EGM 3520 Mechanics of Materials (MoM)

Motivation 1: Important historical case,
RMS Titanic: Why the disaster happen

Curriculum roadmap: The big picture

→ Motivation 2: Simpler cases for MoM course

Important course information

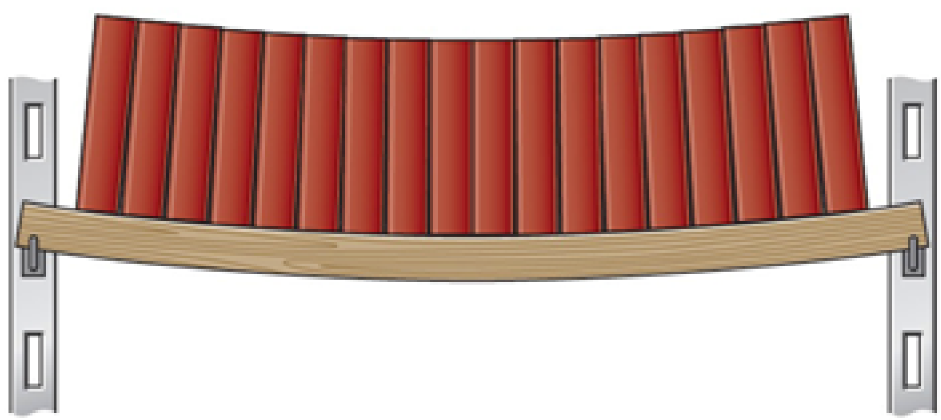
Motivation 3: Important historical case,
Tacoma Narrows Bridge collapse

Beer et al. 2012, Mechanics of Materials, McGraw-Hill.
Steif 2012, Mechanics of Materials, Pearson.

Motivation 2: Simpler cases for MoM course

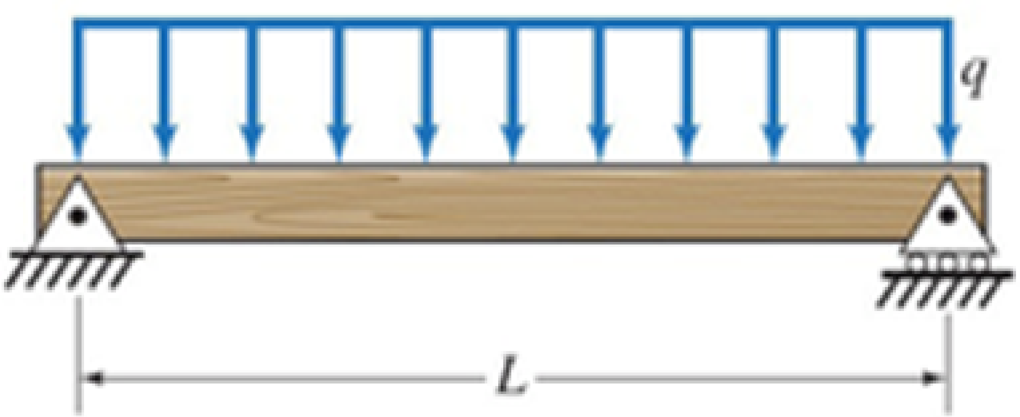
Design a good bookshelf:

Avoid sagging under book weight

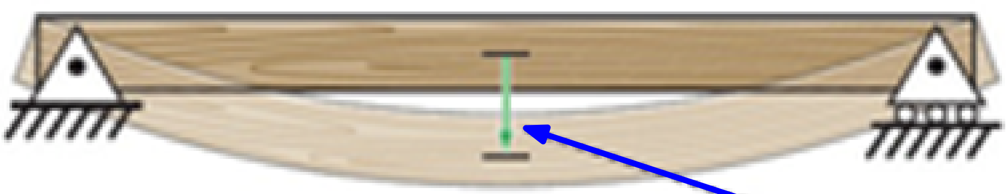


Model book shelf as a simply-supported beam of length L under a uniform dead load q:

Hinge, fixed displacements



Hinge on roller (free horizontal disp, fixed vertical disp)



Mid-span deflection v

Steif 2012, p.2

How to estimate the deflection v in terms of the beam length L and the distributed load q, and the properties of the material used for the shelf ?

