Monad P3 : Existential Types (1D)

Young Won Lim
4/2/21

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Based on

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

Existential Quantification

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Existentials

Existential types, or

Existentials for short,

provide a way of

squashing a group of types

into one, single type.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Existentials

Existentials are part of GHC's type system **extensions**.

But not part of **Haskell98**

have to either compile with a command-line parameter of

-XExistentialQuantification,

or put at the top of your sources that use existentials.

{-# LANGUAGE ExistentialQuantification #-}

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

forall type variables

The forall keyword is to explicitly bring fresh **type variables** into scope

type variables

those variables that begin with a **owercase** letter

the compiler allows **any type** to fill these variables

those variables that are **universally quantified**

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

forall type variables

Example: A polymorphic function

map :: (a -> b) -> [a] -> [b]

a **lowercase type parameter**

implicitly begins with a **forall** keyword,

Example: Explicitly quantifying the type variables

map :: forall a b. (a -> b) -> [a] -> [b]

two type declarations for map are equivalent

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

forall type variables

Example: A polymorphic function

map :: (a -> b) -> [a] -> [b]

Example: Explicitly quantifying the type variables

map :: forall a b. (a -> b) -> [a] -> [b]

instantiating the general type of **map**

to a more specific type

a = Int

b = String

(Int -> String) -> [Int] -> [String]

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Hiding a type variable

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Hiding a type variable (1)

Normally when creating a new type

using **type**, **newtype**, **data**, etc.,

every **type variable** that appears on the right-hand side

must also appear on the left-hand side.

```
newtype ST s a = ST (State# s -> (# State# s, a #))
```

Existential types are a way of escaping this rule

Existential types can be used for several different purposes.

But what they do is to **hide** a **type variable** on the right-hand side.

https://wiki.haskell.org/Existential_type

Hiding a type variable (2)

Normally, any type variable appearing on the right
must also appear on the left:

data Worker x y = Worker {buffer :: b, input :: x, output :: y}

This is an **error**, since the **type** b of the **buffer**
is not specified on the right
(b is a **type variable** rather than a **type**)
but also is not specified on the left
(there's no b in the left part).

In Haskell98, you would have to write

data Worker b x y = Worker {buffer :: b, input :: x, output :: y}

https://wiki.haskell.org/Existential_type

Hiding a type variable (3)

However, suppose that a **Worker** can use any type **b**
so long as it belongs to some particular **class**.
Then every **function** that uses a **Worker** will have a type like

foo :: (Buffer b) => Worker b Int Int

In particular, failing to write an **explicit type signature** **(Buffer b)** will invoke the dreaded **monomorphism restriction**.

Using **existential types**, we can avoid this:

https://wiki.haskell.org/Existential_type

Hiding a type variable (3’)

The "monomorphism restriction" is a counter-intuitive rule
in Haskell type inference.

If you forget to provide a type signature,
sometimes this rule will fill the free type variables
with specific types using "type defaulting" rules.

The resulting type signature is always
less polymorphic than you'd expect,
so often this results in the compiler throwing type errors
when you expected it to infer a perfectly sane type
for a polymorphic expression.

https://wiki.haskell.org/Existential_type

Hiding a type variable (3')

A simple example is **plus = (+)**.

Without an explicit signature for plus,

the compiler will not infer the type

(+) :: (Num a) => a -> a -> a for `plus`,

but will apply defaulting rules to specify

plus :: Integer -> Integer -> Integer.

When applied to plus 3.5 2.7, GHCi will then produce

the somewhat-misleading-looking error,

No instance for (Fractional Integer) arising from the literal '3.5'.

https://wiki.haskell.org/Existential_type

Hiding a type variable (4)

Using existential type :

```
data Worker x y = forall b. Buffer b => Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

The **type** of the **buffer** (**Buffer**) now does not appear in the **Worker** type at all. **Worker x y**

Explicit type signature :

```
data Worker b x y = Worker {buffer :: b, input :: x, output :: y}
foo :: (Buffer b) => Worker b Int Int
```

https://wiki.haskell.org/Existential_type

Hiding a type variable (5)

- it is now impossible for a function
  to demand a **Worker** having a specific type of **buffer**.

- the **type** of **foo** can now be derived automatically
  without needing an explicit **type signature**.
  (No **monomorphism** restriction.)

- since code now has no idea
  what **type** the buffer function returns,
  you are more limited in what you can do to it.

