Trigonometry (3A)

- Quadrant Angle Trigonometry
- Negative Angle Trigonometry
- Reference Angle Trigonometry
- Sinusoidal Waves

Young Won Lim
1/2/15

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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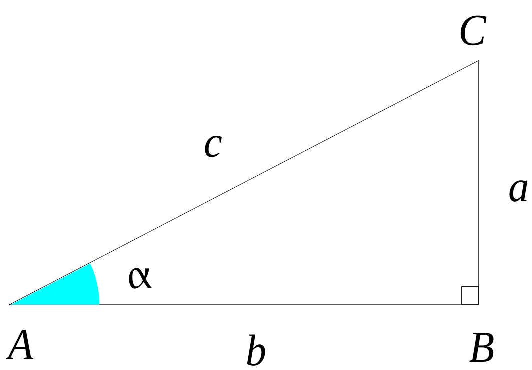
Triangle Trigonometry

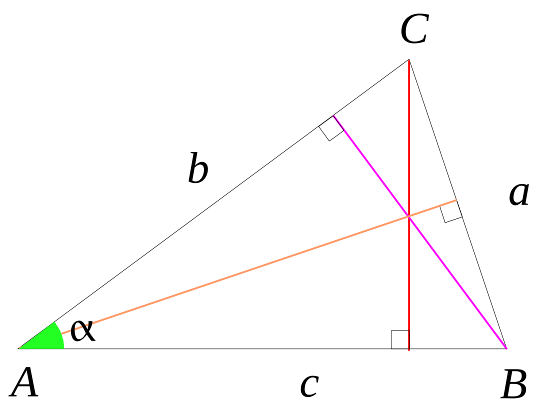
Right Triangle

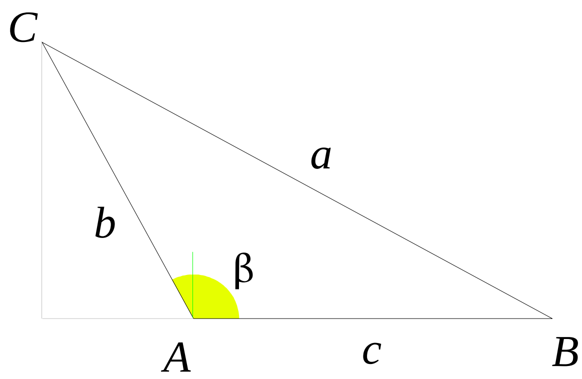
Oblique Triangle

All Acute Angles

One Obtuse Angle







$0^\circ < \alpha < 90^\circ$

$0^\circ < \alpha < 90^\circ$

$90^\circ < \beta < 180^\circ$

$\sin\alpha = a / c$

$\cos\alpha = b / c$

$\tan\alpha = a / b$

$\sin\alpha = ?$

$\cos\alpha = ?$

$\tan\alpha = ?$

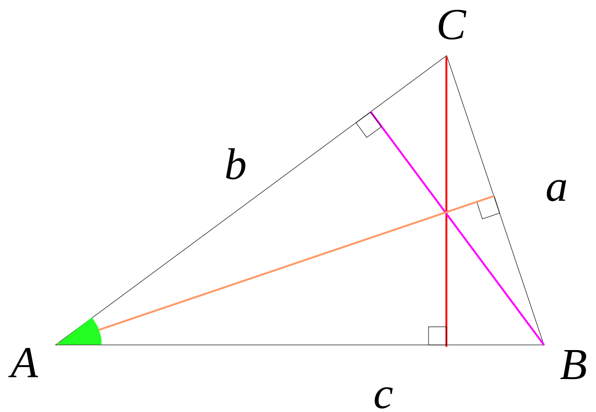
$\sin\beta = ?$

$\cos\beta = ?$

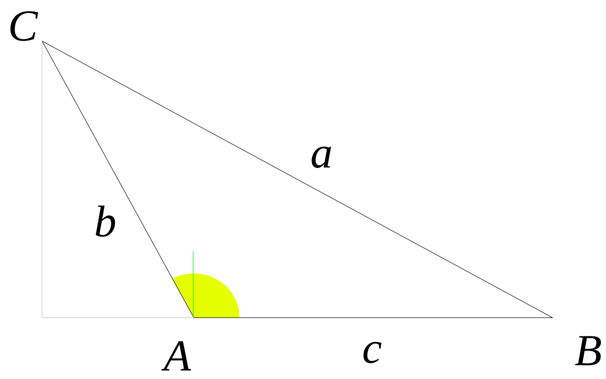
$\tan\beta = ?$

Oblique Triangles Trigonometry

All Acute Angles



One Obtuse Angle



$$\sin\beta = \sin(180^\circ - \alpha) = +\sin\alpha$$

$$\cos\beta = \cos(180^\circ - \alpha) = -\cos\alpha$$

$$\tan\beta = \tan(180^\circ - \alpha) = -\tan\alpha$$

$$0^\circ < \alpha < 90^\circ, \qquad 0^\circ < \beta < 180^\circ$$

The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

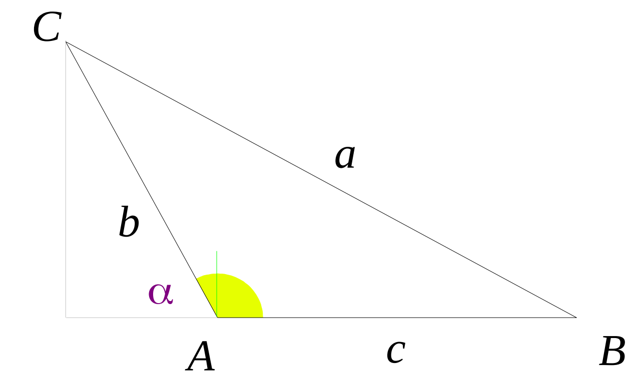
The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

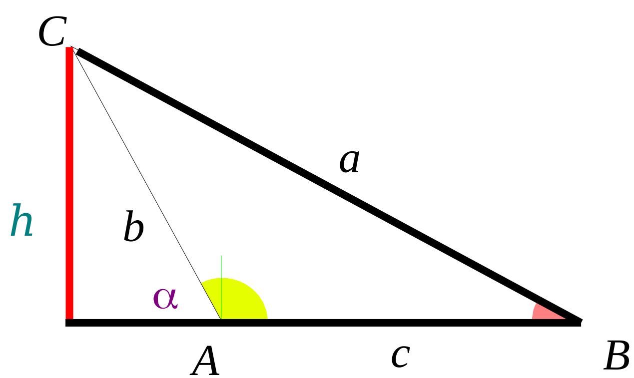
$$c^2 = a^2 + b^2 - 2ab\cos C$$

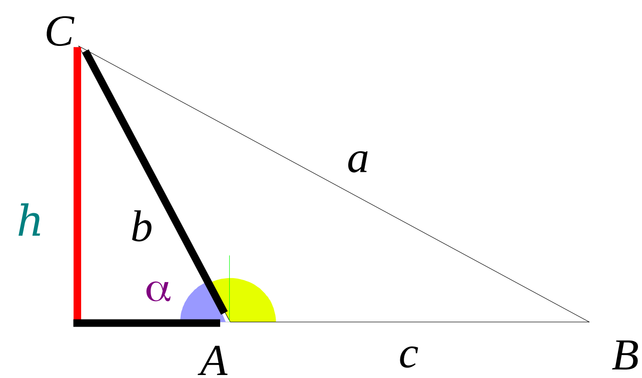
One Obtuse Angle (1)



The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



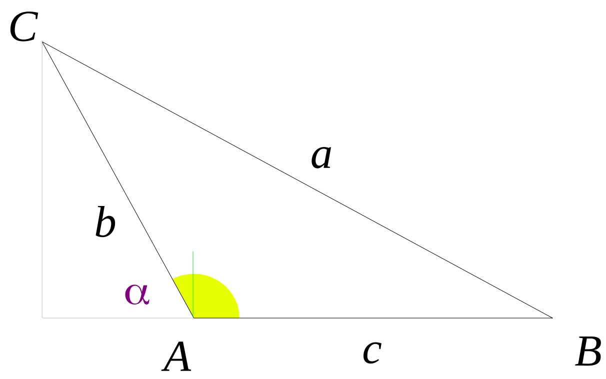


$$\sin A = \frac{a}{b}\sin B = \frac{a}{b}\frac{h}{a} = \frac{h}{b}$$

$$\sin\alpha = \frac{h}{b}$$

$$\sin A = \sin\alpha$$

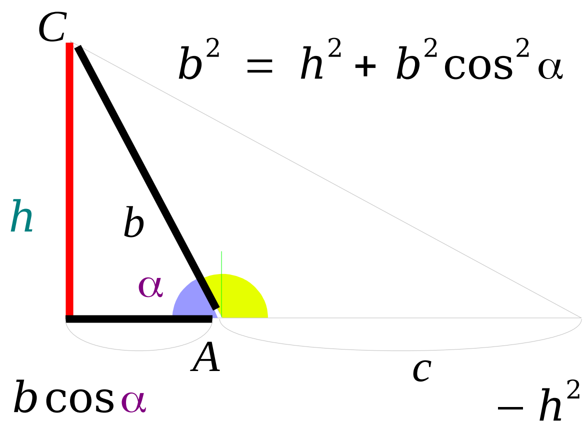
One Obtuse Angle (2)



