Laurent Series and z-Transform - Geometric Series Applications

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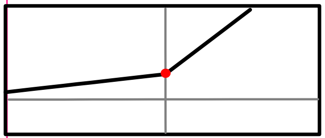
20200708 Wed

a^n



*a

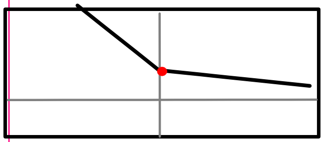
/a

Shifting a sequence

$\frac{1}{1-az}$ $\lvert z\rvert < a^{-1}$	*a	$\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$
$a^n\ u(n)$	/z, *z	$a^{n+1}\ u(n)$
$\frac{az}{1-az}$ $\lvert z\rvert < a^{-1}$	/a	$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$
$a^n\ u(n-1)$		$a^{n-1}\ u(n-1)$
$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	*a	$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$
$a^n\ u(-n-1)$	/z, *z	$a^{n+1}\ u(-n-1)$
$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	/a	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$
$a^n\ u(-n)$		$a^{n-1}\ u(-n)$

Shifting a sequence

a^{-n}



/a

*a

$$\frac{1}{1-a^{-1}z} \quad |z|<a$$

a^{-n} u(n)

/a

$$\frac{a^{-1}}{1-a^{-1}z} \quad |z|<a$$

a^{-n-1} u(n)

/z

*z

$$\frac{a^{-1}z}{1-a^{-1}z} \quad |z|<a$$

*a

$$\frac{z}{1-a^{-1}z} \quad |z|<a$$

a^{-n} u(n-1)

a^{-n+1} u(n-1)

$$-\frac{az^{-1}}{1-az^{-1}} \quad |z|>a$$

/a

$$-\frac{z^{-1}}{1-az^{-1}} \quad |z|>a$$

a^{-n} u(-n-1)

/z

a^{-n-1} u(-n-1)

*z

$$-\frac{1}{1-az^{-1}} \quad |z|>a$$

*a

$$-\frac{a}{1-az^{-1}} \quad |z|>a$$

a^{-n} u(-n)

a^{-n+1} u(-n)

2 formulas   Simple Pole Form

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each   Geometric Series Form

$$\boxed{\frac{1}{z-p}} \to \begin{cases} \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \triangleq \underset{}{\overset{\text{causal}}{f(z)}} = \overset{\text{anti- causal}}{X(z^{-1})} \\ \qquad\qquad\qquad \| \qquad\qquad \| \\ \boxed{\frac{z^{-1}}{1-pz^{-1}}} \triangleq \underset{\text{causal}}{Y(z)} = \underset{\text{anti- causal}}{g(z^{-1})} \end{cases}$$

$$\boxed{\frac{1}{z^{-1}-p}} \to \begin{cases} \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \triangleq \overset{\text{causal}}{X(z)} = \overset{\text{anti- causal}}{f(z^{-1})} \\ \qquad\qquad\qquad \| \qquad\qquad \| \\ \boxed{\frac{z}{1-pz}} \triangleq \underset{\text{causal}}{g(z)} = \underset{\text{anti- causal}}{Y(z^{-1})} \end{cases}$$

Simple Pole Form | Geometric Series Form

Geometric Series (1)

2 formulas   Simple Pole Form

$\boxed{\dfrac{1}{z - p}}$ $\boxed{\dfrac{1}{z^{-1} - p}}$

2 representations each   Geometric Series Form

$$\boxed{\frac{1}{z-p}}$$

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}} \xleftarrow{/p} \boxed{-\frac{1}{1-p^{-1}z}}$$

$p^{-n-1}u(n)$ $p^{-n}u(n)$

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} \xleftarrow{/z} \boxed{\frac{1}{1-pz^{-1}}}$$

$p^{-n-1}u(-n-1)$ $p^{-n}u(-n)$

$$\boxed{\frac{1}{z^{-1}-p}}$$

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \xleftarrow{/p} \boxed{-\frac{1}{1-p^{-1}z^{-1}}}$$

$p^{n-1}u(-n)$ $p^{n}u(-n)$

$$\boxed{\frac{z}{1-pz}} \xleftarrow{*z} \boxed{\frac{1}{1-pz}}$$

$p^{n-1}u(n-1)$ $p^{n}u(n)$

Simple Pole Form | Geometric Series Form

Geometric Series (2)

2 formulas

Simple Pole Form

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each

Geometric Series Form

$$\boxed{\frac{1}{z-p}} \to \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \xleftarrow{/z} \boxed{-\frac{p^{-1}z}{1-p^{-1}z}}$$

$p^{-n-1}u(n)$ $\qquad$ $p^{-n}u(n-1)$

$$\boxed{\frac{1}{z-p}} \to \boxed{\frac{z^{-1}}{1-pz^{-1}}} \xleftarrow{/p} \boxed{\frac{pz^{-1}}{1-pz^{-1}}}$$

$p^{-n-1}u(-n-1)$ $\qquad$ $p^{-n}u(-n-1)$

$$\boxed{\frac{1}{z^{-1}-p}} \to \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \xleftarrow{*z} \boxed{-\frac{p^{-1}z^{-1}}{1-p^{-1}z^{-1}}}$$

$p^{n-1}u(-n)$ $\qquad$ $p^{n}u(-n-1)$

$$\boxed{\frac{1}{z^{-1}-p}} \to \boxed{\frac{z}{1-pz}} \xleftarrow{/p} \boxed{\frac{pz}{1-pz}}$$

$p^{n-1}u(n-1)$ $\qquad$ $p^{n}u(n)$

Simple Pole Form | Geometric Series Form

Geometric Series Form Combinations
with a unit start term

$$-\frac{1}{1-\boxed{az}} \qquad -a^{n}\ u(n)$$

$$-\frac{1}{1-\boxed{az^{-1}}} \qquad -a^{-n}\ u(-n)$$

$$+\frac{1}{1-\boxed{a^{-1}z^{-1}}} \qquad a^{n}\ u(-n)$$

$$+\frac{1}{1-\boxed{a^{-1}z}} \qquad a^{-n}\ u(n)$$

$$-\frac{1}{1-\boxed{a^{-1}z}} \qquad -a^{-n}\ u(n)$$

$$-\frac{1}{1-\boxed{a^{-1}z^{-1}}} \qquad -a^{n}\ u(-n)$$

$$+\frac{1}{1-\boxed{az^{-1}}} \qquad a^{-n}\ u(-n)$$

$$+\frac{1}{1-\boxed{az}} \qquad a^{n}\ u(n)$$

Geometric Series Form Combinations
with a common-ratio start term

$$\boxed{+\frac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}} \quad a^{n}\ u(-n-1)$$

$$\boxed{+\frac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}} \quad a^{-n}\ u(n-1)$$

$$\boxed{-\frac{\boxed{az}}{1-\boxed{az}}} \quad -a^{n}\ u(n-1)$$

$$\boxed{-\frac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}}} \quad -a^{-n}\ u(-n-1)$$

$$\boxed{+\frac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}}} \quad a^{-n}\ u(-n-1)$$

$$\boxed{+\frac{\boxed{az}}{1-\boxed{az}}} \quad a^{n}\ u(n-1)$$

$$\boxed{-\frac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}} \quad -a^{-n}\ u(n-1)$$

$$\boxed{-\frac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}} \quad -a^{n}\ u(-n-1)$$