```
data Worker x y = forall b. Buffer b => Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

https://wiki.haskell.org/Existential_type

Hiding a type variable (6)

In general, when you use a **hidden type** in this way,
you will usually want that **type** to belong to a **specific class**,
or you will want to **pass some functions** along
that can work on that type.

Otherwise you'll have some value belonging
to a **random unknown type**,
and you won't be able to do anything to it!

https://wiki.haskell.org/Existential_type

Less specific types (1)

Note: You can use **existential types**

to **convert** a **more specific type**

into a **less specific one**.

constrained type variables

There is no way to perform the reverse conversion!

https://wiki.haskell.org/Existential_type

Less specific types (2)

This illustrates **creating a heterogeneous list**,
all of whose members implement "**Show**",
and progressing through that list to show these items:

```
data Obj = forall a. (Show a) => Obj a

    xs :: [Obj]
    xs = [Obj 1, Obj "foo", Obj 'c']

    doShow :: [Obj] -> String
    doShow [] = ""
    doShow ((Obj x):xs) = show x ++ doShow xs
```

With output: **doShow xs ==> "1\"foo\"'c'"**

https://wiki.haskell.org/Existential_type

Existentials in terms of **forall** (1)

It is also possible to express existentials with RankNTypes
as **type expressions** directly (without a **data** declaration)

forall r. (forall a. Show a => a -> r) -> r

(the leading forall r. is optional
unless the expression is part of another expression).

the equivalent type **Obj** :

data Obj = forall a. (Show a) => Obj a

https://wiki.haskell.org/Existential_type

Existentials in terms of **forall** (2)

The conversions are:

```
fromObj :: Obj -> forall r. (forall a. Show a => a -> r) -> r
fromObj (Obj x) k = k x

toObj :: (forall r. (forall a. Show a => a -> r) -> r) -> Obj
toObj f = f Obj
```

https://wiki.haskell.org/Existential_type

Heterogeneous Lists

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Type hider

Suppose we have a group of values.

they may not be all the same **type**,

but they are all members of some **class**

thus, they have a certain **property**

It might be useful to throw all these values into a list.

normally this is impossible because lists elements

must be of the same type

(homogeneous with respect to types).

existential types allow us to loosen this requirement

by defining a **type hider** or **type box**:

```
data ShowBox = forall s. Show s => SB s
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (1)

```
data ShowBox = forall s. Show s => SB s          -- type hider
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

[SB (), SB 5, SB True] calls the **constructor**

on three values of different types,

to place them all into a single list

virtually the same type for each one.

Use the **forall** in the constructor

SB :: forall s. Show s => s -> ShowBox.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (2)

```
data ShowBox = forall s. Show s => SB s
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

When passing **heteroList** type parameters to a function

we cannot take out the **values** inside the SB

because their type might **Bool**. **Int**, **Char**, …

But each of the elements can be

converted to a **string** via **show**.

In fact, that's the only thing we know about them.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (3)

```
instance Show ShowBox where
 show (SB s) = show s

f :: [ShowBox] -> IO ()
f xs = mapM_ print xs

main = f heteroList
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (4)

Example: Using our heterogeneous list

```
 instance Show ShowBox where
show (SB s) = show s        -- (*) see the comment in the text below
f :: [ShowBox] -> IO ()
f xs = mapM_ print xs
main = f heteroList
```