The Law of Cosines

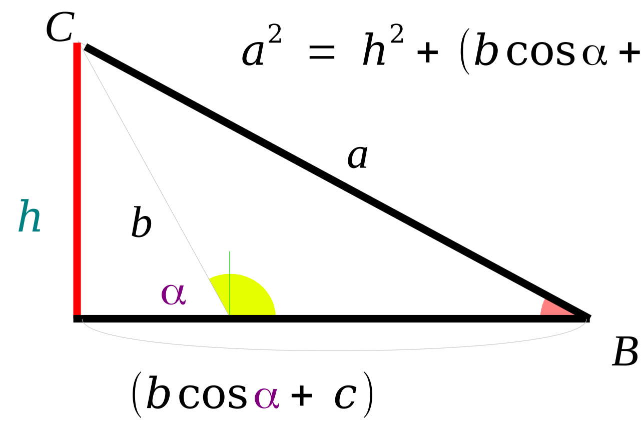
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = (b^2 + c^2 - a^2) / 2bc$$



$$b^2 = h^2 + b^2\cos^2\alpha$$

$$- h^2 - (b\cos\alpha + c)^2$$



$$a^2 = h^2 + (b\cos\alpha + c)^2$$

$$(b^2 + c^2 - a^2) = h^2 + b^2\cos^2\alpha + c^2 - h^2 - (b\cos\alpha + c)^2$$

$$\cos A = -2bc\cos\alpha / 2bc$$

$$\cos A = -\cos\alpha$$

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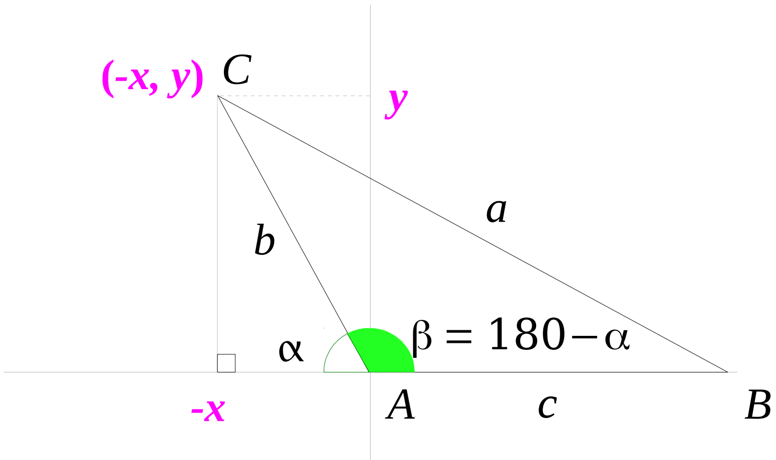
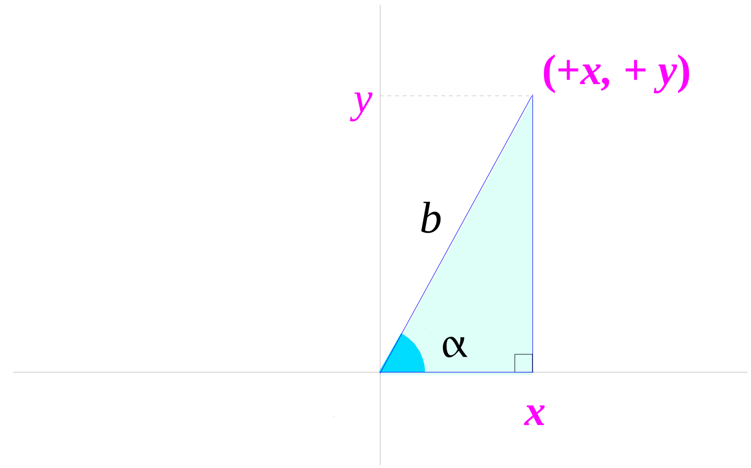
Trigonometry in the 2^{nd} Quadrant Angles (1)



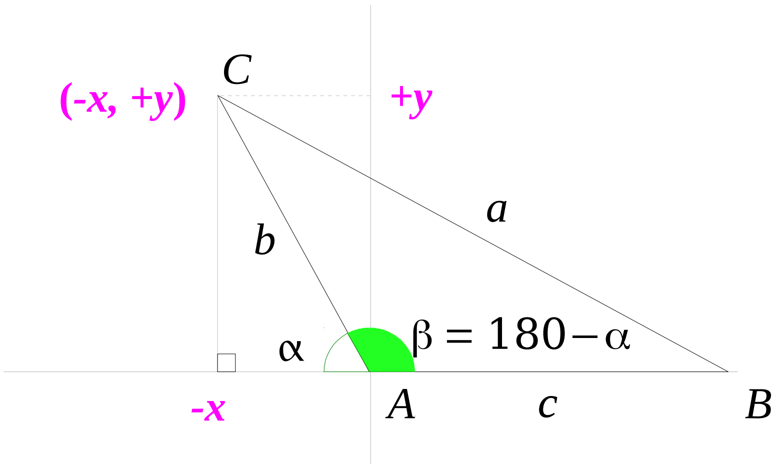


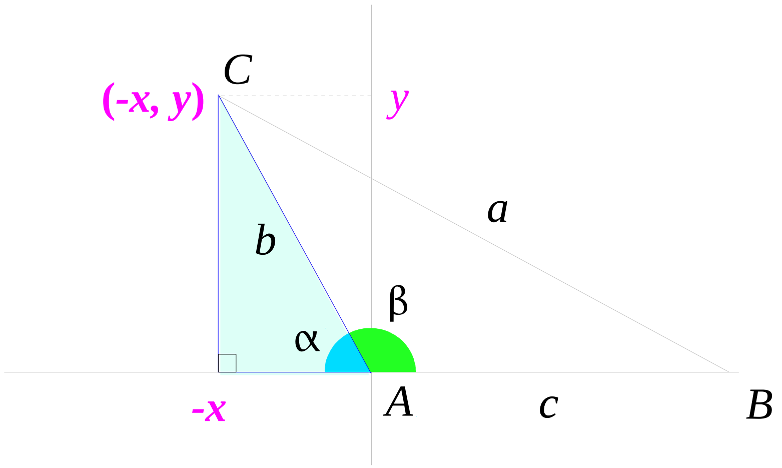
$$\sin\beta = \sin(180^\circ - \alpha) = +\sin\alpha \longleftarrow +(\sin\alpha = y/b)$$

$$\cos\beta = \cos(180^\circ - \alpha) = -\cos\alpha \longleftarrow -(\cos\alpha = x/b)$$

$$\tan\beta = \tan(180^\circ - \alpha) = -\tan\alpha \longleftarrow -(\tan\alpha = y/x)$$

Trigonometry in the 2nd Quadrant Angles (2)





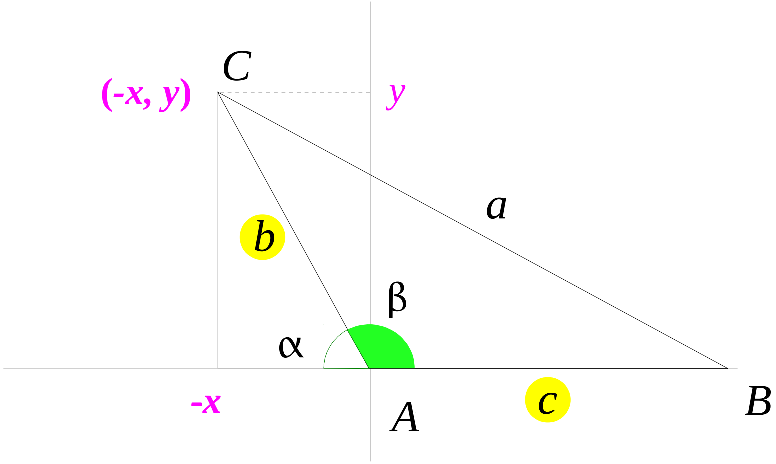
$$\sin\beta = \sin(180^\circ - \alpha) = +\sin\alpha \quad \leftarrow \quad \sin\beta = (+y)/b$$

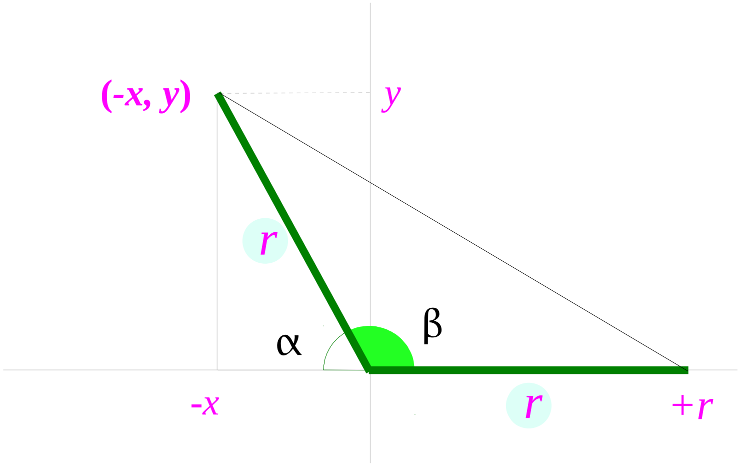
$$\cos\beta = \cos(180^\circ - \alpha) = -\cos\alpha \quad \leftarrow \quad \cos\beta = (-x)/b$$

$$\tan\beta = \tan(180^\circ - \alpha) = -\tan\alpha \quad \leftarrow \quad \tan\beta = (+y)/(-x)$$

Trigonometry in the 2nd Quadrant Angles (3)