Geometric Series with a unit start term

Laurent Series

$a\,z$ $\quad$ $|z| < a^{-1}$

$$-\,(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$$

$a_n = -a^n \quad (n \geq 0)$

$a\,z^{\boxed{-1}}$ $\quad$ $|z| > a$

$$-\,(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$$

$$-\,\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$$

$a_n = -\left(\tfrac{1}{a}\right)^n \quad (n < 1)$

$a^{\boxed{-1}}\,z^{\boxed{-1}}$ $\quad$ $|z| > a^{-1}$

$$(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$$

$a_n = a^n \quad (n < 1)$

$a^{\boxed{-1}}\,z$ $\quad$ $|z| < a$

$$(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots\right)$$

$a_n = \left(\tfrac{1}{a}\right)^n \quad (n \geq 0)$

$a^{\boxed{-1}}\,z$ $\quad$ $|z| < a$

$$-\,(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$$

$$-\,\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots\right)$$

$a_n = -\left(\tfrac{1}{a}\right)^n \quad (n \geq 0)$

$a^{\boxed{-1}}\,z^{\boxed{-1}}$ $\quad$ $|z| > a^{-1}$

$$-\,(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$$

$a_n = -a^n \quad (n < 1)$

$a\,z^{\boxed{-1}}$ $\quad$ $|z| > a$

$$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$$

$a_n = \left(\tfrac{1}{a}\right)^n \quad (n < 1)$

$a\,z$ $\quad$ $|z| < a^{-1}$

$$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$$

$a_n = a^n \quad (n \geq 0)$

Geometric Series with a unit start term

z-Transform

$a z$	$\lvert z\rvert < a^{-1}$

$-(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + \cdots)$

$a_n = -a^{-n}$	$(-n \geq 0)$
$a_n = -(\frac{1}{a})^n$	$(n < 1)$

$a z^{-1}$	$\lvert z\rvert > a$

$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$a_n = -(\frac{1}{a})^{-n}$	$(-n < 1)$
$a_n = -a^n$	$(n \geq 0)$

$a^{-1} z^{-1}$	$\lvert z\rvert > a^{-1}$

$(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \cdots)$

$a_n = a^{-n}$	$(-n < 1)$
$a_n = (\frac{1}{a})^n$	$(n \geq 0)$

$a^{-1} z$	$\lvert z\rvert < a$

$(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

$a_n = (\frac{1}{a})^{-n}$	$(-n \geq 0)$
$a_n = a^n$	$(n < 1)$

$a^{-1} z$	$\lvert z\rvert < a$

$-(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

$a_n = -(\frac{1}{a})^{-n}$	$(-n \geq 0)$
$a_n = -a^n$	$(n < 1)$

$a^{-1} z^{-1}$	$\lvert z\rvert > a^{-1}$

$-(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^{-1} + (\frac{1}{a})^2 z^{-2} + \cdots)$

$a_n = -a^{-n}$	$(-n < 1)$
$a_n = -(\frac{1}{a})^n$	$(n \geq 0)$

$a z^{-1}$	$\lvert z\rvert > a$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$a_n = (\frac{1}{a})^{-n}$	$(-n < 1)$
$a_n = a^n$	$(n \geq 0)$

$a z$	$\lvert z\rvert < a^{-1}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^1 + (\frac{1}{a})^{-2} z^2 + \cdots)$

$a_n = a^{-n}$	$(-n \geq 0)$
$a_n = (\frac{1}{a})^n$	$(n < 1)$

Geometric Series with a unit start term

Laurent Series vs. z-Transform

$a\,z$	$\lvert z\rvert < a^{-1}$

$$-(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^{-1}z^1 + \left(\tfrac{1}{a}\right)^{-2}z^2 + \cdots\right)$$

Laurent Series	$a_n = -a^n$	$(n \geq 0)$
z-Transform	$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$

$a\,z^{\boxed{-1}}$	$\lvert z\rvert > a$

$$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \cdots\right)$$

Laurent Series	$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$
z-Transform	$a_n = -a^n$	$(n \geq 0)$

$a^{\boxed{-1}}z^{\boxed{-1}}$	$\lvert z\rvert > a^{-1}$

$$(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^1z^{-1} + \left(\tfrac{1}{a}\right)^2z^{-2} + \cdots\right)$$

Laurent Series	$a_n = a^n$	$(n < 1)$
z-Transform	$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$

$a^{\boxed{-1}}z$	$\lvert z\rvert < a$

$$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^1z^1 + \left(\tfrac{1}{a}\right)^2z^2 + \cdots\right)$$

Laurent Series	$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$
z-Transform	$a_n = a^n$	$(n < 1)$

$a^{\boxed{-1}}z$	$\lvert z\rvert < a$

$$-(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^1z^1 + \left(\tfrac{1}{a}\right)^2z^2 + \cdots\right)$$

Laurent Series	$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$
z-Transform	$a_n = -a^n$	$(n < 1)$

$a^{\boxed{-1}}z^{\boxed{-1}}$	$\lvert z\rvert > a^{-1}$

$$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^1z^{-1} + \left(\tfrac{1}{a}\right)^2z^{-2} + \cdots\right)$$

Laurent Series	$a_n = -a^n$	$(n < 1)$
z-Transform	$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

$a\,z^{\boxed{-1}}$	$\lvert z\rvert > a$

$$(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \cdots\right)$$

Laurent Series	$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$
z-Transform	$a_n = a^n$	$(n \geq 0)$

$a\,z$	$\lvert z\rvert < a^{-1}$

$$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0z^0 + \left(\tfrac{1}{a}\right)^{-1}z^1 + \left(\tfrac{1}{a}\right)^{-2}z^2 + \cdots\right)$$

Laurent Series	$a_n = a^n$	$(n \geq 0)$
z-Transform	$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

Geometric Series with a non-unit start term

Laurent Series

$\boxed{a^{-1} z^{-1}}$ $\boxed{|z| > a^{-1}}$

$(a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \cdots)$

$\boxed{a_n = a^n \quad (n < 0)}$

$\boxed{a^{-1} z}$ $\boxed{|z| < a}$

$(a^{-1} z^{1} + a^{-2} z^{2} + a^{-3} z^{3} + \cdots)$

$((\frac{1}{a})^{1} z^{1} + (\frac{1}{a})^{2} z^{2} + (\frac{1}{a})^{3} z^{3} + \cdots)$

$\boxed{a_n = (\frac{1}{a})^n \quad (n \geq 1)}$

$\boxed{a\, z}$ $\boxed{|z| < a^{-1}}$

$-(a^{1} z^{1} + a^{2} z^{2} + a^{3} z^{3} + \cdots)$

$\boxed{a_n = -a^n \quad (n \geq 1)}$

$\boxed{a\, z^{-1}}$ $\boxed{|z| > a}$

$-(a^{1} z^{-1} + a^{2} z^{-2} + a^{3} z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \cdots)$

$\boxed{a_n = -(\frac{1}{a})^n \quad (n < 0)}$

$\boxed{a\, z^{-1}}$ $\boxed{|z| > a}$

$(a^{1} z^{-1} + a^{2} z^{-2} + a^{3} z^{-3} + \cdots)$

$((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \cdots)$

$\boxed{a_n = (\frac{1}{a})^n \quad (n < 0)}$

$\boxed{a\, z}$ $\boxed{|z| < a^{-1}}$

$(a^{1} z^{1} + a^{2} z^{2} + a^{3} z^{3} + \cdots)$

$\boxed{a_n = a^n \quad (n \geq 1)}$

$\boxed{a^{-1} z}$ $\boxed{|z| < a}$

$-(a^{-1} z^{1} + a^{-2} z^{2} + a^{-3} z^{3} + \cdots)$

$-((\frac{1}{a})^{1} z^{1} + (\frac{1}{a})^{2} z^{2} + (\frac{1}{a})^{3} z^{3} + \cdots)$

