considered  
→  $2^3 = 8$  possible combinations
- only two cases result in **overflow** for an addition
  - 0 0 1     ( $p + p \rightarrow n$ )
  - 1 1 0     ( $n + n \rightarrow p$ )
- only two cases are considered as **overflow** for an subtraction
  - 0 1 1     ( $p - n \rightarrow n$ )
  - 1 0 0     ( $n - p \rightarrow p$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow condition for an addition

- Overflow in an addition ( $A + B$ )

	$a_{n-1}$	$b_{n-1}$	$c_{n-1}$	
	0	0	0	$p + p \rightarrow p$
OVER	0	0	1	$p + p \rightarrow n$
	0	1	0	$p + n \rightarrow p$
	0	1	1	$p + n \rightarrow n$
	1	0	0	$n + p \rightarrow p$
	1	0	1	$n + p \rightarrow n$
OVER	1	1	0	$n + n \rightarrow p$
	1	1	1	$n + n \rightarrow n$

- adding two positives should be positive
- adding two negatives should be negative

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Overflow conditions for a subtraction

- Overflow in a subtraction ( $A - B$ )

	$a_{n-1}$	$b_{n-1}$	$c_{n-1}$	
	0	0	0	$p - p \rightarrow p$
	0	0	1	$p - p \rightarrow n$
	0	1	0	$p - n \rightarrow p$
OVER	0	1	1	$p - n \rightarrow n$
OVER	1	0	0	$n - p \rightarrow p$
	1	0	1	$n - p \rightarrow n$
	1	1	0	$n - n \rightarrow p$
	1	1	1	$n - n \rightarrow n$

- subtracting a negative is the same as adding a positive
- subtracting a positive is the same as adding a negative

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow in signed computations

- ALU might contain a small logic that sets the **overflow** flag to "1" if and only if any one of the above four **OV conditions** is met.
- in **signed** computations, adding two numbers of the same sign must produce a result of the same sign, otherwise overflow happened.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Method 2 for computing the overflow flag

- Carry into and carry out of the sign bit
- Overflow in 2's complement arithmetic
- Overflow flag =  $C_n \oplus C_{n-1}$
- Examples of 4-bit signed additions
- $C_n$  and  $C_{n-1}$  in a  $n$ -bit addition
- Overflow flag computation
- Examples of computing overflow flag
- Hexadecimal carry, octal carry, decimal carry
- No carry into the sign bit

# Carry into and carry out of the sign bit

- When adding two  $n$ -bit binary values, consider
  - the **carry** *coming into* the most significant bit (msb)  
 $C_{n-1}$  : **carry** into the **sign** bit
  - the **carry** *going out of* the most significant bit (msb)  
 $C_n$  : **carry** out of the **sign** bit  
this is the **carry** flag (**CF**) in the processor

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow in 2's complement arithmetic

- **overflow** in 2's complement happens (**OF=1**) when
  - there is a **carry into** the **sign** bit ( $C_{n-1} = 1$ )  
but no carry out of the **sign** bit ( $C_n = 0$ )
  - there is no carry into the **sign** bit ( $C_{n-1} = 0$ )  
but a **carry out** of the **sign** bit ( $C_n = 1$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

$$\text{Overflow flag} = C_n \oplus C_{n-1}$$

- the **overflow** flag is the **XOR** ( $C_n \oplus C_{n-1}$ ) of
  - of the **carry coming into** the **sign** bit ( $C_{n-1}$ )
  - with the **carry going out of** the **sign** bit ( $C_n$ )
- **overflow** happens when the **carry in** ( $C_{n-1}$ ) does not equal to the **carry out** ( $C_n$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Examples of 4-bit signed additions (1)