Example: Types of the functions involved

```
print :: Show s => s -> IO ()        -- print x = putStrLn (show x)
mapM_ :: (a -> m b) -> [a] -> m ()
mapM_ print :: Show s => [s] -> IO ()
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

mapM, **mapM_**, and **map** (1)

The core idea is that **mapM** maps
an "action" (ie function of type **a -> m b**) over a list and
gives you all the results as **m [b]**

mapM_ does the same thing,
but never collects the results, returning a **m ()**.

If you care about the results
of your **a -> m b** function, use **mapM**.
If you only care about the effect,
but not the resulting value,
use **mapM_**, because it can be more efficient

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

mapM, **mapM_**, and **map** (2)

Always use **mapM_** with functions of the type **a -> m ()**,

like **print** or **putStrLn**.

these functions return **()** to signify that only the effect matters.

If you used **mapM**, you'd get a list of **()** (ie **[(), (), ()]**),

which would be completely useless

but waste some memory.

If you use **mapM_**, you would just get a **()**,

but it would still print everything.

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

mapM, **mapM_**, and **map** (3)

Normal **map** is something different:

it takes a normal function **(a -> b)**

instead of one using a monad **(a -> m b)**.

This means that it cannot have any sort of effect

besides returning the changed list.

You would use it if you want to transform a list

using a normal function.

map_ doesn't exist because, since you don't have any effects,

you always care about the results of using **map**.

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

Quantified types

as products and sums

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Quantified Types as Products and Sums

A **universally quantified type** may be interpreted

as an **infinite product** of **types**.

a **polymorphic function** can be understood

as a **product**, or a **tuple**, of **individual functions**,

one per every possible **type a**.

To construct a **value** of such **type**, we have

to provide all the **components** of the **tuple** at once.

-- one formula generating an infinity of functions

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Example: Identity function

id :: forall a. a -> a

id a = a

a **polymorphic function** can be understood

as a **product**, or a **tuple**, of **individual functions**,

one per every possible **type a**.

Int -> Int, Double -> Double, ...

Char -> Char, [Char] -> [Char], …

…

…

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

To construct a **value** of such **type**, we have
to provide all the **components** of the **tuple** at once.

in case of **numeric types**, one **numeric constant**
may be used to initialize **many types** at once.

Example: Polymorphic value

x :: forall a. Num a => a

x = 0

x may be conceptualized as a **tuple** consisting
of an **Int value**, a **Double value**, etc.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Similarly, an **existentially quantified type** may be interpreted as an **infinite sum**.

Example: Existential type

data ShowBox = forall s. Show s => SB s

may be conceptualized as a **sum**:

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Example: Existential type

data ShowBox = forall s. Show s => SB s

Example: Sum type

data ShowBox = SBUnit | SBInt Int | SBBool Bool | SBIntList [Int] | ...

to construct a **value** of this **type**,

we only have to pick one of the constructors.

A **polymorphic constructor SB**

combines all those constructors into one.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantification as a primitive

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Newtype creates a function (1)

newtype Parser a = Parser { parse :: String -> Maybe (a,String) }

1) A **type** named **Parser**.

2) A **term level constructor** of **Parser's** named **Parser**.

The **type** of this (constructor) function is

Parser :: (String -> Maybe (a, String)) -> Parser a

You give it a function of the type

(String -> Maybe (a, String))

and it wraps it inside a **Parser**

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (2)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
```

3) A **function** named **parse** to remove the **Parser** wrapper and get your function back. The type of this function is:

parse :: Parser a -> String -> Maybe (a, String)

A **term level constructor** named **Parser**

Parser :: (String -> Maybe (a, String)) -> Parser a

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (3)

```
Prelude> newtype
                 Parser a = Parser { parse :: String -> Maybe (a,String) }

Prelude> :t Parser
Parser :: (String -> Maybe (a, String)) -> Parser a

Prelude> :t parse
parse :: Parser a -> String -> Maybe (a, String)
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (4)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
```

the **term level constructor** (**Parser**)

the **function** to remove the wrapper (**parse**)

Both can have arbitrary names

No need to match the type name.

It's common to write:

```
newtype Parser a = Parser { unParser :: String -> Maybe (a,String) }
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (5)

```
newtype Parser a = Parser { unParser :: String -> Maybe (a,String) }
```

this name makes it clear **unParser** removes

the **wrapper** around the parsing function.