$\boxed{a_n = -(\frac{1}{a})^n \quad (n \geq 1)}$

$\boxed{a^{-1} z^{-1}}$ $\boxed{|z| > a^{-1}}$

$-(a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \cdots)$

$\boxed{a_n = -a^n \quad (n < 0)}$

Geometric Series with a non-unit start term

z-Transform

$$a^{\boxed{-1}} z^{\boxed{-1}} \qquad |z| > a^{-1}$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^{1} z^{-1} + \left(\tfrac{1}{a}\right)^{2} z^{-2} + \left(\tfrac{1}{a}\right)^{3} z^{-3} + \cdots\right)$$

$a_n = a^{-n}$	$(-n < 0)$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 1)$

$$a^{\boxed{-1}} z \qquad |z| < a$$

$$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^{1} z^{1} + \left(\tfrac{1}{a}\right)^{2} z^{2} + \left(\tfrac{1}{a}\right)^{3} z^{3} + \cdots\right)$$

$a_n = \left(\frac{1}{a}\right)^{-n}$	$(-n \geq 1)$
$a_n = a^{n}$	$(n < 0)$

$$a\, z \qquad |z| < a^{-1}$$

$$-(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^{-1} z^{1} + \left(\tfrac{1}{a}\right)^{-2} z^{2} + \left(\tfrac{1}{a}\right)^{-3} z^{3} + \cdots\right)$$

$a_n = -a^{-n}$	$(-n \geq 1)$
$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n < 0)$

$$a\, z^{\boxed{-1}} \qquad |z| > a$$

$$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \left(\tfrac{1}{a}\right)^{-3} z^{-3} + \cdots\right)$$

$a_n = -\left(\frac{1}{a}\right)^{-n}$	$(-n < 0)$
$a_n = -a^{n}$	$(n \geq 1)$

$$a\, z^{\boxed{-1}} \qquad |z| > a$$

$$(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \left(\tfrac{1}{a}\right)^{-3} z^{-3} + \cdots\right)$$

$a_n = \left(\frac{1}{a}\right)^{-n}$	$(-n < 0)$
$a_n = a^{n}$	$(n \geq 1)$

$$a\, z \qquad |z| < a^{-1}$$

$$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^{-1} z^{1} + \left(\tfrac{1}{a}\right)^{-2} z^{2} + \left(\tfrac{1}{a}\right)^{-3} z^{3} + \cdots\right)$$

$a_n = a^{-n}$	$(-n \geq 1)$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n < 0)$

$$a^{\boxed{-1}} z \qquad |z| < a$$

$$-(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^{1} z^{1} + \left(\tfrac{1}{a}\right)^{2} z^{2} + \left(\tfrac{1}{a}\right)^{3} z^{3} + \cdots\right)$$

$a_n = -\left(\frac{1}{a}\right)^{-n}$	$(-n \geq 1)$
$a_n = -a^{n}$	$(n < 0)$

$$a^{\boxed{-1}} z^{\boxed{-1}} \qquad |z| > a^{-1}$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^{1} z^{-1} + \left(\tfrac{1}{a}\right)^{2} z^{-2} + \left(\tfrac{1}{a}\right)^{3} z^{-3} + \cdots\right)$$

$a_n = -a^{-n}$	$(-n < 0)$
$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n \geq 1)$

Geometric Series with a non-unit start term

Laurent Series vs. z-Transform

$a^{\boxed{-1}} z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$(a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \cdots)$

$((\frac{1}{a})^{1} z^{-1} + (\frac{1}{a})^{2} z^{-2} + (\frac{1}{a})^{3} z^{-3} + \cdots)$

Laurent Series	$a_n = a^n$	$(n < 0)$
z-Transform	$a_n = (\frac{1}{a})^n$	$(n \geq 1)$

$a^{\boxed{-1}} z$ $\quad |z| < a$

$(a^{-1} z^{1} + a^{-2} z^{2} + a^{-3} z^{3} + \cdots)$

$((\frac{1}{a})^{1} z^{1} + (\frac{1}{a})^{2} z^{2} + (\frac{1}{a})^{3} z^{3} + \cdots)$

Laurent Series	$a_n = (\frac{1}{a})^n$	$(n \geq 1)$
z-Transform	$a_n = a^n$	$(n < 0)$

$a\, z$ $\quad |z| < a^{-1}$

$-(a^{1} z^{1} + a^{2} z^{2} + a^{3} z^{3} + \cdots)$

$-((\frac{1}{a})^{-1} z^{1} + (\frac{1}{a})^{-2} z^{2} + (\frac{1}{a})^{-3} z^{3} + \cdots)$

Laurent Series	$a_n = -a^n$	$(n \geq 1)$
z-Transform	$a_n = -(\frac{1}{a})^n$	$(n < 0)$

$a\, z^{\boxed{-1}}$ $\quad |z| > a$

$-(a^{1} z^{-1} + a^{2} z^{-2} + a^{3} z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \cdots)$

Laurent Series	$a_n = -(\frac{1}{a})^n$	$(n < 0)$
z-Transform	$a_n = -a^n$	$(n \geq 1)$

$a\, z^{\boxed{-1}}$ $\quad |z| > a$

$(a^{1} z^{-1} + a^{2} z^{-2} + a^{3} z^{-3} + \cdots)$

$((\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + (\frac{1}{a})^{-3} z^{-3} + \cdots)$

Laurent Series	$a_n = (\frac{1}{a})^n$	$(n < 0)$
z-Transform	$a_n = a^n$	$(n \geq 1)$

$a\, z$ $\quad |z| < a^{-1}$

$(a^{1} z^{1} + a^{2} z^{2} + a^{3} z^{3} + \cdots)$

$((\frac{1}{a})^{-1} z^{1} + (\frac{1}{a})^{-2} z^{2} + (\frac{1}{a})^{-3} z^{3} + \cdots)$

Laurent Series	$a_n = a^n$	$(n \geq 1)$
z-Transform	$a_n = (\frac{1}{a})^n$	$(n < 0)$

$a^{\boxed{-1}} z$ $\quad |z| < a$

$-(a^{-1} z^{1} + a^{-2} z^{2} + a^{-3} z^{3} + \cdots)$

$-((\frac{1}{a})^{1} z^{1} + (\frac{1}{a})^{2} z^{2} + (\frac{1}{a})^{3} z^{3} + \cdots)$

Laurent Series	$a_n = -(\frac{1}{a})^n$	$(n \geq 1)$
z-Transform	$a_n = -a^n$	$(n < 0)$

$a^{\boxed{-1}} z^{\boxed{-1}}$ $\quad |z| > a^{-1}$

$-(a^{-1} z^{-1} + a^{-2} z^{-2} + a^{-3} z^{-3} + \cdots)$

$-((\frac{1}{a})^{1} z^{-1} + (\frac{1}{a})^{2} z^{-2} + (\frac{1}{a})^{3} z^{-3} + \cdots)$

Laurent Series	$a_n = -a^n$	$(n < 0)$
z-Transform	$a_n = -(\frac{1}{a})^n$	$(n \geq 1)$

