

# Variable Block Adder (1C)

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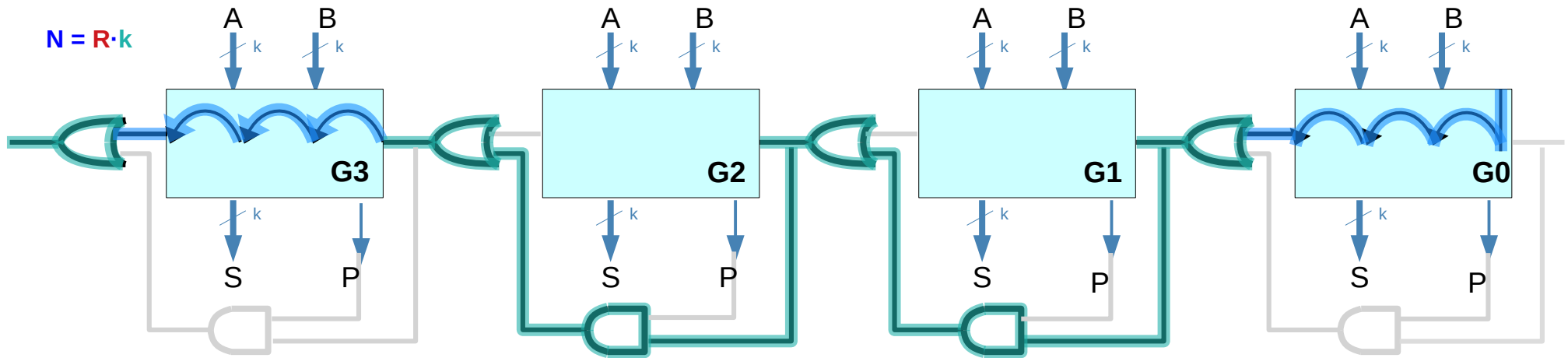
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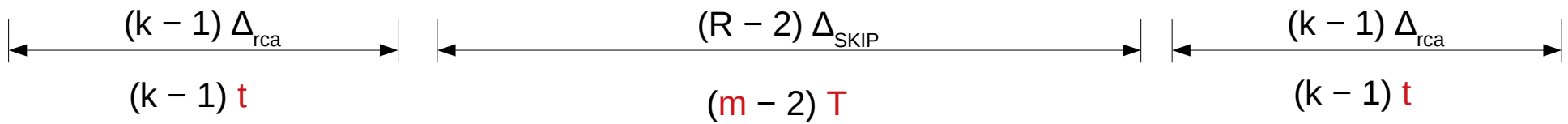
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# Carry Skip Adder



Fixed block size =  $k$  bits

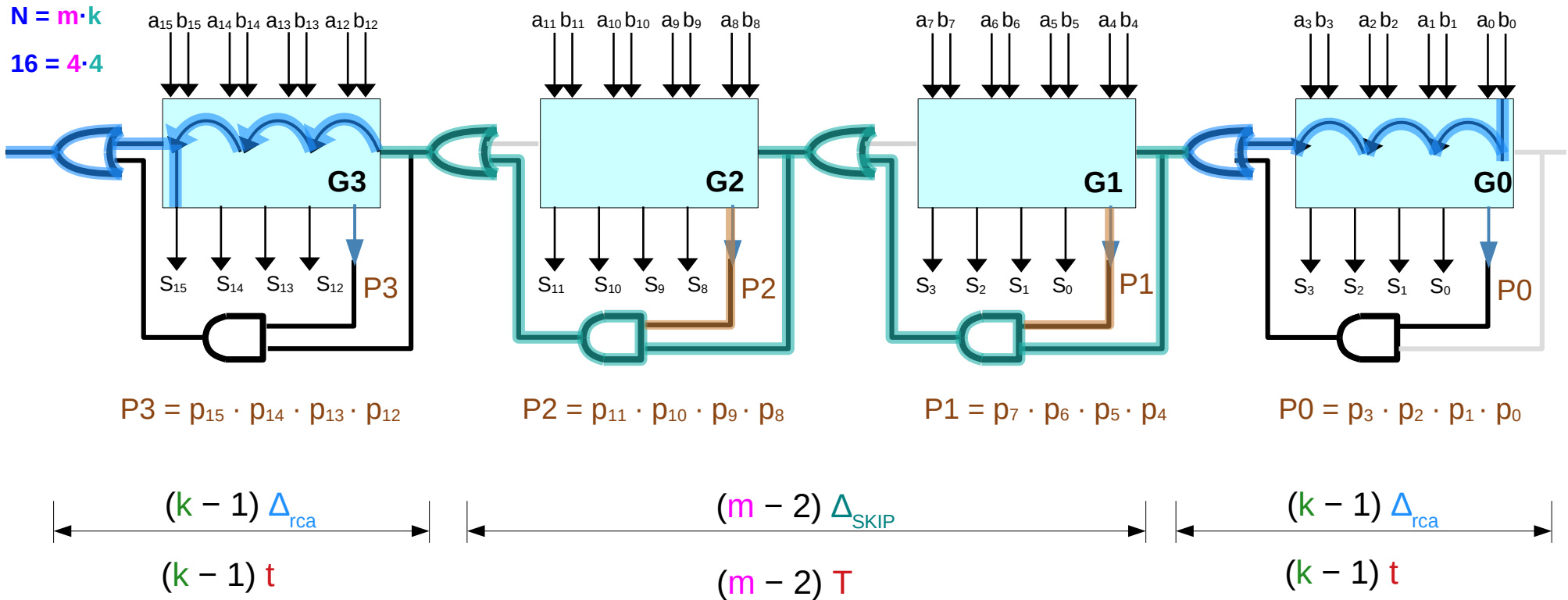


Variable block size =  $x_i$  bits for the  $i$ -th group

$$(x_i - 1) t \qquad (m - 2) T \qquad (x_j - 1) t$$

$t$  denote the time required for a carry signal to ripple across a bit  
 $T$  denote the time required for the signal to skip over a group of bits  
 $m$  denotes the optimal number of groups for an  $n$ -bit carry chain

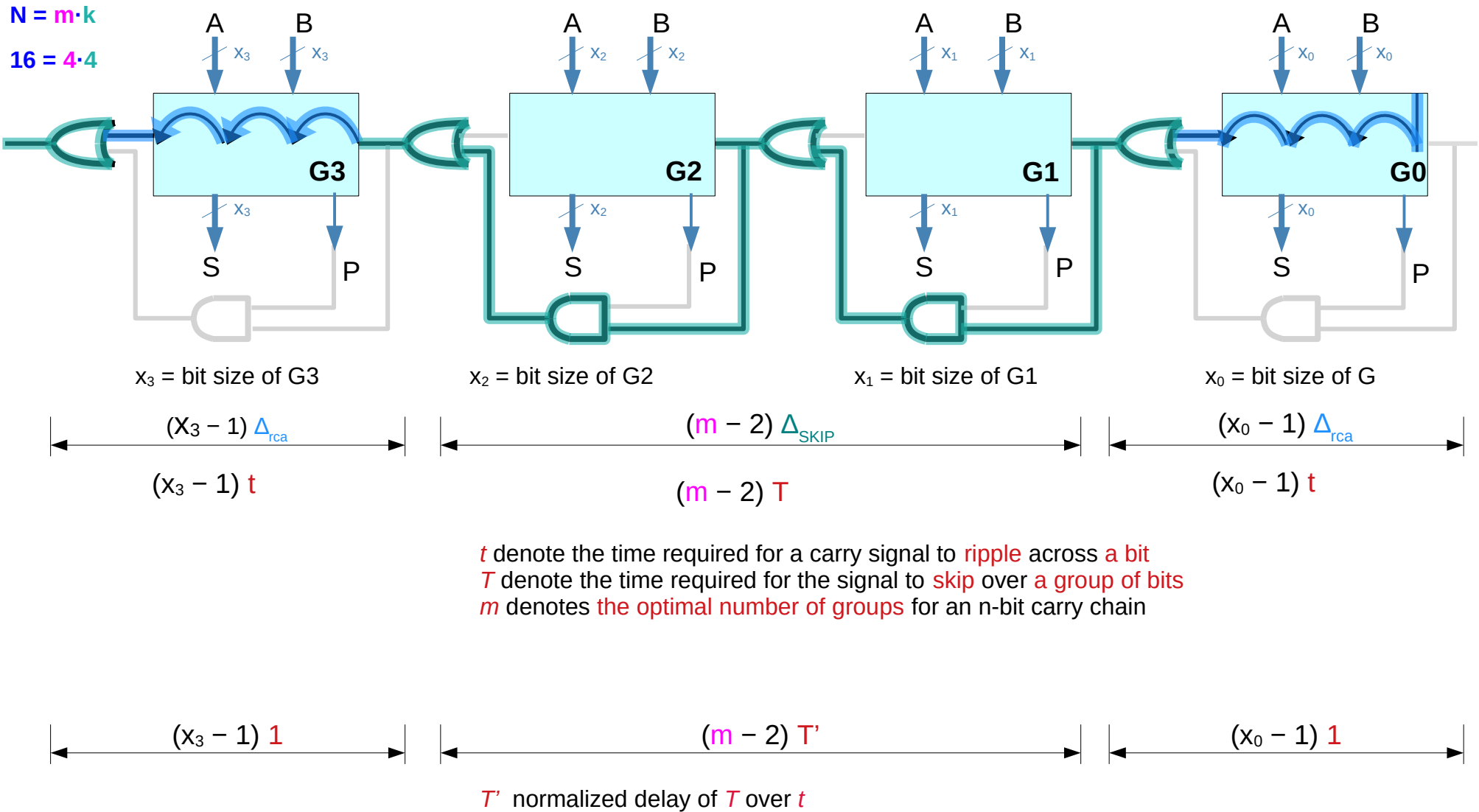
# Carry Skip Adder – fixed block size



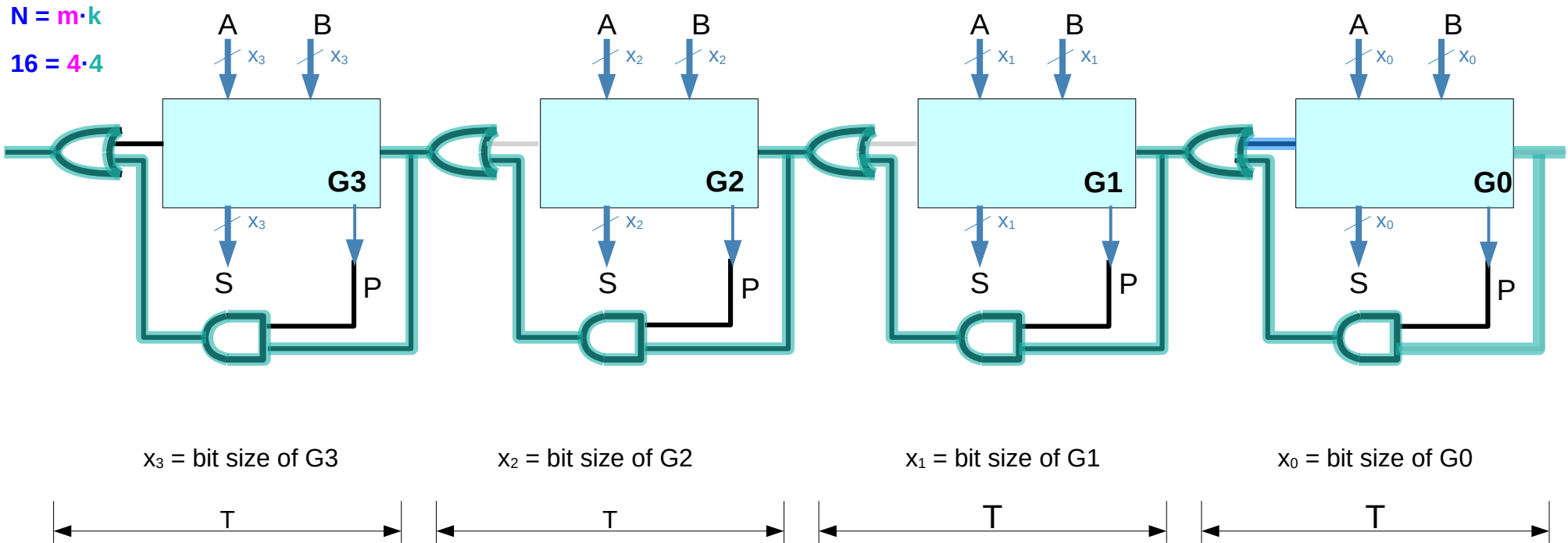
$t$  denote the time required for a carry signal to ripple across a bit  
 $T$  denote the time required for the signal to skip over a group of bits  
 $m$  denotes the optimal number of groups for an  $n$ -bit carry chain

Fixed Block Size  $\Rightarrow$  delay(P3) = delay(P2) = delay(P1) = delay(P0) = Fixed Delay