- 4-bit 2's complement addition examples

```
0000
 0100 (+4) (pos sign 0)
+1000 (-8) (neg sign 1)
=====
01100 (-4) (neg sign 1)
```

```
C4 carry out 0 (1+0+0)
C3 carry in 0 (0+1+0)
0 XOR 0 = NO OVERFLOW
```

```
1100
 1100 (-4) (neg sign 1)
+0100 (+4) (pos sign 0)
=====
10000 ( 0) (pos sign 0)
```

```
C4 carry out 1 (1+0+1)
C3 carry in 1 (1+1+0)
1 XOR 1 = NO OVERFLOW
```

```
0100
 0100 (+4) (pos sign 0)
+0100 (+4) (pos sign 0)
=====
01000 (-8) (neg sign 1)
```

```
C4 carry out 0 (0+0+1)
C3 carry in 1 (1+1+0)
0 XOR 1 = OVERFLOW!
```

```
1000
 1100 (-4) (neg sign 1)
+1000 (-8) (neg sign 1)
=====
10100 (+4) (pos sign 0)
```

```
C4 carry out 1 (1+1+0)
C3 carry in 0 (1+0+0)
1 XOR 0 = OVERFLOW!
```

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Examples of 4-bit signed additions (2)

- same sign addition → possible overflow

----- + +, - -----	----- - -, + -----	----- + +, + -----	----- - -, - -----
+5 +5	-5 -5	+5 +1	-5 -1
----- -6(OF)	----- +6(OF)	----- +6	----- -6

0101 0101 0101	1011 1011 1011	0001 0101 0001	1111 1011 1111
----- 01010	----- 10110	----- 00110	----- 11010
----- C4 = 0	----- C4 = 1	----- C4 = 0	----- C4 = 1
----- C3 = 1	----- C3 = 0	----- C3 = 0	----- C3 = 1
----- OF = 1	----- OF = 1	----- OF = 0	----- OF = 0

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Examples of 4-bit signed additions (3)

- mixed sign addition → no overflow

----- + -, + ----- +5 -1 ----- +4	----- + -, - ----- +5 -6 ----- -1	----- - +, + ----- -5 +6 ----- +1	----- - +, - ----- -5 +1 ----- -4
1111 0101 1111 ----- 10100 ----- C4 = 1 C3 = 1 ----- OF = 0	0000 0101 1010 ----- 01111 ----- C4 = 0 C3 = 0 ----- OF = 0	1110 1011 0110 ----- 10001 ----- C4 = 1 C3 = 1 ----- OF = 0	0011 1011 0001 ----- 01100 ----- C4 = 0 C3 = 0 ----- OF = 0

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# $C_n$ and $C_{n-1}$ in a $n$ -bit addition

## $(n-1)^{th}$ bit – MSB

- adding operations at the  $(n-1)$  bit position
- $\{C_n, S_{n-1}\} =$   
 $a_{n-1} + b_{n-1} + c_{n-1}$

$$\begin{array}{r} \text{msb} \\ a_{n-1} \\ b_{n-1} \\ C_{n-1} \\ \hline C_n \quad S_{n-1} \end{array}$$

- $C_n$  :  
carry coming out of the msb

## $(n-2)^{th}$ bit

- adding operations at the  $(n-2)$  bit position
- $\{C_{n-1}, S_{n-2}\} =$   
 $a_{n-2} + b_{n-2} + c_{n-2}$

$$\begin{array}{r} \text{msb} \\ a_{n-2} \\ b_{n-2} \\ C_{n-2} \\ \hline C_{n-1} \quad S_{n-2} \end{array}$$

- $C_{n-1}$  :  
carry coming into the msb

# Overflow flag computation

---

**ADD** (addition)

---

$$OF = C_n \oplus C_{n-1}$$

a 2's complement addition

$$A + B = A + B + \mathbf{0} \quad (C_0 = 0)$$

$$\begin{aligned} &\{C_n, S_{n-1}\} \\ &= a_{n-1} + b_{n-1} + c_{n-1} \end{aligned}$$

$$\begin{aligned} &\{C_{n-1}, S_{n-2}\} \\ &= a_{n-2} + b_{n-2} + c_{n-2} \end{aligned}$$

