Laurent Series and z-Transform - Geometric Series Simple Pole Examples B

20180216

Causal signal a_n $n \geq 0$

anti-causal signal a_n $n < 0$

Laurent Series $f(z)$

z - Transform $X(z)$

① Causal $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$

$a_n: \quad \underset{n=0}{\left(\frac{1}{2}\right)^0}, \quad \underset{n=1}{\left(\frac{1}{2}\right)^1}, \quad \underset{n=2}{\left(\frac{1}{2}\right)^2}, \quad \cdots \qquad (n \geq 0)$

$$f(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \cdots = \frac{1}{1-\frac{z}{2}} = \frac{2}{2-z}$$

$\frac{|z|}{2} < 1 \qquad |z| < 2$

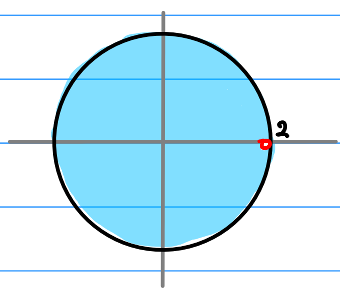
$$X(z) = \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots = \frac{1}{1-\frac{1}{2z}} = \frac{z}{z-0.5}$$

$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$f(z) = \frac{1}{1-\frac{z}{2}} \qquad |z| < 2$

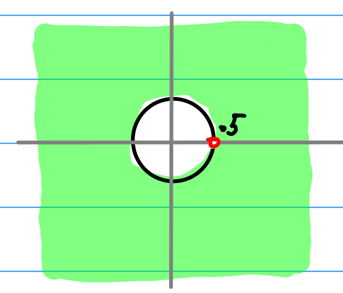
$= \frac{z^{-1}}{z^{-1} - 0.5} = \frac{2}{2-z}$



$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$X(z) = \frac{1}{1-\frac{1}{2z}} \qquad |z| > 0.5$

$= \frac{z}{z-0.5}$



② Causal $(n \geq 0)$ $a_n = (2)^n$

$a_n : \quad (2)^0, \quad (2)^1, \quad (2)^2, \quad \cdots \qquad (n \geq 0)$

$n=0 \qquad n=1 \qquad n=2$

$$f(z) = (2)^0 z^0 + (2)^1 z^1 + (2)^2 z^2 + \cdots = \frac{1}{1-2z} = \frac{0.5}{0.5-z}$$

$2|z| < 1 \qquad |z| < 0.5$

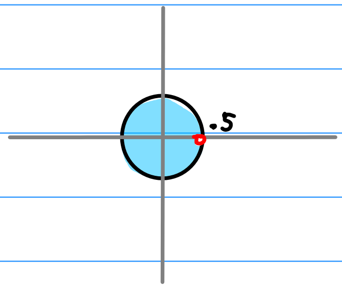
$$X(z) = (2)^0 z^0 + (2)^1 z^{-1} + (2)^2 z^{-2} + \cdots = \frac{1}{1-\frac{2}{z}} = \frac{z}{z-2}$$

$\frac{2}{|z|} < 1 \qquad |z| > 2$

$a_n = (2)^n \qquad (n \geq 0)$

$$f(z) = \frac{1}{1-2z} \qquad |z| < 0.5$$

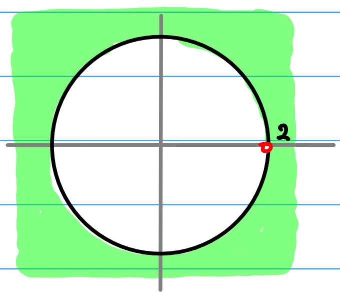
$$= \frac{0.5}{0.5-z}$$



$a_n = (2)^n \qquad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= \frac{z}{z-2}$$



③ Anti-causal $(n<0)$ $a_n = \left(\frac{1}{2}\right)^n$

$a_n: \quad \left(\frac{1}{2}\right)^{-1}, \quad \left(\frac{1}{2}\right)^{-2}, \quad \left(\frac{1}{2}\right)^{-3}, \quad \cdots \qquad (n<0)$

$\quad n=-1 \qquad n=-2 \qquad n=-3$

$$f(z) = (2)^1 z^{-1} + (2)^2 z^{-2} + (2)^3 z^{-3} + \cdots = \frac{\frac{2}{z}}{1-\frac{2}{z}} = \frac{2}{z-2}$$