$$v = \frac{5qL^4}{384\,EI} \qquad (1)$$

v = \frac{5 q L^4}{384 \, EI}

v vertical mid-span deflection of beam

q uniform distributed load per unit beam length

L beam span length between supports

E Young's modulus in relation of stress σ versus strain ϵ

$$\sigma = E\,\epsilon \qquad (2)$$

\sigma = E \, \epsilon

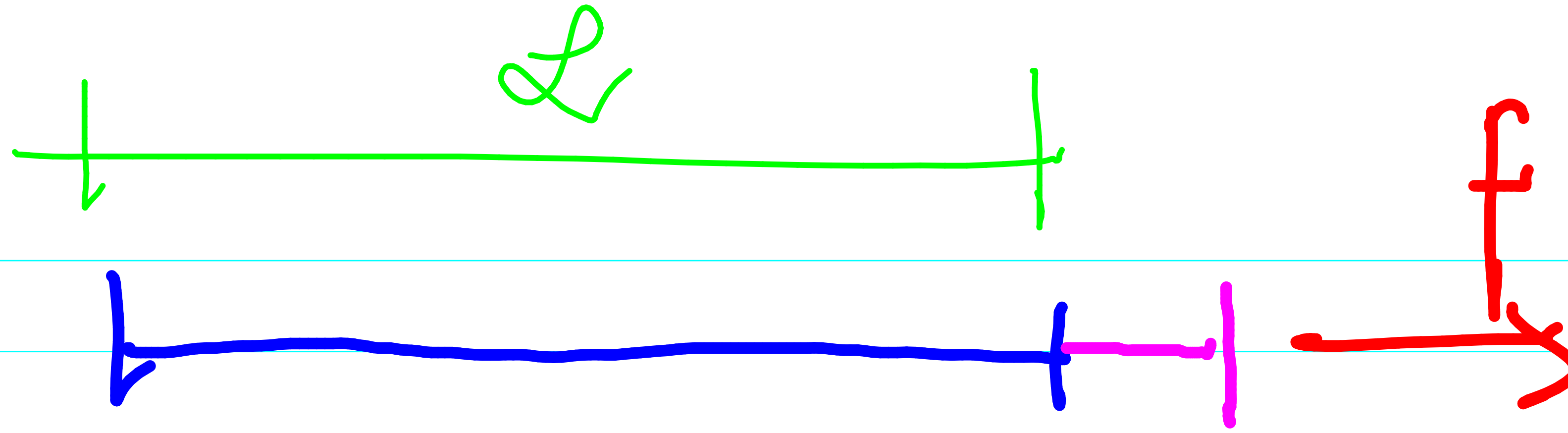
I Second moment of inertia of cross section