---

**SUB** (subtraction)

---

$$OF = C_n \oplus C_{n-1}$$

the transformed addition

$$A - B = A + \overline{B} + \mathbf{1} \quad (C_0 = 1)$$

$$\begin{aligned} &\{C_n, S_{n-1}\} \\ &= a_{n-1} + \overline{b_{n-1}} + c_{n-1} \end{aligned}$$

$$\begin{aligned} &\{C_{n-1}, S_{n-2}\} \\ &= a_{n-2} + \overline{b_{n-2}} + c_{n-2} \end{aligned}$$

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Hexadecimal carry, octal carry, decimal carry

- Note that this XOR method only works with the **binary** carry that goes into the sign **bit**.
- not works with **hexadecimal carry**  
**decimal carry**, **octal carry**
  - the carry doesn't go into the sign **bit**
  - can't XOR that non-binary carry with the outgoing carry.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# No carry into the sign bit

- Hexadecimal addition example  
(showing that XOR doesn't work for hex carry):

```
8Ah
+8Ah
====
114h
```

- The hexadecimal carry of 1 resulting from A+A does not affect the sign bit.
- If you do the math in binary, you'll see that there is **no** carry **into** the sign bit; but, there is carry out of the sign bit. Therefore, the above example sets OVERFLOW on. (The example adds two negative numbers and gets a positive number.)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Summary I

unsigned add/sub			signed addition			signed subtraction			CF	OF
1101	(13)		1101	(-3)		1101	(-3)			
+1110	+(14)	ADD	+1110	+(-2)	ADD	-0010	-(-2)			
-----	-----		-----	-----		-----	-----			
11011	(11)	(+16)	11011	(-5)		11011	(-5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
-1110	-(14)	SUB	+0010	+(+2)		-1110	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
10101	( 5)	(-16)	00101	(+5)		00101	(+5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
+0010	+( 2)	ADD	+0010	+(+2)	ADD	-1110	-(-2)			
-----	-----		-----	-----		-----	-----			
00101	( 5)	(+ 0)	00101	(+5)		00101	(+5)		0	0
1101	(13)		1101	(-3)		1101	(-3)			
-0010	-( 2)	SUB	+1110	+(-2)		-0010	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
11011	(11)	(-16)	11011	(-5)		11011	(-5)		0	0

# Summary II

unsigned add/sub			signed addition			signed subtraction			CF	OF
1011	(11)		1011	(-5)		1011	(-5)			
+1100	+(12)	ADD	+1100	+(-4)	ADD	-0100	-(+4)			
-----	-----		-----	-----		-----	-----			
10111	( 7) (+16)		10111	(+7)		10111	(+7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
-1100	-(12)	SUB	+0100	+(+4)		-1100	-(-4)	SUB		
-----	-----		-----	-----		-----	-----			
11001	( 9) (-16)		01001	(-7)		01001	(-7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
+0100	+( 4)	ADD	+0100	+(+4)	ADD	-1100	-(-4)			
-----	-----		-----	-----		-----	-----			
01001	( 9) (+ 0)		01001	(-7)		01001	(-7)		0	1
1011	(11)		1011	(-5)		1011	(-5)			
-0100	-( 4)	SUB	+1100	+(-4)		-0100	-(+4)	SUB		
-----	-----		-----	-----		-----	-----			
00111	( 7) ( 0)		10111	(+7)		10111	(+7)		0	1

# Cases for setting the overflow flag (1) CF=1, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned addition		* signed addition	signed subtraction
1011 (11)		1000	
+1100 +(12) ADD		1011 (-5)	1011 (-5)
-----		+1100 +(-4) ADD	-0100 -(+4)
10111 ( 7) (+16)		-----	-----
		10111 (+7)	10111 (+7)
OF=1		n + n -> p (OF=1)	n - p -> p (OF=1)
OF meaningless		-> incorrect S	-> incorrect S
S = 0111		S = 0111	S = 0111
* think hand		* OF <- C4 XOR C3 = 1 XOR 0 = 1	
addition		of signed addition	