# Carry Skip Adder – maximum carry delay (3)

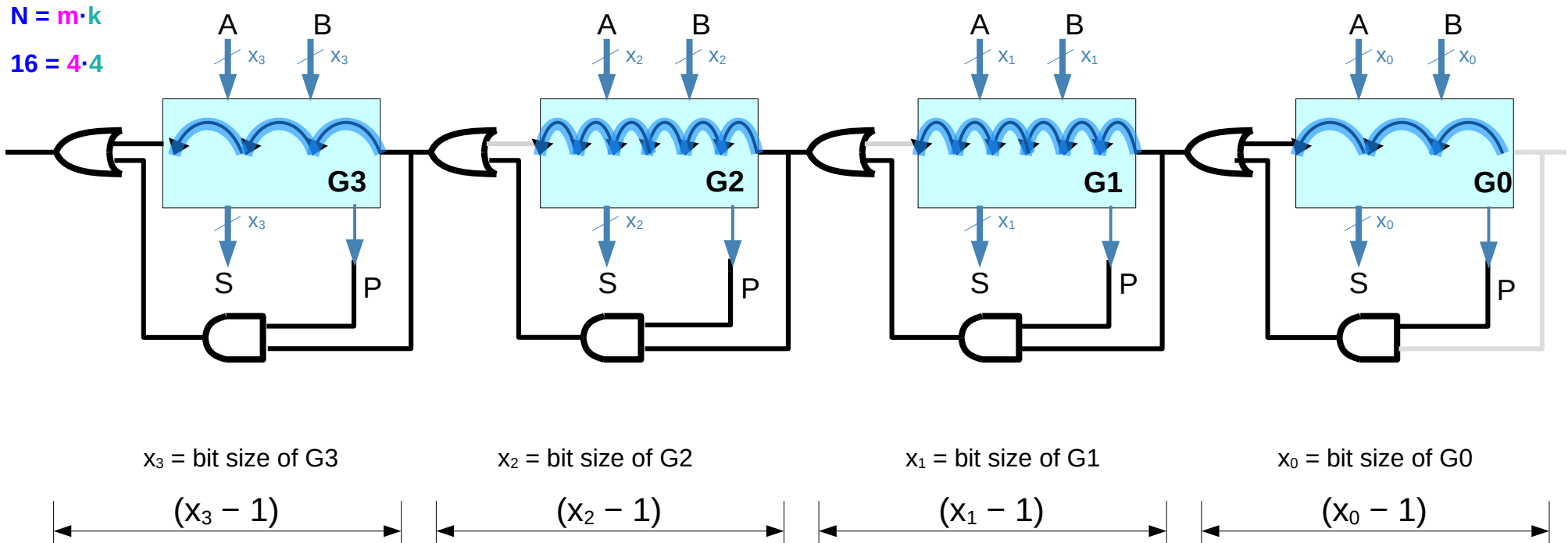


# Carry Skip Adder – maximum carry delay (3)



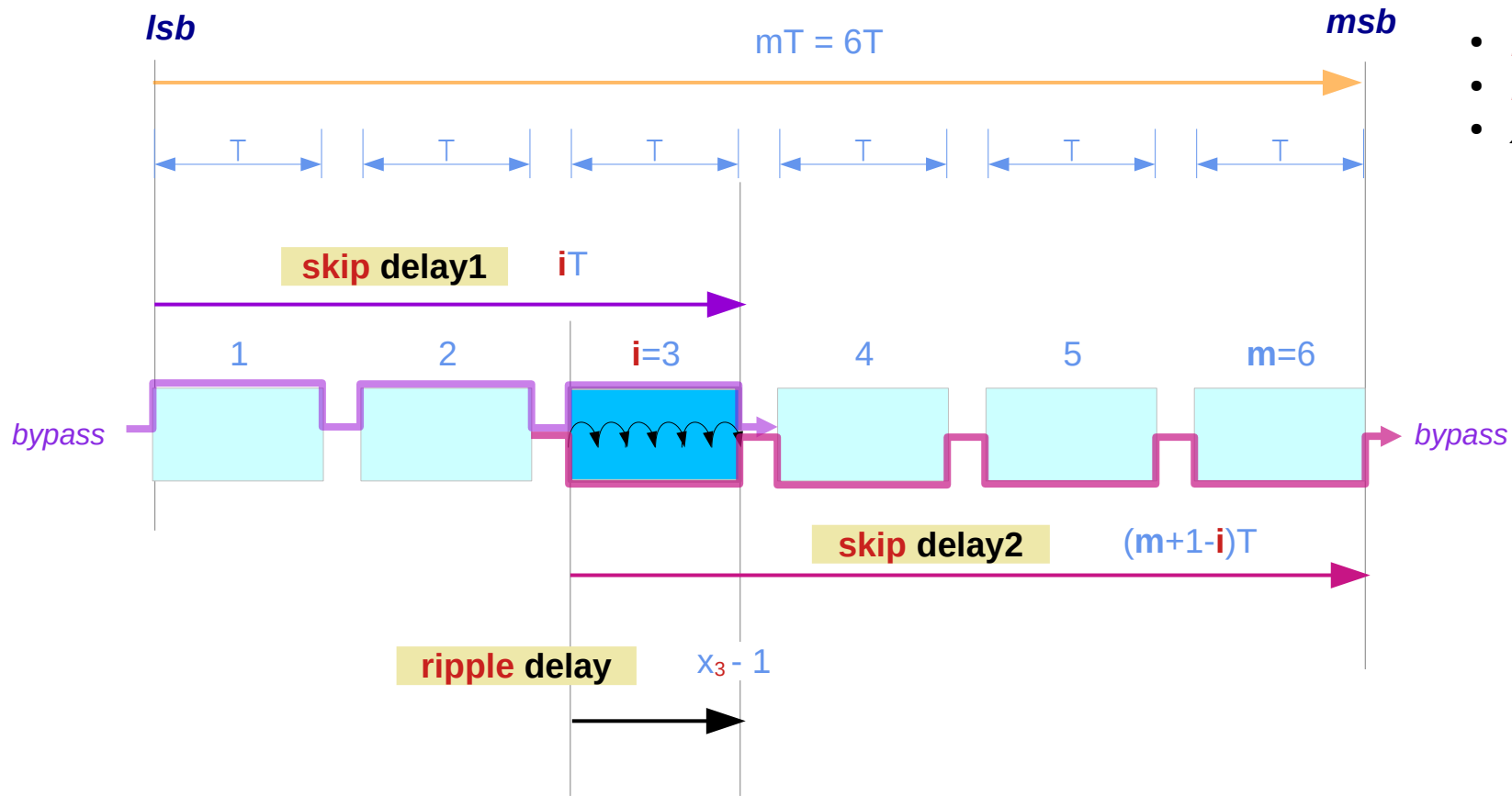
Carry Skip Delays

# Carry Skip Adder – maximum carry delay (3)



Carry Ripple delays

# Skip path delays and ripple delays



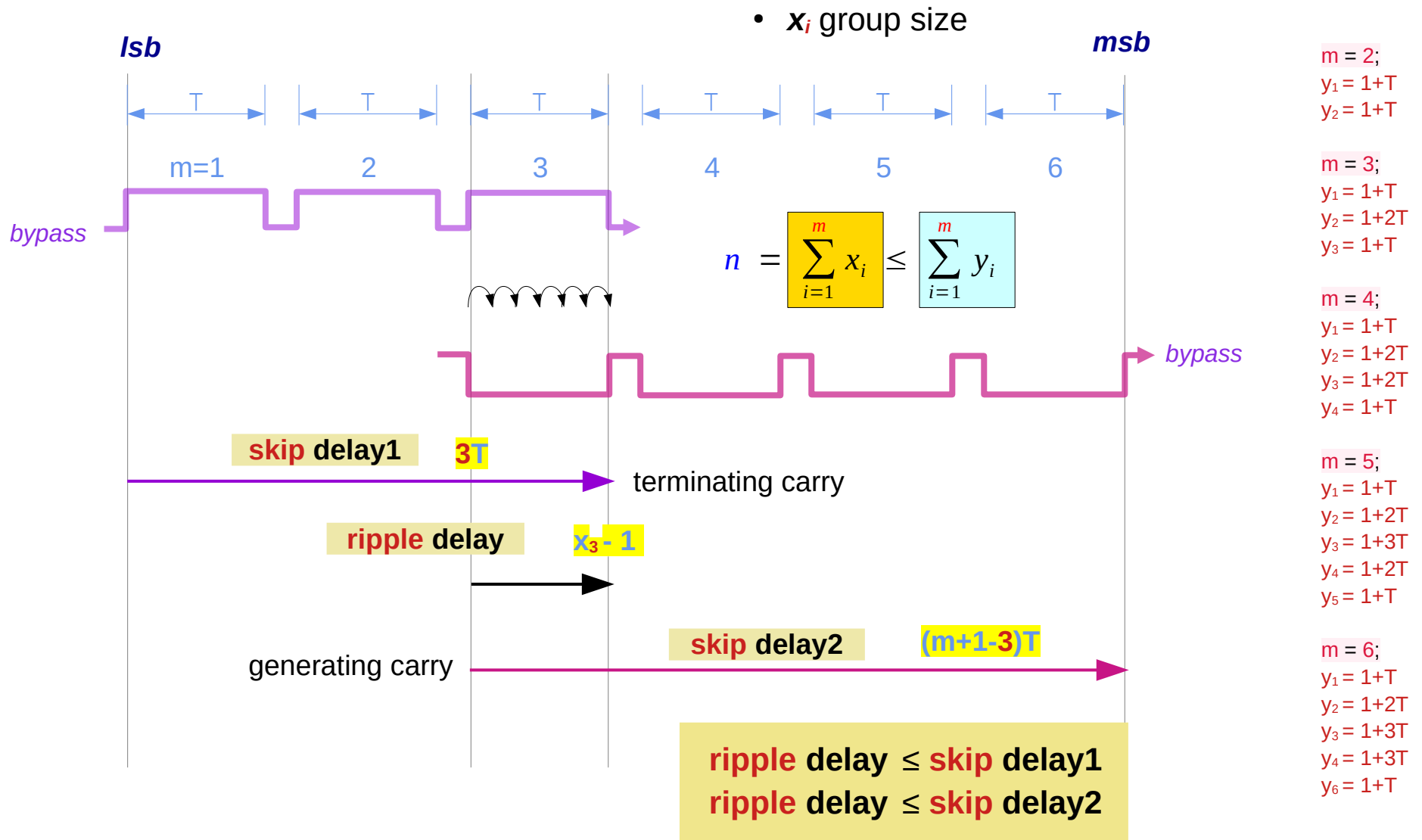
- $n$  bits
- $m$  groups
- $x_i$  group size

$\text{ripple delay} \leq \text{skip delay1}$   
 $\text{ripple delay} \leq \text{skip delay2}$

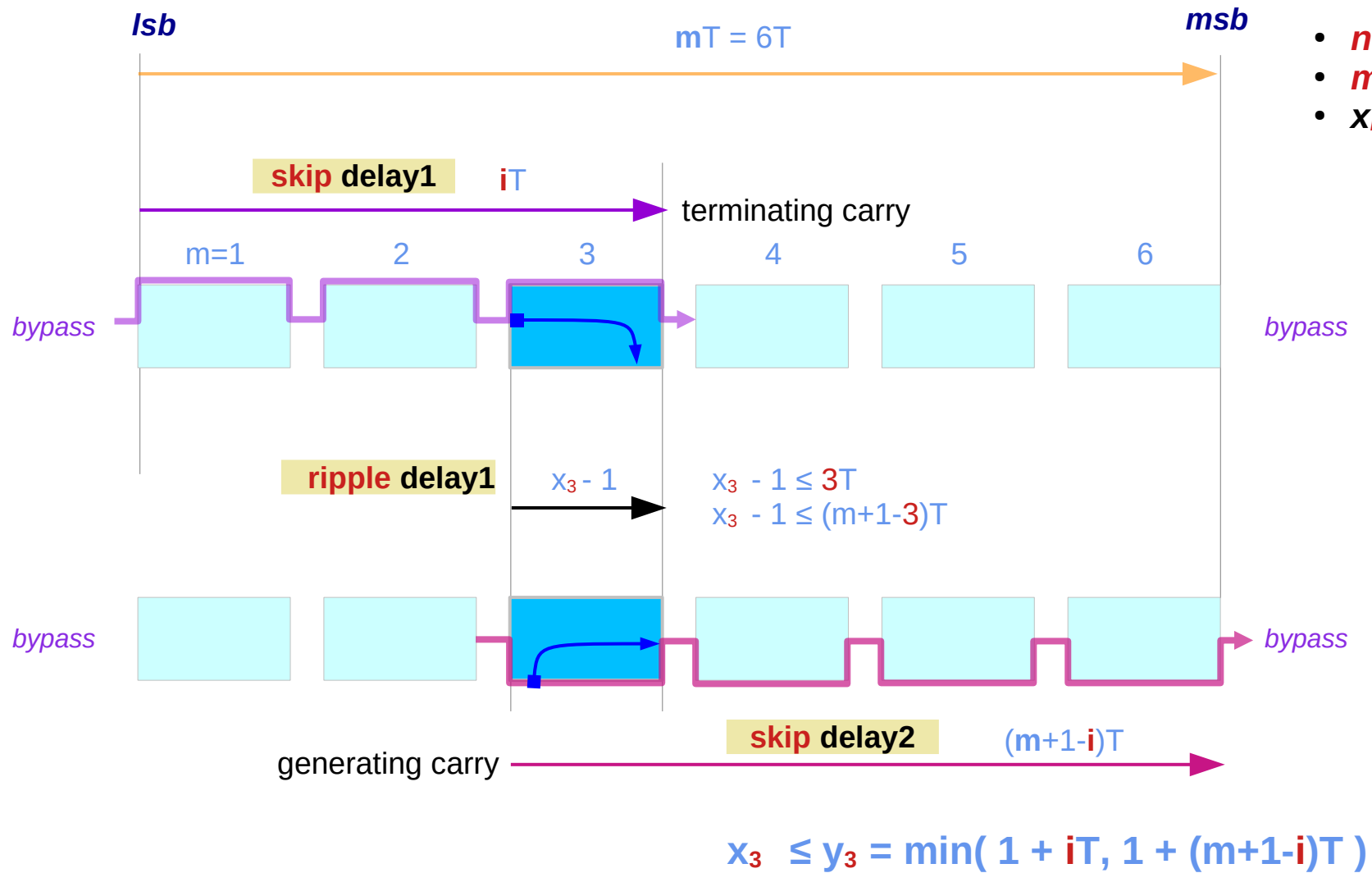
$\text{ripple delay} \leq \min(\text{skip delay1}, \text{skip delay2})$



# Overlapping Delay Paths



# Minimum skip path delay ( $y_i - 1$ ) of the $i^{th}$ group



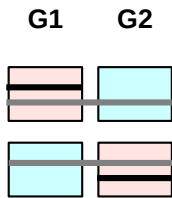
- $n$  bits
- $m$  groups
- $x_i$  group size

# Minimum skip path delay of the $i^{th}$ group (1)

$m = 2;$

$$\min(T, 2T) = T$$

$$\min(2T, T) = T$$

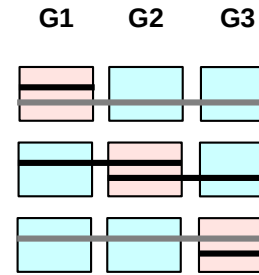


$m = 3;$

$$\min(T, 3T) = T$$

$$\min(2T, 2T) = 2T$$

$$\min(3T, T) = T$$



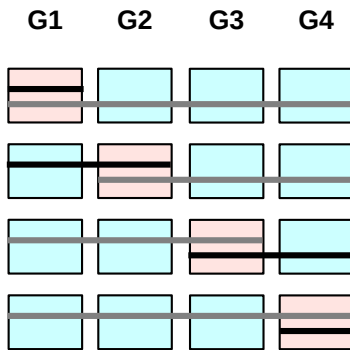
$m = 4;$

$$\min(T, 4T) = T$$

$$\min(2T, 3T) = 2T$$

$$\min(3T, 2T) = 2T$$

$$\min(4T, T) = T$$



$m = 5;$

$$\min(T, 5T) = T$$

$$\min(2T, 4T) = 2T$$

$$\min(3T, 3T) = 3T$$

$$\min(4T, 2T) = 2T$$

$$\min(5T, T) = T$$

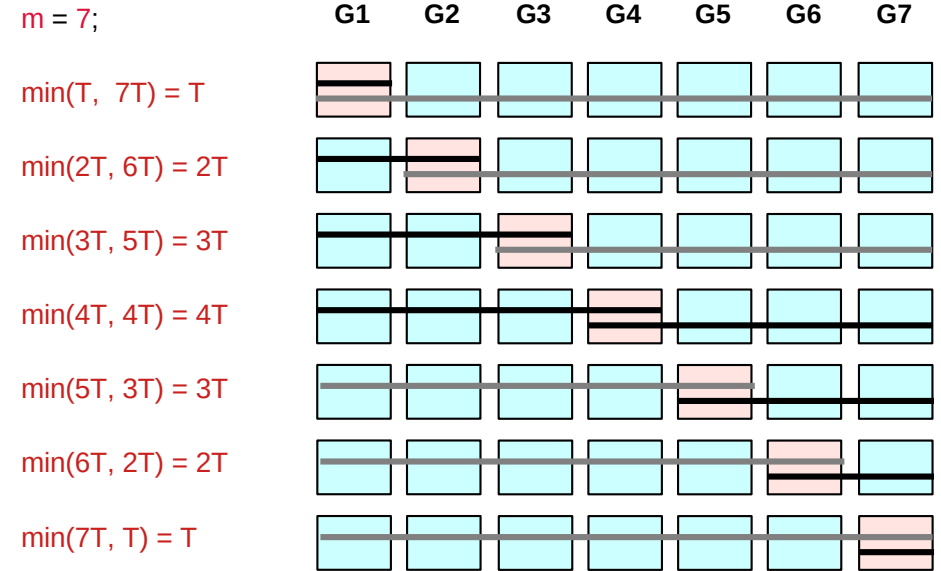
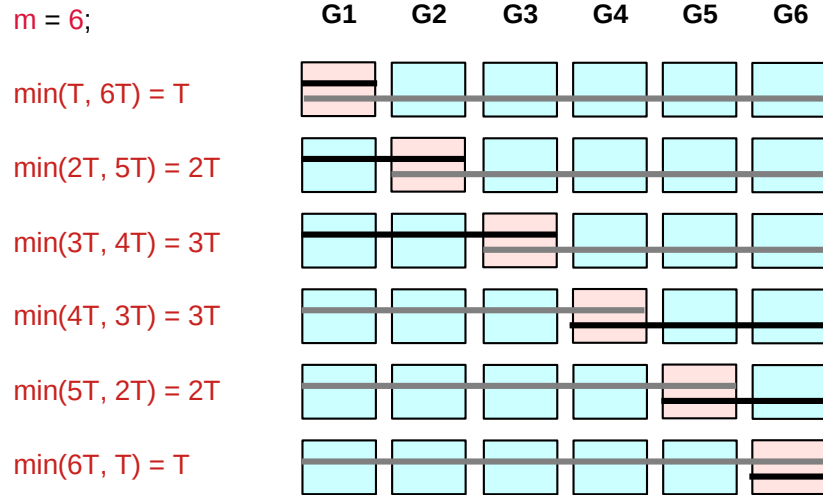


$$y_i = 1 + \min\{iT, (m+1-i)T\}$$



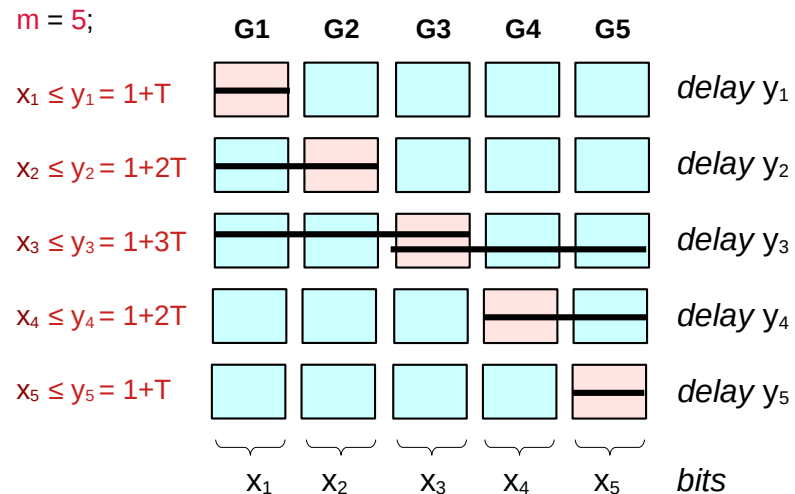
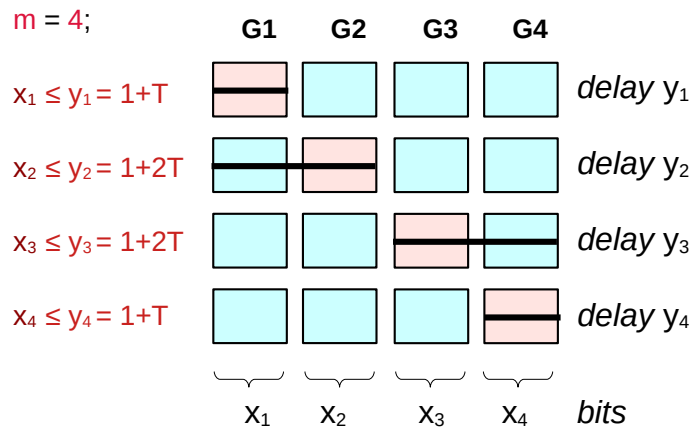
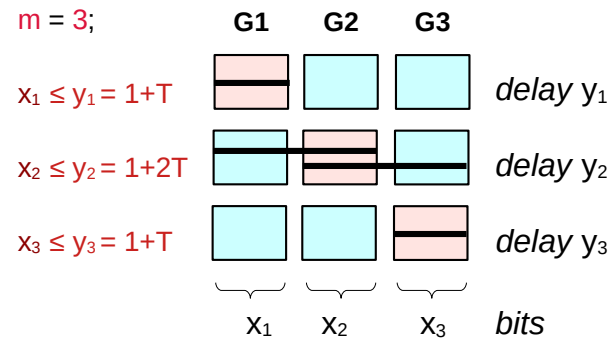
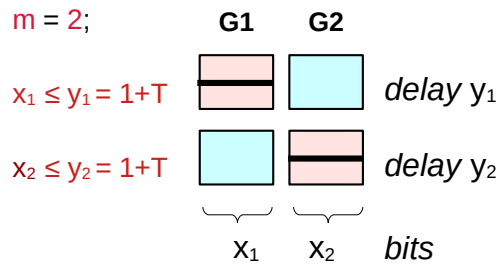
$$y_i = \min\{1+iT, 1+(m+1-i)T\}, \quad i = 1, \dots, m$$