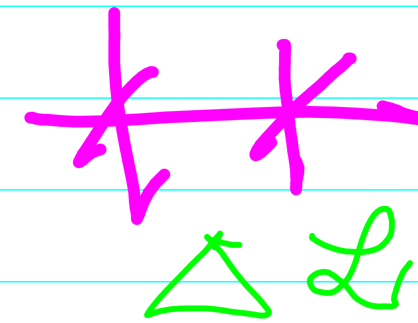
$$I_z = \int_A y^2 dA \qquad (3)$$

dydz

I_z = \int_A y^2 dA

Demo of latex codecogs online equation editor





$$\varepsilon = \frac{\Delta \mathcal{L}}{\mathcal{L}}$$

$$[\varepsilon] = \frac{[\Delta\mathcal{L}]}{[\mathcal{L}]} = \frac{L}{L} = 1$$

dimension of

$$\sigma = F/A \Rightarrow [\sigma] = \frac{[F]}{[A]} = \frac{F}{L^2}$$

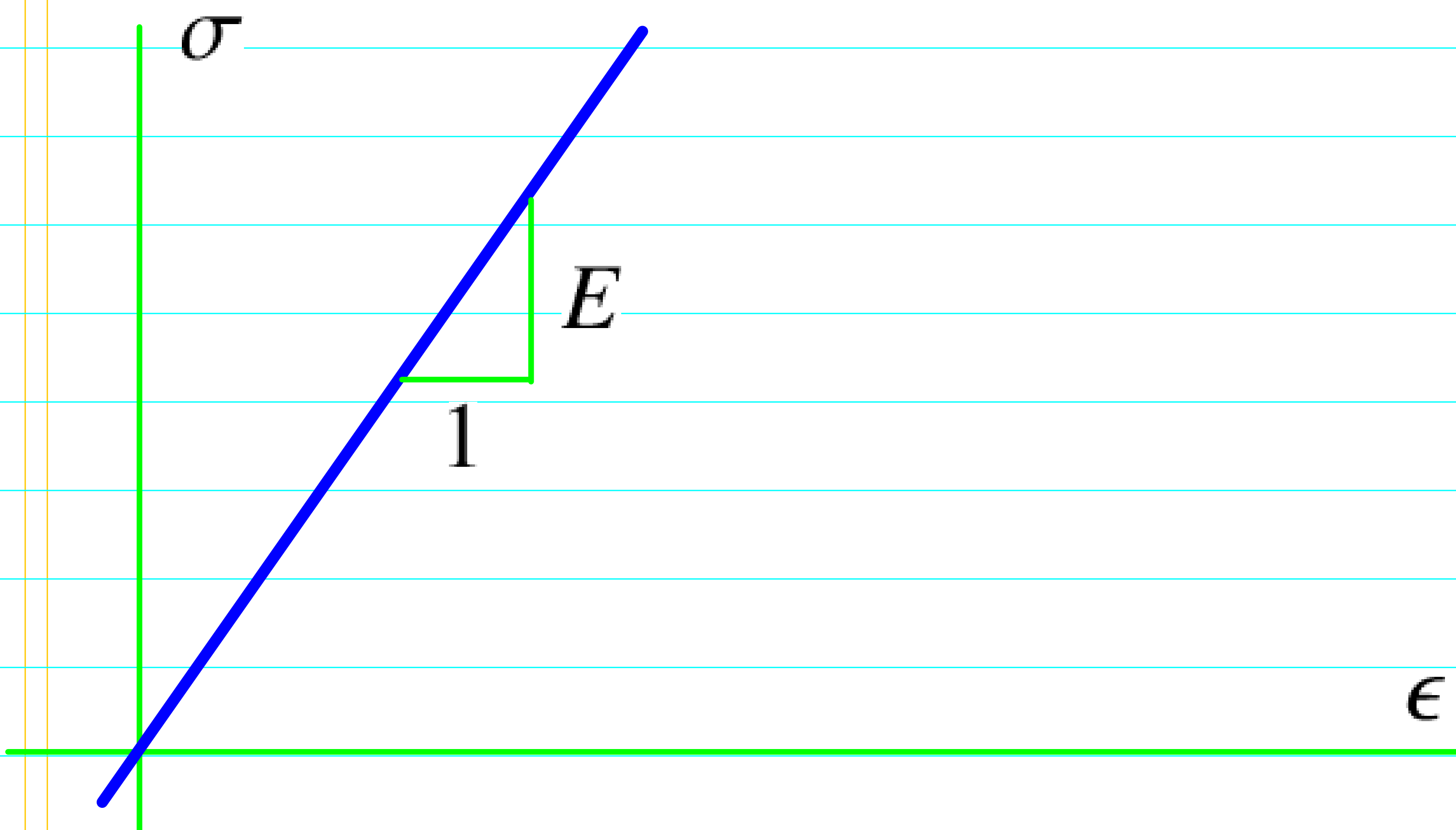
$$\sigma = E\varepsilon \Rightarrow [\sigma] = [E]\underbrace{[\varepsilon]}_{=1}$$

$$\Rightarrow [E] = [\sigma] = \frac{F}{L^2}$$

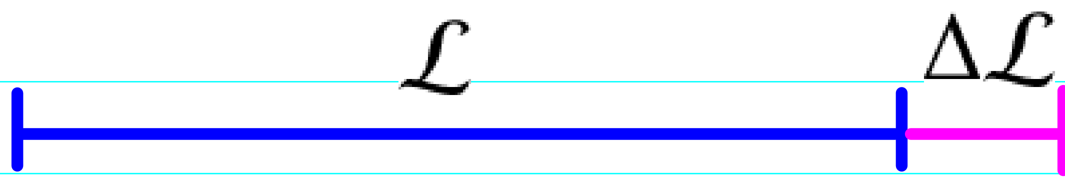
$$[I] = \underbrace{[y^2]}_{L^2}\,\underbrace{[dA]}_{=\,[dy\,dz]} = L^4$$

$$[dy\,dz] = \underbrace{[dy]}_{L}\,\underbrace{[dz]}_{L}$$

$$[v] = \left[\frac{5\,q\,L^4}{384\,E\,I}\right] = L$$



Dimensional analysis



\Delta \mathcal L

Strain:

$$\epsilon = \frac{\Delta \mathcal{L}}{\mathcal{L}} \qquad (1)$$

\epsilon = \frac{\Delta \mathcal L}{\mathcal L}

Dimension of strain:

$$[\epsilon] = \frac{[\Delta \mathcal{L}]}{[\mathcal{L}]} = \frac{L}{L} = 1 \qquad (2)$$

[\epsilon] = \frac{[\Delta \mathcal L]}{[\mathcal L]} = \frac{L}{L} = 1

L here means "length dimension", **not** the length of the beam as in (1) p.1-2; just the same notation for a different meaning.

Dimension of stress (pressure):

$$\sigma = \frac{P}{A} \Rightarrow [\sigma] = \frac{[P]}{[A]} = \frac{F}{L^2} \qquad (3)$$

\sigma = \frac{P}{A} \Rightarrow [\sigma] = \frac{[P]}{[A]} = \frac{F}{L^2}

(2) p.1-2:

$$\sigma = E \epsilon \Rightarrow [\sigma] = [E] \underbrace{[\epsilon]}_{1} = [E] \cdot 1 \quad (1)$$

\sigma = E \epsilon \Rightarrow [\sigma] = [E] \underbrace{[\epsilon]}_{1} = [E] \cdot 1

$$[E] = [\sigma] = \frac{F}{L^2} \quad (2)$$

[E] = [\sigma] = \frac{F}{L^2}

Dimension of 2nd moment of inertia:

$$[I] = [y^2][dA] = L^2[dy][dz] = L^4 \quad (3)$$

[I] = [y^2][dA] = L^2 [dy][dz] = L^4

Principle of dimensional homogeneity: In an equation, the dimension of the lhs must equal that of the rhs.