Isosceles Triangle





$$r = \sqrt{x^2 + y^2}$$

$\sin\beta = +y / b$

$\cos\beta = -x / b$

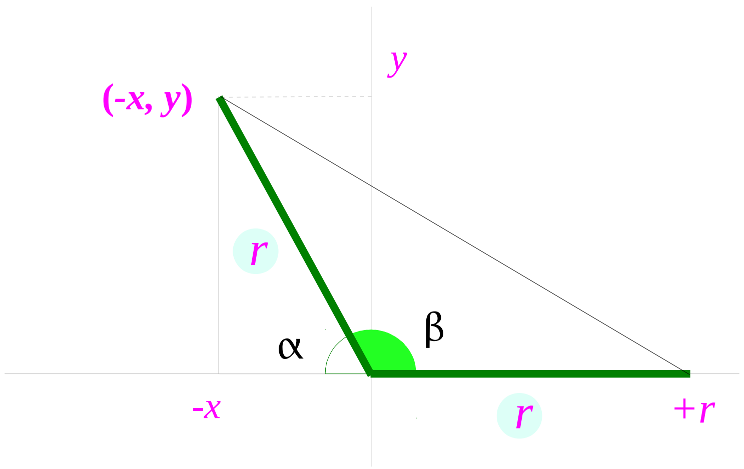
$\tan\beta = -y/x$

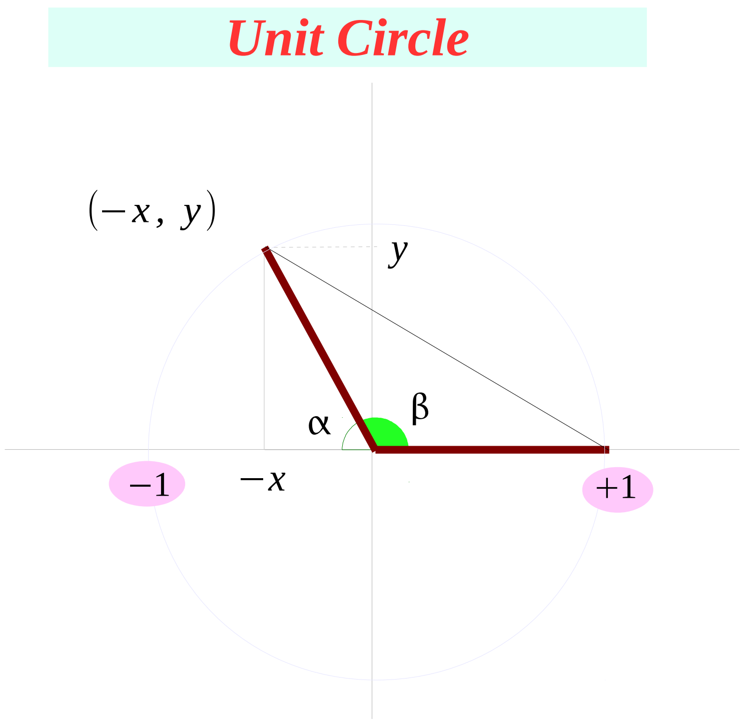
$\sin\beta = (+y) / r$

$\cos\beta = (-x) / r$

$\tan\beta = (+y) / (-x)$

Trigonometry in Quadrant Angles (4)





$$r = \sqrt{x^2 + y^2}$$

$$\sin\beta = +y \,/\, r$$

$$\cos\beta = -x \,/\, r$$

$$\tan\beta = -y \,/\, x$$

$$1 = \sqrt{x^2 + y^2}$$

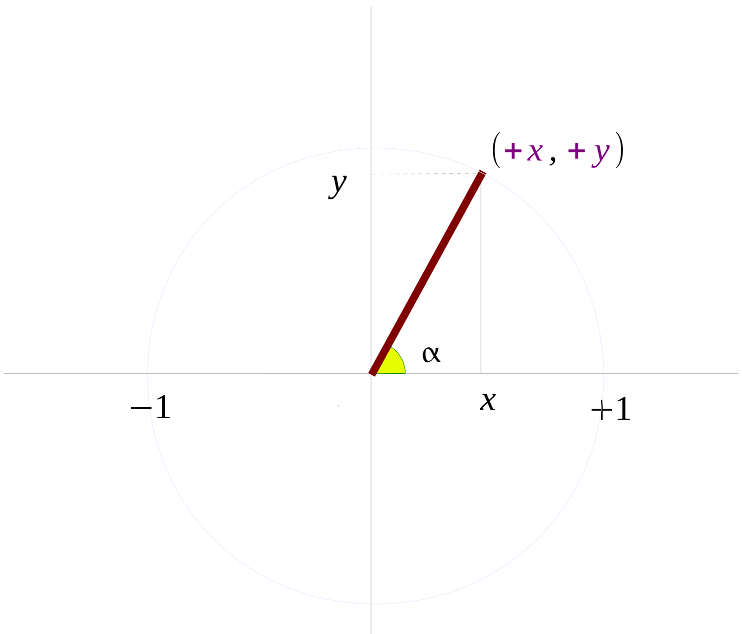
$$\sin\beta = +\sin\alpha = (+y)$$

$$\cos\beta = -\sin\beta = (-x)$$

$$\tan\beta = -\tan\alpha = (+y)/(-x)$$

Negative Angle Trigonometry (1)

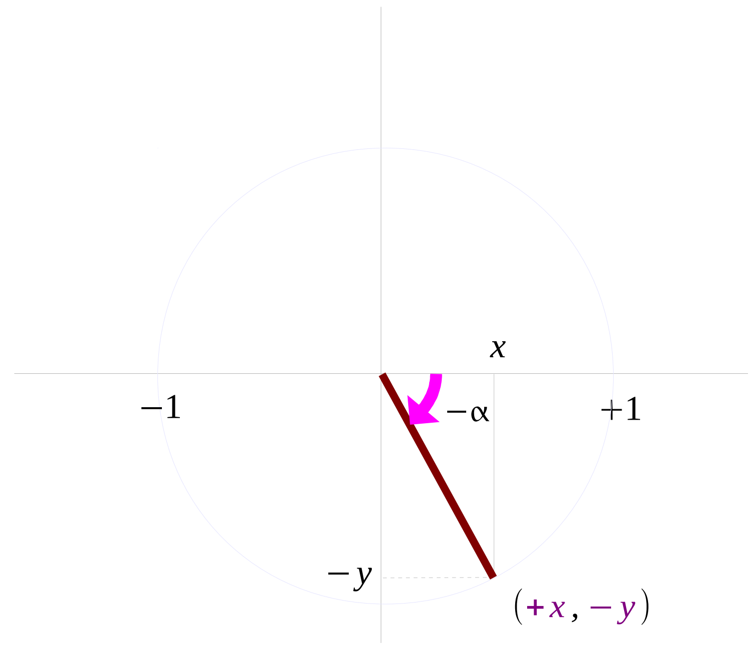
1st Quadrant Angle



$0^\circ < \alpha < 90^\circ$

$$\sin\alpha = (+y)$$
$$\cos\alpha = (+x)$$
$$\tan\alpha = (+y)/(+x)$$

4th Quadrant Angle

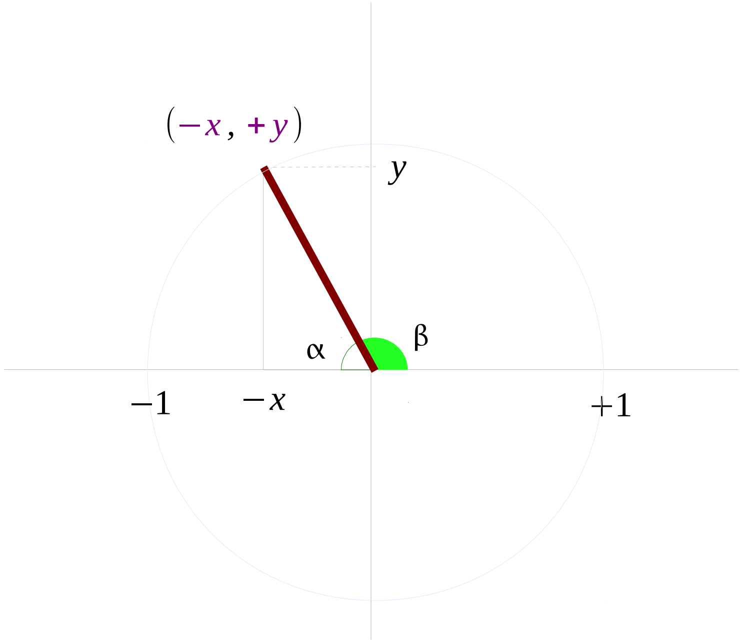


$-90^\circ < -\alpha < 0^\circ$

$$\sin(-\alpha) = -\sin\alpha = (-y)$$
$$\cos(-\alpha) = +\cos\alpha = (+x)$$
$$\tan(-\alpha) = -\tan\alpha = (-y)/(+x)$$

Negative Angle Trigonometry (2)

