Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Conditional Distribution and Density
for a single random variable X

Definition

Let A denote the event $\{X \leq x\}$ in $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$
the conditional distribution function of X is defined as

$$F_X(x \mid B) = P\{X \leq x \mid B\} = \frac{P\{X \leq x \cap B\}}{P(B)}$$

$$f_X(x \mid B) = \frac{dF_x(x \mid B)}{dx}$$

the density function of the random variable X
the derivative of the distribution function $F_x(x \mid B)$

Point Conditioning
for 2 random variables X and Y

Definition

the distribution function of a random variable X conditioned by the fact that a second random variable Y has some specific value y

$$F_X(x|B) = \lim_{\Delta y \to 0} F_X(x|y - \Delta y < Y \leq y + \Delta y)$$

$$F_X(x|B) = \lim_{\Delta y \to 0} \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^{x} f_{X,Y}(\xi_1, \xi_2) d\xi_1 d\xi_2}{\int_{y-\Delta y}^{y+\Delta y} f_Y(\xi) d\xi}$$

where event B is defined as $\{y - \Delta y < Y \leq y + \Delta y\}$

Point Conditioning (1)

for 2 discrete random variables X and Y

Definition

assume X and Y are both discrete random variables
and have values x_i, $i = 1, 2, ..., N$ and y_j, $j = 1, 2, ..., M$.
with the corresponding probabilities $P(x_i)$ and $P(y_i)$, respectively
$P(x_i, y_i)$ denotes the probability of joint occurrence of x_i and y_i

$$f_X(x|y = y_k) = \sum_{i=1}^{N} \frac{P(x_i, y_k)}{P(y_k)} \delta(x - x_i)$$

Point Conditioning (2)

for 2 discrete random variables X and Y

$$f_Y(y) = \sum_{j=1}^{M} P(y_j)\delta(y - y_j)$$

$$f_{X,Y}(x,y) = \sum_{i=1}^{N}\sum_{j=1}^{M} P(x_i, y_j)\delta(x - x_i)\delta(y - y_j)$$

$$B = \{y - \Delta y < Y \leq y + \Delta y\}$$

$$F_X(x|B) = \lim_{\Delta y \to 0} \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^{x} f_{X,Y}(\xi_1, \xi_2) d\xi_1 d\xi_2}{\int_{y-\Delta y}^{y+\Delta y} f_Y(\xi) d\xi}$$

$$F_X(x|B) = \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^{x} \sum_{i=1}^{N}\sum_{j=1}^{M} P(x_i, y_j)\delta(x - x_i)\delta(y - y_j) dxdy}{\int_{y-\Delta y}^{y+\Delta y} \sum_{j=1}^{M} P(y_j)\delta(y - y_j) dy}$$

Point Conditioning (3)

for 2 discrete random variables X and Y

$$F_X(x|B) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M} \int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^{x} P(x_i, y_i)\delta(x - x_i)\delta(y - y_j)dxdy}{\sum_{j=1}^{M} \int_{y-\Delta y}^{y+\Delta y} P(y_j)\delta(y - y_j)dy}$$

$$F_X(x|y = y_k) = \frac{\sum_{i=1}^{N} \int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^{x} P(x_i, y_k)\delta(x - x_i)\delta(y - y_k)dxdy}{\int_{y-\Delta y}^{y+\Delta y} P(y_j)\delta(y - y_k)dy}$$

$$F_X(x|y = y_k) = \frac{\sum_{i=1}^{N} \int_{-\infty}^{x} P(x_i, y_k)\delta(x - x_i)dx}{P(y_k)}$$

Point Conditioning (4)
for 2 discrete random variables X and Y

$$F_X(x|y = y_k) = \sum_{i=1}^{N} \frac{P(x_i, y_k)}{P(y_k)} u(x - x_i)$$

$$f_X(x|y = y_k) = \sum_{i=1}^{N} \frac{P(x_i, y_k)}{P(y_k)} \delta(x - x_i)$$

Marginal Density Functions
for continuous N random variable $X_1, X_2, \cdots, X_n$

Definition

$$f_{X_1,X_2,\cdots,X_k}(x_1, x_2, \cdots, x_k) =$$

$$\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_{X_1,X_2,\cdots,X_N}(x_1, x_2, \cdots, x_N) dx_{k+1} dx_{k+2} \cdots dx_N$$