Complemnt ROC Pairs -
Original Geometric Series Form Combinations

unit	$-\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$-a^n$ $(n \geq 0)$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-(\frac{1}{a})^n$ $(n<1)$
non-unit	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	a^n $(n<0)$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$(\frac{1}{a})^n$ $(n \geq 1)$

unit	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	a^n $(n<1)$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$(\frac{1}{a})^n$ $(n \geq 0)$
non-unit	$-\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$-a^n$ $(n \geq 1)$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$-(\frac{1}{a})^n$ $(n<0)$

unit	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-(\frac{1}{a})^n$ $(n \geq 0)$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ $(n<1)$
non-unit	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$(\frac{1}{a})^n$ $(n<0)$	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n $(n \geq 1)$

unit	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$(\frac{1}{a})^n$ $(n<1)$	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	a^n $(n \geq 0)$
non-unit	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-(\frac{1}{a})^n$ $(n \geq 1)$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^n$ $(n<0)$

start term

Complemnt ROC Pairs -
Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$-a^{n+1}$ $(n\geq 0)$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$	$-(\frac{1}{a})^{n-1}$ $(n<1)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	a^{n+1} $(n<0)$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$(\frac{1}{a})^{n-1}$ $(n\geq 1)$

$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	a^{n+1} $(n<0)$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$(\frac{1}{a})^{n-1}$ $(n\geq 1)$
$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$-a^{n+1}$ $(n\geq 0)$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$	$-(\frac{1}{a})^{n-1}$ $(n<1)$

$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-(\frac{1}{a})^{n+1}$ $(n\geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^{n-1}$ $(n<1)$
$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$(\frac{1}{a})^{n+1}$ $(n<0)$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$	a^{n-1} $(n\geq 1)$

$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$(\frac{1}{a})^{n+1}$ $(n<0)$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$	a^{n-1} $(n\geq 1)$
$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-(\frac{1}{a})^{n+1}$ $(n\geq 0)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-a^{n-1}$ $(n<1)$

Complemnt ROC Pairs - Reduced
Shifted Geometric Series Form Combinations

$-\dfrac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$-a^{n+1}$ $(n \geq 0)$	$-\dfrac{a}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\left(\dfrac{1}{a}\right)^{n-1}$ $(n < 1)$
$\dfrac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	a^{n+1} $(n < 0)$	$\dfrac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\left(\dfrac{1}{a}\right)^{n-1}$ $(n \geq 1)$

$-\dfrac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\left(\dfrac{1}{a}\right)^{n+1}$ $(n \geq 0)$	$-\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-a^{n-1}$ $(n < 1)$
$\dfrac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\left(\dfrac{1}{a}\right)^{n+1}$ $(n < 0)$	$\dfrac{z}{1-az}$ $\lvert z\rvert < a^{-1}$	a^{n-1} $(n \geq 1)$

$$\boxed{2z} \qquad \boxed{2z^{-1}} \qquad \boxed{2^{-1}z^{-1}} \qquad \boxed{2^{-1}z}$$

$$\boxed{|z|<0.5} \qquad \boxed{|z|>2} \qquad \boxed{|z|>0.5} \qquad \boxed{|z|<2}$$

$$-\frac{2}{1-2z} \xleftrightarrow{z^{-1}} -\frac{2}{1-2z^{-1}}$$

$$\cdot\frac{(2z)^{-1}}{(2z)^{-1}} \Big\downarrow \Big\uparrow \cdot\frac{(2z)}{(2z)} \qquad\qquad \cdot\frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}} \Big\downarrow \Big\uparrow \cdot\frac{(2z^{-1})}{(2z^{-1})}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} \xleftrightarrow{z^{-1}} +\frac{z}{1-0.5z}$$

		scale(a)		scale(a)
	$-\frac{1}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$

		scale(1/z)		scale(z)
	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert < a$
Comp.ROC	$-\frac{az}{1-az}$ $\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert > a$

		scale(1/a)		scale(1/a)
	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{az}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$

		scale(1/z)		scale(z)
	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert > a$	$\frac{1}{1-az}$ $\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert < a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert > a^{-1}$

	scale(a)		scale(a)	
	$-\frac{1}{1-az}$ $\quad |z|<a^{-1}$	$-\frac{a}{1-az}$ $\quad |z|<a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\quad |z|>a$	$-\frac{a}{1-az^{-1}}$ $\quad |z|>a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\quad |z|<a$	$\frac{z}{1-a^{-1}z}$ $\quad |z|<a$

	scale(1/z)		scale(z)	
	$\frac{1}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\quad |z|<a$	$\frac{z}{1-a^{-1}z}$ $\quad |z|<a$
Comp.ROC	$-\frac{az}{1-az}$ $\quad |z|<a^{-1}$	$-\frac{a}{1-az}$ $\quad |z|<a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\quad |z|>a$	$\frac{a}{1-az^{-1}}$ $\quad |z|>a$

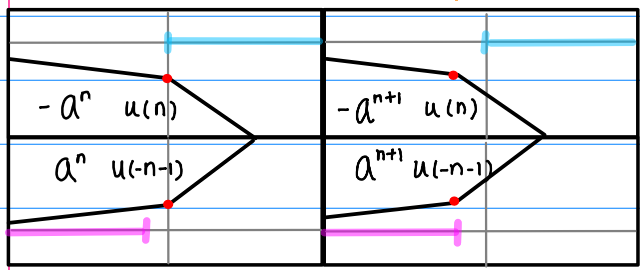
	scale(1/a)		scale(1/a)	
	$-\frac{1}{1-a^{-1}z}$ $\quad |z|<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\quad |z|<a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\quad |z|>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\quad |z|>a$	$\frac{az}{1-az}$ $\quad |z|<a^{-1}$	$\frac{z}{1-az}$ $\quad |z|<a^{-1}$

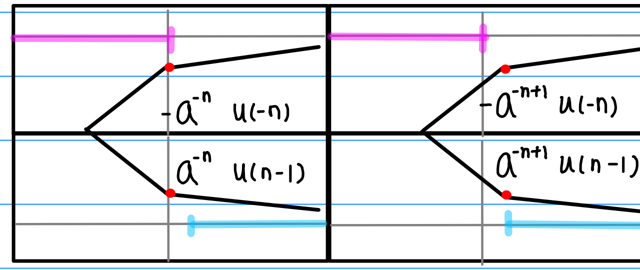
	scale(1/z)		scale(z)	
	$\frac{1}{1-az^{-1}}$ $\quad |z|>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\quad |z|>a$	$\frac{1}{1-az}$ $\quad |z|<a^{-1}$	$\frac{z}{1-az}$ $\quad |z|<a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\quad |z|<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\quad |z|<a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$