$\frac{2}{|z|} < 1 \qquad |z| > 2$

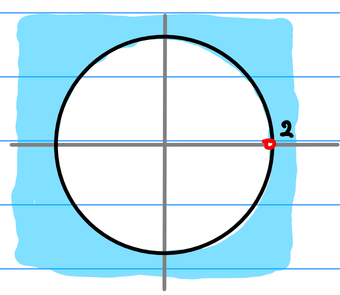
$$X(z) = (2)^1 z^1 + (2)^2 z^2 + (2)^3 z^3 + \cdots = \frac{2z}{1-2z} = \frac{z}{0.5-z}$$

$2|z| < 1 \qquad |z| < 0.5$

$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$

$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| > 2$

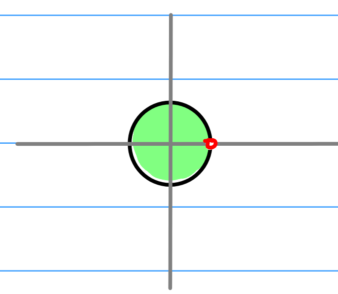
$= -\frac{2}{2-z}$



$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$

$X(z) = \frac{2z}{1-2z} \qquad |z| < 0.5$

$= -\frac{z}{z-0.5}$



④ Anti-causal $(n<0)$ $a_n = (2)^n$

$a_n : \quad (2)^{-1}, \quad (2)^{-2}, \quad (2)^{-3}, \quad \cdots \qquad (n < 0)$

$n=-1 \qquad n=-2 \qquad n=-3$

$$f(z) = \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \left(\frac{1}{2}\right)^3 z^{-3} + \cdots = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} = \frac{0.5}{z-0.5}$$

$$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$$

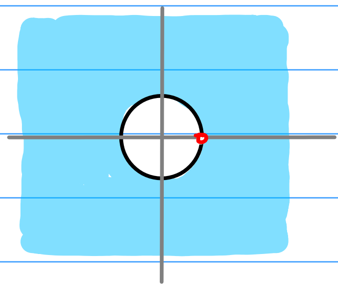
$$X(z) = \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \left(\frac{1}{2}\right)^3 z^3 + \cdots = \frac{\frac{z}{2}}{1-\frac{z}{2}} = \frac{z}{2-z}$$

$$\frac{|z|}{2} < 1 \qquad |z| < 2$$

$a_n = (2)^n \quad (n<0)$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

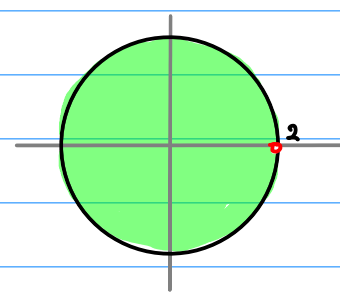
$$= -\frac{0.5}{0.5 - z}$$



$a_n = (2)^n \quad (n<0)$

$$X(z) = \frac{\frac{z}{2}}{1-\frac{z}{2}} \qquad |z| < 2$$

$$= -\frac{z}{z-2}$$



⑤ Causal $(n \geq 1)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$

a_n : $\cancel{\left(\frac{1}{2}\right)^{-1}}$, $\left(\frac{1}{2}\right)^{0}$, $\left(\frac{1}{2}\right)^{1}$, $\cdots$ $(n > 0)$

$n=0$ $n=1$ $n=2$

$$f(z) = \left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \cdots = \frac{z}{1-\frac{z}{2}} = \frac{2z}{2-z}$$

$\frac{|z|}{2} < 1$ $|z| < 2$

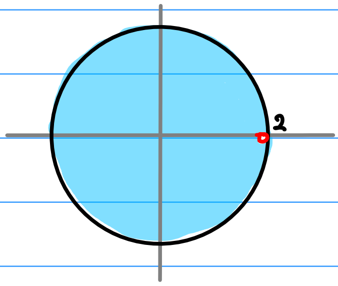
$$X(z) = \left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \cdots = \frac{\frac{1}{z}}{1-\frac{1}{2z}} = \frac{1}{z-0.5}$$