```
unParser :: Parser a -> String -> Maybe (a, String)
```

however, it is recommended that the **type** and **constructor**

have the same name when using **newtypes**.

(**Parser**, **Parser**)

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (6)

newtype Parser a = Parser { parser :: String -> Maybe (a,String) }

1) **Parser** is declared as a **type** with a **type parameter a**

2) can instantiate **Parser** by providing a **parser** function

p = Parser (\s -> Nothing)

3) a function name **parser** defined and

it is capable of *running Parser's*.

unwrap the function

then apply the function

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (7)

```
newtype Parser a = Parser { parser :: String -> Maybe (a,String) }

parser :: Parser a -> String -> Maybe (a, String)
parser (Parser  (\s -> Nothing)) "my input"
(\s -> Nothing)) "my input"
Nothing
```

You are unwrapping the function using **parse** and

then calling the unwrapped function with "myInput".

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (8)

First, let’s have a look at a parser **newtype** without **record** syntax:

```
newtype Parser' a = Parser' (String -> Maybe (a,String))
```

it stores a **function** **String -> Maybe (a,String)**.

To run this parser, we will need to make a **new function:**

```
runParser' :: Parser' a -> String -> Maybe (a,String)
runParser' (Parser' f) i = f i
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (9)

```
runParser' :: Parser' a -> String -> Maybe (a,String)
runParser' (Parser' f) i = f i

runParser' (Parser' $ \s -> Nothing) "my input".
```

But now note that, since Haskell functions are curried,
we can simply remove the reference to the input **i** to get:

```
runParser'' :: Parser' -> (String -> Maybe (a,String))
runParser'' (Parser' f') = f'
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (10)

runParser'' :: Parser' -> (String -> Maybe (a,String))

runParser'' (Parser' f') = f'

This function is exactly equivalent to **runParser'**,

but you could think about it differently:

instead of applying the parser function to the value explicitly,

it simply takes a parser and fetches the parser function from it;

(Parser' f') → **f'**

however, thanks to **currying**, **runParser''**

can still be used with two arguments.

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (11)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
newtype Parser' a = Parser' (String -> Maybe (a,String))
```

difference : record syntax with only one field

this record syntax automatically defines a function

```
parse :: Parser a  -> (String -> Maybe (a,String)),
```

which extracts the **String -> Maybe (a,String)** function
from the **Parser a**.

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (12)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
```

parse can be used with two arguments thanks to **currying**,
and this simply has the effect of **running** the function stored
within the **Parser a**.

equivalent definition to the following code:

```
newtype Parser a = Parser (String -> Maybe (a,String))

parse :: Parser a -> (String -> Maybe (a,String))
parse (Parser p) = p
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Access functions in a record type (1)

```
data Person = Person {  firstName  :: String  ,
                        lastName   :: String  ,
                        age        :: Int  ,
                        height     :: Float  ,
                        phoneNo    :: String  ,
                        flavor     :: String
                     } deriving (Show)

ghci> :t flavor
flavor :: Person -> String
ghci> :t firstName
firstName :: Person -> String
```

return types of access functions

Person :: the input type of access functions

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Access functions in a record type (2)

```
data Car = Car String String Int deriving (Show)

ghci> Car "Ford" "Mustang" 1967
Car "Ford" "Mustang" 1967



data Car = Car {company :: String,
                model :: String,
                year :: Int} deriving (Show)

ghci> Car {company="Ford", model="Mustang", year=1967}
Car {company = "Ford", model = "Mustang", year = 1967}
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Pair type example (1)

Universal quantification is useful

for defining **data types** that aren't already defined.

Suppose there was no such thing as **pairs** built into haskell.

Quantification could be used to define them.

```
{-# LANGUAGE ExistentialQuantification, RankNTypes #-}

newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)