# Minimum skip path delay of the $i^{th}$ group (2)

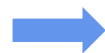


$$y_i = 1 + \min\{iT, (m+1-i)T\} \rightarrow y_i = \min\{1+iT, 1+(m+1-i)T\}, \quad i = 1, \dots, m$$

# The $i^{th}$ group has $x_i$ bits for a given $m$ (1)

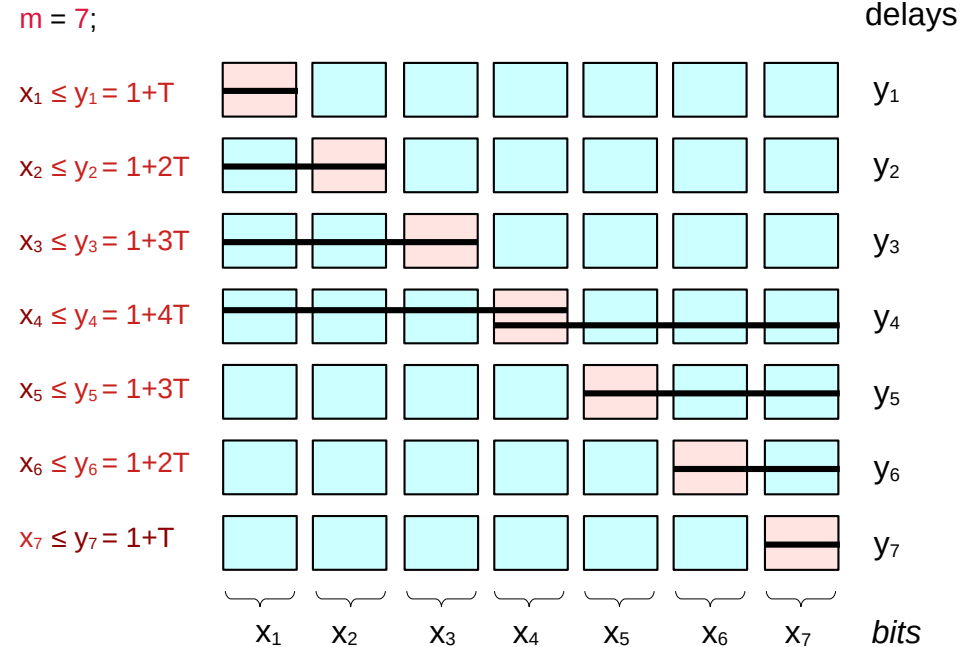
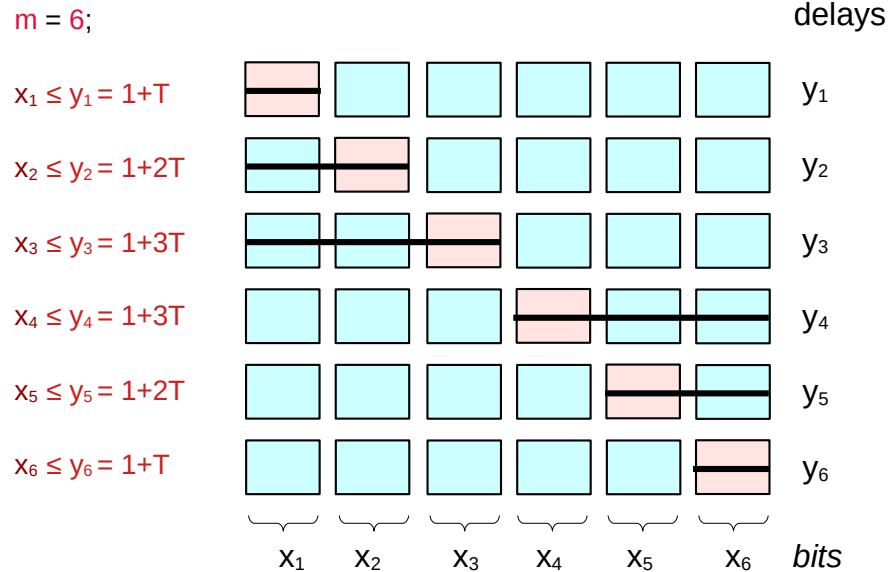


$$x_i - 1 \leq \min( iT, (m+1-i)T )$$

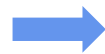


$$x_i \leq y_i = \min( 1 + iT, 1 + (m+1-i)T )$$

# The $i^{th}$ group has $x_i$ bits for a given $m$ (2)



$$x_i - 1 \leq \min( iT, (m+1-i)T )$$



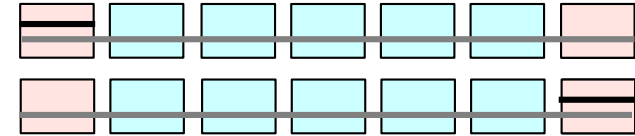
$$x_i \leq y_i = \min( 1 + iT, 1 + (m+1-i)T )$$

# Symmetric histograms of $y_i$ 's

$$y_1 = \min\{1+1\cdot T, 1+(m+1-1)T\} = 1+T$$

$$y_m = \min\{1+m\cdot T, 1+(m+1-m)T\} = 1+T$$

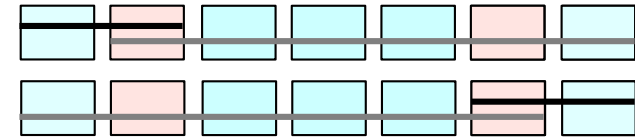
$$\begin{aligned} x_1 &\leq y_1 = 1+T \quad (\text{bits}) \\ x_m &\leq y_m = 1+T \quad (\text{bits}) \end{aligned}$$



$$y_2 = \min\{1+2\cdot T, 1+(m+1-2)T\} = 1+2T$$

$$y_{m-1} = \min\{1+(m-1)\cdot T, 1+(m+1-(m-1))T\} = 1+2T$$

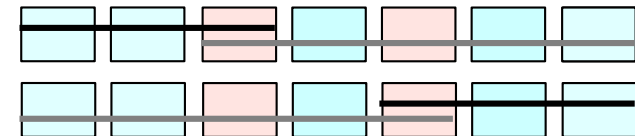
$$\begin{aligned} x_2 &\leq y_2 = 1+2T \quad (\text{bits}) \\ x_{m-1} &\leq y_{m-1} = 1+2T \quad (\text{bits}) \end{aligned}$$



$$y_3 = \min\{1+3\cdot T, 1+(m+1-3)T\} = 1+3T$$

$$y_{m-2} = \min\{1+(m-2)\cdot T, 1+(m+1-(m-2))T\} = 1+3T$$

$$\begin{aligned} x_3 &\leq y_3 = 1+3T \quad (\text{bits}) \\ x_{m-2} &\leq y_{m-2} = 1+3T \quad (\text{bits}) \end{aligned}$$



# Procedure

(I) Let  $m$  be the smallest positive integer

$$n \leq \sum_{i=1}^m y_i \quad i = 1, \dots, m$$

(II) Let

$$y_i = \min\{1+iT, 1+(m+1-i)T\}$$

(III) Let  $x_i, i = 1, \dots, m$

starting with the first row, row by row

$$n = \sum_{i=1}^m x_i \leq \sum_{i=1}^m y_i$$

Variable block size =  $x_i$  bits for the  $i$ -th group

find the smallest  $m$

$$n \leq \sum_{i=1}^m y_i = \sum_{i=1}^m \min\{1+iT, 1+(m+1-i)T\}$$

$m = 2$ ;  
while  $(y_1 + \dots + y_m < n)$   $m = m + 1$  ;

$m = 2$ ;	$m = 4$ ;	$m = 5$ ;	$m = 6$ ;	$m = 7$ ;
$y_1 = 1+T$	$y_1 = 1+T$	$y_1 = 1+T$	$y_1 = 1+T$	$y_1 = 1+T$
$y_2 = 1+T$	$y_2 = 1+2T$	$y_2 = 1+2T$	$y_2 = 1+2T$	$y_2 = 1+2T$
	$y_3 = 1+2T$	$y_3 = 1+3T$	$y_3 = 1+3T$	$y_3 = 1+3T$
$m = 3$ ;	$y_4 = 1+T$	$y_4 = 1+2T$	$y_4 = 1+3T$	$y_4 = 1+4T$
$y_1 = 1+T$		$y_5 = 1+T$	$y_5 = 1+2T$	$y_5 = 1+3T$
$y_2 = 1+2T$			$y_6 = 1+T$	$y_6 = 1+2T$
$y_3 = 1+T$				$y_7 = 1+T$

the scheme (i), (ii), (iii)  
gives the **max propagation time**  $mT$



# Determining $m$ the number of groups (1)

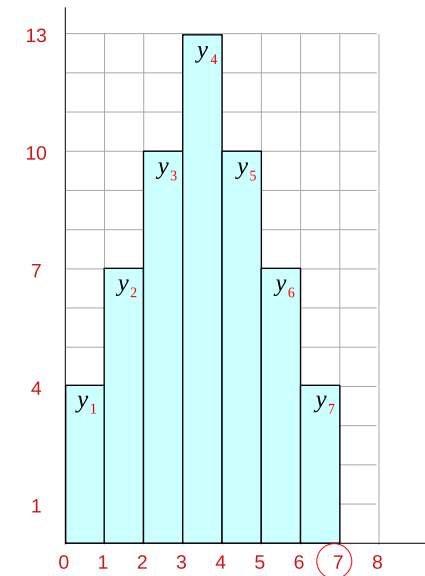
**Method 1** – using a histogram

Let  $m$  be the smallest positive integer such that

$$n \leq \sum_{i=1}^{(m)} y_i$$

$m = 2;$   
while  $(y_1 + \dots + y_m < n)$   $m = m + 1;$

$$y_i = \min\{1 + iT, 1 + (m + 1 - i)T\}, \quad i = 1, \dots, m$$



$m = 2; T = 3$

$$y_1 = \min\{1+T, 1+2T\} = 1+T = 4$$

$$y_2 = \min\{1+2T, 1+T\} = 1+T = 4$$

$m = 3; T = 3$

$$y_1 = \min\{1+T, 1+3T\} = 1+T = 4$$

$$y_2 = \min\{1+2T, 1+2T\} = 1+2T = 7$$

$$y_3 = \min\{1+3T, 1+T\} = 1+T = 4$$

$m = 4; T = 3$

$$y_1 = \min\{1+T, 1+4T\} = 1+T = 4$$

$$y_2 = \min\{1+2T, 1+3T\} = 1+2T = 7$$

$$y_3 = \min\{1+3T, 1+2T\} = 1+2T = 7$$

$$y_4 = \min\{1+4T, 1+T\} = 1+T = 4$$

$m = 5; T = 3$

$$y_1 = \min\{1+T, 1+5T\} = 1+T = 4$$

$$y_2 = \min\{1+2T, 1+4T\} = 1+2T = 7$$

$$y_3 = \min\{1+3T, 1+3T\} = 1+3T = 10$$

$$y_4 = \min\{1+4T, 1+2T\} = 1+2T = 7$$

$$y_5 = \min\{1+5T, 1+T\} = 1+T = 4$$

$m = 6; T = 3$

$$y_1 = \min\{1+T, 1+6T\} = 1+T = 4$$

$$y_2 = \min\{1+2T, 1+5T\} = 1+2T = 7$$

$$y_3 = \min\{1+3T, 1+4T\} = 1+3T = 10$$

$$y_4 = \min\{1+4T, 1+3T\} = 1+3T = 10$$

$$y_5 = \min\{1+5T, 1+2T\} = 1+2T = 7$$

$$y_6 = \min\{1+6T, 1+T\} = 1+T = 4$$

$m = 7; T = 3$

$$y_1 = \min\{1+T, 1+7T\} = 1+T = 4$$

$$y_2 = \min\{1+2T, 1+6T\} = 1+2T = 7$$

$$y_3 = \min\{1+3T, 1+5T\} = 1+3T = 10$$

$$y_4 = \min\{1+4T, 1+4T\} = 1+4T = 13$$

$$y_5 = \min\{1+5T, 1+3T\} = 1+3T = 10$$

$$y_6 = \min\{1+6T, 1+2T\} = 1+2T = 7$$

$$y_7 = \min\{1+7T, 1+1T\} = 1+T = 4$$

$$4+4 = 8$$

$$4+7+4 = 15$$

$$4+7+7+4 = 22$$

$$4+7+10+7+4 = 32$$

$$4+7+10+10+7+4 = 42$$

$$4+7+10+13+10+7+4 = 55$$

# Determining $m$ the number of groups (2)

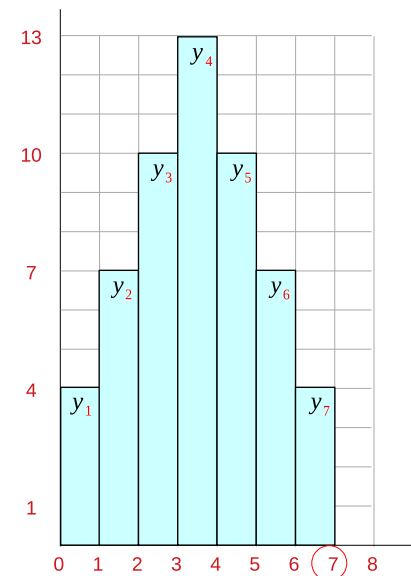
**Method 1** – using a histogram

Let  $m$  be the smallest positive integer such that

$$n \leq \sum_{i=1}^{(m)} y_i$$

$(m) = 2;$   
while  $(y_1 + \dots + y_{(m)} < n)$   $(m) = m + 1;$

$$y_i = \min\{1 + iT, 1 + ((m) + 1 - i)T\}, \quad i = 1, \dots, m$$

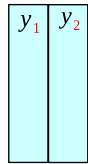


$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = (7)$
$\sum_{i=1}^2 y_i$	$\sum_{i=1}^3 y_i$	$\sum_{i=1}^4 y_i$	$\sum_{i=1}^5 y_i$	$\sum_{i=1}^6 y_i$	$\sum_{i=1}^7 y_i$
$2 + 2 \cdot T$	$3 + 4 \cdot T$	$4 + 6 \cdot T$	$5 + 9 \cdot T$	$6 + 12 \cdot T$	$7 + 16 \cdot T$
$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$2 + 2 \cdot T < n$	$3 + 4 \cdot T < n$	$4 + 6 \cdot T < n$	$5 + 9 \cdot T < n$	$6 + 12 \cdot T < n$	$7 + 16 \cdot T > n$
$2 + 2 \cdot 3 < 48$	$3 + 4 \cdot 3 < 48$	$4 + 6 \cdot 3 < 48$	$5 + 9 \cdot 3 < 48$	$6 + 12 \cdot 3 < 48$	$7 + 16 \cdot 3 > 48$
$8 < 48$	$15 < 48$	$22 < 48$	$32 < 48$	$42 < 48$	$55 > 48$
					$m = 7$

$n = 48$   
 $T = 3$   $\rightarrow$   $m = 7$

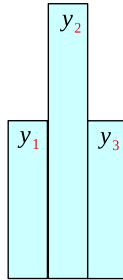
# $n$ the number of bits, $m$ the number of groups

$4+4 = 8$  bits  
 $4+7+4 = 15$  bits  
 $4+7+7+4 = 22$  bits  
 $4+7+10+7+4 = 32$  bits  
 $4+7+10+10+7+4 = 42$  bits  
 $4+7+10+13+10+7+4 = 55$  bits