$\frac{1}{2|z|} < 1$ $|z| > 0.5$

$a_n = \left(\frac{1}{2}\right)^{n-1}$ $(n \geq 0)$

$$f(z) = \frac{z}{1-\frac{z}{2}} \quad |z| < 2$$

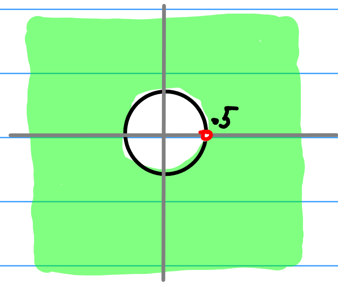
$$= \frac{1}{z^{-1}-0.5} = \frac{2z}{2-z}$$



$a_n = \left(\frac{1}{2}\right)^{n-1}$ $(n \geq 0)$

$$X(z) = \frac{\frac{1}{z}}{1-\frac{1}{2z}} \quad |z| > 0.5$$

$$= \frac{1}{z-0.5}$$



6 **Causal** $(n \geq 1)$ $a_n = (2)^{n-1}$

$a_n: \underset{n=0}{\cancel{(2)}}, \quad \underset{n=1}{(2)^0}, \quad \underset{n=2}{(2)^1}, \quad \cdots \qquad (n > 0)$

$$f(z) = (2)^0 z^1 + (2)^1 z^2 + (2)^2 z^3 + \cdots = \frac{z}{1-2z} = \frac{0.5z}{0.5-z}$$

$2|z| < 1 \qquad |z| < 0.5$

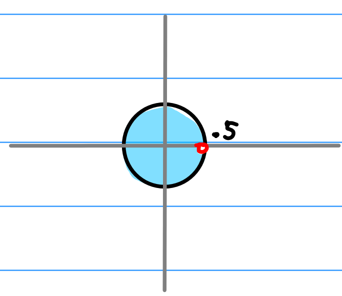
$$X(z) = (2)^0 z^{-1} + (2)^1 z^{-2} + (2)^2 z^{-3} + \cdots = \frac{\frac{1}{z}}{1-\frac{2}{z}} = \frac{1}{z-2}$$

$\frac{2}{|z|} < 1 \qquad |z| > 2$

$a_n = (2)^{n-1} \quad (n \geq 0)$

$$f(z) = \frac{z}{1-2z} \qquad |z| < 0.5$$

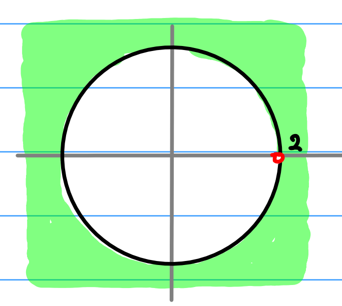
$$= \frac{0.5z}{0.5-z}$$



$a_n = (2)^{n-1} \quad (n \geq 0)$

$$X(z) = \frac{\frac{1}{z}}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= \frac{1}{z-2}$$



⑦ Anti-causal $(n < 1)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$

$a_n :$ $\left(\frac{1}{2}\right)^{-1}$, $\left(\frac{1}{2}\right)^{-2}$, $\left(\frac{1}{2}\right)^{-3}$, $\cdots$ $(n \leq 0)$

$n=0$ $\quad n=-1$ $\quad n=-2$

$$f(z) = (2)^1 z^0 + (2)^2 z^{-1} + (2)^3 z^{-2} + \cdots = \frac{\frac{2}{1}}{1-\frac{2}{z}} = \frac{2z}{z-2}$$

$\frac{2}{|z|} < 1$ $\qquad |z| > 2$

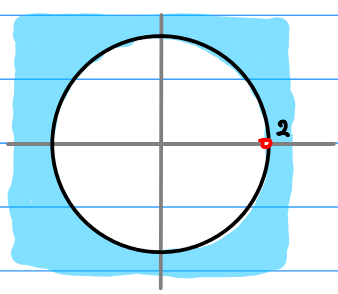
$$X(z) = (2)^1 z^0 + (2)^2 z^1 + (2)^3 z^2 + \cdots = \frac{2}{1-2z} = \frac{1}{0.5-z}$$