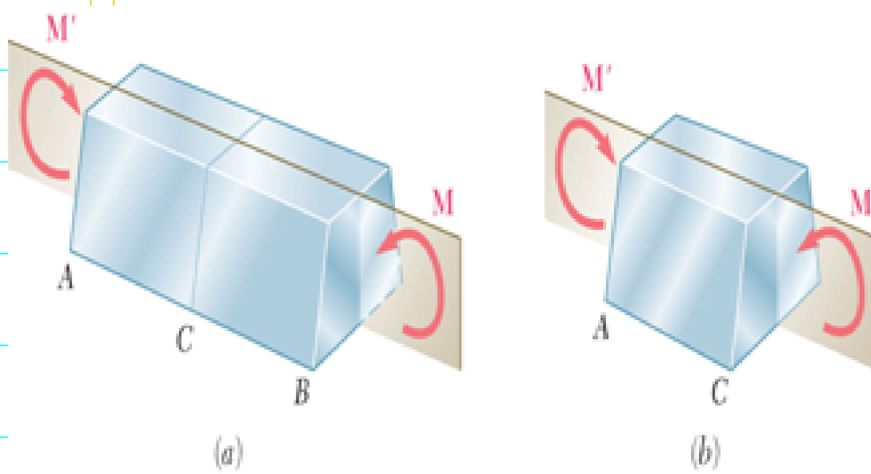
(1) p.1-2:

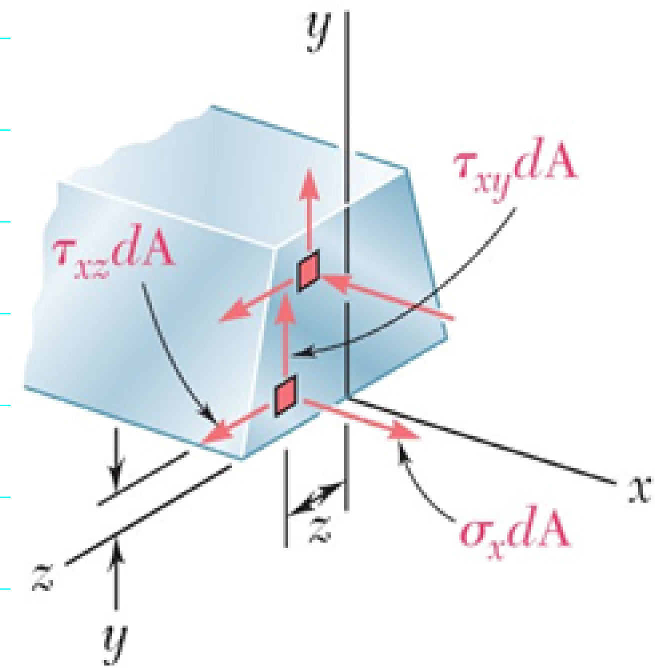
$$[v] = \left[\frac{5qL^4}{384\, EI} \right] = \frac{[5][q][L^4]}{[384][E][I]} = \frac{1 \cdot (F/L) \cdot L^4}{1 \cdot (F/L^2) \cdot L^4} \quad (4)$$

[v] = \left[\frac{5 q L^4}{384 \, EI} \right] = \frac{[5][q][L^4]}{[384][E][I]} =
\frac{1 \cdot (F/L) \cdot L^4}{1 \cdot (F/L^2) \cdot L^4}

$$[lhs] = [rhs] = L \quad (5)$$

[lhs] = [rhs] = L





Beer et al. 2012, p.225

http://tappthatgraphic.files.wordpress.com/2009/11/shelf1.jpg

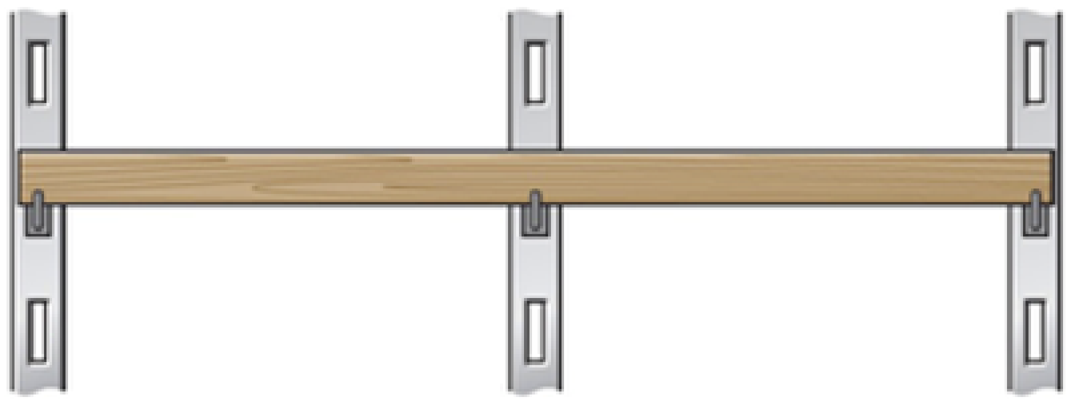
Methods of reducing mid-span deflection v