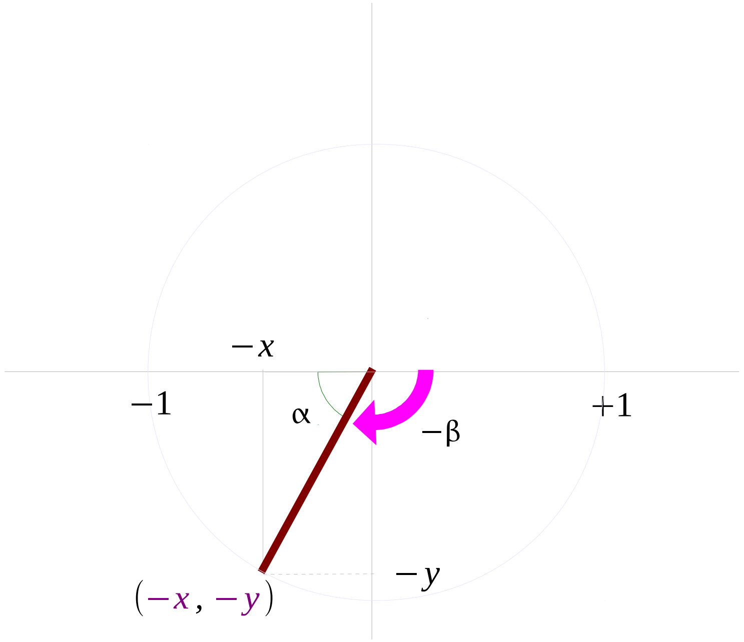
2nd Quadrant Angle



$90^\circ < \beta < 180^\circ$

$$\sin\beta = +\sin\alpha = (+y)$$
$$\cos\beta = -\cos\alpha = (-x)$$
$$\tan\beta = -\tan\alpha = (+y)/(-x)$$

3rd Quadrant Angle

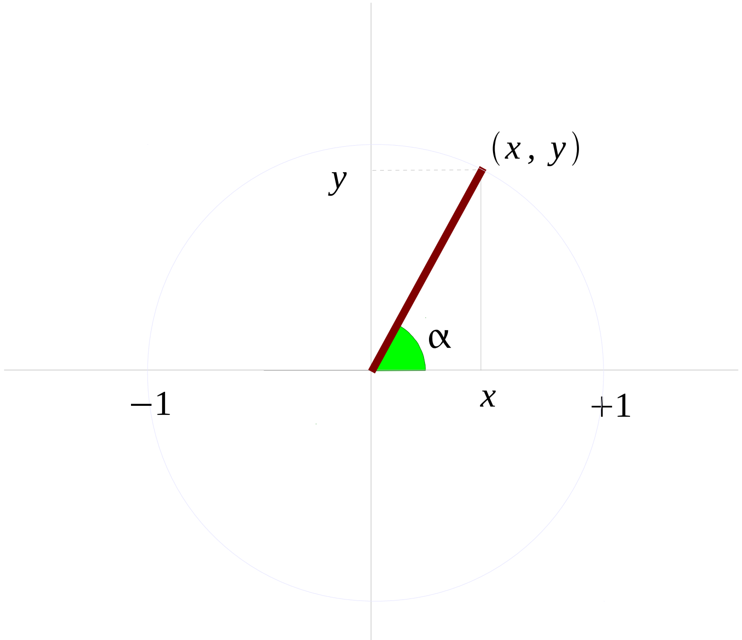


$-180^\circ < -\beta < -90^\circ$

$$\sin(-\beta) = -\sin\alpha = (-y)$$
$$\cos(-\beta) = -\cos\alpha = (-x)$$
$$\tan(-\beta) = +\tan\alpha = (-y)/(-x)$$

Reference Angle (1)

1st Quadrant Angle θ



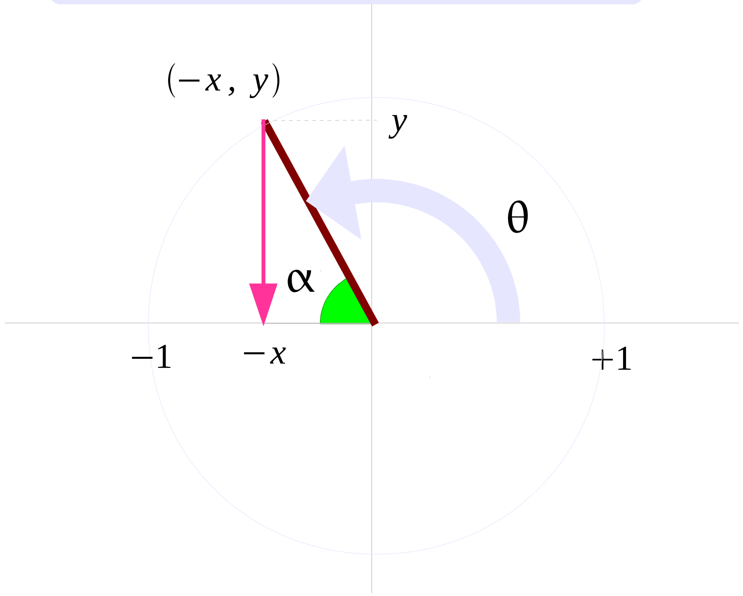
$\sin\alpha = y$

$\cos\alpha = x$

$\tan\alpha = y/x$

2nd Quadrant Angle θ

$90^\circ < \theta < 180^\circ$



Reference Angle α

$\alpha = 180^\circ - \theta$

$\sin\theta = +\sin\alpha = (+y)$

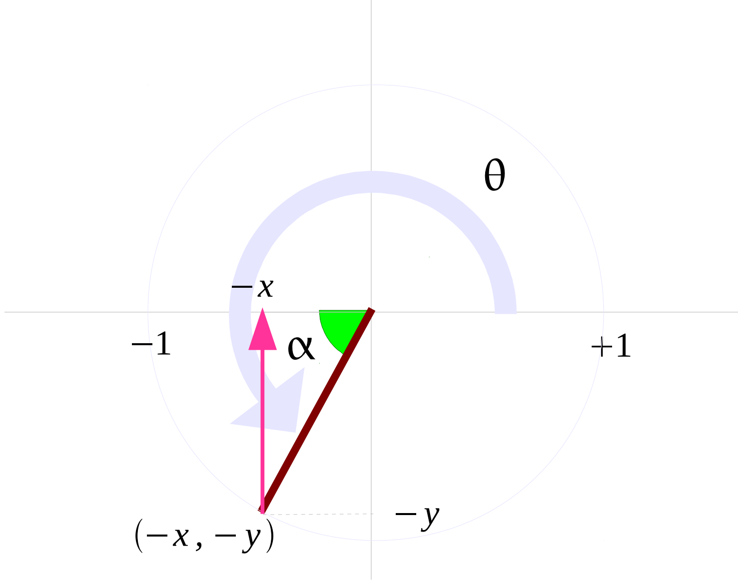
$\cos\theta = -\cos\alpha = (-x)$

$\tan\theta = -\tan\alpha = (+y)/(-x)$

Reference Angle (2)