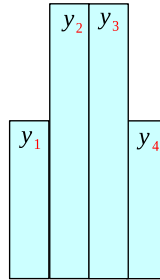
no of groups  
 $m = 2$ ;  $T = 3$   
 $y_1 = 1+T = 4$   
 $y_2 = 1+T = 4$   
 --  
 8

no of bits  
 $n \leq 8$



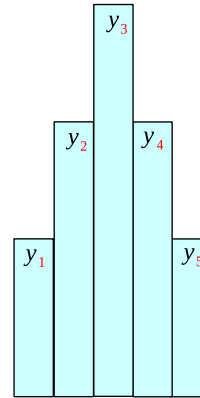
no of groups  
 $m = 3$ ;  $T = 3$   
 $y_1 = 1+T = 4$   
 $y_2 = 1+2T = 7$   
 $y_3 = 1+T = 4$   
 --  
 15

no of bits  
 $8 < n \leq 15$



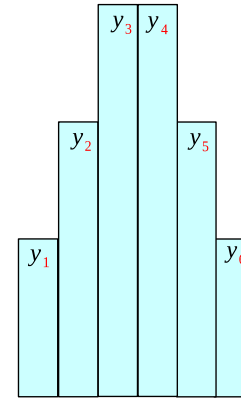
no of groups  
 $m = 4$ ;  $T = 3$   
 $y_1 = 1+T = 4$   
 $y_2 = 1+2T = 7$   
 $y_3 = 1+2T = 7$   
 $y_4 = 1+T = 4$   
 --  
 22

no of bits  
 $15 < n \leq 22$



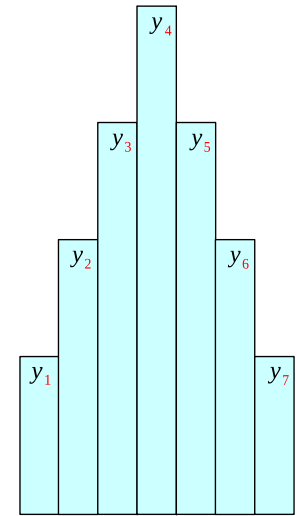
no of groups  
 $m = 5$ ;  $T = 3$   
 $y_1 = 1+T = 4$   
 $y_2 = 1+2T = 7$   
 $y_3 = 1+3T = 10$   
 $y_4 = 1+2T = 7$   
 $y_5 = 1+T = 4$   
 --  
 32

no of bits  
 $22 < n \leq 32$



no of groups  
 $m = 6$ ;  $T = 3$   
 $y_1 = 1+T = 4$   
 $y_2 = 1+2T = 7$   
 $y_3 = 1+3T = 10$   
 $y_4 = 1+3T = 10$   
 $y_5 = 1+2T = 7$   
 $y_6 = 1+T = 4$   
 --  
 42

no of bits  
 $32 < n \leq 42$

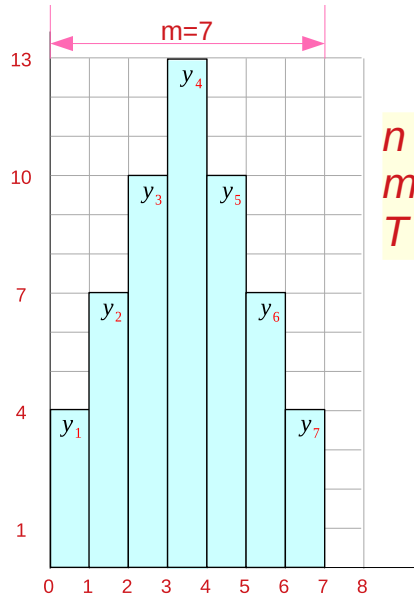


no of groups  
 $m = 7$ ;  $T = 3$   
 $y_1 = 1+T = 4$   
 $y_2 = 1+2T = 7$   
 $y_3 = 1+3T = 10$   
 $y_4 = 1+4T = 13$   
 $y_5 = 1+3T = 10$   
 $y_6 = 1+2T = 7$   
 $y_7 = 1+T = 4$   
 --  
 55

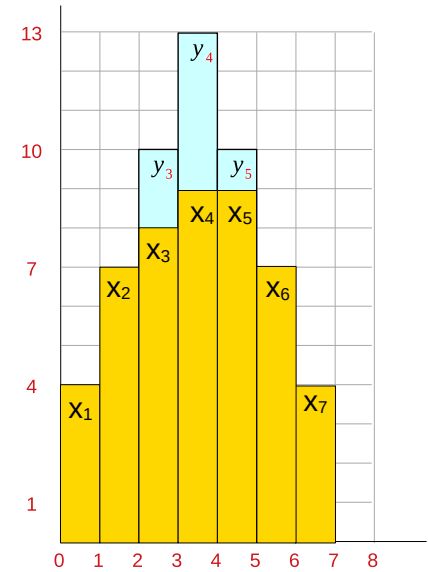
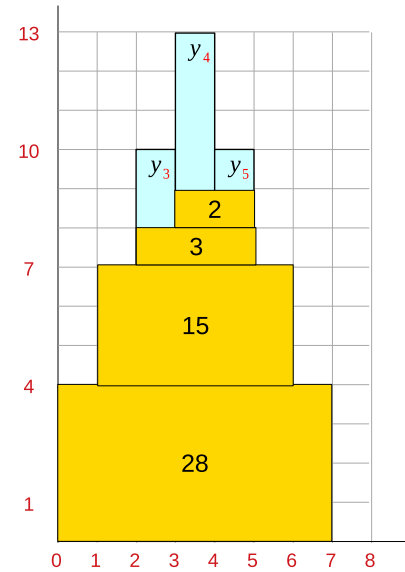
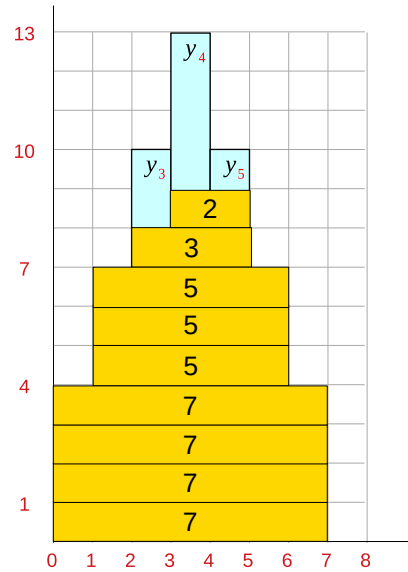
no of bits  
 $42 < n \leq 55$

$n = 48$   
 $T = 3$   $\rightarrow$   $m = 7$

# Determining $x_i$ the group size of the $i^{\text{th}}$ group



$n = 48$   
 $m = 7$   
 $T = 3$



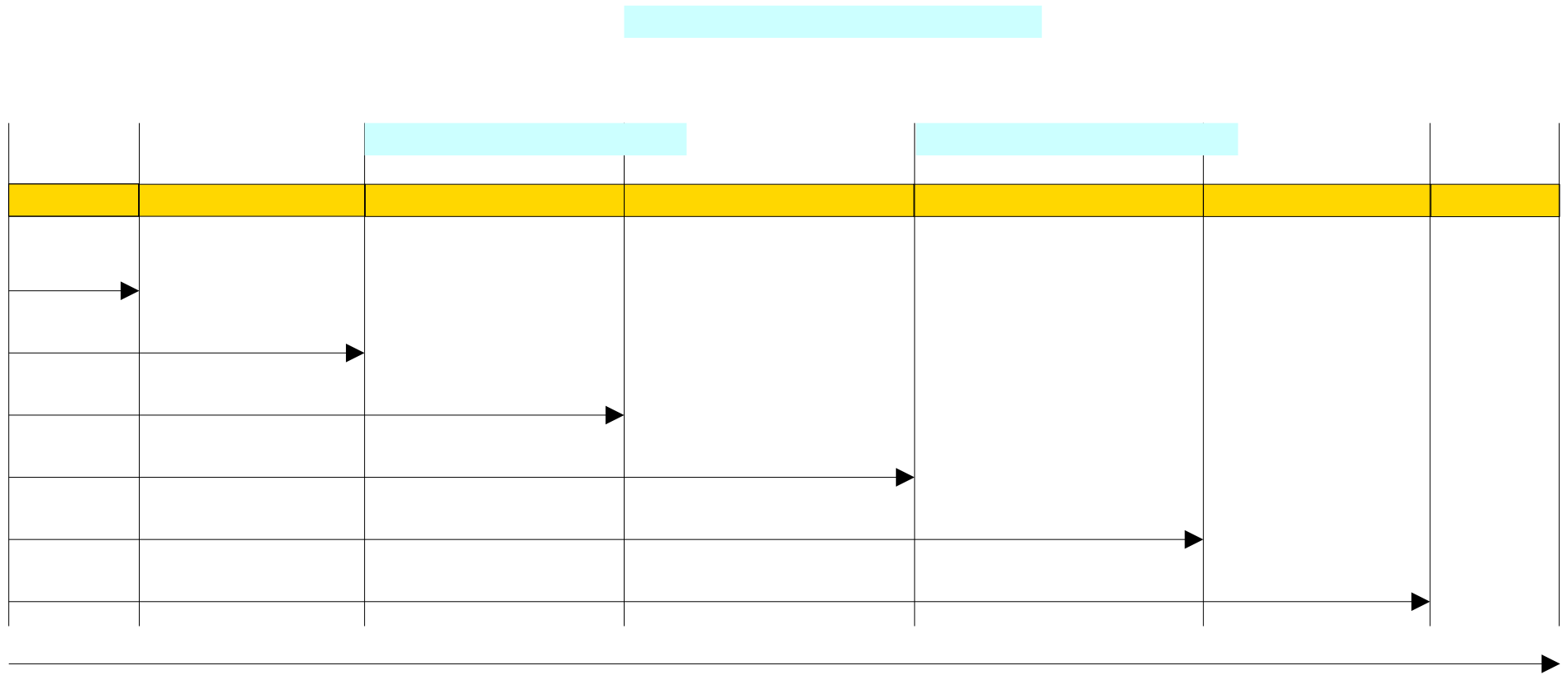
7	7
7	14
7	21
7	28
5	33
5	38
5	43
3	46
2	48

$4 * 7 = 28$	28
$3 * 5 = 15$	$43 = 28 + 15$
$1 * 3 = 3$	$46 = 28 + 15 + 3$
$1 * 2 = 2$	$48 = 28 + 15 + 3 + 2$

- $x_1 = 4 \leq y_1 = 4$
- $x_2 = 7 \leq y_2 = 7$
- $x_3 = 8 < y_3 = 10$
- $x_4 = 9 < y_4 = 13$
- $x_5 = 9 < y_5 = 10$
- $x_6 = 7 \leq y_6 = 7$
- $x_7 = 4 \leq y_7 = 4$

$m = 7$  groups

# Determining $x_i$ the group size of the $i^{\text{th}}$ group



# Procedure

(I) Let  $m$  be the smallest positive integer such that

$$n \leq m + \frac{1}{2}mT + \frac{1}{4}m^2T + (1 - (-1)^m)\frac{1}{8}T = \sum_{i=1}^m y_i$$

- total  $n = 48$  bits
- $m = 7$  groups
- $i$ -th group has  $x_i$  bits (size)
- constant skip delay  $T = T(x_i) = 3$

(II) Let

$$y_i = \min\{1 + iT, 1 + (m + 1 - i)T\}, \quad i = 1, \dots, m$$

and construct a **histogram** whose  $i$ -th column has height  $y_i$   
for example, for  $T=3$ , and  $n=48$ , we have  $m=7$

(III) It is easily verified that the area of the histogram in (II) is

$$\sum_{i=1}^m y_i = m + \frac{1}{2}mT + \frac{1}{4}m^2T + (1 - (-1)^m)\frac{1}{8}T \geq n$$

so these are at least  $n$  unit squares in the histogram  
starting with the first row, shade in  $n$  of the squares, row by row  
Let  $x_i$  denote the number of shaded squares in column  $i$  of the histogram,  
 $i = 1, \dots, m$

$$n = \sum_{i=1}^m x_i \leq \sum_{i=1}^m y_i$$

# Procedure - determining $m$ the number of groups

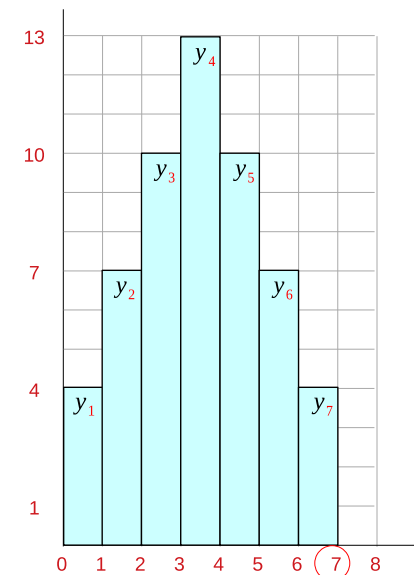
## Method 1 – using a histogram

Let  $m$  be the smallest positive integer such that

$$n \leq \sum_{i=1}^{(m)} y_i$$

$m = 2;$   
while  $(y_1 + \dots + y_m < n)$   $m = m + 1;$

$$y_i = \min\{1 + iT, 1 + (m + 1 - i)T\}, \quad i = 1, \dots, m$$



## Method 2 – using a *closed formula*

Let  $m$  be the smallest positive integer such that

$$n \leq m + \frac{1}{2}mT + \frac{1}{4}m^2T + (1 - (-1)^m) \frac{1}{8}T$$



$$m = 2k$$

$$\frac{m}{2} = k$$

$$y_i = \min\{1 + iT, 1 + (m + 1 - i)T\}, \quad i = 1, \dots, m$$

$y_1 = \min\{1 + 1 \cdot T, 1 + (m - 0) \cdot T\}$	$0 \leq x_1 \leq 1 + 1 \cdot T$
$y_2 = \min\{1 + 2 \cdot T, 1 + (m - 1) \cdot T\}$	$0 \leq x_2 \leq 1 + 2 \cdot T$
$y_3 = \min\{1 + 3 \cdot T, 1 + (m - 2) \cdot T\}$	$0 \leq x_3 \leq 1 + 3 \cdot T$
$y_k = \min\{1 + k \cdot T, 1 + (k + 1) \cdot T\}$	$0 \leq x_k \leq 1 + k \cdot T$
$y_{k+1} = \min\{1 + (k + 1) \cdot T, 1 + k \cdot T\}$	$0 \leq x_{k+1} \leq 1 + k \cdot T$
$y_{m-2} = \min\{1 + (m - 2) \cdot T, 1 + 3 \cdot T\}$	$0 \leq x_{m-2} \leq 1 + 3 \cdot T$
$y_{m-1} = \min\{1 + (m - 1) \cdot T, 1 + 2 \cdot T\}$	$0 \leq x_{m-1} \leq 1 + 2 \cdot T$
$y_{m-0} = \min\{1 + (m - 0) \cdot T, 1 + 1 \cdot T\}$	$0 \leq x_{m-0} \leq 1 + 1 \cdot T$

Oklobdzija: High-Speed VLSI arithmetic units: adders and multipliers

$0 \leq x_i \leq y_i, i = 1, \dots, m$

$\frac{1}{2} \cdot k(k+1)$

# Processing - determining $x_i$ the group size of the $i^{\text{th}}$ group

construct a **histogram**  
whose  $i$ -th column has height  $y_i$

so these  $y_i$ 's are at least  $n$  unit squares  
in the histogram, starting with the first row,  
shade in  $n$  of the squares, row by row

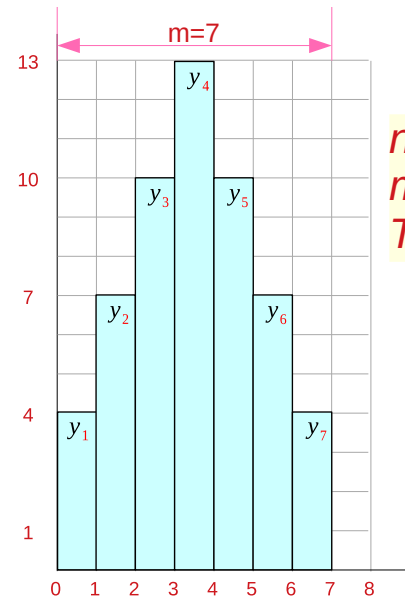
let  $x_i$  denote the number of shaded squares  
in column  $i$  of the histogram,  
 $i = 1, \dots, m$

$$0 \leq x_i \leq y_i, \quad i=1, \dots, m$$

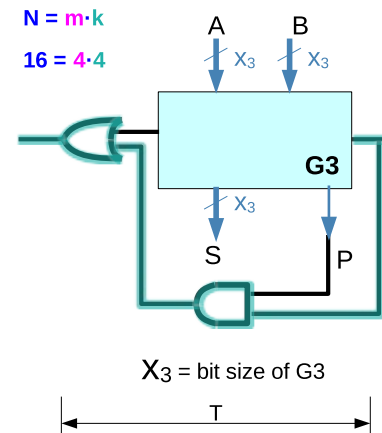
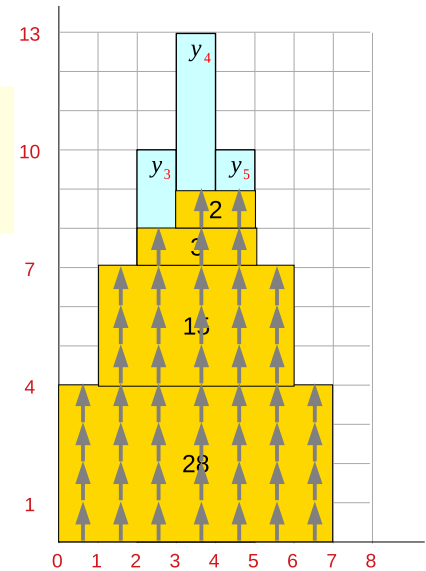
$$n = \sum_{i=1}^m x_i \leq \sum_{i=1}^m y_i$$

$$n = \sum_{i=1}^7 x_i \leq 4+7+8+9+9+7+4=48$$

$$\leq 4+7+10+13+10+7+4=55$$



$n = 48$   
 $m = 7$   
 $T = 3$



$m = 7; \quad T = 3$   
 $x_1 = 4 \leq y_1 = 4$   
 $x_2 = 7 \leq y_2 = 7$   
 $x_3 = 8 < y_3 = 10$   
 $x_4 = 9 < y_4 = 13$   
 $x_5 = 9 < y_5 = 10$   
 $x_6 = 7 \leq y_6 = 7$   
 $x_7 = 4 \leq y_7 = 4$