1. reduce load q

(1) p.1-2: $q \to q/2 \Rightarrow v \to v/2$ (1)

q \to q/2 \Rightarrow v \to v/2

2. reduce beam length L



(1) p.1-2: $L \to L/2 \Rightarrow v \to v/16$ (2)

L \to L/2 \Rightarrow v \to v/16

3. increase Young's modulus E (use stronger material)

Timber

Ponderosa Pine, 1.3x10^6 psi, 0.015 lb/in^3
Hickory, 2.2x10^6 psi, 0.026 lb/in^3

Steel ASTM A36

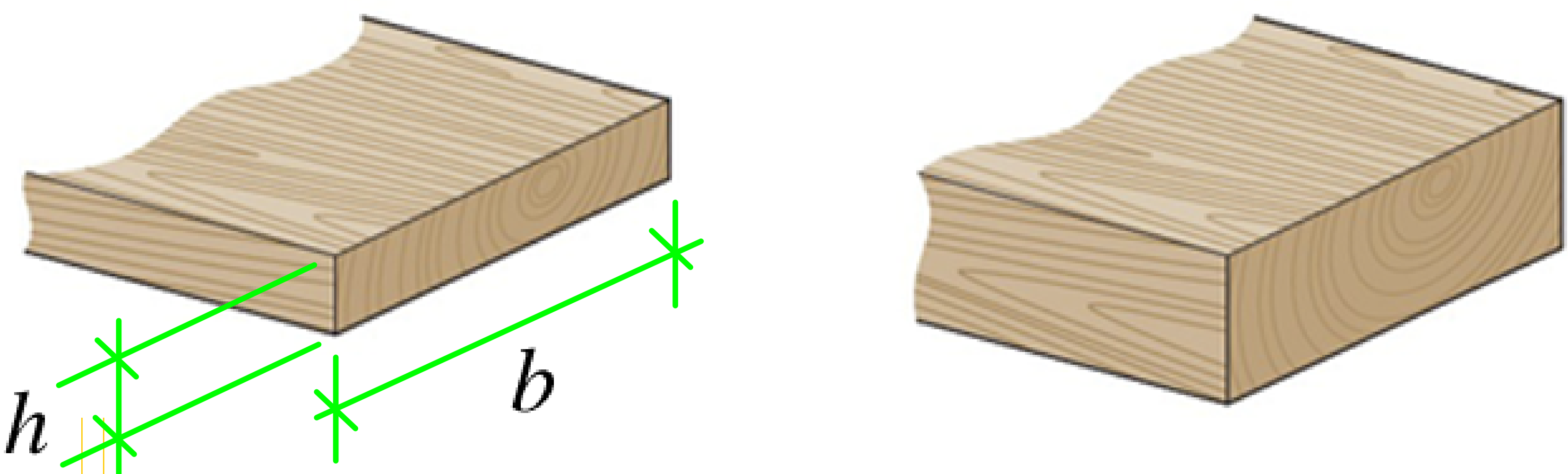
Structural, 29x10^6 psi, 0.284 lb/in^3

Beer et al. 2012, p.A12

So steel is 18-20 times stronger than timber, but is also 11-18 times heavier than timber.

4. increase the 2nd moment of inertia I

   4.1. use deeper beam (increase height of cross section)