SHL.Seq

SHR.Seq

-Comp.Rng

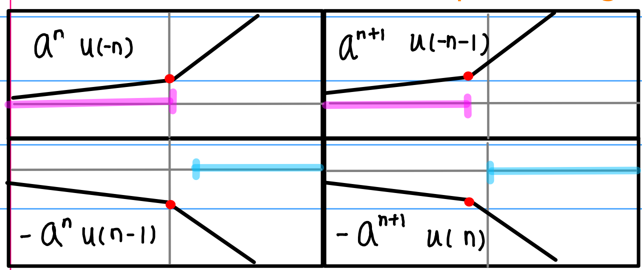


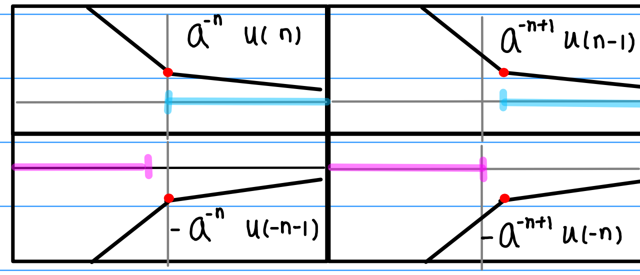


SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

-Comp.Rng

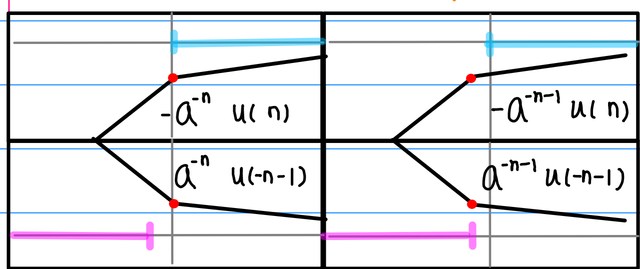


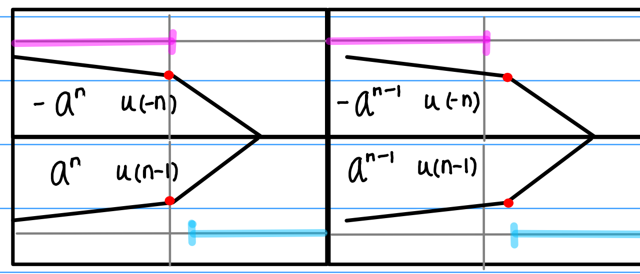


SHL.Seq

SHR.Seq

-Comp.Rng

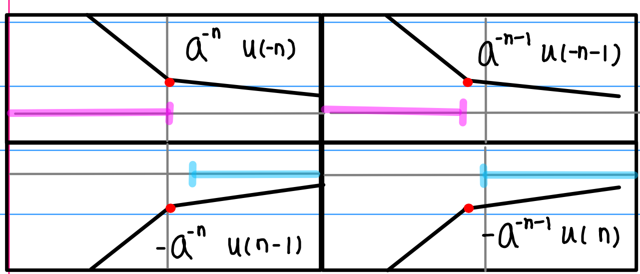


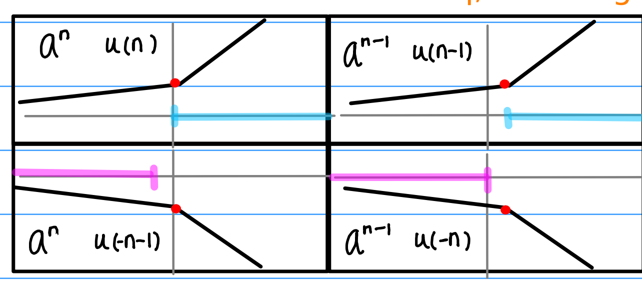


SHL.Seq, SHL.Rng

SHR.Seq, SHR.Rng

-Comp.Rng





Left Shifted Sequence

Right Shifted Sequence

		scale(a)		scale(a)
	$-\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$-\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$

		scale(1/z)		scale(z)
	$\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$\frac{z}{1-a^{-1}z}$ $\lvert z\rvert<a$
Comp.ROC	$-\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$-\frac{a}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{a}{1-az^{-1}}$ $\lvert z\rvert>a$

		scale(1/a)		scale(1/a)
	$-\frac{1}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{1}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{az}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$

		scale(1/z)		scale(z)
	$\frac{1}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{z^{-1}}{1-az^{-1}}$ $\lvert z\rvert>a$	$\frac{1}{1-az}$ $\lvert z\rvert<a^{-1}$	$\frac{z}{1-az}$ $\lvert z\rvert<a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}}{1-a^{-1}z}$ $\lvert z\rvert<a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\lvert z\rvert>a^{-1}$

SHL.Seq

	$-a^n \quad (n \geq 0)$	$-a^{n+1} \quad (n \geq 0)$
	$-(a^0, a^1, a^2, \cdots)$	$-(a^1, a^2, a^3, \cdots)$
-Comp.Rng	$a^n \quad (n < 0)$	$a^{n+1} \quad (n < 0)$
	$(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$

SHR.Seq

	$-(\frac{1}{a})^n \quad (n < 1)$	$-(\frac{1}{a})^{n-1} \quad (n < 1)$
	$-(\cdots, a^2, a^1, a^0)$	$-(\cdots, a^3, a^2, a^1)$
-Comp.Rng	$(\frac{1}{a})^n \quad (n \geq 1)$	$(\frac{1}{a})^{n-1} \quad (n \geq 1)$
	$(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3} \cdots)$	$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2} \cdots)$

SHL.Seq, SHL.Rng

	$a^n \quad (n < 1)$	$a^{n+1} \quad (n < 0)$
	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$
-Comp.Rng	$-a^n \quad (n \geq 1)$	$-a^{n+1} \quad (n \geq 0)$
	$-(a^1, a^2, a^3, \cdots)$	$-(a^1, a^2, a^3, \cdots)$

SHR.Seq, SHR.Rng

	$(\frac{1}{a})^n \quad (n \geq 0)$	$(\frac{1}{a})^{n-1} \quad (n \geq 1)$
	$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2} \cdots)$	$(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2} \cdots)$
-Comp.Rng	$-(\frac{1}{a})^n \quad (n < 0)$	$-(\frac{1}{a})^{n-1} \quad (n < 1)$
	$(\cdots, a^3, a^2, a^1)$	$(\cdots, a^3, a^2, a^1)$

SHL.Seq

	$-(\frac{1}{a})^n \quad (n \geq 0)$	$-(\frac{1}{a})^{n+1} \quad (n \geq 0)$
	$-(\frac{1}{a^0}, \frac{1}{a^1}, \frac{1}{a^2} \cdots)$	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3} \cdots)$
-Comp.Rng	$(\frac{1}{a})^n \quad (n < 0)$	$(\frac{1}{a})^{n+1} \quad (n < 0)$
	$(\cdots, a^3, a^2, a^1)$	$(\cdots, a^2, a^1, a^0)$

SHR.Seq

	$-a^n \quad (n < 1)$	$-a^{n-1} \quad (n < 1)$
	$-(\cdots, \frac{1}{a^2}, \frac{1}{a^1}, \frac{1}{a^0})$	$-(\cdots, \frac{1}{a^2}, \frac{1}{a^3}, \frac{1}{a^1})$
-Comp.Rng	$a^n \quad (n \geq 1)$	$a^{n-1} \quad (n \geq 1)$
	$(a^1, a^2, a^3, \cdots)$	$(a^0, a^1, a^2, \cdots)$