3rd Quadrant Angle θ

$180^\circ < \theta < 270^\circ$



Reference Angle α

$\alpha = \theta - 180^\circ$

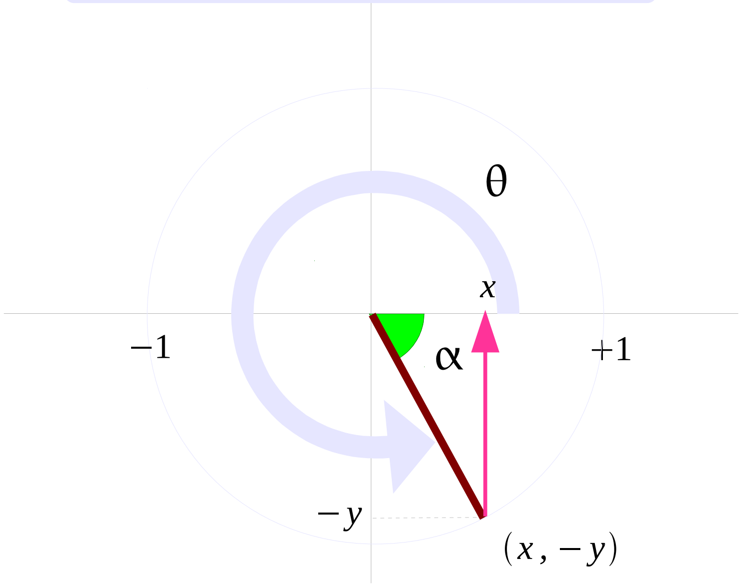
$\sin\theta = -\sin\alpha = (-y)$

$\cos\theta = -\cos\alpha = (-x)$

$\tan\theta = +\tan\alpha = (-y)/(-x)$

4th Quadrant Angle θ

$270^\circ < \theta < 360^\circ$



Reference Angle α

$\alpha = 360^\circ - \theta$

$\sin\theta = -\sin\alpha = (-y)$

$\cos\theta = +\cos\alpha = (+x)$

$\tan\theta = -\tan\alpha = (-y)/(+x)$

A Quadrant Angle θ

Reference Angle α

$\alpha = \pi - \theta$

$\sin\theta = +\sin\alpha$

$\cos\theta = -\cos\alpha$

$\tan\theta = -\tan\alpha$

$\alpha = \theta$

$\sin\theta = \sin\alpha$

$\cos\theta = \cos\alpha$

$\tan\theta = \tan\alpha$

$\sin\theta = -\sin\alpha$

$\cos\theta = -\cos\alpha$

$\tan\theta = +\tan\alpha$

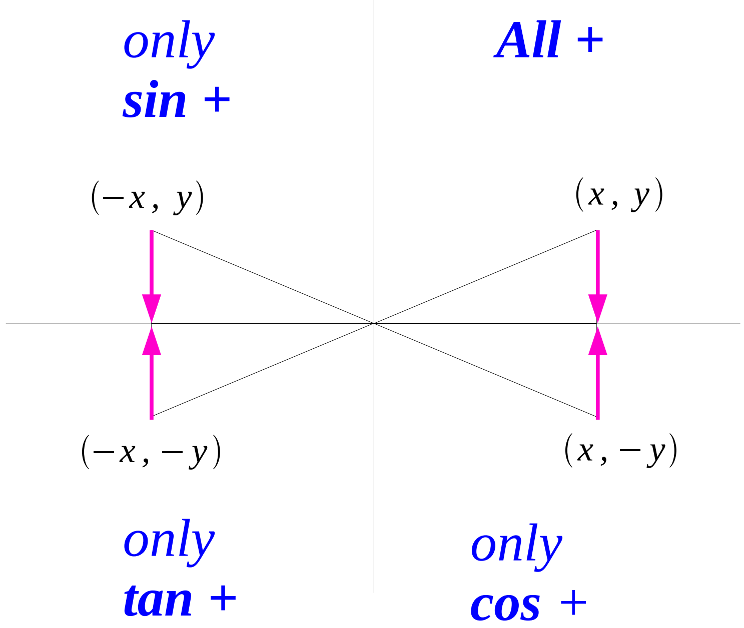
$\alpha = \theta - \pi$

$\sin\theta = -\sin\alpha$

$\cos\theta = +\cos\alpha$

$\tan\theta = -\tan\alpha$

$\alpha = 2\pi - \theta$



Trigonometric Functions

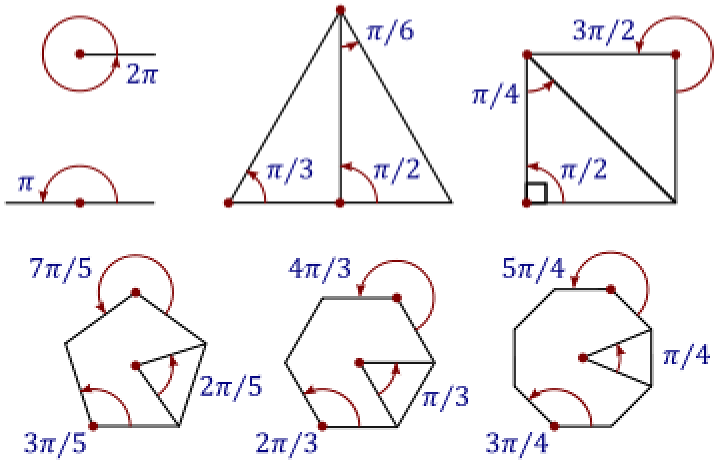
$$\tan\theta = \frac{\sin\theta}{\cos\theta}.$$

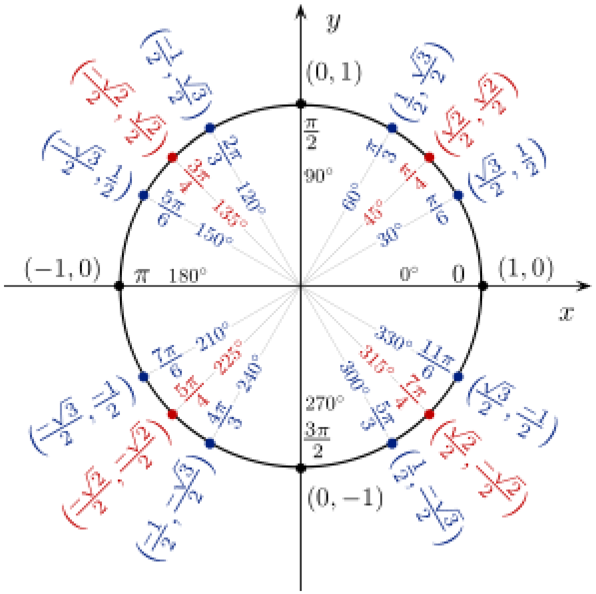
$$\sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}.$$

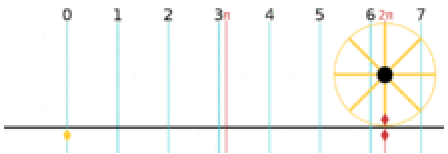
Function	sin	cos	tan	sec	csc	cot
Inverse	arcsin	arccos	arctan	arcsec	arccsc	arccot

http://en.wikipedia.org/wiki/Derivative

Radian







Degrees	30°	60°	120°	150°	210°	240°	300°	330°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$
Grads	33⅓ grad	66⅔ grad	133⅓ grad	166⅔ grad	233⅓ grad	266⅔ grad	333⅓ grad	366⅔ grad

Degrees	45°	90°	135°	180°	225°	270°	315°	360°
Radians	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
Grads	50 grad	100 grad	150 grad	200 grad	250 grad	300 grad	350 grad	400 grad

http://en.wikipedia.org/wiki/Derivative

Pythagorean identity

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta} \quad \text{and} \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}.$$

$$1 + \tan^2\theta = \sec^2\theta \quad \text{and} \quad 1 + \cot^2\theta = \csc^2\theta.$$

http://en.wikipedia.org/wiki/Derivative

Inverse Relations

Each trigonometric function in terms of the other five.[2]

in terms of	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\csc\theta}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\frac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

http://en.wikipedia.org/wiki/Derivative

Symmetry

Reflected in $\theta = 0$[4]	Reflected in $\theta = \pi/2$ (co-function identities)[5]	Reflected in $\theta = \pi$
$\sin(-\theta) = -\sin\theta$	$\sin(\frac{\pi}{2} - \theta) = +\cos\theta$	$\sin(\pi - \theta) = +\sin\theta$
$\cos(-\theta) = +\cos\theta$	$\cos(\frac{\pi}{2} - \theta) = +\sin\theta$	$\cos(\pi - \theta) = -\cos\theta$
$\tan(-\theta) = -\tan\theta$	$\tan(\frac{\pi}{2} - \theta) = +\cot\theta$	$\tan(\pi - \theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$	$\csc(\frac{\pi}{2} - \theta) = +\sec\theta$	$\csc(\pi - \theta) = +\csc\theta$
$\sec(-\theta) = +\sec\theta$	$\sec(\frac{\pi}{2} - \theta) = +\csc\theta$	$\sec(\pi - \theta) = -\sec\theta$
$\cot(-\theta) = -\cot\theta$	$\cot(\frac{\pi}{2} - \theta) = +\tan\theta$	$\cot(\pi - \theta) = -\cot\theta$

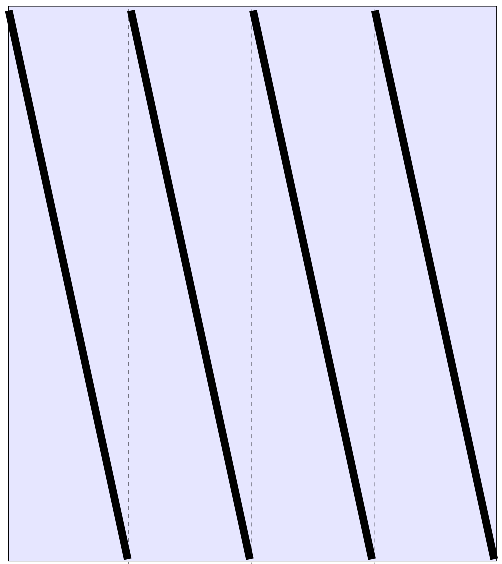
http://en.wikipedia.org/wiki/Derivative

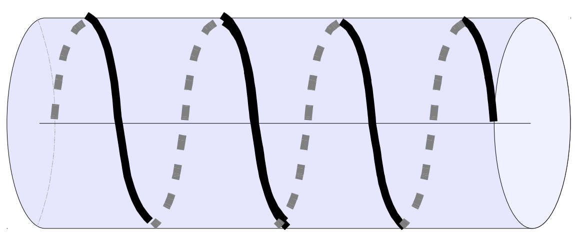
Shifts and periodicity

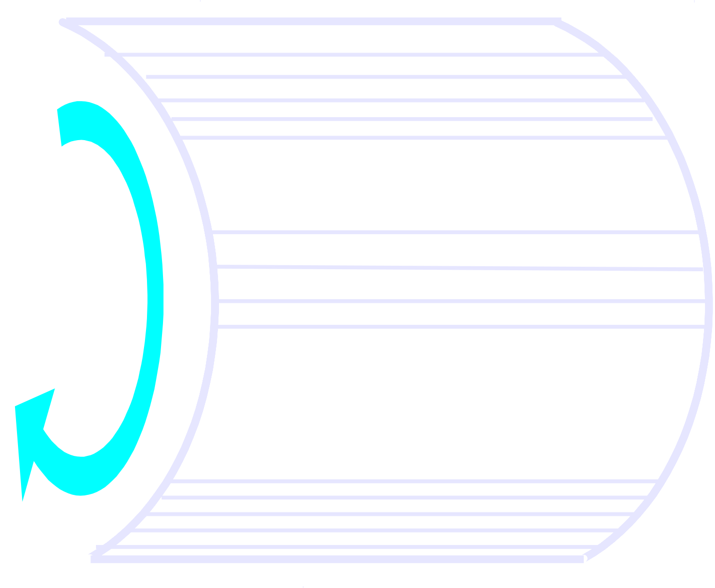
Shift by π/2	Shift by π Period for tan and cot[6]	Shift by 2π Period for sin, cos, csc and sec[7]
$\sin(\theta + \frac{\pi}{2}) = +\cos\theta$	$\sin(\theta + \pi) = -\sin\theta$	$\sin(\theta + 2\pi) = +\sin\theta$
$\cos(\theta + \frac{\pi}{2}) = -\sin\theta$	$\cos(\theta + \pi) = -\cos\theta$	$\cos(\theta + 2\pi) = +\cos\theta$
$\tan(\theta + \frac{\pi}{2}) = -\cot\theta$	$\tan(\theta + \pi) = +\tan\theta$	$\tan(\theta + 2\pi) = +\tan\theta$
$\csc(\theta + \frac{\pi}{2}) = +\sec\theta$	$\csc(\theta + \pi) = -\csc\theta$	$\csc(\theta + 2\pi) = +\csc\theta$
$\sec(\theta + \frac{\pi}{2}) = -\csc\theta$	$\sec(\theta + \pi) = -\sec\theta$	$\sec(\theta + 2\pi) = +\sec\theta$
$\cot(\theta + \frac{\pi}{2}) = -\tan\theta$	$\cot(\theta + \pi) = +\cot\theta$	$\cot(\theta + 2\pi) = +\cot\theta$

