

# Complex Random Processes

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi



# Complex Random Processes

$N$  Gaussian random variables

## Definition

$$Z(t) = X(t) + jY(t)$$

$$E[Z(t)] = E[X(t)] + jE[Y(t)]$$

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[\{Z(t) - E[Z(t)]\}\{Z(t + \tau) - E[Z(t + \tau)]\}^*]$$

# Pseudo-correlation and covariance functions

$N$  Gaussian random variables

## Definition

- auto-correlation  $R_{ZZ}(t, t + \tau)$
- auto-covariance  $C_{ZZ}(t, t + \tau)$
- pseudo auto-correlation  $\tilde{R}_{ZZ}(t, t + \tau)$
- pseudo auto-covariance  $\tilde{C}_{ZZ}(t, t + \tau)$

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[\{Z(t) - E[Z(t)]\}\{Z(t + \tau) - E[Z(t + \tau)]\}^*]$$

$$\tilde{R}_{ZZ}(t, t + \tau) = E[Z(t)Z(t + \tau)]$$

$$\tilde{C}_{ZZ}(t, t + \tau) = E[\{Z(t) - E[Z(t)]\}\{Z(t + \tau) - E[Z(t + \tau)]\}]$$

# Proper Random Processes

$N$  Gaussian random variables

## Definition

A complex random process  $Z(t)$  is said to be **proper** if the pseudo-autocovariance function is identically zero. If  $Z(t)$  is at least **wide-sense stationary**, the **mean** value becomes a constant

$$\bar{Z} = \bar{X} + j\bar{Y}$$

# Proper Random Processes

$N$  Gaussian random variables

## Definition

If  $Z(t)$  is at least **wide-sense stationary**,  
the correlation and pseudo-correlation functions  
are independent of absolute time

$$R_{ZZ}(t, t + \tau) = R_{ZZ}(\tau)$$

$$C_{ZZ}(t, t + \tau) = C_{ZZ}(\tau)$$

$$\tilde{R}_{ZZ}(t, t + \tau) = \tilde{R}_{ZZ}(\tau)$$

$$\tilde{C}_{ZZ}(t, t + \tau) = \tilde{C}_{ZZ}(\tau)$$

# Cross / Pseudo-cross, -corelation / -covariance

$N$  Gaussian random variables

## Definition

for two complex processes  $Z_i(t)$ ,  $Z_j(t)$ ,

- cross-correlation  $R_{Z_i Z_j}(t, t + \tau)$
- cross-covariance  $C_{Z_i Z_j}(t, t + \tau)$
- pseudo cross-correlation  $\tilde{R}_{Z_i Z_j}(t, t + \tau)$
- pseudo cross-covariance  $\tilde{C}_{Z_i Z_j}(t, t + \tau)$

$$R_{Z_i Z_j}(t, t + \tau) = E [Z_i(t) Z_j^*(t + \tau)]$$

$$C_{Z_i Z_j}(t, t + \tau) = E [\{Z_i(t) - E[Z_i(t)]\} \{Z_j(t + \tau) - E[Z_j(t + \tau)]\}^*]$$

$$\tilde{R}_{Z_i Z_j}(t, t + \tau) = E [Z_i(t) Z_j(t + \tau)]$$

$$\tilde{C}_{Z_i Z_j}(t, t + \tau) = E [\{Z_i(t) - E[Z_i(t)]\} \{Z_j(t + \tau) - E[Z_j(t + \tau)]\}]$$



# Cross / Pseudo-cross, -corelation / -covariance

$N$  Gaussian random variables

## Definition

for two complex processes  $Z_i(t)$ ,  $Z_j(t)$ ,

- auto-correlation  $R_{Z_i Z_i}(t, t + \tau)$ ,  $R_{Z_j Z_j}(t, t + \tau)$
  - auto-covariance  $C_{Z_i Z_i}(t, t + \tau)$ ,  $C_{Z_j Z_j}(t, t + \tau)$
  - pseudo auto-correlation  $\tilde{R}_{Z_i Z_i}(t, t + \tau)$ ,  $\tilde{R}_{Z_j Z_j}(t, t + \tau)$
  - pseudo auto-covariance  $\tilde{C}_{Z_i Z_i}(t, t + \tau)$ ,  $\tilde{C}_{Z_j Z_j}(t, t + \tau)$
- 
- cross-correlation  $R_{Z_i Z_j}(t, t + \tau)$
  - cross-covariance  $C_{Z_i Z_j}(t, t + \tau)$
  - pseudo cross-correlation  $\tilde{R}_{Z_i Z_j}(t, t + \tau)$
  - pseudo cross-covariance  $\tilde{C}_{Z_i Z_j}(t, t + \tau)$

# Jointly Wide Sense Stationary Process

$N$  Gaussian random variables

## Definition

If the two processes are at least jointly wide-sense stationary

$$R_{Z_i Z_j}(t, t + \tau) = R_{Z_i Z_j}(\tau)$$

$$C_{Z_i Z_j}(t, t + \tau) = C_{Z_i Z_j}(\tau)$$

$$\tilde{R}_{Z_i Z_j}(t, t + \tau) = R_{Z_i Z_j}(\tau)$$

$$\tilde{C}_{Z_i Z_j}(t, t + \tau) = C_{Z_i Z_j}(\tau)$$

# Uncorrelated / Orthogonal / Jointly Proper Process

$N$  Gaussian random variables

## Definition

$Z_i(t)$  and  $Z_j(t)$  are **uncorrelated** processes

if  $C_{Z_i Z_j}(t, t + \tau) = 0$  and  $\tilde{C}_{Z_i Z_j}(t, t + \tau) = 0$

$Z_i(t)$  and  $Z_j(t)$  are **orthogonal** processes

if  $\tilde{R}_{Z_i Z_j}(t, t + \tau) = 0$  and  $\tilde{R}_{Z_j Z_i}(t, t + \tau) = 0$

$Z_i(t)$  and  $Z_j(t)$  are **jointly proper** processes

if  $\tilde{C}_{Z_i Z_j}(t, t + \tau) = 0$