$2|z| < 1$ $\qquad |z| < 0.5$

$a_n = \left(\frac{1}{2}\right)^{n-1}$ $(n < 1)$

$$f(z) = \frac{\frac{2}{1}}{1-\frac{2}{z}} \qquad |z| > 2$$

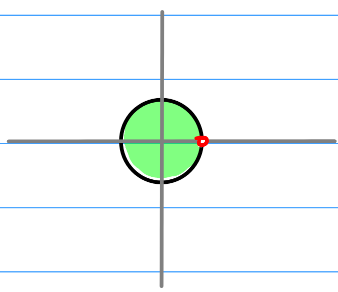
$$= -\frac{2z}{2-z}$$



$a_n = \left(\frac{1}{2}\right)^{n-1}$ $(n < 1)$

$$X(z) = \frac{2}{1-2z} \qquad |z| < 0.5$$

$$= -\frac{1}{z-0.5}$$



⑧ Anti-causal $(n<1)$ $a_n = (2)^{n-1}$

$a_n : \quad (2)^{-1}, \quad (2)^{-2}, \quad (2)^{-3}, \quad \cdots \qquad (n<1)$

$\quad\quad n=0 \qquad n=-1 \qquad n=-2$

$$f(z) = \left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots = \frac{\frac{1}{2}}{1-\frac{1}{2z}} = \frac{0.5z}{z-0.5}$$

$$\frac{1}{2|z|} < 1 \qquad |z| > 0.5$$

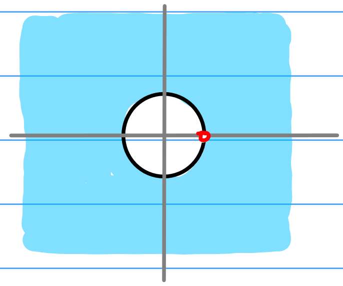
$$X(z) = \left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots = \frac{\frac{1}{2}}{1-\frac{z}{2}} = \frac{1}{2-z}$$

$$\frac{|z|}{2} < 1 \qquad |z| < 2$$

$a_n = (2)^{n-1} \quad (n<1)$

$$f(z) = \frac{\frac{1}{2}}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

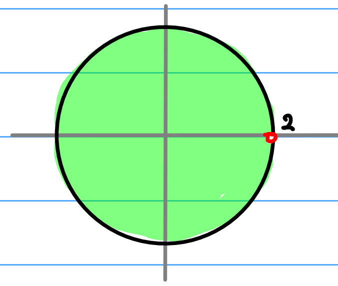
$$= -\frac{0.5z}{0.5-z}$$



$a_n = (2)^{n-1} \quad (n<1)$

$$X(z) = \frac{\frac{1}{2}}{1-\frac{z}{2}} \qquad |z| < 2$$

$$= -\frac{1}{z-2}$$



$$2 \longleftrightarrow \frac{1}{2}$$

1	$(n \geq 0)$	$a_n = \left(\frac{1}{2}\right)^n$	$f(z) = \frac{2}{2-z}$	$X(z) = \frac{z}{z-0.5}$
2	$(n \geq 0)$	$a_n = (2)^n$	$f(z) = \frac{0.5}{0.5-z}$	$X(z) = \frac{z}{z-2}$
3	$(n < 0)$	$a_n = \left(\frac{1}{2}\right)^n$	$f(z) = -\frac{2}{2-z}$	$X(z) = -\frac{z}{z-0.5}$
4	$(n < 0)$	$a_n = (2)^n$	$f(z) = -\frac{0.5}{0.5-z}$	$X(z) = -\frac{z}{z-2}$
5	$(n \geq 1)$	$a_n = \left(\frac{1}{2}\right)^{n-1}$	$f(z) = \frac{2z}{2-z}$	$X(z) = \frac{1}{z-0.5}$
6	$(n \geq 1)$	$a_n = (2)^{n-1}$	$f(z) = \frac{0.5z}{0.5-z}$	$X(z) = \frac{1}{z-2}$
7	$(n < 1)$	$a_n = \left(\frac{1}{2}\right)^{n-1}$	$f(z) = -\frac{2z}{2-z}$	$X(z) = -\frac{1}{z-0.5}$
8	$(n < 1)$	$a_n = (2)^{n-1}$	$f(z) = -\frac{0.5z}{0.5-z}$	$X(z) = -\frac{1}{z-2}$
9	$(n \geq -1)$	$a_n = \left(\frac{1}{2}\right)^{n+1}$	$f(z) = \frac{2}{(2-z)z}$	$X(z) = \frac{z}{z-0.5}$
10	$(n \geq -1)$	$a_n = (2)^{n+1}$	$f(z) = \frac{0.5}{(0.5-z)z}$	$X(z) = \frac{z}{z-2}$
11	$(n < -1)$	$a_n = \left(\frac{1}{2}\right)^{n+1}$	$f(z) = -\frac{2}{(2-z)z}$	$X(z) = -\frac{z}{z-0.5}$
12	$(n < -1)$	$a_n = (2)^{n+1}$	$f(z) = -\frac{0.5}{(0.5-z)z}$	$X(z) = -\frac{z}{z-2}$