http://en.wikipedia.org/wiki/Derivative

Making a Helix

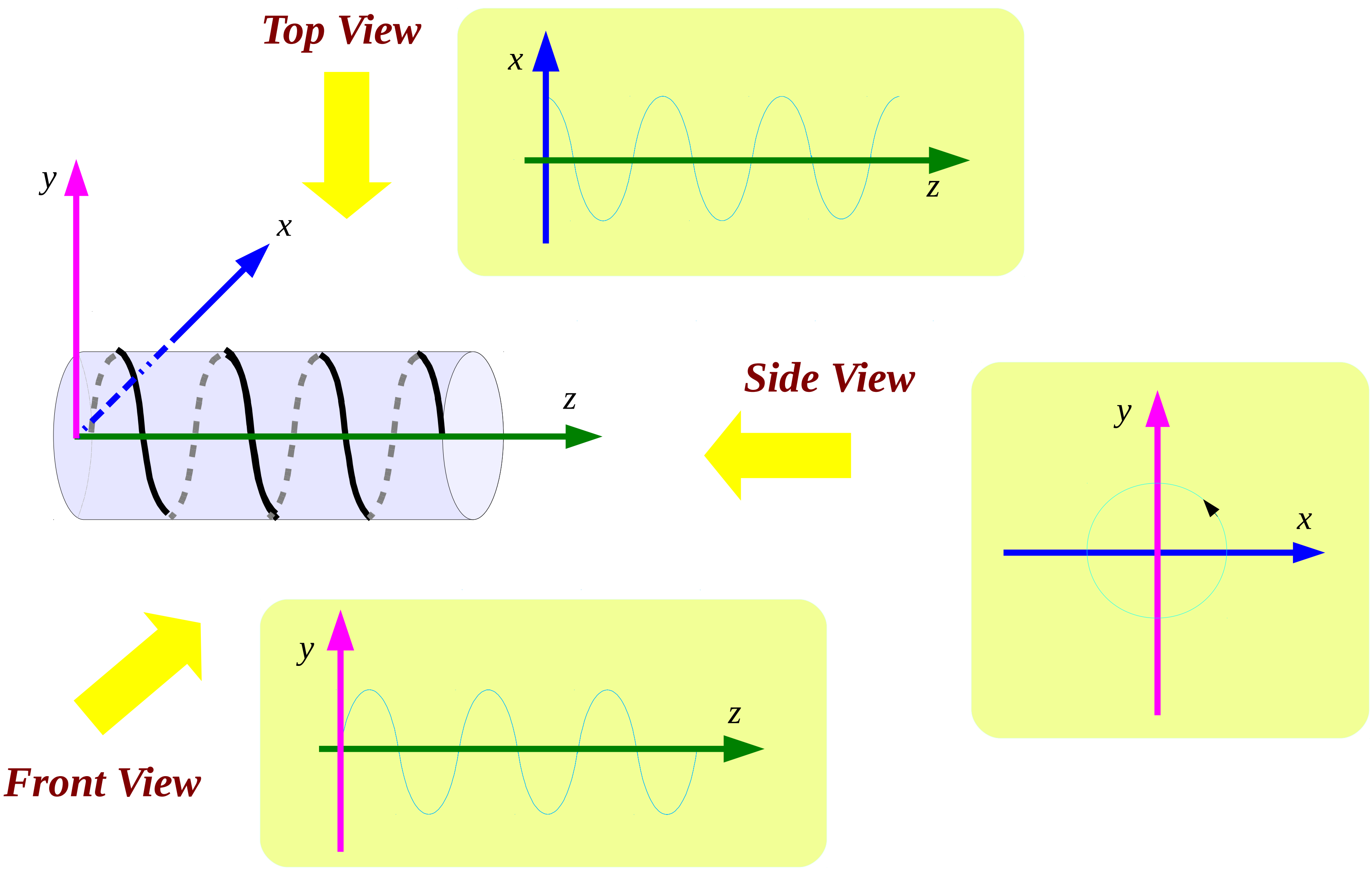




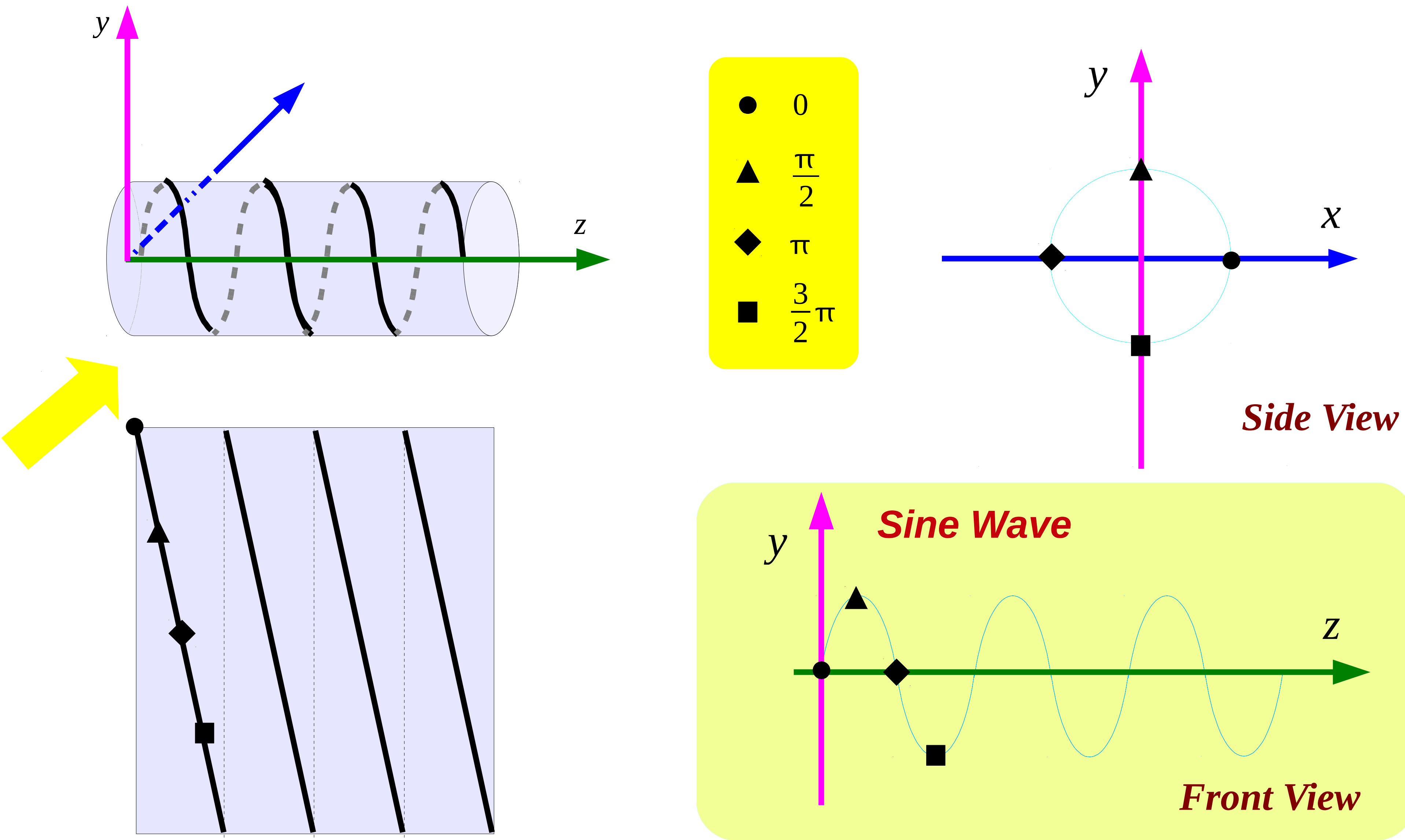


Transparent OHP Film

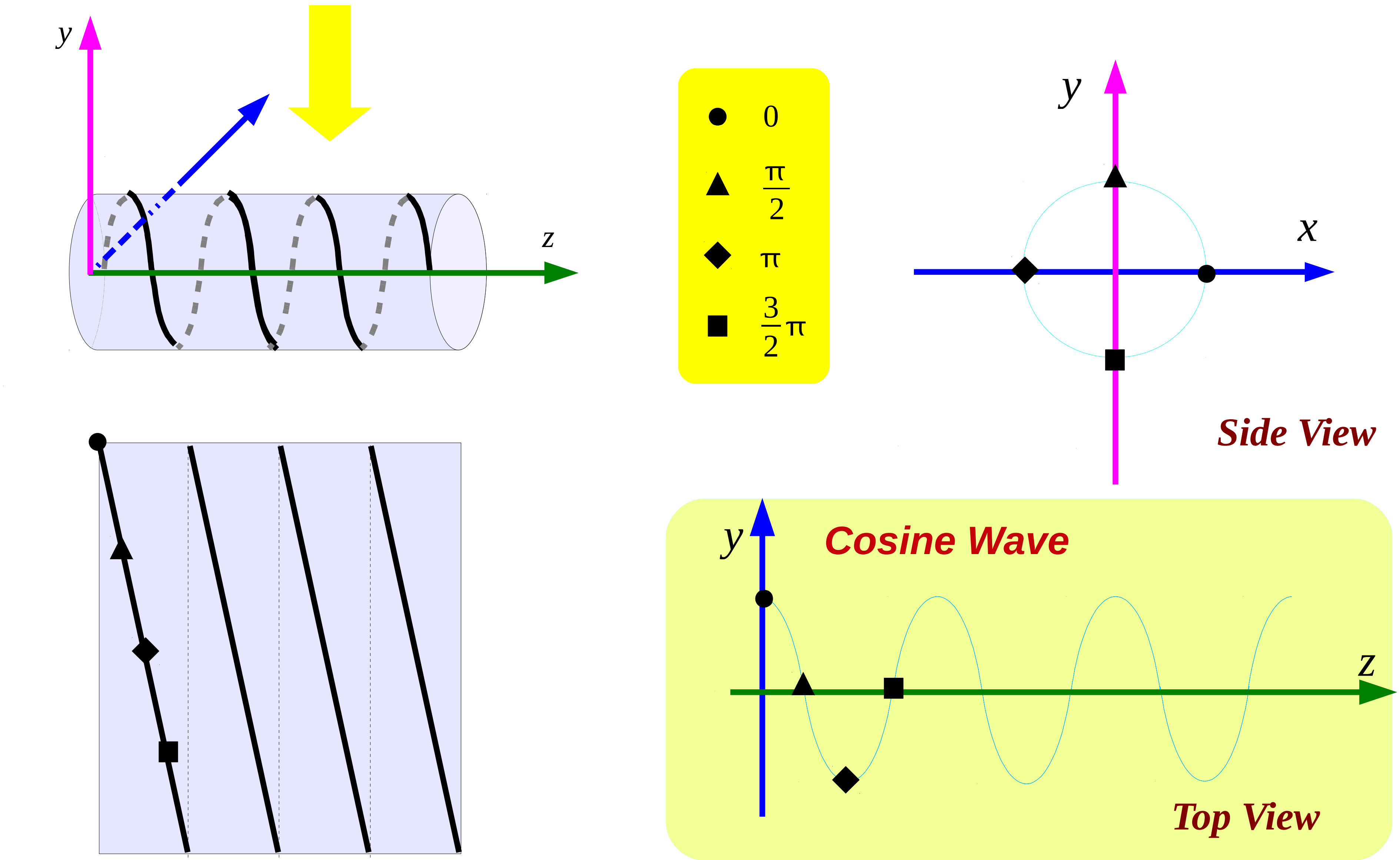
A Helix and Viewpoints



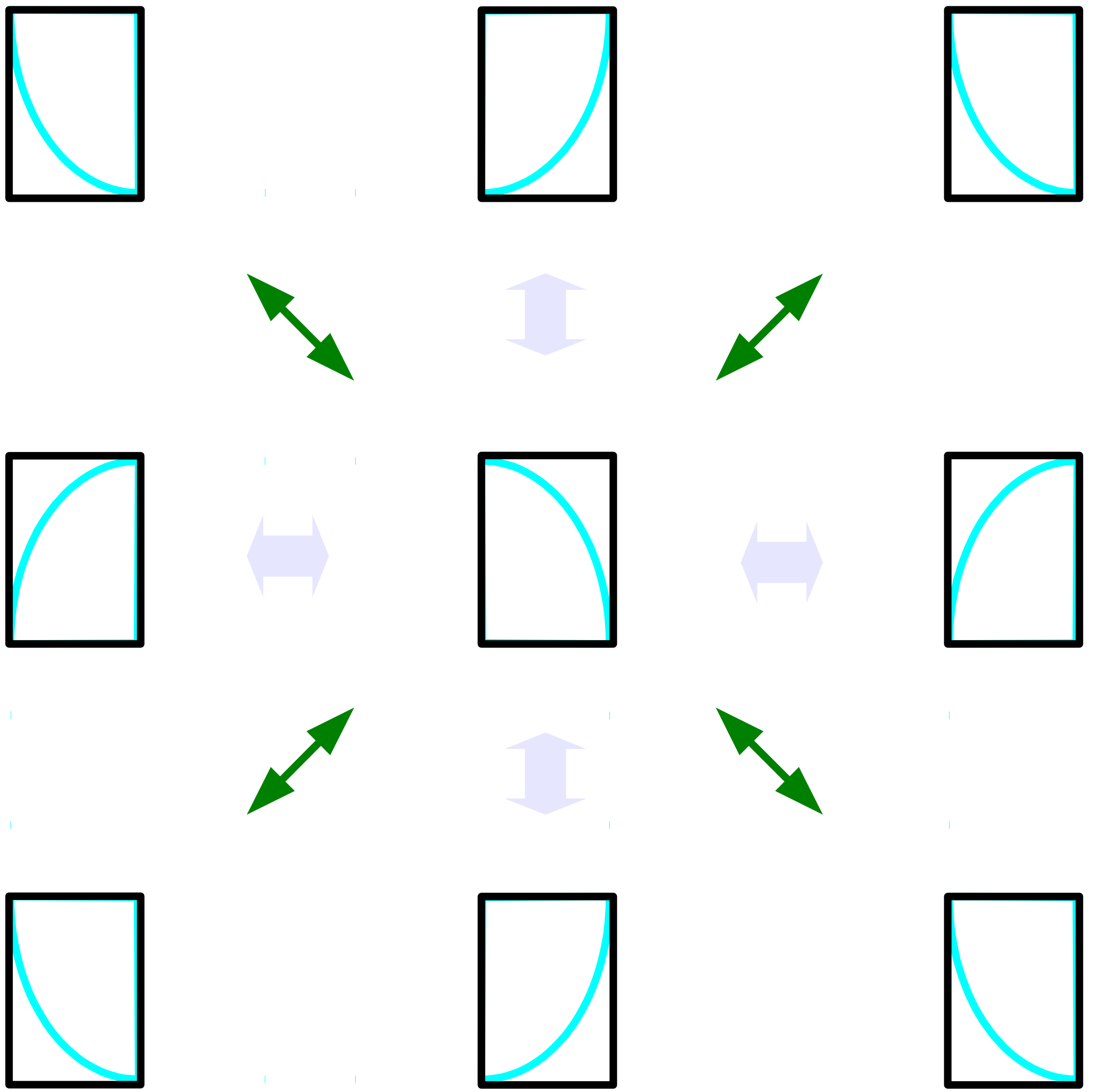
