Complex Random Processes

Young W Lim

October 22, 2020

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Definition

$$Z(t) = X(t) + jY(t)$$
$$E[Z(t)] = E[X(t)] + jE[Y(t)]$$
$$R_{ZZ}(t, t+\tau) = E[Z(t)Z^*(t+\tau)]$$
$$C_{ZZ}(t, t+\tau) = E\left[\{Z(t) - E[Z(t)]\}\{Z(t+\tau) - E[Z(t+\tau)]\}^*\right]$$

—

Definition

- auto-correlation $R_{ZZ}(t, t+\tau)$
- auto-covariance $C_{ZZ}(t, t+\tau)$
- pseudo auto-correlation $\widetilde{R}_{ZZ}(t, t+\tau)$
- pseudo auto-covariance $\widetilde{C}_{ZZ}(t, t+\tau)$

$$R_{ZZ}(t, t+\tau) = E[Z(t)Z^*(t+\tau)]$$
$$C_{ZZ}(t, t+\tau) = E\left[\{Z(t) - E[Z(t)]\}\{Z(t+\tau) - E[Z(t+\tau)]\}^*\right]$$
$$\widetilde{R}_{ZZ}(t, t+\tau) = E[Z(t)Z(t+\tau)]$$
$$\widetilde{C}_{ZZ}(t, t+\tau) = E[\{Z(t) - E[Z(t)]\}\{Z(t+\tau) - E[Z(t+\tau)]\}]$$

Definition

A complex random process Z(t) is said to be **proper**
if the pseudo-autocovariance function is identically zero.
If $Z(t)$ is at least **wide-sense stationary**,
the **mean** value becomes a constant

$$\overline{Z} = \overline{X} + j\overline{Y}$$

Definition

If Z(t) is at least **wide-sense stationary**,
the correlation and pseudo-correlation functions
are independent of absolute time

$$R_{ZZ}(t, t+\tau) = R_{ZZ}(\tau)$$

$$C_{ZZ}(t, t+\tau) = C_{ZZ}(\tau)$$

$$\widetilde{R}_{ZZ}(t, t+\tau) = \widetilde{R}_{ZZ}(\tau)$$

$$\widetilde{C}_{ZZ}(t, t+\tau) = \widetilde{C}_{ZZ}(\tau)$$

Definition

for two complex processes $Z_i(t)$, $Z_j(t)$,

- cross-correlation $R_{Z_iZ_j}(t, t+\tau)$
- cross-covariance $C_{Z_iZ_j}(t, t+\tau)$
- pseudo cross-correlation $\widetilde{R}_{Z_iZ_j}(t, t+\tau)$
- pseudo cross-covariance $\widetilde{C}_{Z_iZ_j}(t, t+\tau)$

$$R_{Z_iZ_j}(t, t+\tau) = E\left[Z_i(t)Z_j^*(t+\tau)\right]$$
$$C_{Z_iZ_j}(t, t+\tau) = E\left[\{Z_i(t) - E[Z_i(t)]\}\{Z_j(t+\tau) - E[Z_j(t+\tau)]\}^*\right]$$
$$\widetilde{R}_{Z_iZ_j}(t, t+\tau) = E[Z_i(t)Z_j(t+\tau)]$$
$$\widetilde{C}_{Z_iZ_j}(t, t+\tau) = E[\{Z_i(t) - E[Z_i(t)]\}\{Z_j(t+\tau) - E[Z_j(t+\tau)]\}]$$

Definition

for two complex processes $Z_i(t)$, $Z_j(t)$,

- auto-correlation $R_{Z_iZ_i}(t, t+\tau)$, $R_{Z_jZ_j}(t, t+\tau)$
- auto-covariance $C_{Z_iZ_i}(t, t+\tau)$, $C_{Z_jZ_j}(t, t+\tau)$
- pseudo auto-correlation $\widetilde{R}_{Z_iZ_i}(t, t+\tau)$, $\widetilde{R}_{Z_jZ_j}(t, t+\tau)$
- pseudo auto-covariance $\widetilde{C}_{Z_iZ_i}(t, t+\tau)$, $\widetilde{C}_{Z_jZ_j}(t, t+\tau)$

- cross-correlation $R_{Z_iZ_j}(t, t+\tau)$
- cross-covariance $C_{Z_iZ_j}(t, t+\tau)$
- pseudo cross-correlation $\widetilde{R}_{Z_iZ_j}(t, t+\tau)$
- pseudo cross-covariance $\widetilde{C}_{Z_iZ_j}(t, t+\tau)$

Definition

If the two processes are at least jointly wide-sense stationary

$$R_{Z_iZ_j}(t, t+\tau) = R_{Z_iZ_j}(\tau)$$
$$C_{Z_iZ_j}(t, t+\tau) = C_{Z_iZ_j}(\tau)$$
$$\widetilde{R}_{Z_iZ_j}(t, t+\tau) = R_{Z_iZ_j}(\tau)$$
$$\widetilde{C}_{Z_iZ_j}(t, t+\tau) = C_{Z_iZ_j}(\tau)$$

Definition

$Z_i(t)$and $Z_j(t)$ are **uncorrelated** processes

if $C_{Z_i Z_j}(t, t+\tau) = 0$ and $\widetilde{C}_{Z_i Z_j}(t, t+\tau) = 0$

$Z_i(t)$and $Z_j(t)$ are **orthogonal** processes

if $\widetilde{R}_{Z_i Z_j}(t, t+\tau) = 0$ and $\widetilde{R}_{Z_i Z_j}(t, t+\tau) = 0$

$Z_i(t)$and $Z_j(t)$ are **jointly proper** processes

if $\widetilde{C}_{Z_i Z_j}(t, t+\tau) = 0$