① $(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = \frac{2}{2-z}$ $X(z) = \frac{z}{z-0.5}$

② $(n \geq 0)$ $a_n = (2)^n$ $f(z) = \frac{0.5}{0.5-z}$ $X(z) = \frac{z}{z-2}$

shift to the right → $\times z$ $\times z^{-1}$

delete a_0

⑤ $(n \geq 1)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = \frac{2z}{2-z}$ $X(z) = \frac{1}{z-0.5}$

⑥ $(n \geq 1)$ $a_n = (2)^{n-1}$ $f(z) = \frac{0.5z}{0.5-z}$ $X(z) = \frac{1}{z-2}$

③ $(n < 0)$ $a_n = \left(\frac{1}{2}\right)^n$ $f(z) = -\frac{2}{2-z}$ $X(z) = -\frac{z}{z-0.5}$

④ $(n < 0)$ $a_n = (2)^n$ $f(z) = -\frac{0.5}{0.5-z}$ $X(z) = -\frac{z}{z-2}$

shift to the right → $\times z$ $\times z^{-1}$

insert a_0

⑦ $(n < 1)$ $a_n = \left(\frac{1}{2}\right)^{n-1}$ $f(z) = -\frac{2z}{2-z}$ $X(z) = -\frac{1}{z-0.5}$

⑧ $(n < 1)$ $a_n = (2)^{n-1}$ $f(z) = -\frac{0.5z}{0.5-z}$ $X(z) = -\frac{1}{z-2}$

① $(n \geq 0) \quad a_n = \left(\frac{1}{2}\right)^n \qquad f(z) = \frac{2}{2-z} \qquad X(z) = \frac{z}{z-0.5}$

② $(n \geq 0) \quad a_n = (2)^n \qquad f(z) = \frac{0.5}{0.5-z} \qquad X(z) = \frac{z}{z-2}$

shift to the left $\leftarrow$ $\qquad \times z^{-1} \qquad \times z$

delete a_0

⑨ $(n \geq -1) \quad a_n = \left(\frac{1}{2}\right)^{n+1} \qquad f(z) = \frac{2}{(2-z)z} \qquad X(z) = \frac{z}{z-0.5}$

⑩ $(n \geq -1) \quad a_n = (2)^{n+1} \qquad f(z) = \frac{0.5}{(0.5-z)z} \qquad X(z) = \frac{z}{z-2}$

③ $(n < 0) \quad a_n = \left(\frac{1}{2}\right)^n \qquad f(z) = -\frac{2}{2-z} \qquad X(z) = -\frac{z}{z-0.5}$

④ $(n < 0) \quad a_n = (2)^n \qquad f(z) = -\frac{0.5}{0.5-z} \qquad X(z) = -\frac{z}{z-2}$

shift to the left $\leftarrow$ $\qquad \times z^{-1} \qquad \times z$

insert a_0

⑪ $(n < -1) \quad a_n = \left(\frac{1}{2}\right)^{n+1} \qquad f(z) = -\frac{2}{(2-z)z} \qquad X(z) = -\frac{z}{z-0.5}$

⑫ $(n < -1) \quad a_n = (2)^{n+1} \qquad f(z) = -\frac{0.5}{(0.5-z)z} \qquad X(z) = -\frac{z}{z-2}$