# Maximum Propagation Time $P_{\max}$

find the smallest  $m$

$$n \leq \sum_{i=1}^m y_i = \sum_{i=1}^m \min\{1+iT, 1+(m+1-i)T\}$$

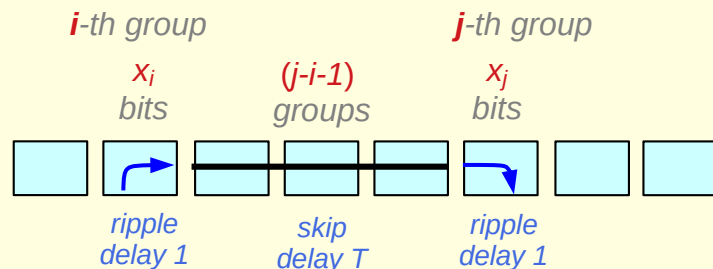
$\textcircled{m} = 2$ ;  
while  $(y_1 + \dots + y_{\textcircled{m}} < n)$   $\textcircled{m} = m+1$  ;

the scheme (i), (ii), (iii)  
gives the max propagation time  $mT$



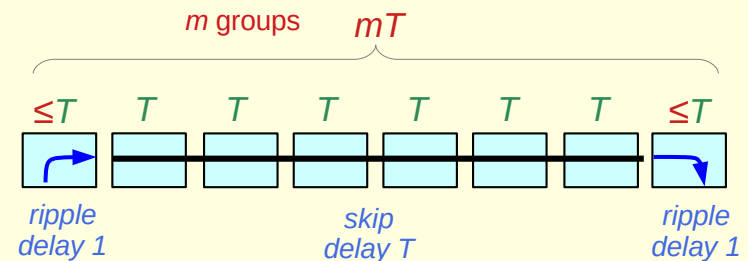
propagation time of a carry signal  $\leq mT$   
the maximum propagation time =  $mT$

**propagation time**  $P = P_{i,j}$



$$P = P_{i,j} \quad \forall i, \forall j \quad 1 \leq i, j \leq m$$

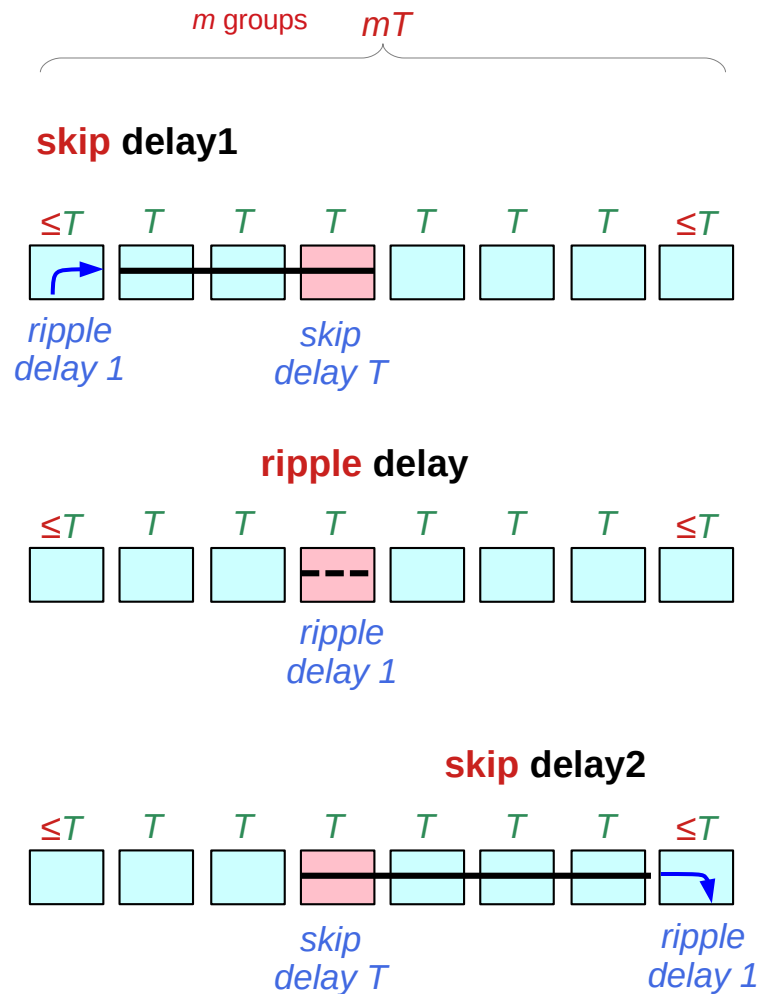
**maximum propagation time**  $P_{\max} = P_{1,m}$



$$P_{\max} = P_{1,m} \leq mT$$

Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

# Maximum propagation time P



the scheme (i), (ii), (iii)  
gives the max prop time  $mT$

<b>skip delay1</b>	$iT$	generating carry
<b>ripple delay</b>	$x_i - 1$	
<b>skip delay2</b>	$(m+1-i)T$	terminating carry

$$\begin{cases} x_i - 1 \leq iT \\ x_i - 1 \leq (m+1-i)T \end{cases}$$

$$\Rightarrow \begin{cases} x_i \leq 1 + iT \\ x_i \leq 1 + (m+1-i)T \end{cases}$$

$$\Rightarrow x_i \leq \min \{1 + iT, 1 + (m+1-i)T\} = y_i$$

$$\Rightarrow x_i \leq y_i$$

Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

# Maximum propagation time P

$$y_i = \min\{1+iT, 1+(m+1-i)T\}, \quad i = 1, \dots, m$$

$$y_1 = \min\{1+1\cdot T, 1+(m+1-1)T\} = 1+T$$

$$y_m = \min\{1+m\cdot T, 1+(m+1-m)T\} = 1+T$$

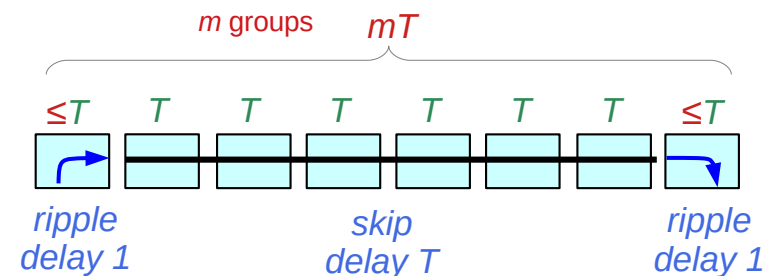
$$x_1 \leq y_1 = 1+T$$

$$x_m \leq y_m = 1+T$$

$x_1 - 1 \leq 1T$	$x_1 \leq T+1$
$x_1 - 1 \leq (m+1-1)T$	$x_1 \leq mT + 1$
$x_2 - 1 \leq 2T$	$x_2 \leq 2T + 1$
$x_2 - 1 \leq (m+1-2)T$	$x_2 \leq (m-1)T + 1$
• • •	• • •
$x_{m-1} - 1 \leq (m-1)T$	$x_{m-1} \leq (m-1)T + 1$
$x_{m-1} - 1 \leq (m+1-(m-1))T$	$x_{m-1} \leq 2T + 1$
$x_m - 1 \leq mT$	$x_m - 1 \leq mT$
$x_m - 1 \leq (m+1-m)T$	$x_m - 1 \leq T$

the scheme (i), (ii), (iii)  
gives the max prop time  $mT$

## maximum propagation time



$$P_{max} = P_{1,m} \leq mT$$

$$P = P_{i,j} \leq mT$$

Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

# Maximum propagation time P

**Lemma 1** When the bits of a **carry skip adder** are grouped according to the scheme (i)-(iii), the **maximum propagation time** of a carry signal is  $mT$

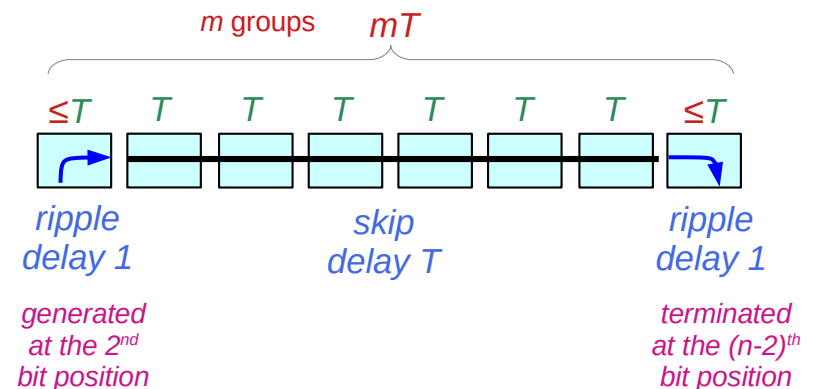
The carry generated at the  $2^{\text{nd}}$  bit position and terminating at the  $(n-1)^{\text{th}}$  bit position clearly has **propagation time**  $mT$ .

We must show that *any other* carry signal has propagation time smaller than or equal to  $mT$

propagation time of a carry signal  $\leq mT$   
the maximum propagation time =  $mT$

the scheme (i), (ii), (iii) gives the max prop time  $mT$

## maximum propagation time



$$P_{max} = P_{1,m} \leq mT$$

# Delay range of a carry signal

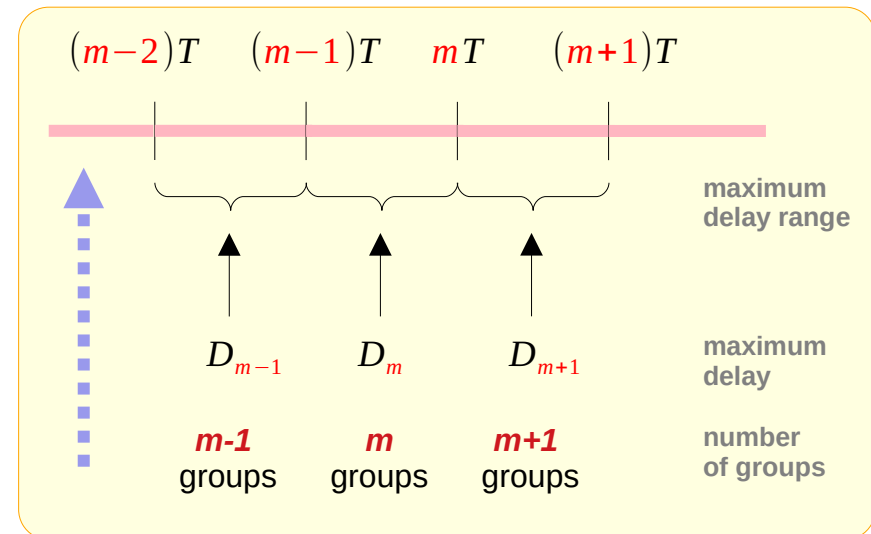
**Lemma 2** Let  $D$  denote the **maximum delay** of a carry signal in a  $n$  bit carry skip adder with **group sizes** chosen **optimally**. Then

- $n$  bits
- $r$  groups

$$(m-1)T \leq D \leq mT$$

Since we have exhibited a division of the carry chain into **groups** in such a way that the **maximum delay** of a carry signal is  $mT$  We clearly have  $D \leq mT$

the **maximum delay** =  $D$   
the **optimal group size** =  $m$   
 $(m-1)T \leq D \leq mT$



Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

# Maximum delay and optimal group size

the maximum propagation time  
 $\propto$  the number of groups

$$D \propto m$$

- not an optimal division
- larger number of groups  $\rightarrow$
- larger delays  $\rightarrow$

- when group size  $m$  is not optimal

then there is an optimal group size  $= r$

- the maximum delay with the group size  $m$   $D_m = mT$
- the maximum delay with the group size  $r$   $D_r = rT$
- $r$  must be smaller than  $m$   $r \leq m$

$$D_r < D_m$$

$$\rightarrow rT < mT$$

$$\rightarrow r < m$$

# Maximum delay of a carry signal

$$(m-1)T \leq D \leq mT$$

Assume there are  $r$  groups  
then 2 cases : **even  $r$** , **odd  $r$**   
for each of these 2 cases

prove  $mT - D < T + 1$

→  $mT - D \leq T$

→  $(m-1)T \leq D$

**$P$** : the propagation delay of  
any carry signal path  $\leq mT$

**$D$** : the max of  $P$

the upper bound of  $D$

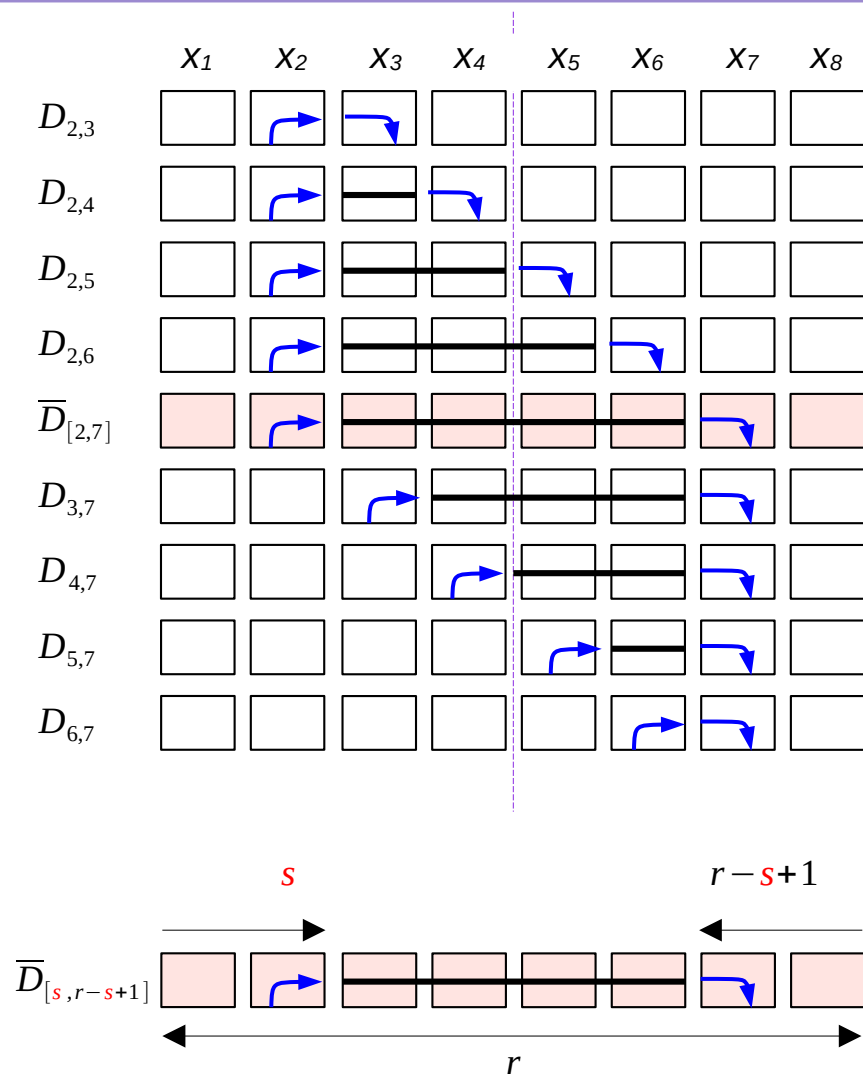
→  $\text{diff}(mT, D) \leq T$

→  $\text{diff}(mT, \max P) \leq T$

the lower bound of  $D$

$$(m-1)T \leq D$$

# Maximum delays of carry signals ( $r = 2k$ )



$\bar{D}_{[2,7]}$  = the maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group and terminated in the  $j$ -th group such that  $2 \leq i, j \leq 7$

$$\bar{D}_{[2,7]} = \max \left\{ \begin{array}{c} D_{2,3}, D_{2,4}, D_{2,5}, D_{2,6}, \\ D_{2,7}, \\ D_{3,7}, D_{4,7}, D_{5,7}, D_{6,7} \end{array} \right\}$$