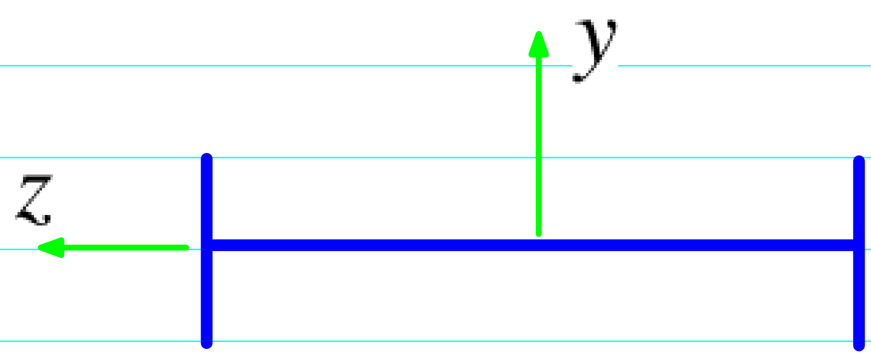
$$I_z = \frac{bh^3}{12} \qquad (1)$$

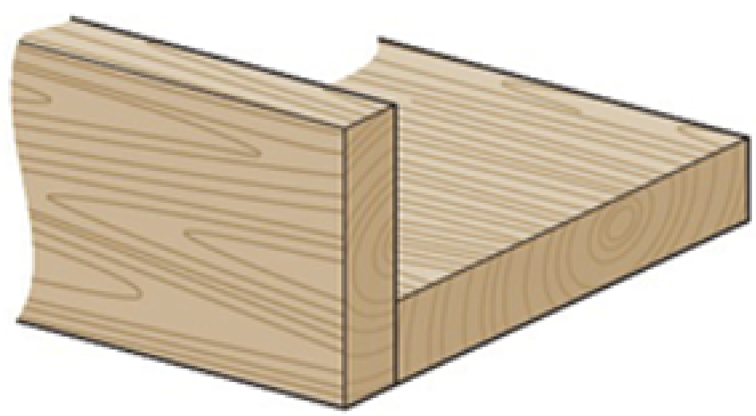
I_z = \frac{b h^3}{12}

$$h \to 2h \Rightarrow I_z \to 8I_z \qquad (2)$$

h \to 2h \Rightarrow I_z \to 8 I_z

4.2. use reinforcement that distributes material away from the z axis





Steif 2012, p.3

Optimization: Design trade-off

Esthetics: Shelf thin and unimposing.

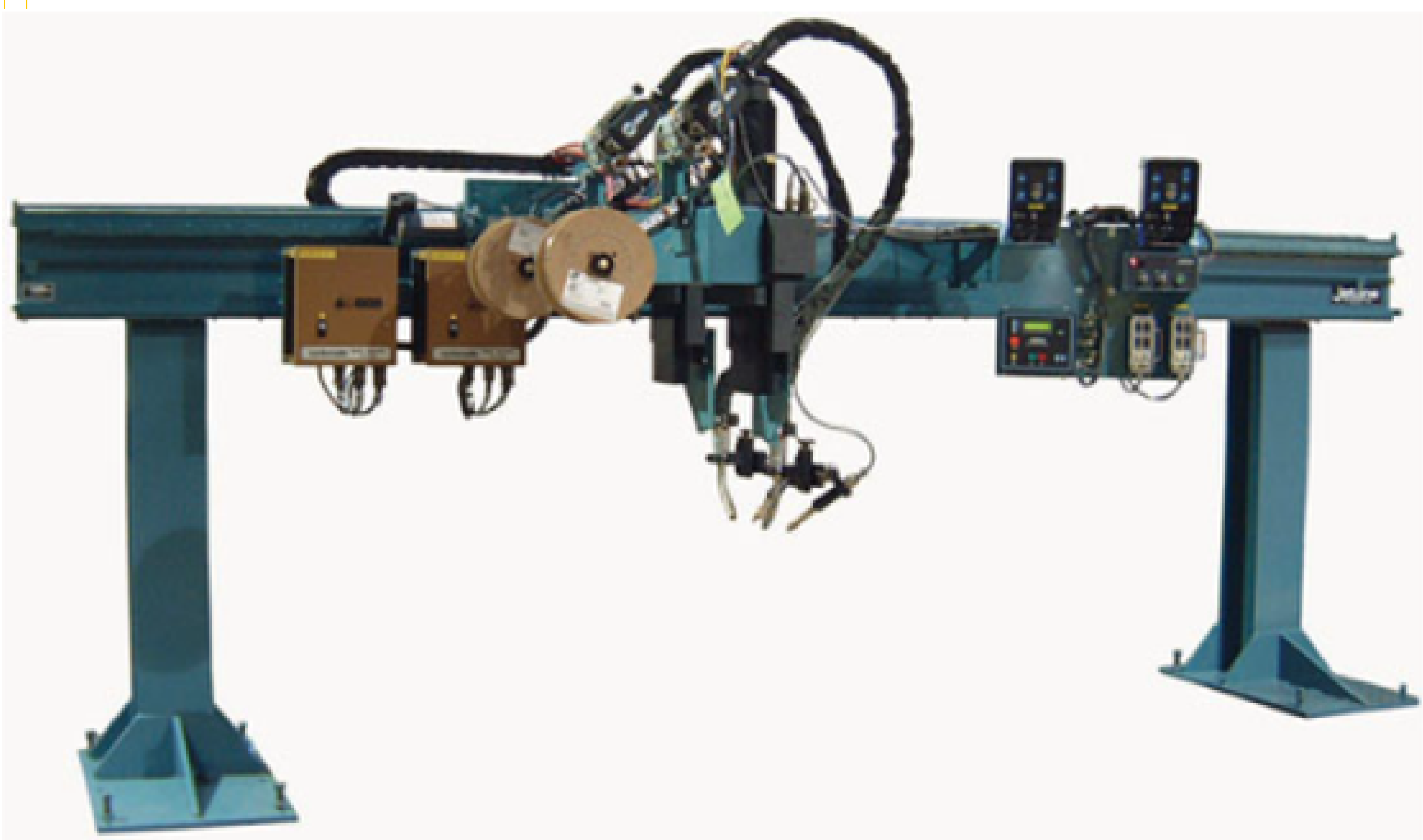
Reliability, perform intended functions: Shelf as thick and strong as possible.