$n=-4$	$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n+1} $n=-2,-3,-4,\cdots$

b^{n+1} $n=-1,0,1,\cdots$

$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$	
b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n} $n=-1,-2,-3,\cdots$

b^{n} $n=0,1,2,\cdots$

$n=-3$	$n=-2$	$n=-1$	$n=0$	$n=1$	$n=2$	$n=3$		
	b^{-3}	b^{-2}	b^{-1}	b^{0}	b^{1}	b^{2}	b^{3}	

b^{n-1} $n=0,-1,-2,\cdots$

b^{n-1} $n=1,2,3,\cdots$

Causal $(n \geq 0)$ $\left(\frac{1}{2}\right)^n$, $(2)^n$

	$f(z)$		$X(z)$
$\left(\frac{1}{2}\right)^n$	$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$		$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$
$(2)^n$	$a_n = (2)^n \quad (n \geq 0)$		$a_n = (2)^n \quad (n \geq 0)$

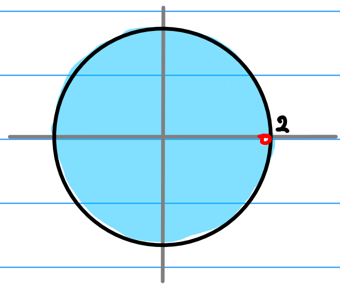
$\left(\frac{1}{2}\right)^n$

$f(z)$:

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-\frac{z}{2}} \quad |z| < 2$$

$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$



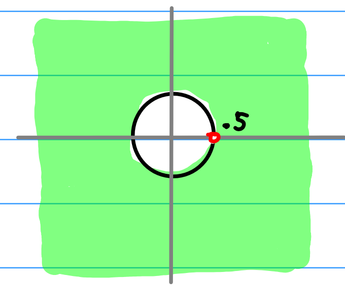
$\xleftrightarrow{z^{-1}}$

$X(z)$:

$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{1}{2z}} \quad |z| > 0.5$$

$$= \frac{z}{z-0.5}$$



$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

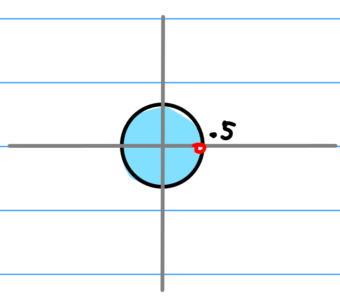
$(2)^n$

$f(z)$:

$a_n = (2)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-2z} \quad |z| < 0.5$$

$$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$$



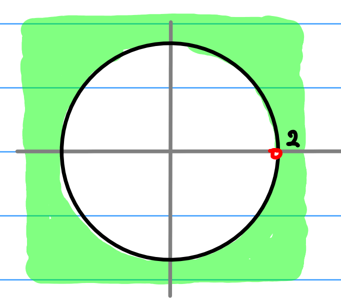
$\xleftrightarrow{z^{-1}}$

$X(z)$:

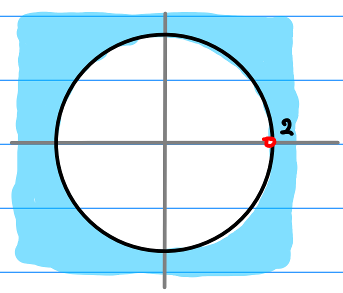
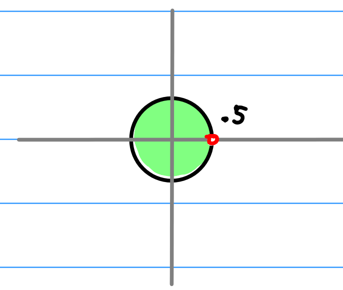
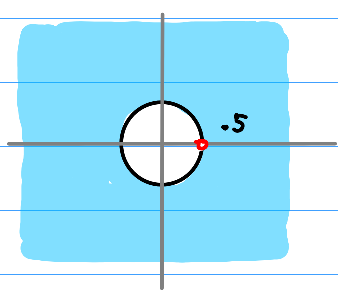
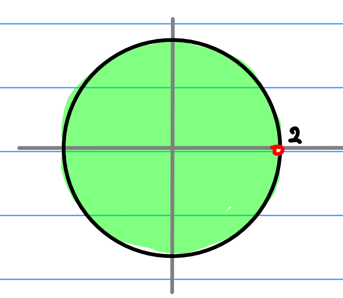
$a_n = (2)^n \quad (n \geq 0)$

$$X(z) = \frac{1}{1-\frac{2}{z}} \quad |z| > 2$$

$$= \frac{z}{z-2}$$



Anti-causal $(n<0)$ $\left(\frac{1}{2}\right)^n$, $(2)^n$

	$f(z)$		$X(z)$
$\left(\frac{1}{2}\right)^n$	$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$	$\xleftrightarrow{z^{-1}}$	$a_n = \left(\frac{1}{2}\right)^n \quad (n<0)$
	$f(z) = \dfrac{\frac{2}{z}}{1-\frac{2}{z}} \quad \lvert z\rvert > 2$		$X(z) = \dfrac{2z}{1-2z} \quad \lvert z\rvert < 0.5$
	$= -\dfrac{2}{2-z}$		$= -\dfrac{z}{z-0.5}$
			