SHL.Seq, SHL.Rng

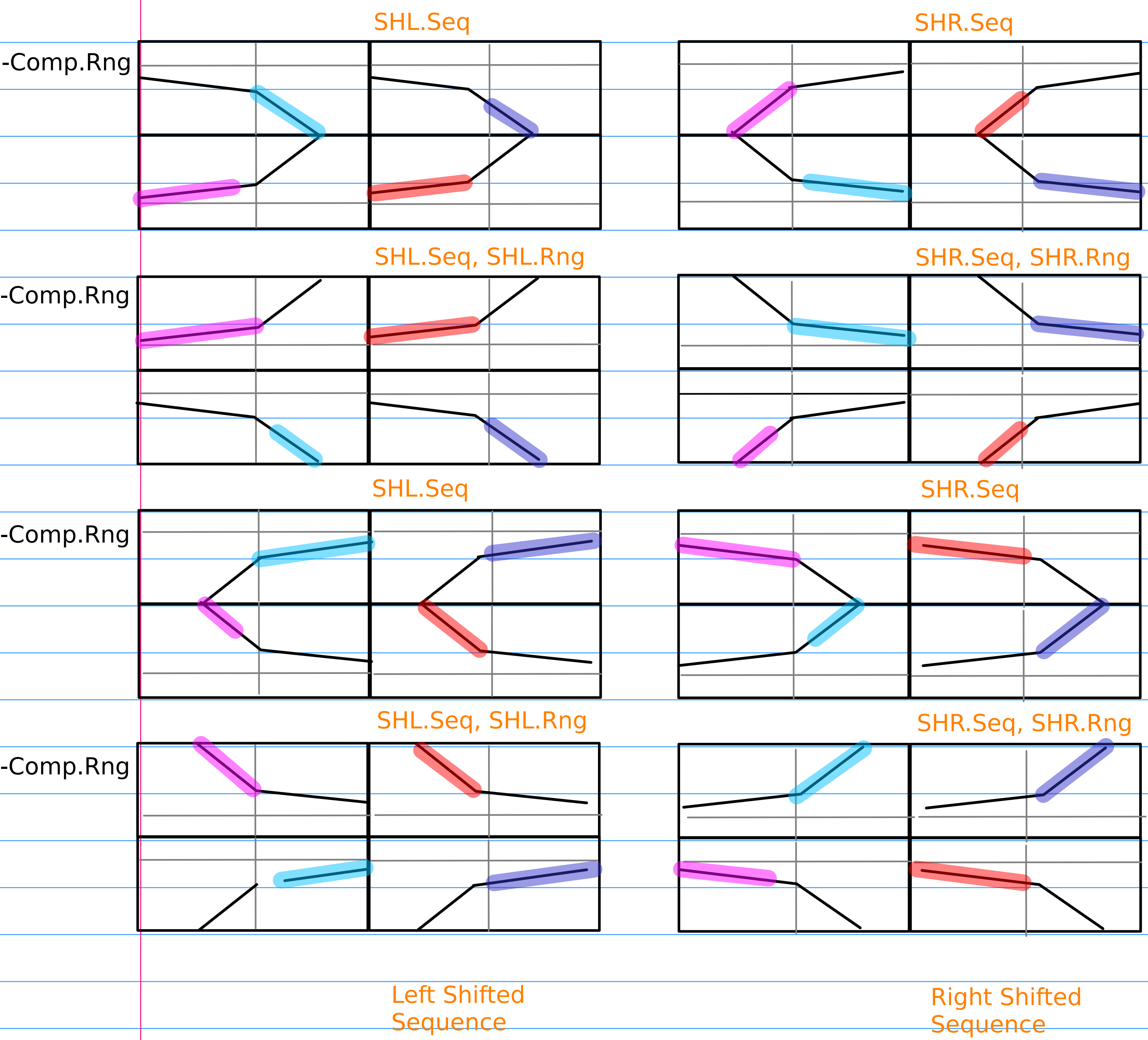
	$(\frac{1}{a})^n \quad (n < 1)$	$(\frac{1}{a})^{n+1} \quad (n < 0)$
	$(\cdots, a^2, a^1, a^0)$	$(\cdots, a^2, a^1, a^0)$
-Comp.Rng	$-(\frac{1}{a})^n \quad (n \geq 1)$	$-(\frac{1}{a})^{n+1} \quad (n \geq 0)$
	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3} \cdots)$	$-(\frac{1}{a^1}, \frac{1}{a^2}, \frac{1}{a^3} \cdots)$

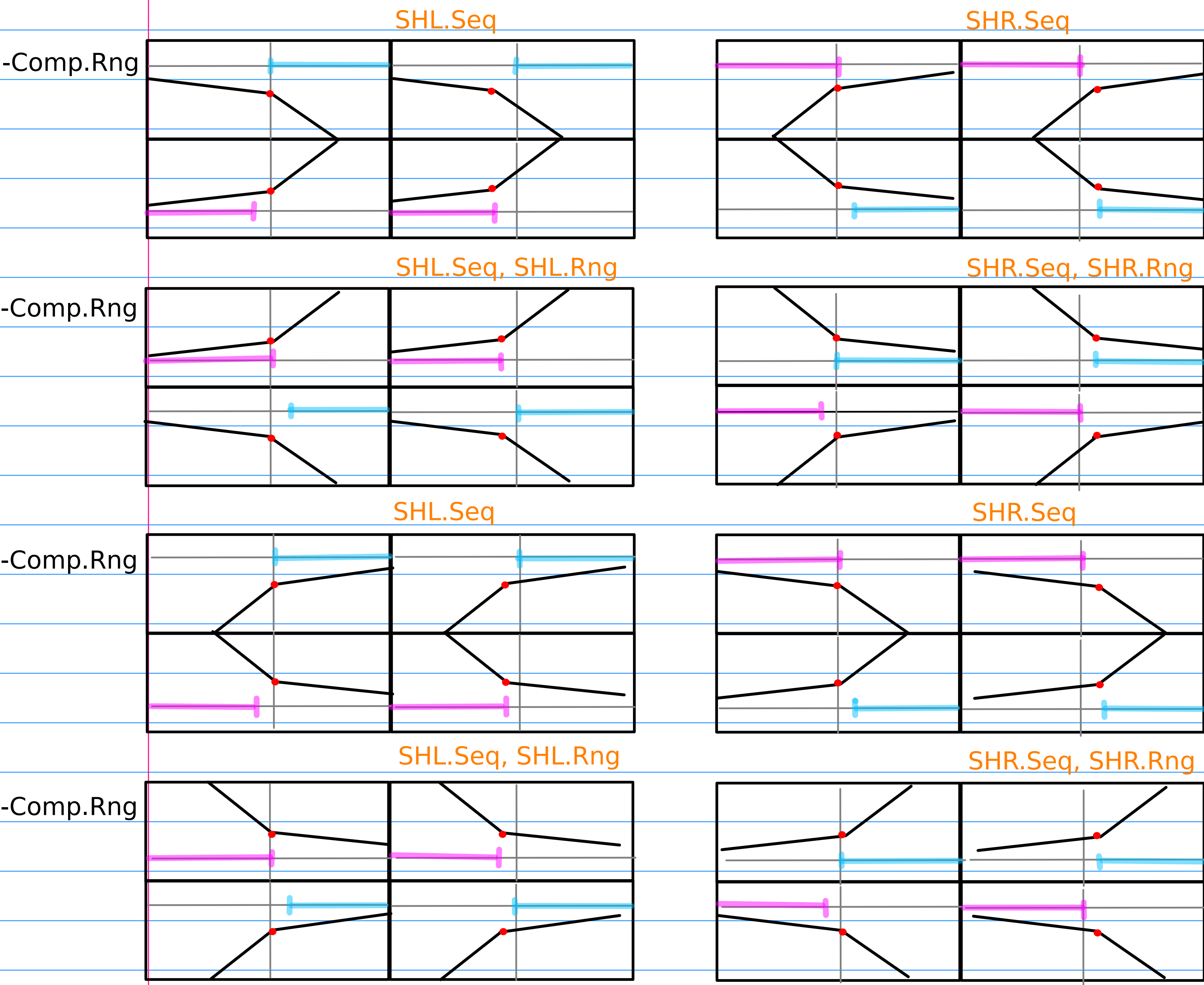
SHR.Seq, SHR.Rng

	$a^n \quad (n \geq 0)$	$a^{n-1} \quad (n \geq 1)$
	$(a^0, a^1, a^2, \cdots)$	$(a^0, a^1, a^2, \cdots)$
-Comp.Rng	$-a^n \quad (n < 0)$	$-a^{n-1} \quad (n < 1)$
	$-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$	$-(\cdots, \frac{1}{a^3}, \frac{1}{a^2}, \frac{1}{a^1})$