Pushing esthetics to the limit, without considering other (unforeseen) forces, may lead to catastrophic failure: Tacoma Narrows Bridge collapse, 7 Nov 1940

http://startingupanengineer.com/engineering/structural-engineering/epicfails-3-structural-engineering/

Unforeseen forces: Aerodynamic self-excitation (negative damping)

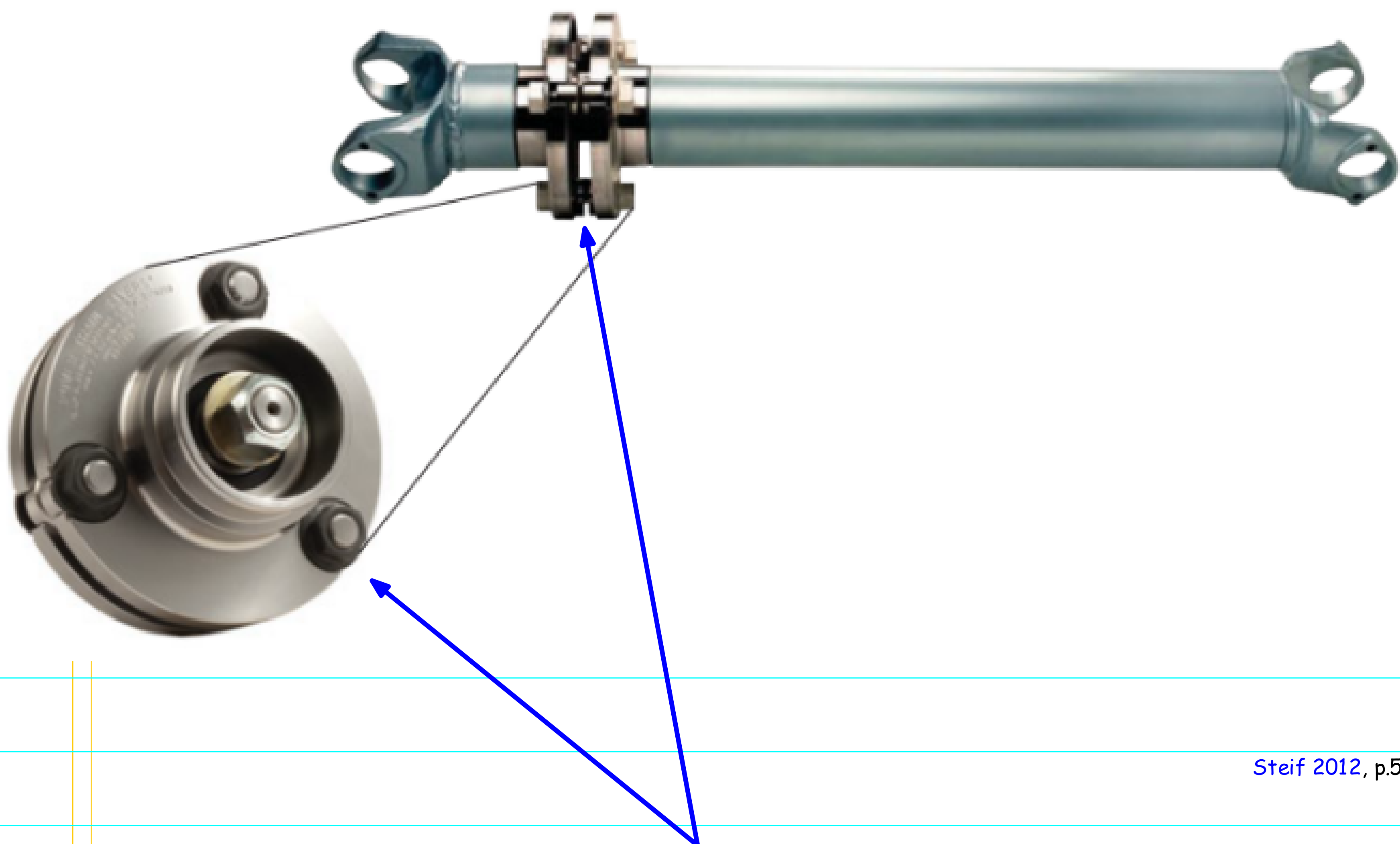
High-precision machinery requires very small deformation, e.g., Computerized welding machine



Steif 2012, p.4

Deliberate, controlled failure is sometimes desired, e.g., similar to an electrical fuse, we want a mechanical fuse to break when the load is above an allowable load to protect a more expensive part.