	$\updownarrow p^{-1}$		$\updownarrow p^{-1}$
$(2)^n$	$a_n = (2)^n \quad (n<0)$	$\xleftrightarrow{z^{-1}}$	$a_n = (2)^n \quad (n<0)$
	$f(z) = \dfrac{\frac{1}{2z}}{1-\frac{1}{2z}} \quad \lvert z\rvert > 0.5$		$X(z) = \dfrac{\frac{2}{z}}{1-\frac{2}{z}} \quad \lvert z\rvert < 2$
	$= -\dfrac{0.5}{0.5-z}$		$= -\dfrac{z}{z-2}$
			

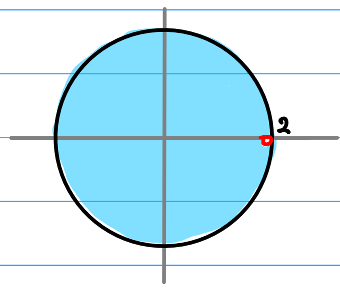
$f(z)$	Causal $(n \geq 0)$		Anti-causal $(n < 0)$
$\left(\frac{1}{2}\right)^n$	$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$		$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$
$(2)^n$	$a_n = (2)^n \quad (n \geq 0)$		$a_n = (2)^n \quad (n < 0)$

$\left(\frac{1}{2}\right)^n$

Causal: $a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-\frac{z}{2}} \qquad |z| < 2$$

$$= \frac{z^{-1}}{z^{-1}-0.5} = \frac{2}{2-z}$$

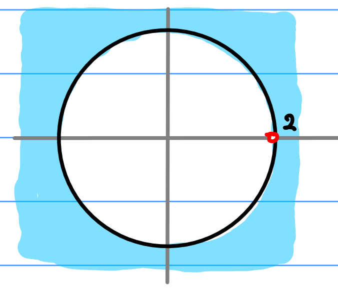


$\xleftrightarrow{-1}$

Anti-causal: $a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$

$$f(z) = \frac{\frac{2}{z}}{1-\frac{2}{z}} \qquad |z| > 2$$

$$= -\frac{2}{2-z}$$



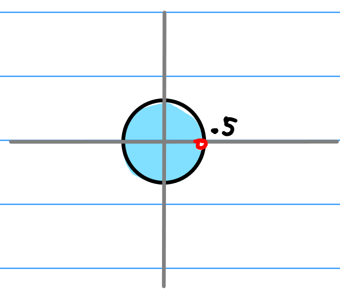
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(2)^n$

Causal: $a_n = (2)^n \quad (n \geq 0)$

$$f(z) = \frac{1}{1-2z} \qquad |z| < 0.5$$

$$= \frac{z^{-1}}{z^{-1}-2} = \frac{0.5}{0.5-z}$$

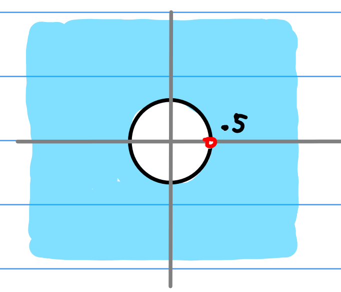


$\xleftrightarrow{-1}$

Anti-causal: $a_n = (2)^n \quad (n < 0)$

$$f(z) = \frac{\frac{1}{2z}}{1-\frac{1}{2z}} \qquad |z| > 0.5$$

$$= -\frac{0.5}{0.5-z}$$



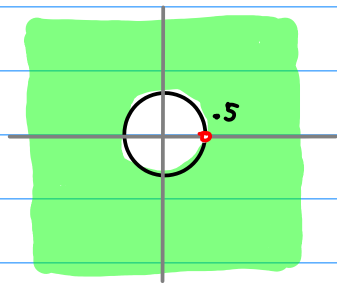
$X(z)$ | Causal $(n \geq 0)$ | Anti-causal $(n < 0)$