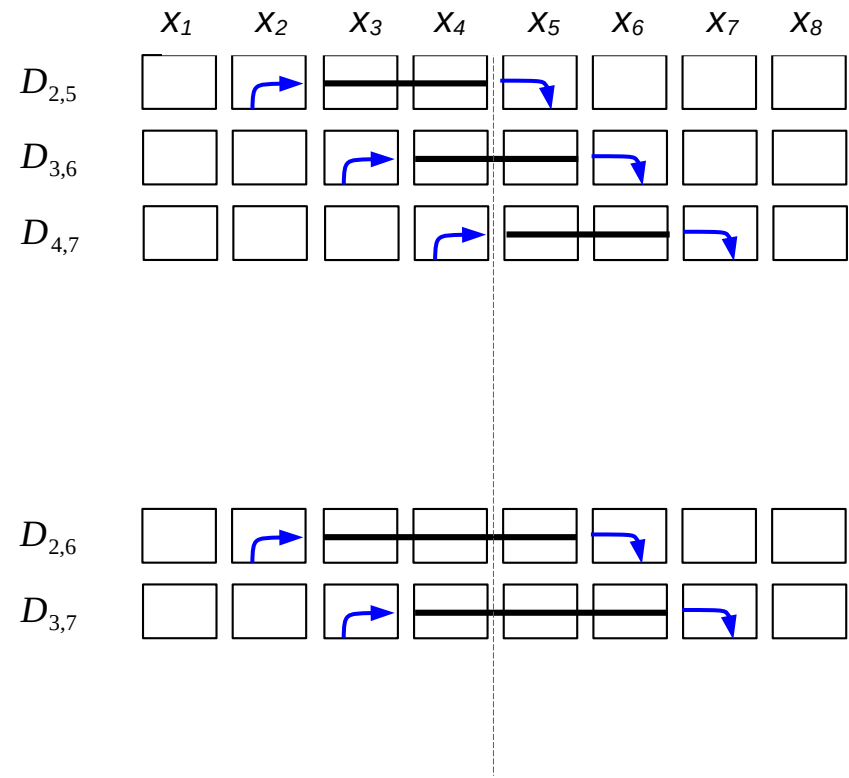
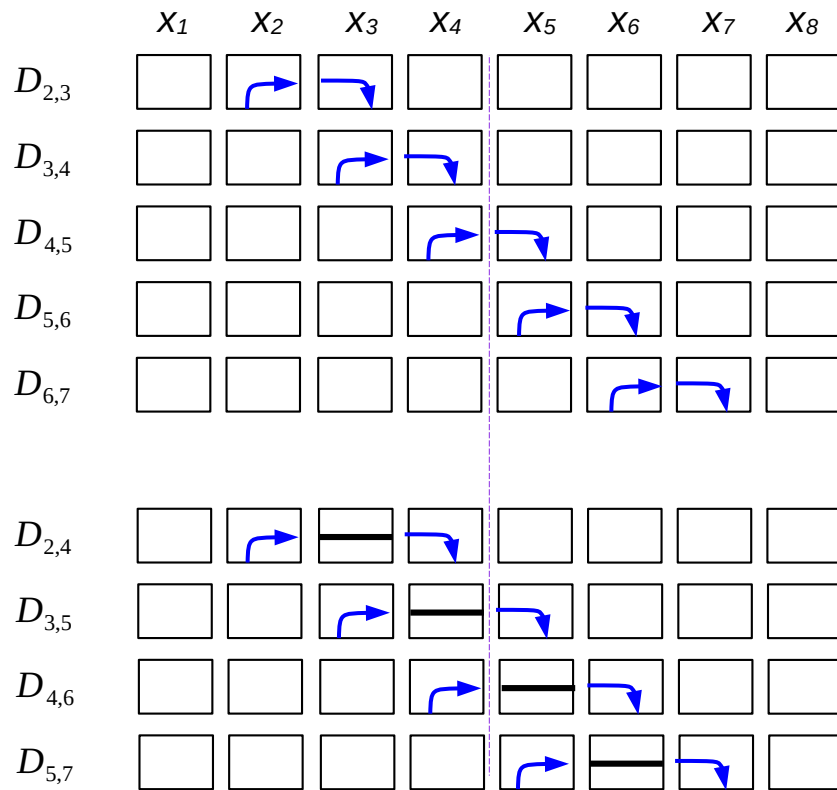
$\bar{D}_{[s, r-s+1]}$  = the maximum delay of carry signals generated in the  $i$ -th group and terminated in the  $j$ -th group such that  $s \leq i, j \leq r-s+1$

$$\bar{D}_{[2,7]} = \bar{D}_{[2,8-2+1]} = \bar{D}_{[s, 8-s+1]} \quad \leftarrow s = 2$$

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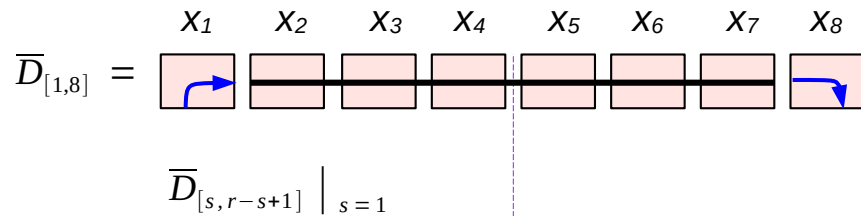


# Maximum delays of carry signals ( $r = 2k$ )

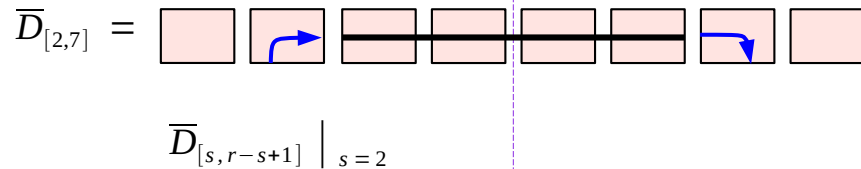


Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

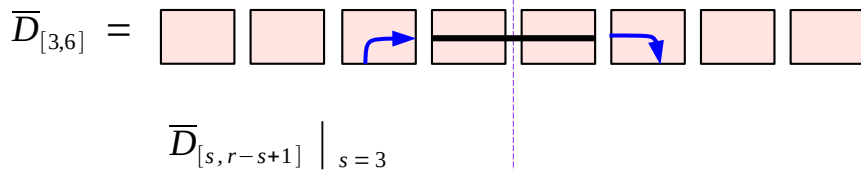
# Maximum delays of carry signals ( $r = 2k$ )



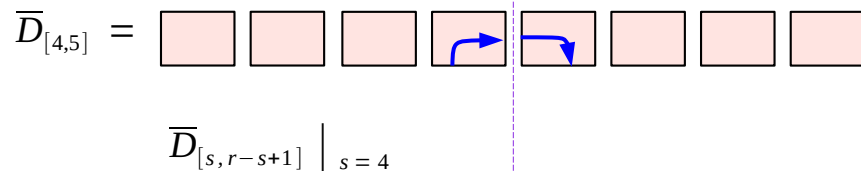
$\bar{D}_{[1,8]} =$  The maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group or terminated in the  $j$ -th group such that  $1 \leq i, j \leq 8$



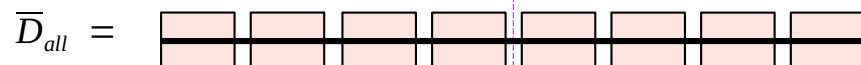
$\bar{D}_{[2,7]} =$  The maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group or terminated in the  $j$ -th group such that  $2 \leq i, j \leq 7$



$\bar{D}_{[3,6]} =$  The maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group or terminated in the  $j$ -th group such that  $3 \leq i, j \leq 6$



$\bar{D}_{[4,5]} =$  The maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group or terminated in the  $j$ -th group such that  $4 \leq i, j \leq 5$



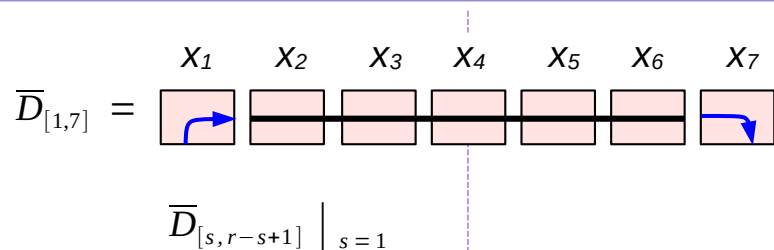
$\bar{D}_{all} =$  All skip delay  $\leq D$

$$D = \max\{\bar{D}_{[1,8]}, \bar{D}_{[2,7]}, \bar{D}_{[3,6]}, \bar{D}_{[4,5]}\}$$

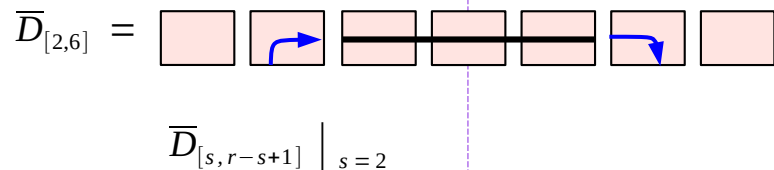
**Max delay of all carry signals**

Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

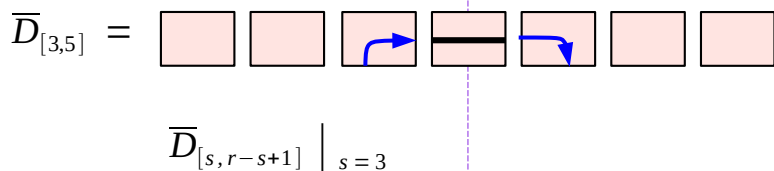
# Maximum delays of carry signals ( $r = 2k+1$ )



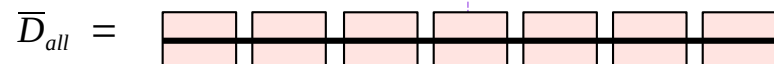
$\bar{D}_{[1,7]} =$  The maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group or terminated in the  $j$ -th group such that  $1 \leq i, j \leq 8$



$\bar{D}_{[2,6]} =$  The maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group or terminated in the  $j$ -th group such that  $2 \leq i, j \leq 7$



$\bar{D}_{[3,5]} =$  The maximum delay of carry signals  $\leq D$  generated in the  $i$ -th group or terminated in the  $j$ -th group such that  $3 \leq i, j \leq 6$



$\bar{D}_{all} =$  All skip delay



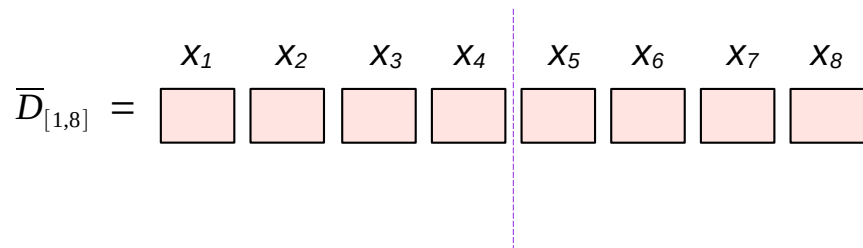
$\tilde{D}_{all} =$  Comparable to all skip delay  $\leq D$

$$D = \max\{\bar{D}_{[1,8]}, \bar{D}_{[2,7]}, \bar{D}_{[3,6]}, \bar{D}_{[4,5]}\}$$

**Max delay of all carry signals**

Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

# Maximum delays of carry signals ( $r = 2k$ )



$$D = \max_{s=1}^{r/2} \bar{D}_{[s, r-s+1]}$$

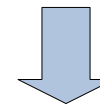
$$= \max_{s=1}^k \bar{D}_{[s, 2k+1-s]}$$

$$= \max_{s=1}^4 \bar{D}_{[s, 9-s]}$$

**Max delay of  
all carry signals**

$$\begin{aligned} \bar{D}_{[1,r]} &\leq D \\ \bar{D}_{[2,r-1]} &\leq D \\ &\vdots \\ \bar{D}_{[k,k+1]} &\leq D \end{aligned}$$

$$\bar{D}_{all} \leq D$$

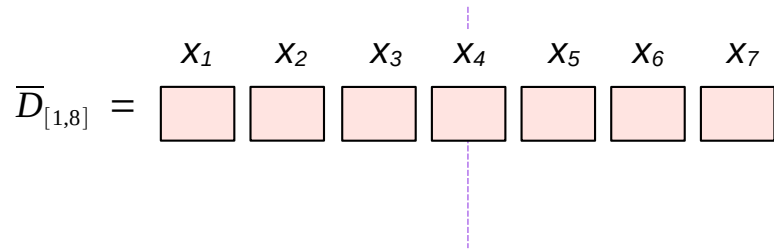


$$(m-1)T \leq D$$

**Lower bound of D**

Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

# Maximum delays of carry signals ( $r = 2k+1$ )



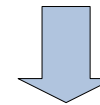
$$D = \max_{s=1}^{\text{floor}(r/2)} \bar{D}_{[s, r-s+1]}$$

$$= \max_{s=1}^k \bar{D}_{[s, 2k+2-s]}$$

$$= \max_{s=1}^3 \bar{D}_{[s, 8-s]}$$

**Max delay of  
all carry signals**

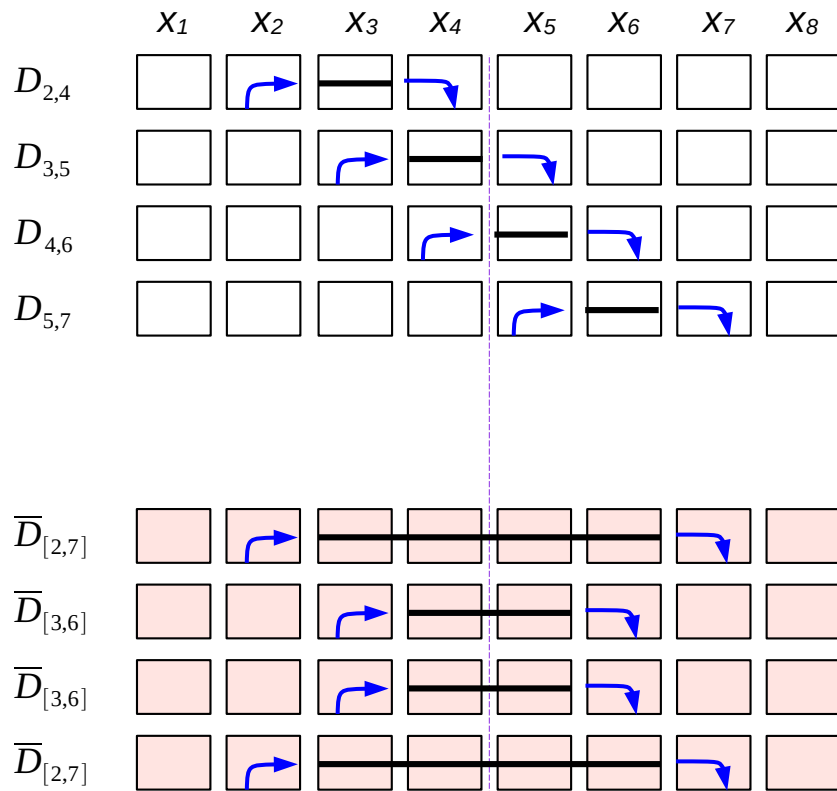
$$\begin{aligned} \bar{D}_{[1,r]} &\leq D \\ \bar{D}_{[2,r-1]} &\leq D \\ &\vdots \\ \bar{D}_{[k,k+1]} &\leq D \\ \tilde{D}_{all} &\leq D \end{aligned}$$



$$(m-1)T \leq D$$

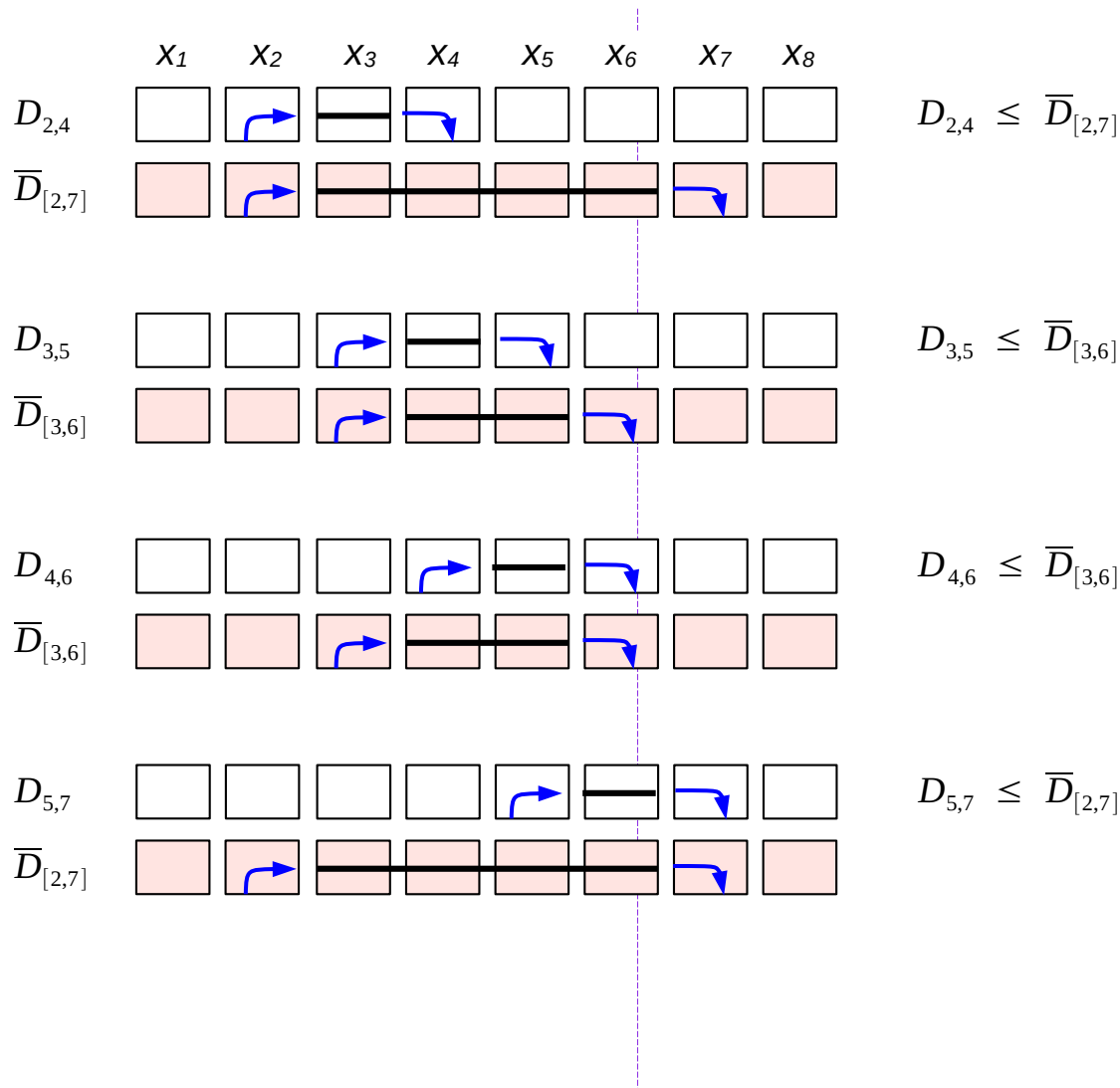
**Lower bound of D**

# Example delays of carry signals ( $r = 2k$ ) (1)



Oklobdzija: High-Speed VLSI arithmetic units : adders and multipliers

# Example delays of carry signals ( $r = 2k$ ) (2)



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# Optimal division into groups (1-1)

## Theorem 1

The scheme 2(i) – 2(iii) given above for dividing the bits of a carry skip adder into **groups** is **optimal** for  $2 \leq T \leq 7$

dividing the bits into groups by the scheme 2(i) – 2(iii) gives  **$m$  groups**

**propagation time** of a carry signal  $\leq mT$   
**the maximum propagation time =  $mT$**

**the maximum delay =  $D$**   
**the optimal group size =  $m$**

$$(m-1)T \leq D \leq mT$$

(I) Let  $m$  be the smallest positive integer such that

$$n \leq m + \frac{1}{2}mT + \frac{1}{4}m^2T + (1 - (-1)^m)\frac{1}{8}T$$

(II) Let  $y_i = \min\{1+iT, 1+(m+1-i)T\}$ ,  
 $i = 1, \dots, m$

and construct a **histogram** whose  $i$ -th column has height  $y_i$

(III) the area of the histogram in (II) is

$$m + \frac{1}{2}mT + \frac{1}{4}m^2T + (1 - (-1)^m)\frac{1}{8}T \geq n$$

so these are at least  $n$  unit squares in the histogram starting with the first row, shade in  $n$  of the squares, row by row  
Let  $x_i$  denote the number of shaded squares in column  $i$  of the histogram,  
 $i = 1, \dots, m$



# Optimal division into groups (1-2)

## Assume

- the scheme by 2(i) – 2(iii) ( $m$  groups) is not optimal
- let  $D$  be the maximum delay corresponding to an optimal division of the bits into groups
- there are  $r$  groups in the optimal division.

