

# Multiple Random Variables

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles,Jr. and B. Shi

# Outline

## 1 Conditional Distribution and Density

## Conditional Distribution and Density

for a single random variable  $X$ 

## Definition

Let  $A$  denote the event  $\{X \leq x\}$  in  $P(A | B) = \frac{P(A \cap B)}{P(B)}$   
the conditional distribution function of  $X$  is defined as

$$F_X(x | B) = P\{X \leq x | B\} = \frac{P\{X \leq x \cap B\}}{P(B)}$$

$$f_X(x | B) = \frac{dF_X(x | B)}{dx}$$

the density function of the random variable  $X$

the derivative of the distribution function  $F_X(x | B)$

# Point Conditioning

for 2 random variables  $X$  and  $Y$

## Definition

the distribution function of a random variable  $X$  conditioned by the fact that a second random variable  $Y$  has some specific value  $y$

$$F_X(x|B) = \lim_{\Delta y \rightarrow 0} F_X(x|y - \Delta y < Y \leq y + \Delta y)$$

$$F_X(x|B) = \lim_{\Delta y \rightarrow 0} \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x f_{X,Y}(\xi_1, \xi_2) d\xi_1 d\xi_2}{\int_{y-\Delta y}^{y+\Delta y} f_Y(\xi) d\xi}$$

where event  $B$  is defined as  $\{y - \Delta y < Y \leq y + \Delta y\}$

# Point Conditioning (1)

for 2 **discrete** random variables  $X$  and  $Y$

## Definition

assume  $X$  and  $Y$  are both discrete random variables and have values  $x_i, i = 1, 2, \dots, N$  and  $y_j, j = 1, 2, \dots, M$ . with the corresponding probabilities  $P(x_i)$  and  $P(y_j)$ , respectively  $P(x_i, y_j)$  denotes the probability of joint occurrence of  $x_i$  and  $y_j$

$$f_X(x|y = y_k) = \sum_{i=1}^N \frac{P(x_i, y_k)}{P(y_k)} \delta(x - x_i)$$

## Point Conditioning (2)

for 2 **discrete** random variables  $X$  and  $Y$ 

$$f_Y(y) = \sum_{j=1}^M P(y_j) \delta(y - y_j)$$

$$f_{X,Y}(x,y) = \sum_{i=1}^N \sum_{j=1}^M P(x_i, y_j) \delta(x - x_i) \delta(y - y_j)$$

$$B = \{y - \Delta y < Y \leq y + \Delta y\}$$

$$F_X(x|B) = \lim_{\Delta y \rightarrow 0} \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x f_{X,Y}(\xi_1, \xi_2) d\xi_1 d\xi_2}{\int_{y-\Delta y}^{y+\Delta y} f_Y(\xi) d\xi}$$

$$F_X(x|B) = \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x \sum_{i=1}^N \sum_{j=1}^M P(x_i, y_j) \delta(x - x_i) \delta(y - y_j) dx dy}{\int_{y-\Delta y}^{y+\Delta y} \sum_{j=1}^M P(y_j) \delta(y - y_j) dy}$$

## Point Conditioning (3)

for 2 **discrete** random variables  $X$  and  $Y$ 

$$F_X(x|B) = \frac{\sum_{i=1}^N \sum_{j=1}^M \int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x P(x_i, y_j) \delta(x - x_i) \delta(y - y_j) dx dy}{\sum_{j=1}^M \int_{y-\Delta y}^{y+\Delta y} P(y_j) \delta(y - y_j) dy}$$

$$F_X(x|y = y_k) = \frac{\sum_{i=1}^N \int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x P(x_i, y_k) \delta(x - x_i) \delta(y - y_k) dx dy}{\int_{y-\Delta y}^{y+\Delta y} P(y_k) \delta(y - y_k) dy}$$

$$F_X(x|y = y_k) = \frac{\sum_{i=1}^N \int_{-\infty}^x P(x_i, y_k) \delta(x - x_i) dx}{P(y_k)}$$



## Point Conditioning (4)

for 2 **discrete** random variables  $X$  and  $Y$ 

$$F_X(x|y = y_k) = \sum_{i=1}^N \frac{P(x_i, y_k)}{P(y_k)} u(x - x_i)$$

$$f_X(x|y = y_k) = \sum_{i=1}^N \frac{P(x_i, y_k)}{P(y_k)} \delta(x - x_i)$$

# Marginal Density Functions

for **continuous**  $N$  random variable  $X_1, X_2, \dots, X_n$

## Definition

$$f_{X_1, X_2, \dots, X_k}(x_1, x_2, \dots, x_k) =$$

$$\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_{X_1, X_2, \dots, X_N}(x_1, x_2, \dots, x_N) dx_{k+1} dx_{k+2} \cdots dx_N$$