makePair :: a -> b -> Pair a b
makePair a b = Pair $ \f -> f a b
```

Pair $ \f -> f a b :: Pair a b

f :: a -> b -> c

f a b :: c

f is not yet defined

c can be any type (**forall c**)

defining data type **c**

that aren't already defined

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (2)

newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)

makePair :: a -> b -> Pair a b

makePair a b = Pair $ \f -> f a b

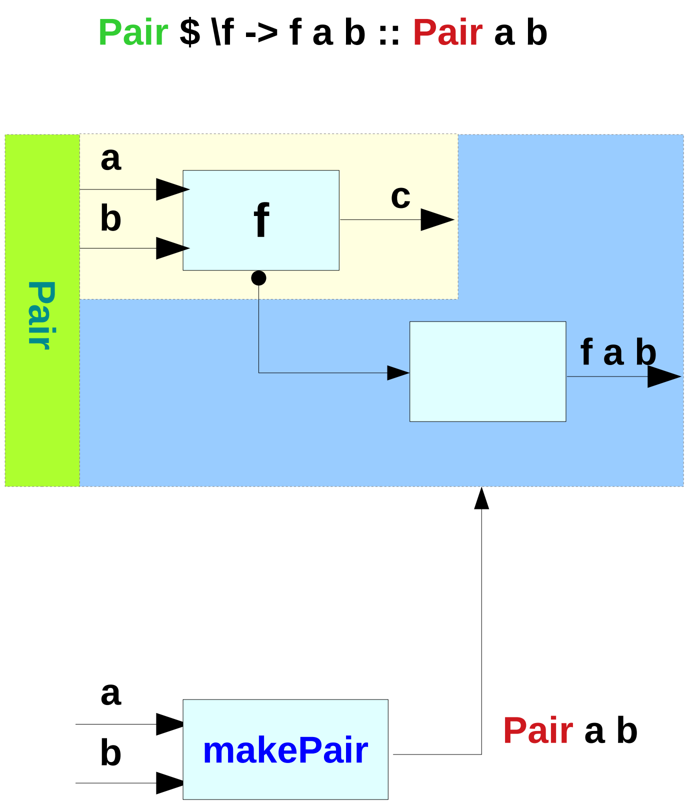
using a record type with a single field

λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}

runPair is an access function

takes an input of the type **Pair a b**

returns an output of the type **forall c. (a -> b -> c) -> c**

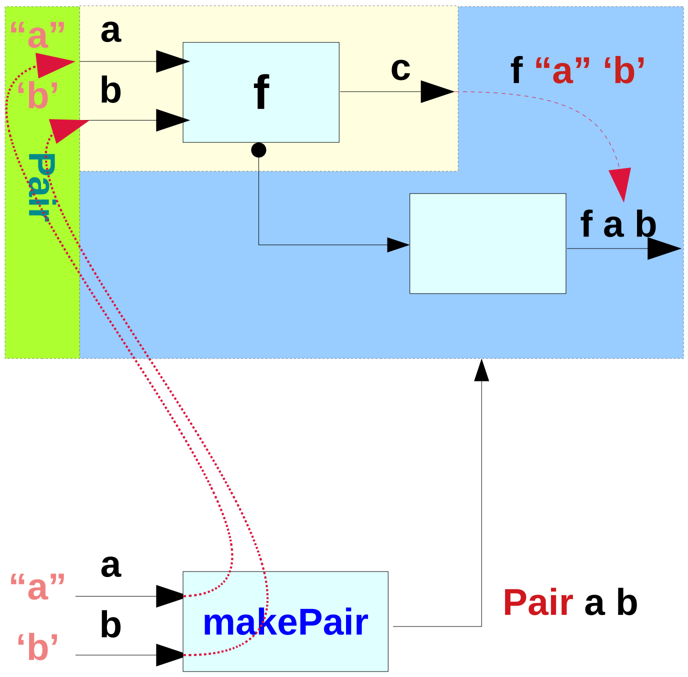


https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (3)

```
In GHCI
λ> :set -XExistentialQuantification
λ> :set -XrankNTypes
λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
λ> makePair a b = Pair $ \f -> f a b
λ> pair = makePair "a" 'b'