Since a carry in signal to the least significant bit group can skip over each group

we have  $rT \leq D \leq mT$  so  $r \leq m$

*if  $m$  is not optimal, but  $r$  is  
then  $mT \geq rT$  (smaller delay  $rT$ )  
thus  $m \geq r$  (smaller  $r$  exists)*

$m$  groups

- not optimal division
- $D$  = maximum delay
- $mT$  skip delay

$r$  groups

- optimal division
- $rT$  skip delay

skip delay

$$rT \leq D \leq mT$$

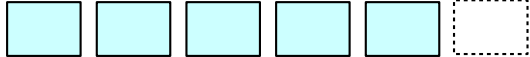
$$r \leq m$$

$D$  = max delay is assumed  
To be greater than all skip  
delay  $rT$  of the optimal division

# Optimal division into groups (1-2)

If the optimal division gives  $m$  groups

$m$  groups   $mT$

$(m-1)$  groups   $(m-1)T$

$$D \leq mT$$

$$(m-1)T \leq D$$

Normally, by 2(i) – 2(iii) ( $m$  groups) is optimal and its maximum delay  $D$  is less than all skip delay  $mT$

$$D \leq mT$$

To prove this, first, negate that

- $m$  is not by the optimal division, but  $r$  is
- $D$  is greater than all skip delay of the optimal division

- when optimal group size =  $m$   
the maximum delay  $D_m \leq mT$
- when optimal group size =  $(m-1)$   
the maximum delay  $D_{m-1} \leq (m-1)T$

$D =$  maximum delay

$$rT \leq D \leq mT$$

$$r \leq m \quad \rightarrow \quad r < (m-1)$$

# Optimal division into groups (1-2)

$$rT \leq D \leq mT \quad \text{so } r \leq m$$

Optimal division :  $r$  groups

$D' \leq$  all skip delay  $rT$  ( $r$  groups)

$D =$  maximum delay

Non-optimal division :  $m$  groups

$D \leq$  all skip delay  $mT$  ( $m$  groups)

too many partitions  $m \quad r \leq m$

$$rT \leq D \leq mT$$

$$r \leq m \quad \rightarrow \quad r < (m-1)$$

Assume max delay  $D$  is greater than all skip delay  $rT$  of the optimal division

if  $m$  is not optimal, but  $r$  is  
then  $mT \geq rT$  (smaller delay  $rT$ )  
thus  $m \geq r$  (smaller  $r$  exists)

$D$  is max delay for  $m$  groups  
 $D'$  is max delay for  $r$  groups  
then  $D' \leq rT \leq D \leq mT$

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# Optimal division into groups (2)

we have  $rT \leq D \leq mT$  so  $r \leq m$

If  $r = m$

then  $D = mT \Rightarrow D = rT \quad rT = D$

If  $r = m-1, (r < m)$

$D \geq (m-1)T \Rightarrow D \geq (m-1)T = rT \quad rT \leq D$

if  $r < m-1, (r < m)$

$D \geq (m-1)T \Rightarrow D \geq (m-1)T > rT \quad rT < D$

# Optimal division into groups (3)

we have  $rT \leq D \leq mT$  so  $r \leq m$

If  $r = m$  then  $D = mT$

and the **theorem** holds by **lemma 1**

When the bits of a carry skip adder are grouped according to the scheme (i)-(iii), the maximum propagation time of a carry signal is  $mT$

$$(m-1)T \leq D \leq mT$$

**Lemma 1** When the bits of a *carry skip adder* are *grouped* according to the scheme (i)-(iii), the *maximum propagation time* of a *carry signal* is  $mT$

**Theorem 1** The scheme 2(i) – 2(iii) given above for dividing the bits of a carry skip adder into *groups* is *optimal* for  $2 \leq T \leq 7$

(5)  $r = 2k$   $X = 4 - T^2$

$$mT - D \leq T + \frac{-8(T/n) + 4}{\sqrt{4(T/n) + 8(T/n^2)} + \sqrt{4(T/n) + 4/n^2}}$$

$r = 2k + 1$   $X = 4$

$$mT - D \leq T + \frac{(T-2)^2/n}{\sqrt{4(T/n) + 4(T/n^2)} + \sqrt{4(T/n) + (T/n)^2 + 4/n^2}}$$

$m$  groups – not optimal division  
 $r$  groups – optimal division

$D$  = maximum delay

$$rT \leq D \leq mT$$

$$r \leq m$$

# Optimal division into groups (3)

If  $r = m-1$ , ( $r < m$ )  
 $m$  and  $r$  have different parities and  
it follows from (5)  
that  $mT - D \leq T$  for  $2 \leq T \leq 7$

so that  $D \geq (m-1)T$   
since  $r = m-1$ ,  
 $D \geq (m-1)T = rT$       $rT \leq D$

This means that a signal which  
skips over each of the  $r$  groups ( $rT$ )  
has delay less than the maximum  $D$ .

$$rT \leq D \leq mT$$

$m$  is not optimal division  
 $r$  is optimal division

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**Lemma 1** When the bits of a *carry skip adder* are *grouped* according to the scheme (i)-(iii), the *maximum propagation time* of a *carry signal* is  $mT$

**Theorem 1** The scheme 2(i) – 2(iii) given above for dividing the bits of a carry skip adder into *groups* is *optimal* for  $2 \leq T \leq 7$

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$r = 2k + 1$       $X = 4$

$$mT - D \leq T + \frac{(T-2)^2/n}{\sqrt{4(T/n) + 4(T/n^2)} + \sqrt{4(T/n) + (T/n)^2 + 4/n^2}}$$

$m$  groups – not optimal division  
 $r$  groups – optimal division

$D$  = maximum delay

$$rT \leq D \leq mT$$

$$r \leq m$$

# Optimal division into groups (4)

Similarly,

if  $r < m-1$ , ( $r < m$ )

$$(m-1)T \leq D$$

since  $r < m-1$ ,

$$rT < (m-1)T \leq D$$

so that a signal which skips over each group has delay  $rT < D$ .

$$rT < D \leq mT$$

$m$  is not optimal division

$r$  is optimal division

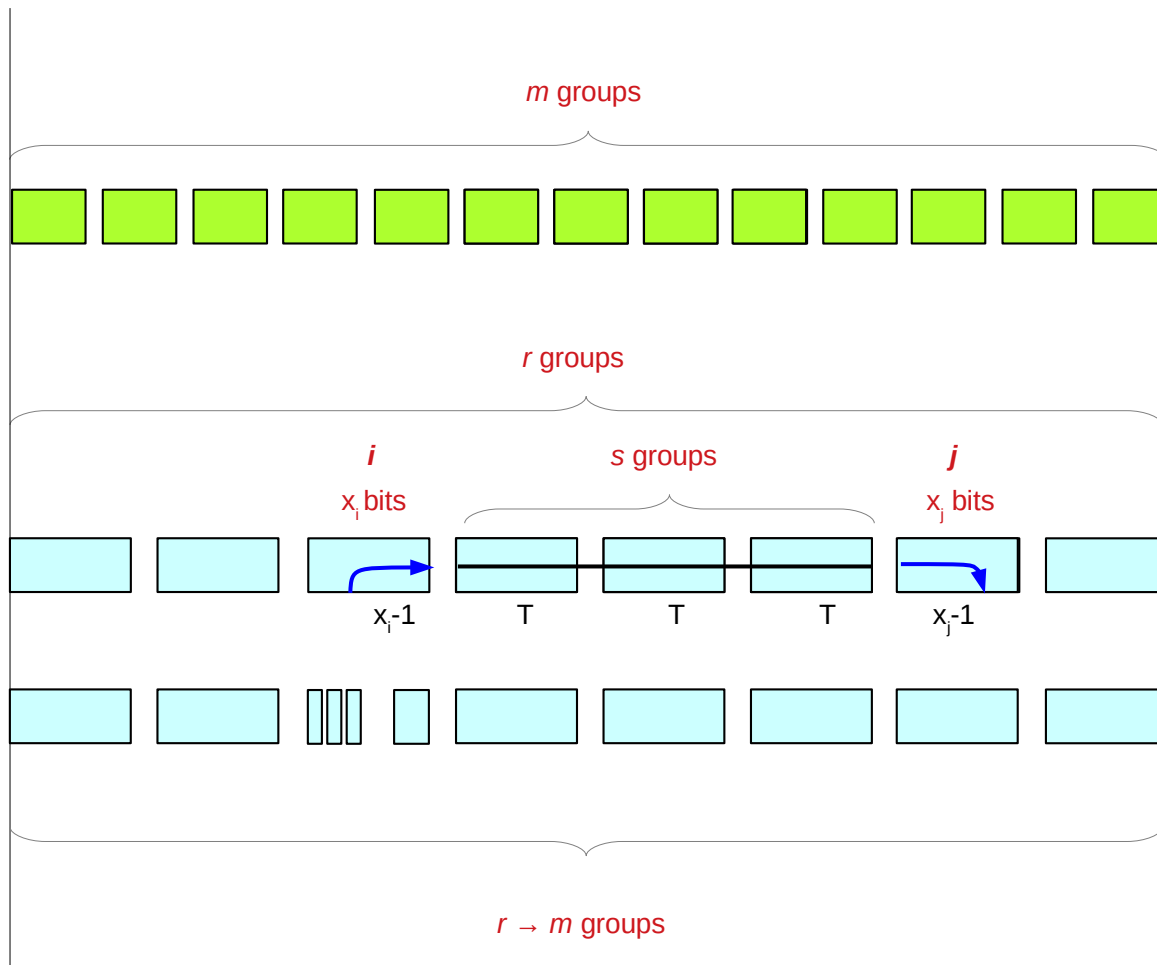
$m$  groups – not optimal division  
 $r$  groups – optimal division

$D$  = maximum delay

$$rT \leq D \leq mT$$

$$r \leq m$$

# Optimal division into groups (5)



$m$  groups – not optimal division  
 $r$  groups – optimal division

$D = \text{maximum delay}$

$$rT \leq D \leq mT$$

$$r \leq m$$

if  $m$  is not optimal, but  $r$  is

$((r + 1) + 1) + 1) \dots \rightarrow m$   
 contradiction!  $r$  must be  $m$

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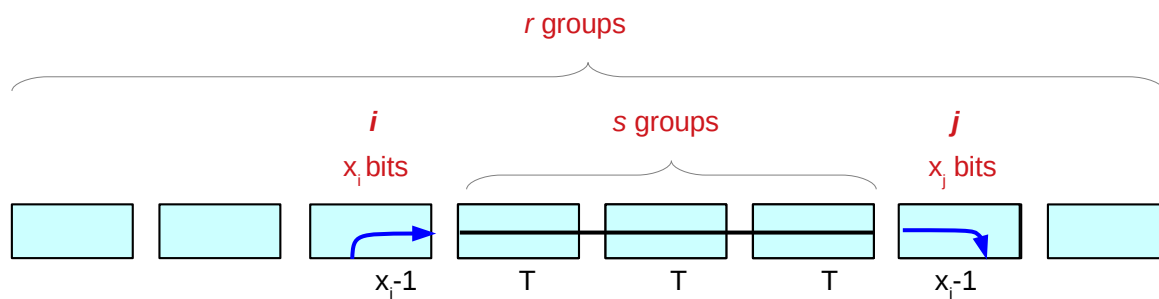
# Optimal division into groups (5)

It follows that a signal with delay  $D$

must start in a group  $i$ ,  
ripple to the end of group  $i$ ,

then skip over  $s < r$  groups and

either terminate, or ripple through  
the first few bits of a group  $j > i$ .



$m$  groups – not optimal division  
 $r$  groups – optimal division

$D = \text{maximum delay}$

$$rT \leq D \leq mT$$

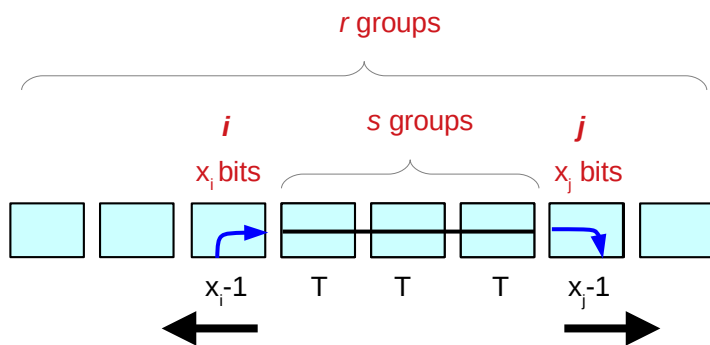
$$r \leq m$$

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# Optimal division into groups (6-1)

Let  $x_i$  and  $x_j$  denote the lengths of the  $i$ -th and  $j$ -th groups respectively.

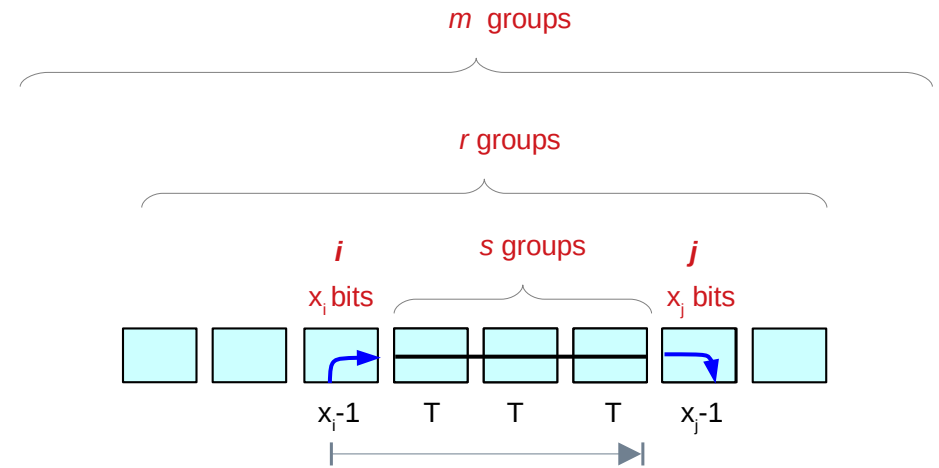
Assume that  $i$  is chosen as small as possible and  $j$  as large as possible. (longer path)



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# Optimal division into groups (6-2)

A signal originating in group  $i$ ,  
rippling to the end of this group  $i$  and  
 then skipping over the next  $s$  group  
 has delay  $(x_i - 1) + sT$



$$\begin{aligned}
 D &\leq (x_i - 1) + sT \\
 &\leq (x_i - 1) + (r - 1)T \\
 &\leq (x_i - 1) + (m - 2)T.
 \end{aligned}$$

$s < r$  groups  $\Rightarrow$   
 $r < m$  groups  $\Rightarrow$

$$\begin{aligned}
 &\Rightarrow s \leq (r - 1) \\
 &\Rightarrow r \leq (m - 1)
 \end{aligned}$$

$$\begin{aligned}
 D &\leq (x_i - 1) + sT \\
 &< (x_i - 1) + rT \\
 &< (x_i - 1) + mT.
 \end{aligned}$$

if  $m$  is not optimal, but  $r$  is

$$\begin{aligned}
 &s < r < m \\
 &s \leq (r - 1) < (m - 1) \\
 &s \leq (r - 1) \leq (m - 2)
 \end{aligned}$$

# Optimal division into groups (6-3)

$$\begin{aligned} D &\leq (x_i - 1) + sT \\ &\leq (x_i - 1) + (r - 1)T \\ &\leq (x_i - 1) + (m - 2)T \end{aligned}$$

$$(m - 1)T \leq D$$

$$D \leq (x_i - 1) + (m - 2)T$$

$$(m - 1)T \leq D \leq (x_i - 1) + (m - 2)T$$

$$(m - 1)T \leq (x_i - 1) + (m - 2)T$$

Since  $D \geq (m - 1)T$   
this implies that  $x_i \geq T + 1$

$$T \leq (x_i - 1)$$

$$T + 1 \leq x_i$$

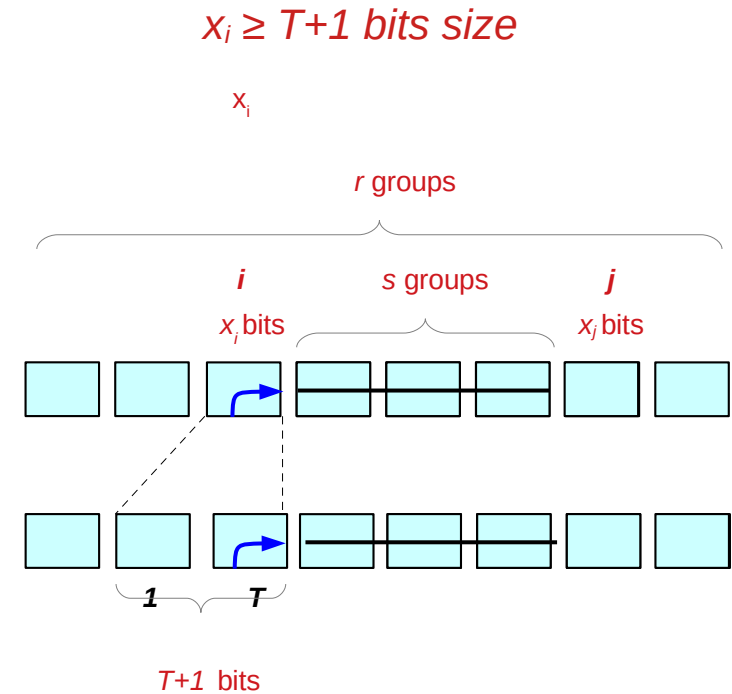
# Optimal division into groups (7)

Divide group  $i$  into **two** groups such that the group containing the **msb** has size  $T$ .