\* CF=1, S=0111, OF=1 for all three interpretations



# Cases for setting the overflow flag (2) CF=1, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned subtraction		signed addition		* signed subtraction
0101 (5)		0100		0101 (+5)
-1100 -(12) SUB		+0100 +(4)		-1100 -(-4) SUB
-----		-----		-----
11001 (9) (-16)		01001 (-7)		01001 (-7)
OF=1		p + p -> n (OF=1)		p - n -> n (OF=1)
OF meaningless		-> incorrect S		-> incorrect S
S = 1001		S = 1001		S = 1001
* think hand subtraction		* OF <- C4 XOR C3 = 0 XOR 1 = 1		
		of signed addition		

\* CF=1, S=1001, OF=1 for all three interpretations

# Cases for setting the overflow flag (3) CF=0, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned addition		* signed addition	signed subtraction
0101 ( 5)		0100	
+0100 +( 4) ADD		0101 (+5)	0101 (+5)
-----		+0100 +(4) ADD	-1100 -(-4)
01001 ( 9) (+ 0)		-----	-----
		01001 (-7)	01001 (-7)
OF=1		p + p -> n (OF=1)	p - n -> n (OF=1)
OF meaningless		-> incorrect S	-> incorrect S
S = 1001		S = 1001	S = 1001
* think hand		* OF <- C4 XOR C3 = 0 XOR 1 = 1	
addition		of signed addition	

\* CF=0, S=1001, OF=1 for all three interpretations

# Cases for setting the overflow flag (4) CF=0, OF=1

- signed integer overflow (OF=1 means incorrect S)

* unsigned subtraction		signed addition		* signed subtraction
1011 (11)		1000		1011 (-5)
-0100 -(4) SUB		+1100 +(-4)		-0100 -(+4) SUB
-----		-----		-----
00111 (7) (0)		10111 (+7)		10111 (+7)
OF=1		n + n -> p (OF=1)		n - p -> p (OF=1)
OF meaningless		-> incorrect S		-> incorrect S
S = 0111		S = 0111		S = 0111
* think hand subtraction		* OF <- C4 XOR C3 = 1 XOR 0 = 1		of signed addition

\* CF=0, S=0111, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (1) CF=1, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1100		
+1110 +(14) ADD		1101 (-3)		1101 (-3)
-----		+1110 +(-2) ADD		-0010 -(+2)
-----		-----		-----
11011 (11) (+16)		11011 (-5)		11011 (-5)
OF=0		n + n -> n (OF=0)		n - p -> n (OF=0)
OF meaningless		-> correct S		-> correct S
S = 0000		S = 0000		S = 0000
* think hand		* OF <- C4 XOR C3 = 1 XOR 1 = 0		
addition		of signed addition		

\* CF=1, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the overflow flag (2) CF=1, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned subtraction		signed addition		* signed subtraction
0011 (3)		0010		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		p + p -> p (OF=0)		p - n -> p (OF=0)
OF meaningless		-> correct S		-> correct S
S = 0101		S = 0101		S = 0101
* think hand subtraction		* OF <- C4 XOR C3 = 0 XOR 0 = 0		of signed addition

\* CF=1, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (3) CF=0, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned addition		* signed addition		signed subtraction
0011 (3)		0010		
+0010 +(2) ADD		0011 (+3)		0011 (+3)
-----		+0010 +(2) ADD		-1110 -(-2)
00101 (5) (+0)		-----		-----
		00101 (+5)		00101 (+5)
OF=0		p + p -> p (OF=0)		p - n -> p (OF=0)
OF meaningless		-> correct S		-> correct S
S = 0101		S = 0101		S = 0101
* think hand		* OF <- C4 XOR C3 = 0 XOR 0 = 0		
addition		of signed addition		