$\left(\frac{1}{2}\right)^n$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1 - \frac{1}{2z}} \qquad |z| > 0.5$$

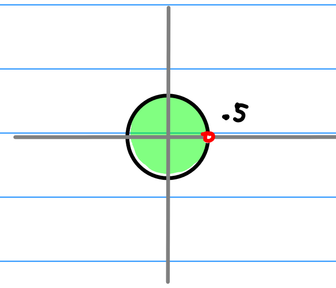
$$= \frac{z}{z - 0.5}$$



$$a_n = \left(\frac{1}{2}\right)^n \quad (n < 0)$$

$$X(z) = \frac{2z}{1 - 2z} \qquad |z| < 0.5$$

$$= -\frac{z}{z - 0.5}$$



$\xleftrightarrow{-1}$

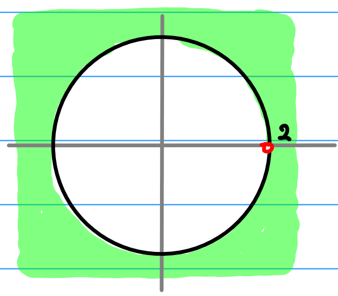
$\updownarrow p^{-1}$ $\qquad$ $\updownarrow p^{-1}$

$(2)^n$

$$a_n = (2)^n \quad (n \geq 0)$$

$$X(z) = \frac{1}{1 - \frac{2}{z}} \qquad |z| > 2$$

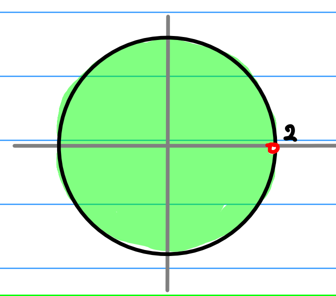
$$= \frac{z}{z - 2}$$



$$a_n = (2)^n \quad (n < 0)$$

$$X(z) = \frac{\frac{2}{z}}{1 - \frac{2}{z}} \qquad |z| < 2$$

$$= -\frac{z}{z - 2}$$



$\xleftrightarrow{-1}$

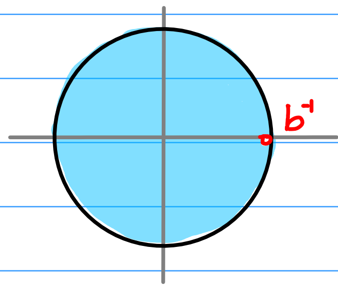
Causal b^n | Anti-causal b^n

$f(z)$

$$a_n = (b)^n \quad (n \geqslant 0)$$

$$f(z) = \frac{1}{1 - bz} \quad |z| < b^{-1}$$

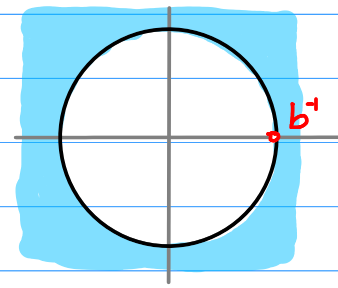
$$= \frac{b^{-1}}{b^{-1} - z}$$



$$a_n = (b)^n \quad (n < 0)$$

$$f(z) = \frac{b^{-1} z^{-1}}{1 - b^{-1} z^{-1}} \quad |z| > b^{-1}$$

$$= - \frac{b^{-1}}{b^{-1} - z}$$

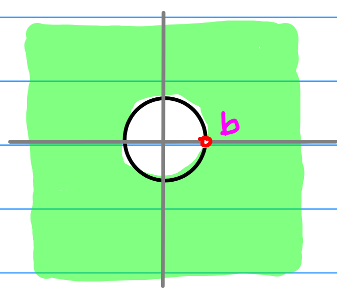


$X(z)$

$$a_n = (b)^n \quad (n \geqslant 0)$$

$$X(z) = \frac{1}{1 - b z^{-1}} \quad |z| > b$$

$$= \frac{z}{z - b}$$



$$a_n = (b)^n \quad (n < 0)$$

$$X(z) = \frac{b^{-1} z}{1 - b^{-1} z} \quad |z| < b$$

$$= - \frac{z}{z - b}$$