Since the  $i$ -th group is the **first** group in which a signal having maximum delay can originate,

this subdivision does not increase the delay of any carry signal of maximum delay

However, it increases the number of **groups** by **1**



$$\begin{aligned}
 D &\leq (x_i - 1) + sT & (m-1)T &< D \\
 &\leq (x_i - 1) + (r-1)T & D &< (x_i - 1) + (m-2)T \\
 &\leq (x_i - 1) + (m-2)T. & x_i &\geq (T+1)
 \end{aligned}$$

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# Optimal division into groups (8-1)

Suppose now that a carry signal originates in a group  $i$ , ripples to its end, skips over  $s \leq r-2$  groups and finally ripples through the first few bits of a group  $j$  and terminates.

We then have

$$\begin{aligned} D &\leq (x_i - 1) + sT + (x_j - 1) \\ &\leq x_i + x_j - 2 + (m - 3)T \end{aligned}$$

So that **either**  $x_i \geq T+1$  **or**  $x_j \geq T+1$

$s < r$  groups

$s \leq (r-1)$  groups

$r < m$  groups

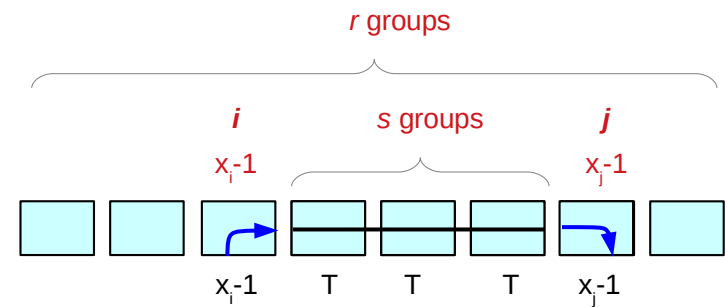
$r \leq (m-1)$  groups

$s < r$  groups

$s \leq (r-2)$  groups

$r < m$  groups

$r \leq (m-2)$  groups



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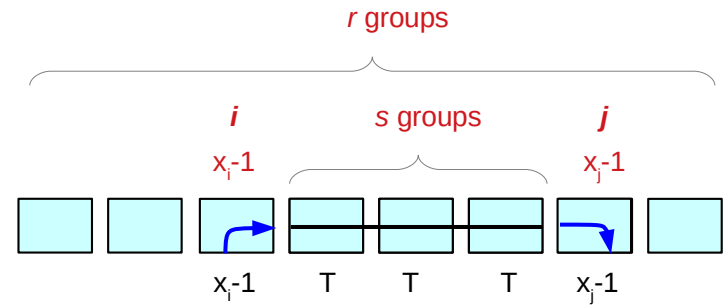
# Optimal division into groups (8-2)

$$s < r < m$$

$$s \leq (r-1) < (m-1)$$

$$s \leq (r-1) \leq (m-2)$$

$$s \leq (r-2) \leq (m-3)$$

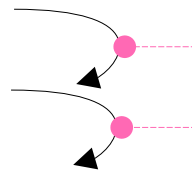


$$s \leq (r-2)$$

$$D \leq (x_i-1) + sT + (x_j-1)$$

$$\leq x_i + x_j - 2 + (r-2)T$$

$$\leq x_i + x_j - 2 + (m-3)T$$



$$(m-1)T < D$$

$$D < (x_i-1) + (m-2)T \iff x_i \geq (T+1)$$

$$D < (x_j-1) + (m-2)T \iff x_j \geq (T+1)$$

So that either  $x_i \geq T+1$  or  $x_j \geq T+1$

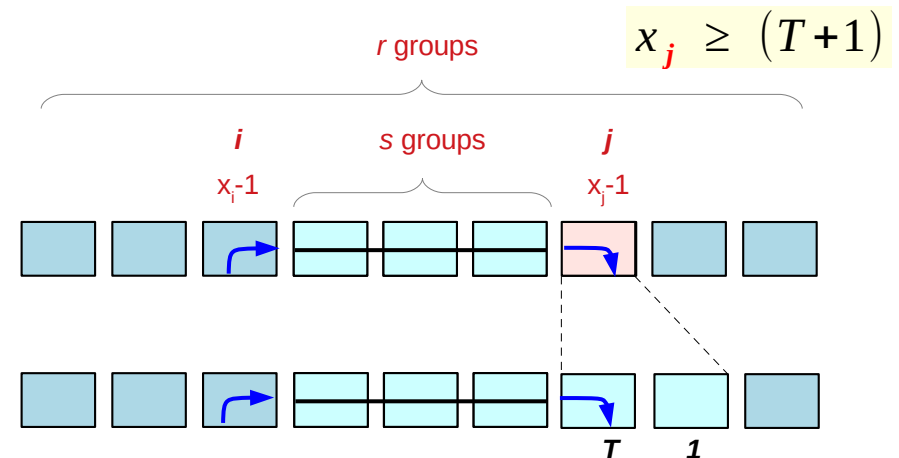
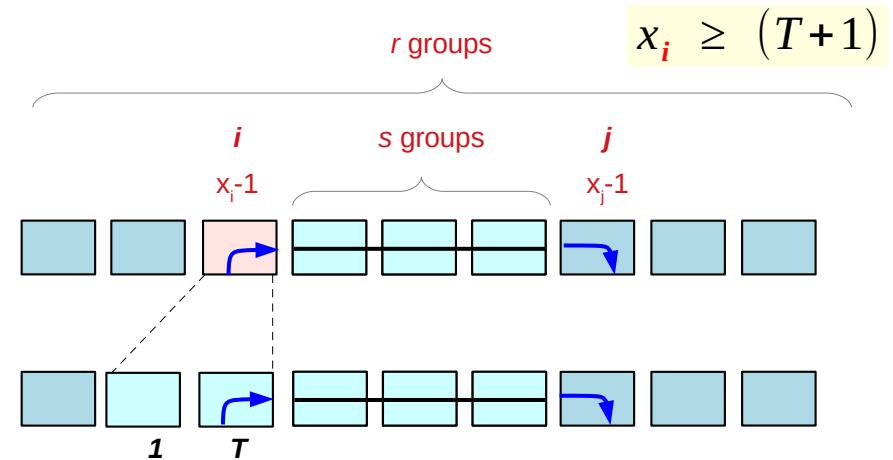
# Optimal division into groups (9)

So that either  $x_i \geq T+1$  or  $x_j \geq T+1$

This means that we can subdivide one of the groups  $i, j$  without increasing  $D$  not both of them

Continuing in this way, we can always increase the number  $r$  of group in an optimal division of a carry chain by 1 without increasing  $D$  if  $r < m$

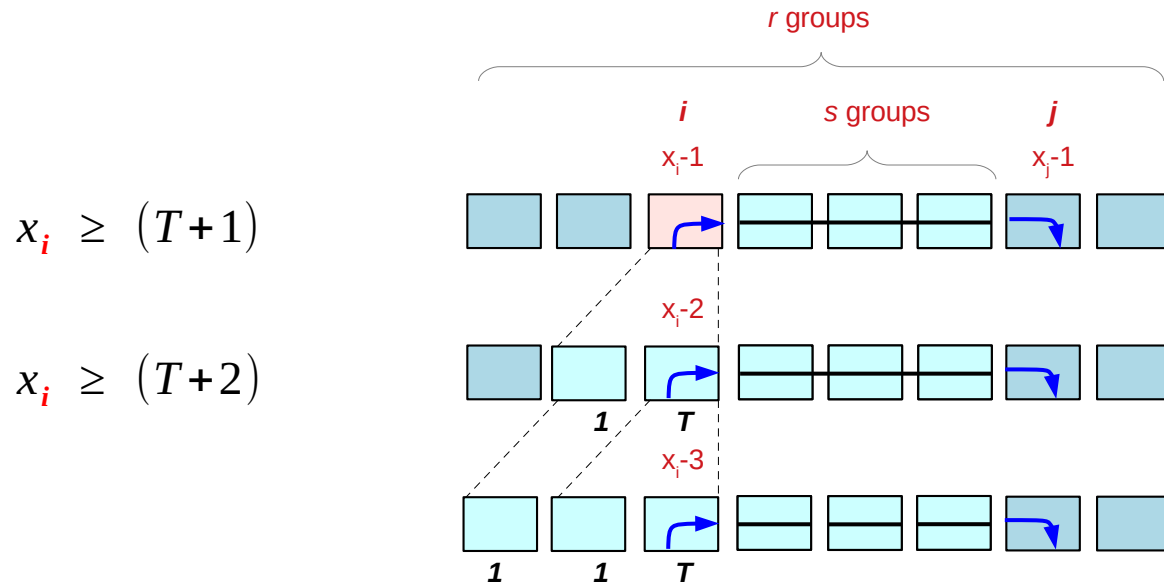
This means that we can arrive at an optimal division of the carry chain into  $m$  groups.





# Optimal division into groups (9)

$$x_i \geq (T+1) > (T+2) \dots$$



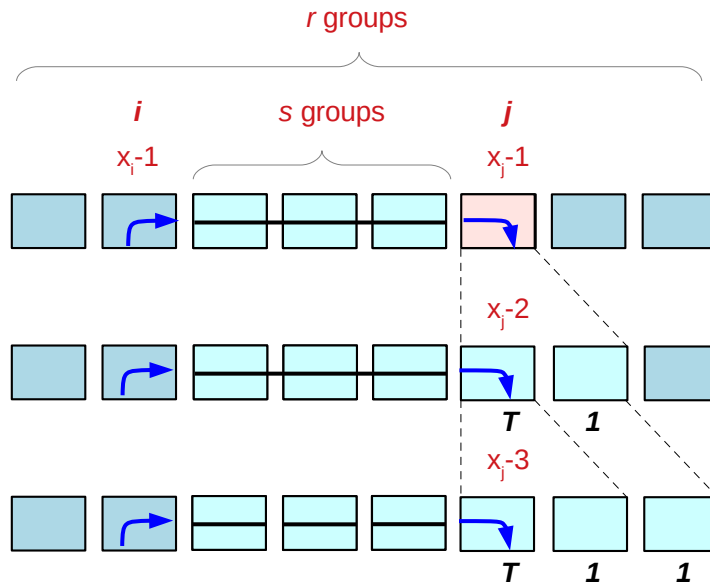
$$\begin{aligned} D &\leq (x_i - 1) + sT \\ &\leq (x_i - 1) + (r - 1)T \\ &\leq (x_i - 1) + (m - 2)T \end{aligned}$$

$$(m - 1)T \leq D \leq (x_i - 1) + (m - 2)T$$

if  $m$  is not optimal, but  $r$  is

$((r + 1) + 1) + 1) \dots \rightarrow m$   
 contradiction!  $m$  must be  $r$

# Optimal division into groups (9)



if  $m$  is not optimal, but  $r$  is

$((r+1)+1)+1) \dots \rightarrow m$   
 contradiction!  $m$  must be  $r$

$$x_j \geq (T+1) > (T+2) \dots$$

$$x_j \geq (T+1)$$

$$x_j \geq (T+2)$$

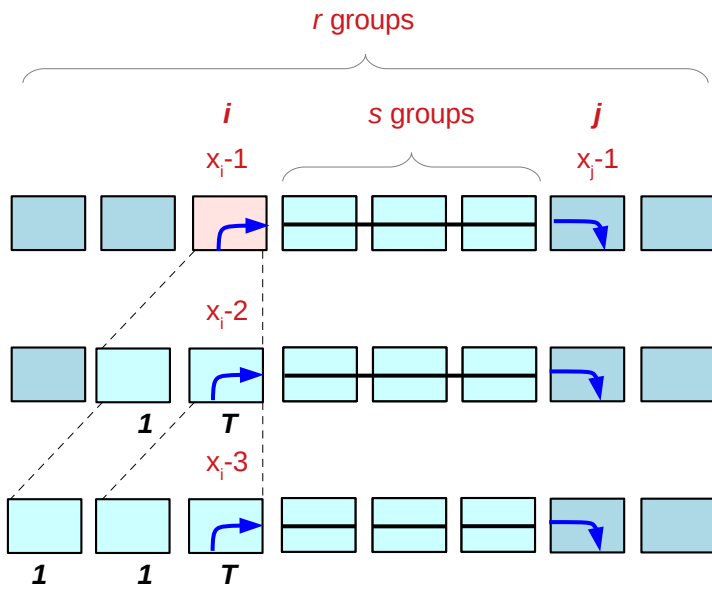
$$\begin{aligned} D &\leq (x_i-1) + sT \\ &\leq (x_i-1) + (r-1)T \\ &\leq (x_i-1) + (m-2)T \end{aligned}$$

$$(m-1)T \leq D \leq (x_i-1) + (m-2)T$$

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# Optimal division into groups (9)

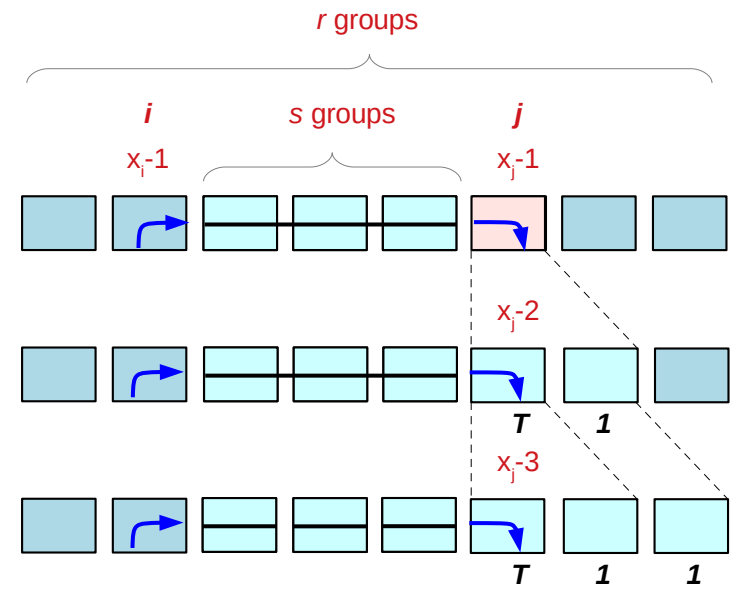
$$x_i \geq (T+1) > (T+2) \dots$$



if  $m$  is not optimal, but  $r$  is

$((r+1)+1)+1 \dots \rightarrow m$   
 contradiction!  $m$  must be  $r$

$$x_j \geq (T+1) > (T+2) \dots$$



if  $m$  is not optimal, but  $r$  is

$((r+1)+1)+1 \dots \rightarrow m$   
 contradiction!  $m$  must be  $r$

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# Optimal division into groups (9)

Normally, by 2(i) – 2(iii) ( $m$  groups) is optimal  
and its maximum delay  $D$  is less than all skip delay  $mT$

$$D \leq mT$$

To prove this, first, negate that

- $m$  is not by the optimal division, but  $r$  is
- $D$  is greater than all skip delay of the optimal division

## Assume

- the scheme by 2(i) – 2(iii) ( $m$  groups) is not optimal
- let  $D$  be the maximum delay corresponding to an optimal division
- there are  $r$  groups in the optimal division.

$$(\dots(((r+1)+1)+1) \dots +1) \rightarrow m : \text{optimal}$$

if  $m$  is not optimal, but  $r$  is

$$(\dots((r+1)+1)+1) \dots \rightarrow m$$

contradiction!  $m$  must be  $r$

# Optimal division into groups (11)

We must then have  $D \geq mT$   
which, together with **Lemma 2**,  
Implies  $D = mT$

This completes the proof of the theorem

$m$  groups – not optimal division  
 $r$  groups – optimal division

$D =$  maximum delay

$$rT \leq D \leq mT$$

$$r \leq m$$

## Lemma 2

Let  $D$  denote the maximum delay of a carry signal in a  $n$  bit carry skip adder with group sizes chosen optimally.

$$(m-1)T \leq D \leq mT$$

## Theorem 1

The scheme 2(i) – 2(iii) given above for dividing the bits of a carry skip adder into groups is optimal for  $2 \leq T \leq 7$