λ> :t pair
 pair :: Pair [Char] Char
λ> runPair pair (\x y -> x)    -- unwrap (a -> b -> c) -> c then apply
 "a"
λ> runPair pair (\x y -> y)   -- unwrap (a -> b -> c) -> c then apply
 'b'
```

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

Pair $ \f -> f "a" 'b' :: Pair a b

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (4)

λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}

λ> makePair a b = Pair $ \f -> f a b

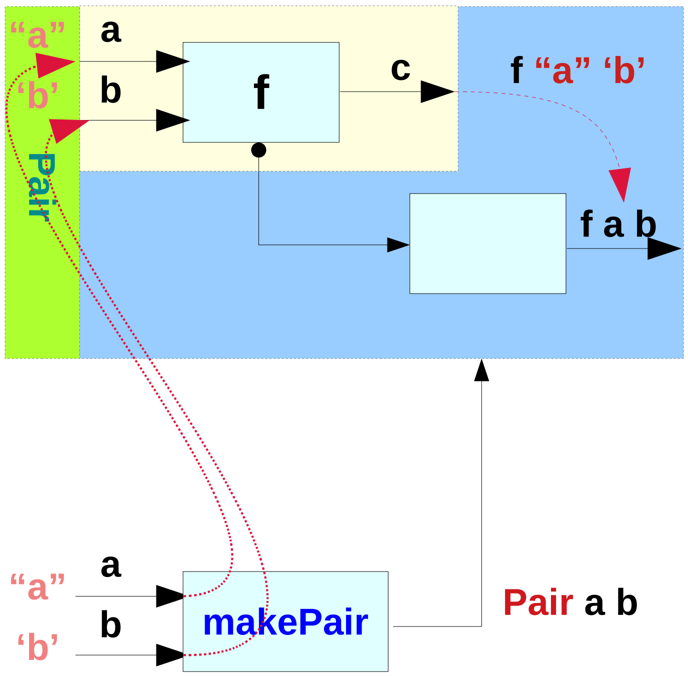
λ> pair = makePair "a" 'b'

Pair $ \f -> f "a" 'b'

\f : function itself   f :: a -> b -> c

f "a" 'b' : the result of applying the function

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

Pair $ \f -> f "a" 'b' :: Pair a b

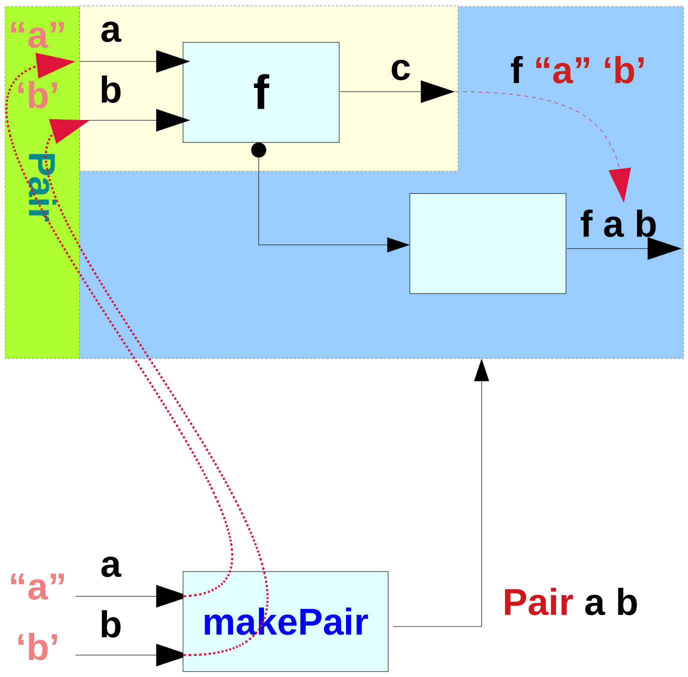
https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (5)

```
newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
runPair ::  Pair a b -> forall c. (a -> b -> c) -> c

makePair a b = Pair $ \f -> f a b
runPair makePair a b = \f -> f a b              -- unwrapping

makePair "a" 'b'  = Pair $ \f -> f "a" 'b'
runPair makePair "a" 'b' = \f -> f "a" 'b'

pair = makePair                              :: Pair [Char] Char

runPair  pair  (\x y -> x) = (\x y -> x) "a" 'b'
runPair  pair  (\x y -> y) = (\x y -> y) "a" 'b'
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

