

# Random Noise

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi



# System Evaluation Using Random Noise

$N$  Gaussian random variables

## Definition

$$R_{XX}(\tau) = \left(\frac{N_0}{2}\right) \delta(t)$$

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} \left(\frac{N_0}{2}\right) \delta(t - \xi) h(\xi) d\xi$$

$$= \left(\frac{N_0}{2}\right) \delta(\tau)$$

$$h(\tau) \cong \left(\frac{2}{N_0}\right) R_{XX}(\tau)$$

# System Evaluation Using Random Noise

$N$  Gaussian random variables

## Definition

$$h(\tau) \cong \left( \frac{2}{N_0} \right) R_{XX}(\tau)$$

$$\tilde{h}(\tau) \cong \left( \frac{2}{N_0} \right) \hat{R}_{XX}(\tau)$$

# Total average power

$N$  Gaussian random variables

## Definition

$$R_{YY} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( \frac{N_0}{2} \right) |H(\omega)|^2 d\omega$$

$$R_{YY} = \frac{N_0}{2\pi} \int_0^{\infty} |H(\omega)|^2 d\omega$$

# Noise Bandwidth

$N$  Gaussian random variables

## Definition

$$|H_I(\omega)|^2 = \begin{cases} |H(0)|^2 & |\omega| < W_N \\ 0 & |\omega| > W_N \end{cases}$$

$$\frac{N_0}{2\pi} \int_0^{\infty} |H(\omega)|^2 d\omega = \frac{N_0}{2\pi} \int_0^{W_N} |H(0)|^2 d\omega = \frac{N_0 |H(0)|^2 W_N}{2\pi}$$

$$W_N = \frac{\int_0^{\infty} |H(\omega)|^2 d\omega}{|H(0)|^2}$$

# Noise Bandwidth - bandpass

$N$  Gaussian random variables

## Definition

$$W_N = \frac{\int_0^\infty |H(\omega)|^2 d\omega}{|H(\omega_0)|^2}$$

$$P_{YY} = \frac{N_0 |H(\omega_0)|^2 W_N}{2\pi}$$



# Resistive (Thermal) Noise Source

$N$  Gaussian random variables

## Definition

$$\overline{e_n^2(t)} = \frac{2kTRd\omega}{\pi}$$

$$\overline{i_n^2(t)} = \overline{e_n^2(t)/R^2} = \frac{2kTd\omega}{\pi R}$$

$$dN_L = \frac{\overline{e_n^2(t)}R_L}{(R + R_L)^2} = \frac{2kTRR_Ld\omega}{\pi(R + R_L)^2}$$

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R} = \frac{kTd\omega}{2\pi}$$

# Effective Noise Temperature

$N$  Gaussian random variables

## Definition

$$dN_{as} = \frac{\overline{e_n^2(t)}}{4R_o(\omega)}$$

$$\overline{e_n^2(t)} = 2kT_s R_o(\omega) \frac{d\omega}{\pi}$$

$$dN_{as} = kT_s \frac{d\omega}{2\pi}$$