Sine Wave



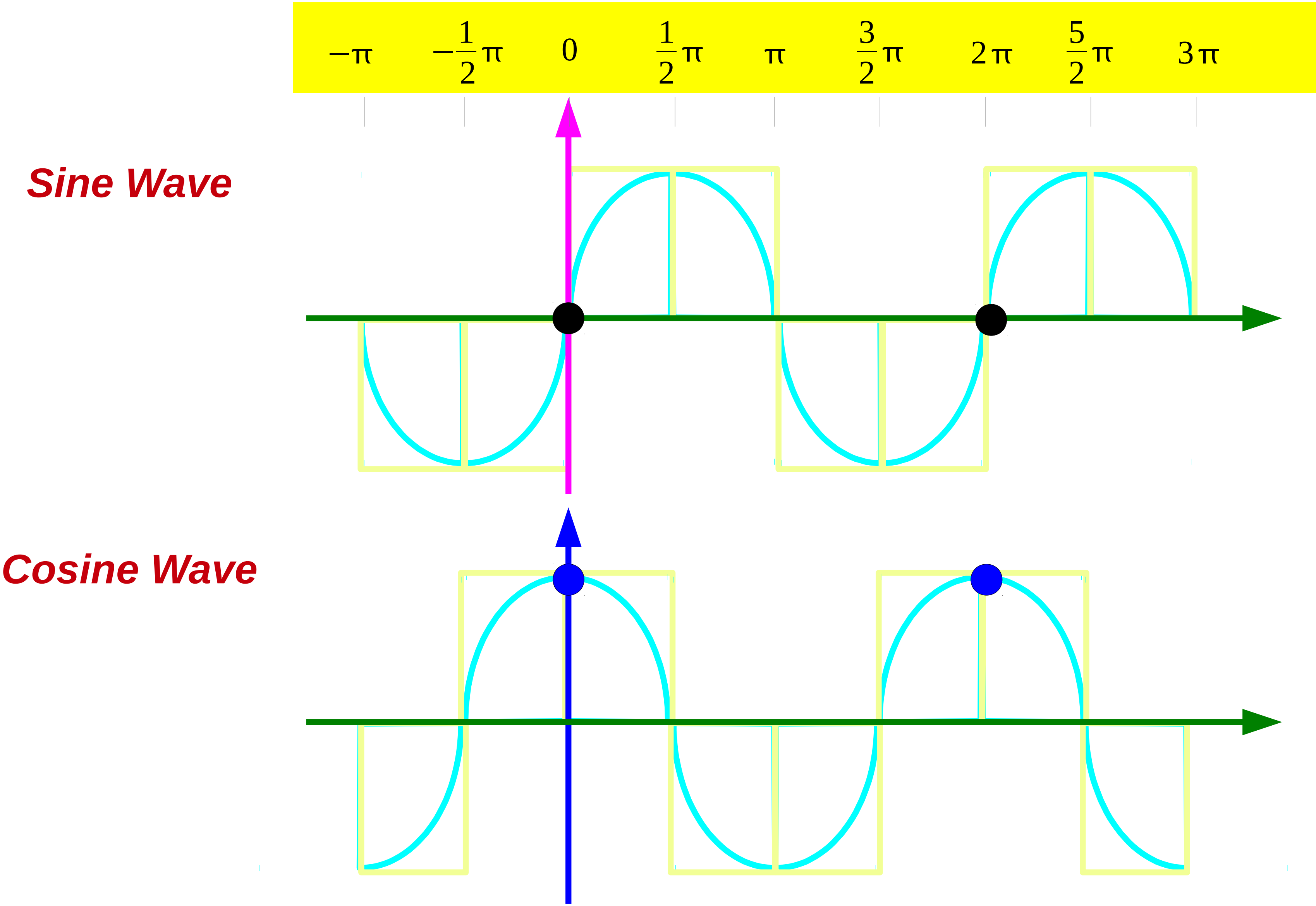
Cosine Wave



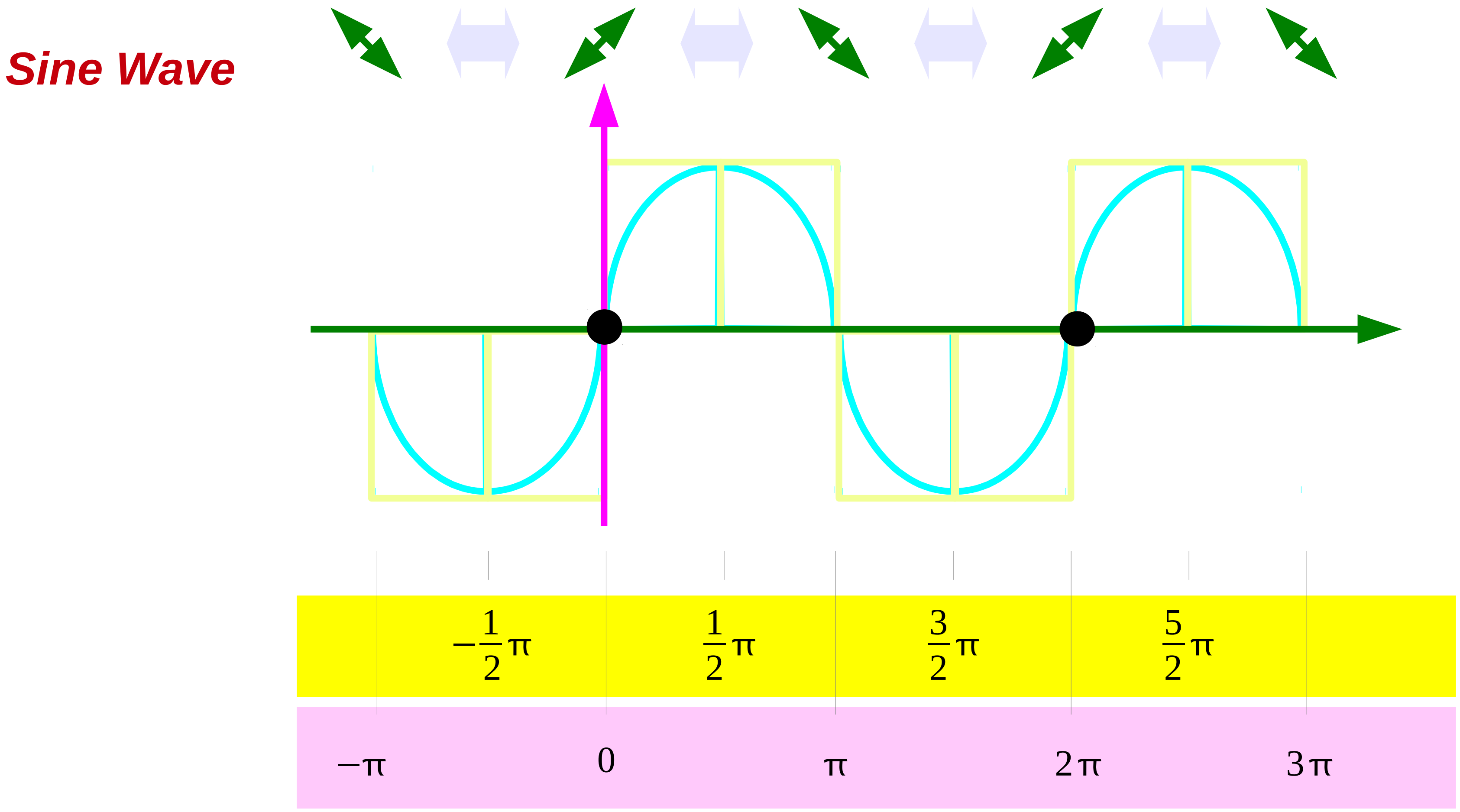
Symmetry in Sinusoid



Sine and Cosine Waves

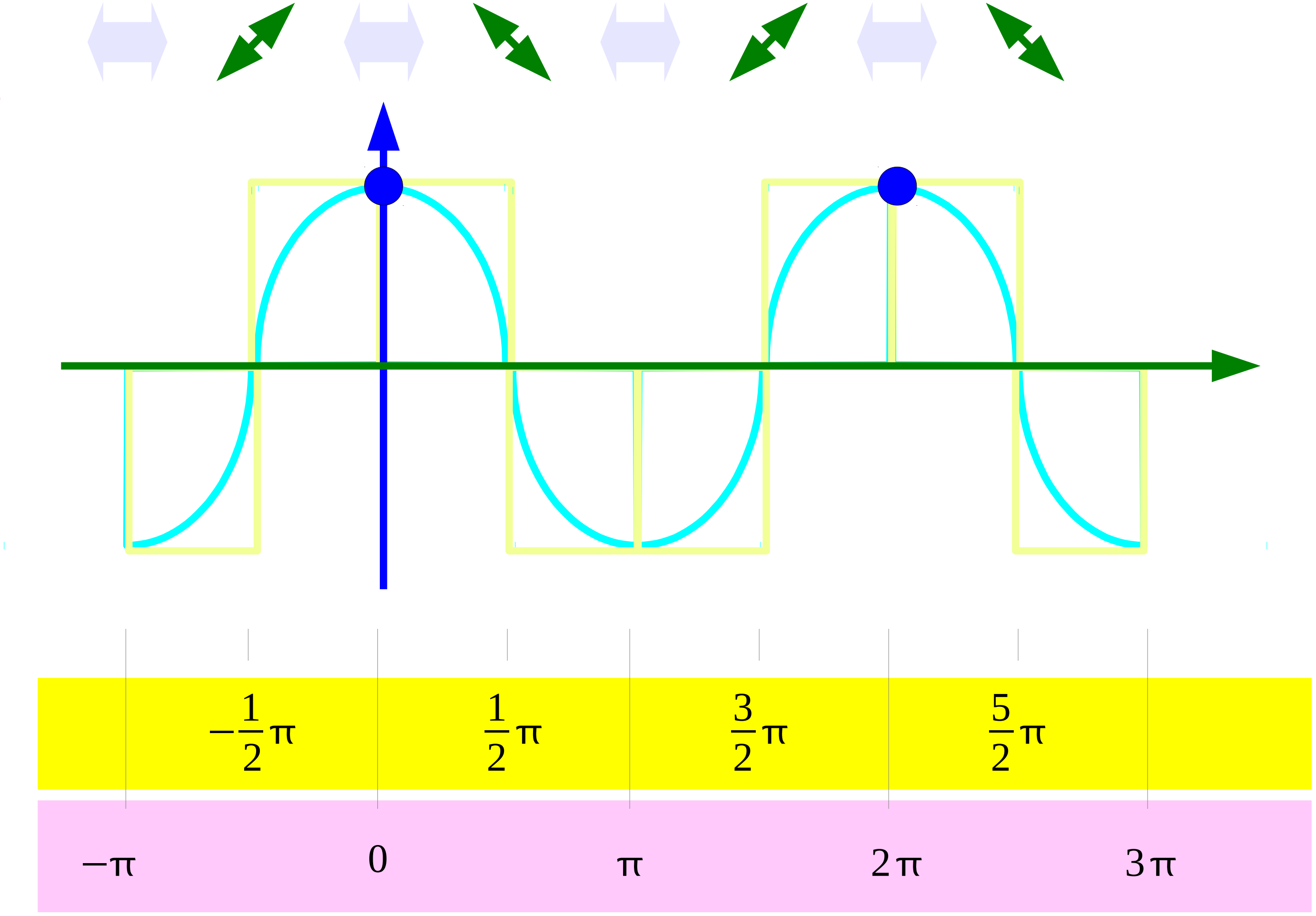


Sine Wave Symmetry



Cosine Wave Symmetry

Cosine Wave



References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] Blitzer, R. “Algebra & Trigonometry.” 3rd ed, Prentice Hall
[4] Smith, R. T., Minton, R. B. “Calculus: Concepts & Connections,” Mc Graw Hill
[5] 홍성대, “기본/실력 수학의 정석,”성지출판