Left Shifted Sequence

Right Shifted Sequence


SHL.Seq
SHR.Seq
-Comp.Rng
SHL.Seq, SHL.Rng
SHR.Seq, SHR.Rng
-Comp.Rng
SHL.Seq
SHR.Seq
-Comp.Rng
SHL.Seq, SHL.Rng
SHR.Seq, SHR.Rng
-Comp.Rng
Left Shifted Sequence
Right Shifted Sequence



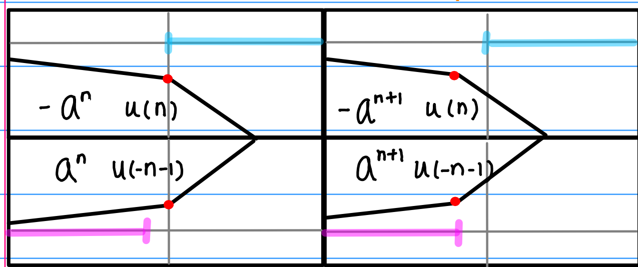
Left Shifted Sequence

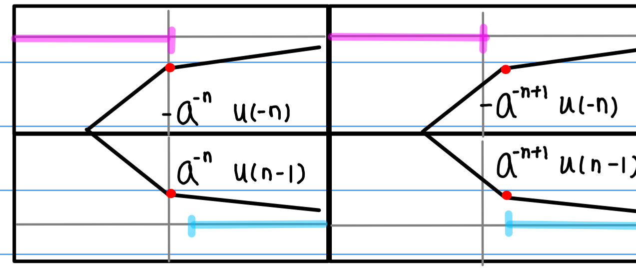
Right Shifted Sequence

SHL.Seq

SHR.Seq

-Comp.Rng





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SHR.Seq, SHR.Rng

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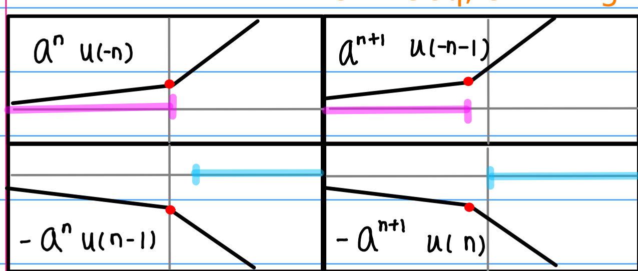

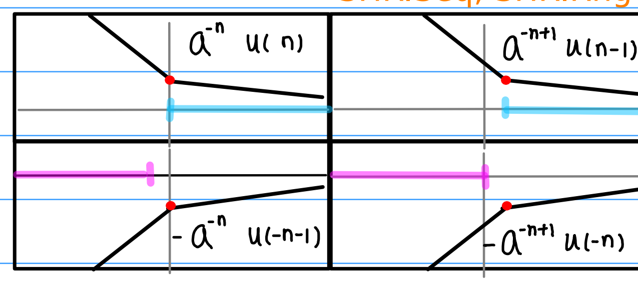

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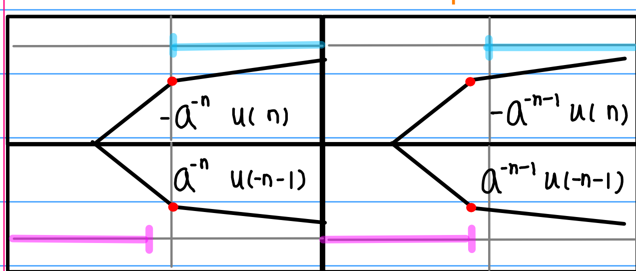

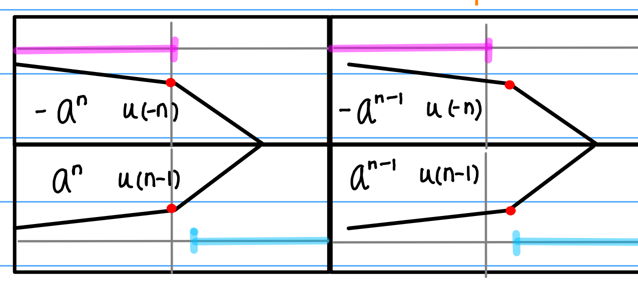

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SHR.Seq, SHR.Rng

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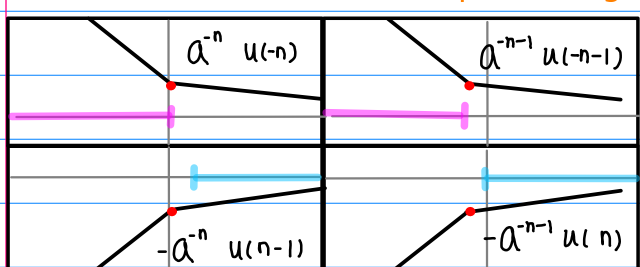
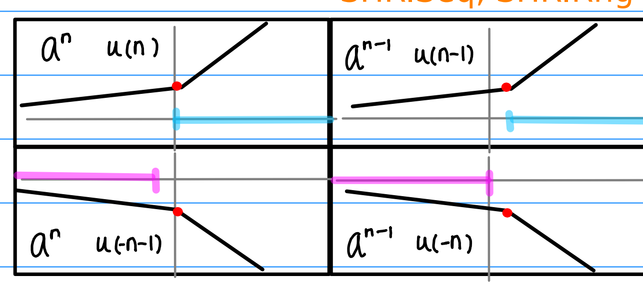




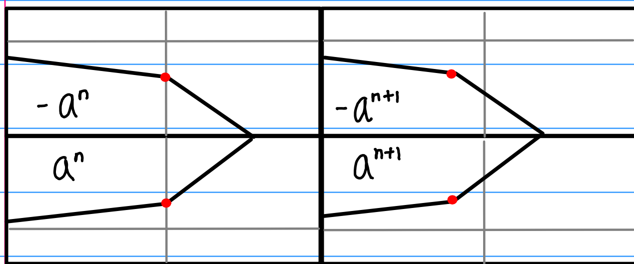
Left Shifted Sequence

Right Shifted Sequence

a Sequence Function

-Comp.Rng

SHL.Seq



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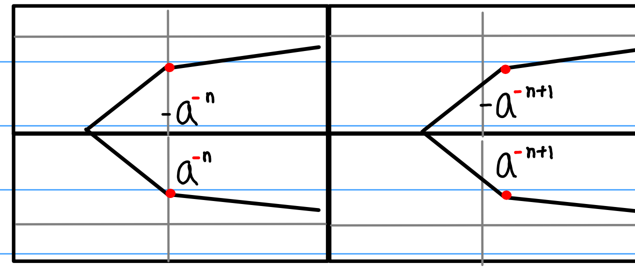

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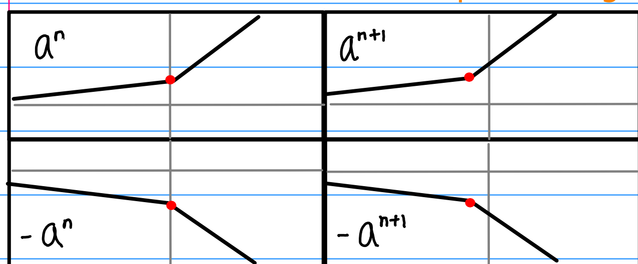