Pair $ \f -> f "a" 'b'   :: Pair a b

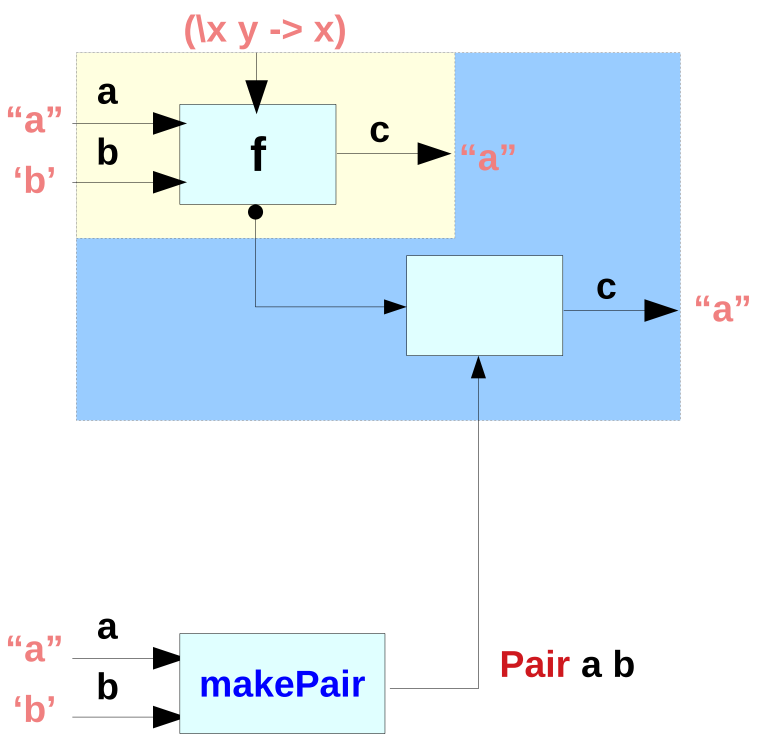
Pair type example (6)

```
runPair  pair  (\x y -> x) = (\x y -> x) "a" 'b'
runPair  pair  (\x y -> y) = (\x y -> y) "a" 'b'

runPair makePair "a" 'b' (\x y -> x)
(\x y -> x) "a" 'b'
 "a"

runPair makePair "a" 'b' (\x y -> y)
(\x y -> y) "a" 'b'
 'b'
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (6)

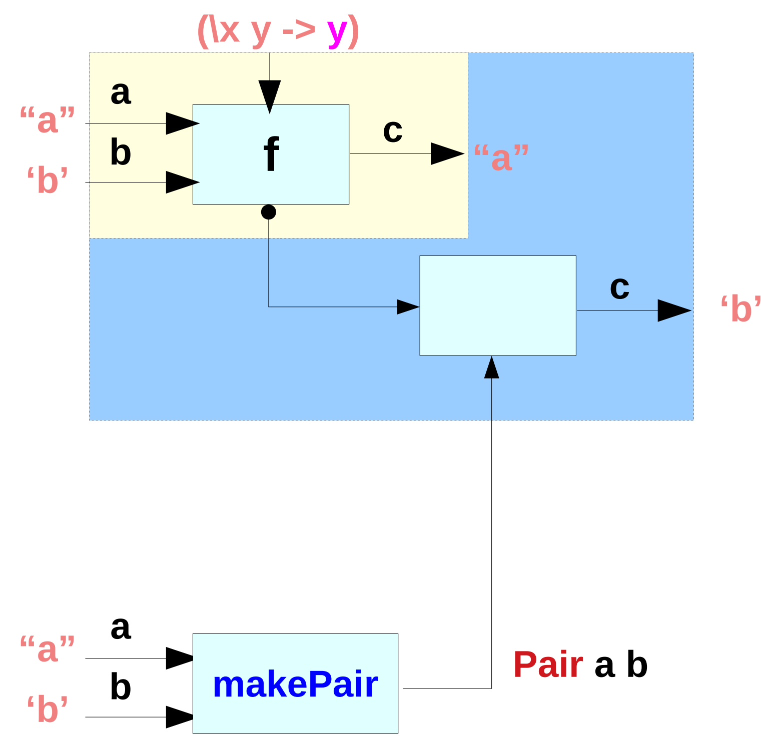
Pair $ \f -> f a b :: Pair a b



pair (\x y -> x)

makePair "a" 'b' (\x y -> x)

Pair $ \f -> f a b :: Pair a b



pair (\x y -> y)

makePair "a" 'b' (\x y -> y)

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf

[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf

Existentials

Existential types, or '**existentials**' for short, provide a way of 'squashing' a group of types into one, single type.

Existentials are part of GHC's type system **extensions**.
They aren't part of Haskell98, and as such you'll have

to either compile any code that contains them
with an extra command-line parameter of
-XExistentialQuantification,
or put at the top of your sources that use existentials.
{-# LANGUAGE ExistentialQuantification #-}

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types