\* CF=0, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (4) CF=0, OF=0

- no signed integer overflow (CF=0 means correct S)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1100		
-0010 -(2) SUB		1101 (-3)		1101 (-3)
-----		+1110 +(-2)		-0010 -(+2) SUB
-----		-----		-----
11011 (11) (-16)		11011 (-5)		11011 (-5)
OF=0		n + n -> n (OF=0)		n - p -> n (OF=0)
OF meaningless		-> correct S		-> correct S
S = 1011		S = 1011		S = 1011
* think hand		* OF <- C4 XOR C3 = 1 XOR 1 = 0		
subtraction		of signed addition		

\* CF=0, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# 4-bit addition table (1)

	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)
0000 (0)	0000 (0)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)
0001 (+1)	0001 (+1)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8)
0010 (+2)	0010 (+2)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8)	1001 (-7)
0011 (+3)	0011 (+3)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8)	1001 (-7)	1010 (-6)
0100 (+4)	0100 (+4)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8)	1001 (-7)	1010 (-6)	1011 (-5)
0101 (+5)	0101 (+5)	0110 (+6)	0111 (+7)	1000 (-8)	1001 (-7)	1010 (-6)	1011 (-5)	1100 (-4)
0110 (+6)	0110 (+6)	0111 (+7)	1000 (-8)	1001 (-7)	1010 (-6)	1011 (-5)	1100 (-4)	1101 (-3)
0111 (+7)	0111 (+7)	1000 (-8)	1001 (-7)	1010 (-6)	1011 (-5)	1100 (-4)	1101 (-3)	1110 (-2)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# 4-bit addition table (2)

111 0111 1001 0000

	0000	0001	0010	0011	0100	0101	0110	0111	
	(0)	(+1)	(+2)	(+3)	(+4)	(+5)	(+6)	(+7)	
1000	1000	1001	1010	1011	1100	1101	1110	1111	
	(-8)	(-8)	(-7)	(-6)	(-5)	(-4)	(-3)	(-2)	(-1)
1001	1001	1010	1011	1100	1101	1110	1111	0000	
	(-7)	(-7)	(-6)	(-5)	(-4)	(-3)	(-2)	(-1)	(0)
1010	1010	1011	1100	1101	1110	1111	0000	0001	
	(-6)	(-6)	(-5)	(-4)	(-3)	(-2)	(-1)	(0)	(+1)
1011	1011	1100	1101	1110	1111	0000	0001	0010	
	(-5)	(-5)	(-4)	(-3)	(-2)	(-1)	(0)	(+1)	(+2)
1100	1100	1101	1110	1111	0000	0001	0010	0011	
	(-4)	(-4)	(-3)	(-2)	(-1)	(0)	(+1)	(+2)	(+3)
1101	1101	1110	1111	0000	0001	0010	0011	0100	
	(-3)	(-3)	(-2)	(-1)	(0)	(+1)	(+2)	(+3)	(+4)
1110	1110	1111	0000	0001	0010	0011	0100	0101	
	(-2)	(-2)	(-1)	(0)	(+1)	(+2)	(+3)	(+4)	(+5)
1111	1111	0000	0001	0010	0011	0100	0101	0110	
	(-1)	(-1)	(0)	(+1)	(+2)	(+3)	(+4)	(+5)	(+6)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## 4-bit addition table (3)

	1000	1001	1010	1011	1100	1101	1110	1111
	(-8)	(-7)	(-6)	(-5)	(-4)	(-3)	(-2)	(-1)
0000								
(0)								
0001								
(+1)								
0010								
(+2)								
0011								
(+3)								
0100								
(+4)								
0101								
(+5)								
0110								
(+6)								
0111								
(+7)								

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# 4-bit addition table (4)

	1000	1001	1010	1011	1100	1101	1110	1111
	(-8)	(-7)	(-6)	(-5)	(-4)	(-3)	(-2)	(-1)
1000								
(-8)								
1001								
(-7)								
1010								
(-6)								
1011								
(-5)								
1100								
(-4)								
1101								
(-3)								
1110								
(-2)								
1111								
(-1)								

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