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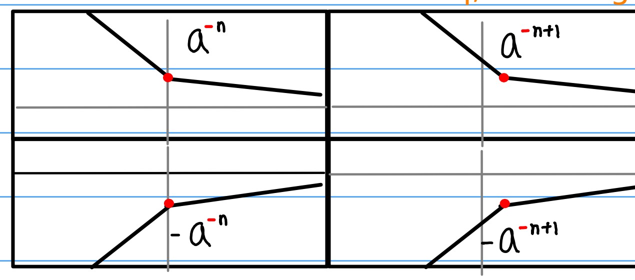

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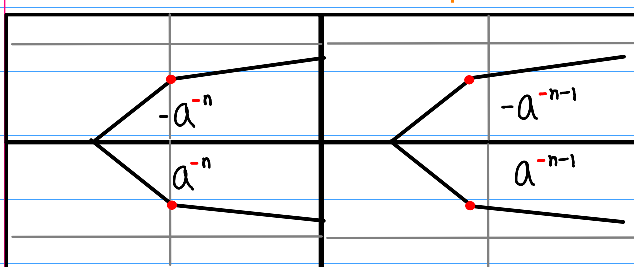

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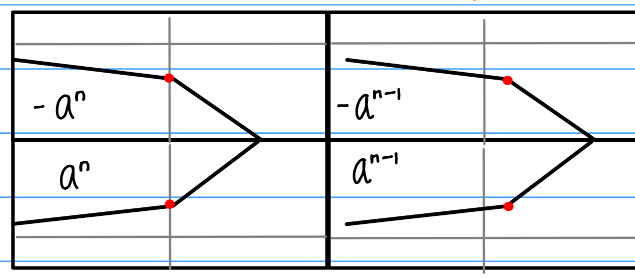

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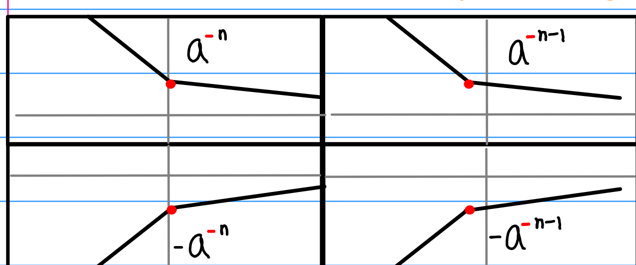

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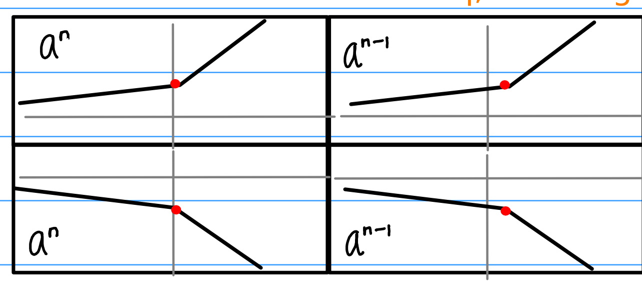


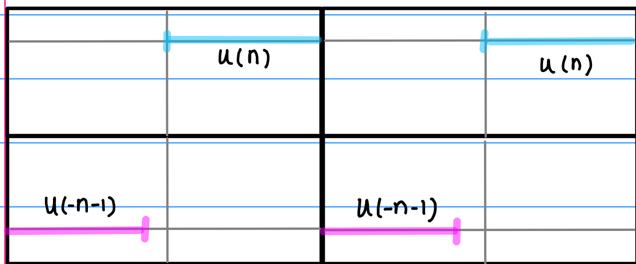
Left Shifted Sequence

Right Shifted Sequence

Range of a Sequence

-Comp.Rng

SHL.Seq



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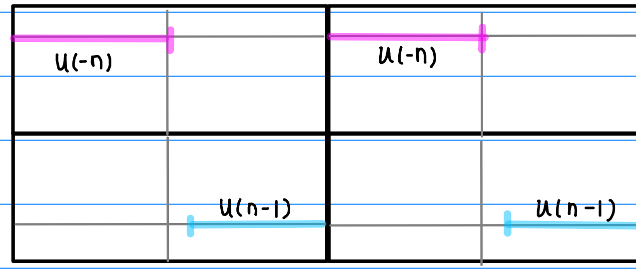

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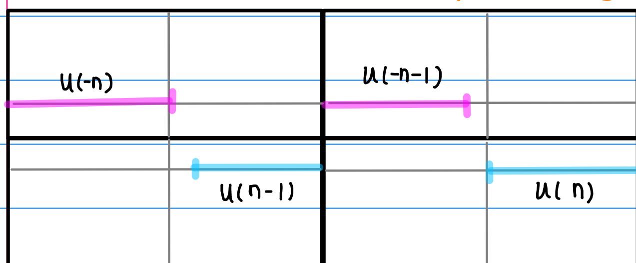

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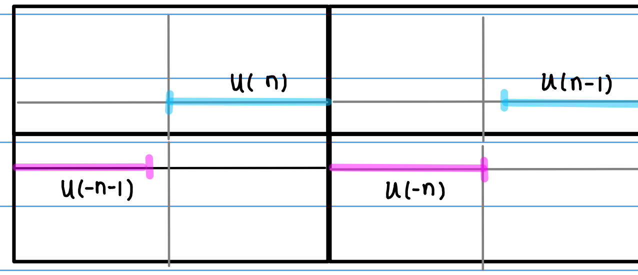

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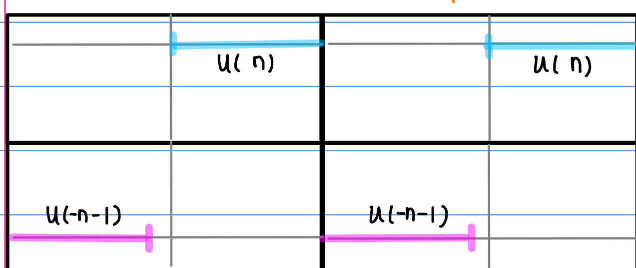

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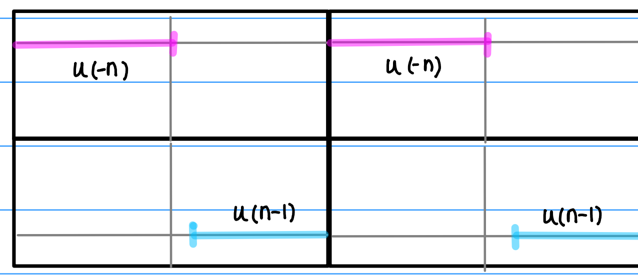

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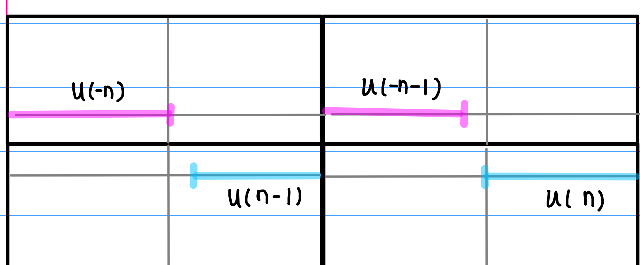
SHL.Seq, SHL.Rng



SHR.Seq, SHR.Rng