Example: Torque fuse to protect the transmission shaft in a drive train:



Steif 2012, p.5

Pins break when torque is above a predetermined value (failure by shear force).

Mechanics of materials:

Formulation of structural theories (bars, beams)

Loadings: axial force, torque, transversal force, bending moment

Boundary conditions: simply supported, clamped

Find deflections (deformation, strains), e.g., (1) p.1-2.

Find stresses.

Pb.1-1:

The textbook by Beer et al. 2012 as listed on amazon.com has dimensions 8.2 in x 1.2 in x 9.9 in, and weighs 3.6 lbs.

Consider a bookshelf lined up with the above books, from support to support. The bookshelf, made of Ponderosa Pine (p.1-4), has a rectangular cross section (p.1-5):

$$L = 100\, in,\ b = 9\, in,\ h = 0.5\, in \qquad (1)$$

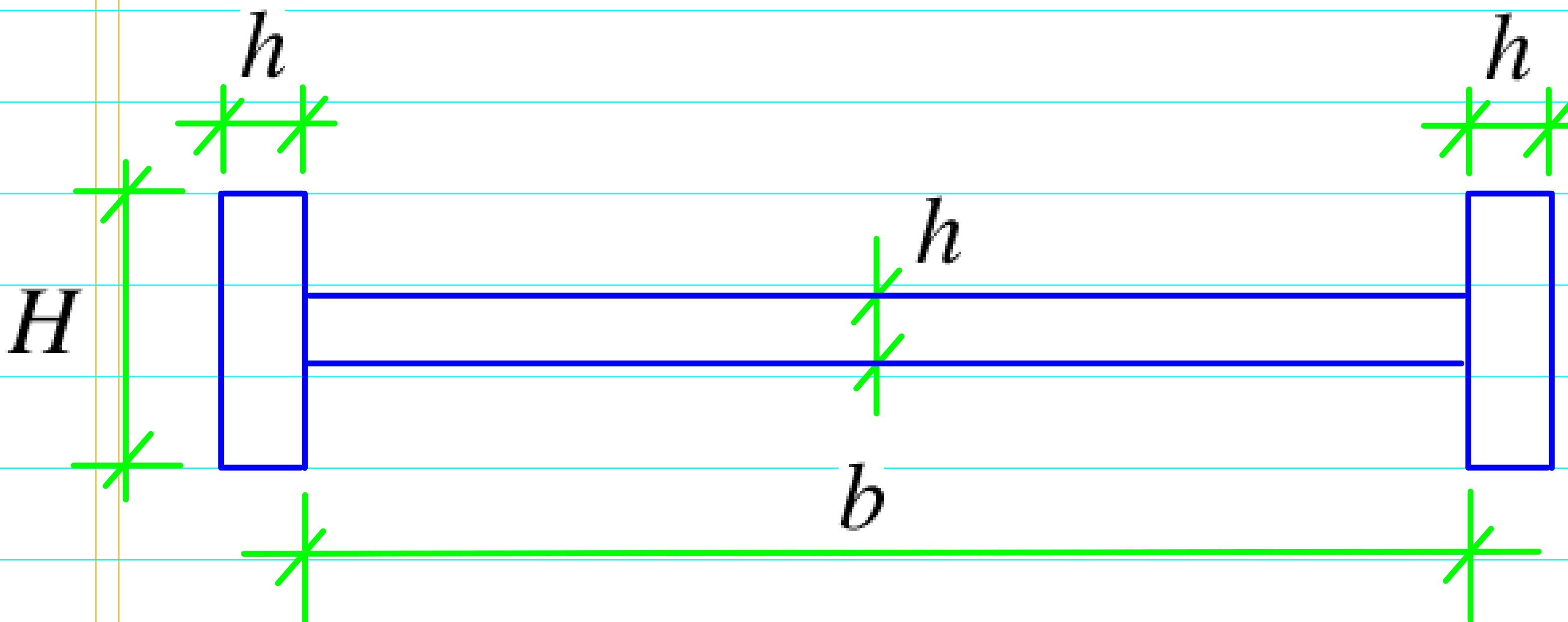
L = 100 \, in , \ b = 9 \, in , \ h = 0.5 \, in

1. Find the vertical mid-span deflection v under the weight of the books and the shelf itself.

2. Increase the shelf thickness to 1 in, find the vertical mid-span deflection.

3. Repeat Parts 1 and 2 with the shelf made from structural steel ASTM-A36.

4. Next, consider reinforcing the bookshelf with 2 side strips so to have the H cross section (p.1-6), with the following geometry:



Consider the following values:

$$b = 9 \, in, \ h = 0.3 \, in, \ H = 1 \, in \tag{1}$$

b = 9 \, in , \ h = 0.3 \, in , \ H = 1 \, in

Find the vertical mid-span deflection for both materials: Ponderosa Pine and A36 steel.

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