Temporal Characteristics of Random Processes

Young W Lim

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Based on Probability, Random Variables and Random Signal Principles, P.Z. Peebles, Jr. and B. Shi



Random Variable Definition

A random variable

a real function over a sample space $S = \{s_1, s_2, s_3, ..., s_n\}$

 $s \to X(s)$ x = X(s)

a function of the possible outcomes s of an experiment

Random Variable Definition

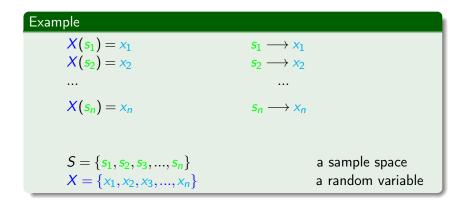
A random variable

 $s \to X(s)$ x = X(s)

$s \rightarrow x$

- a random variable : a capital letter X
- a particular value : a lowercase letter x
- a sample space $S = \{s_1, s_2, s_3, ..., s_n\}$
- an element of S : s

Random Variable Example



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Random Process

A random process

a function of both outcome s and time t

X(t,s)

assigning a time function to every outcome s_i

 $s_i \rightarrow x(t, s_i)$

the family of such time functions is called a random process

 $x(t,s_i) = X(t,s_i)$ x(t,s) = X(t,s)

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Ensemble of time functions

Time functions

A random process X(t,s) represents a family or ensemble of time functions

X(t, s) represents

- a single time function x(t,s)
- when *t* is a variable and *s* is fixed at an outcome

x(t, s) represents

- a sample function,
- an ensemble member,
- a realization of the process

Short-form notation for time functions

The short-form notation x(t)

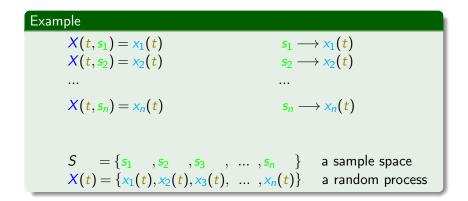
to represent a specific waveform of a random process X(t) for a given **outcome** s_i

 $\mathbf{x}(t) = \mathbf{x}(t,s)$

X(t) = X(t,s)

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Random Process Example



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Random variables with time

a random process X(t,s) represents a single time function when t is a variable and s is fixed at an outcome

a random process X(t,s) represents a single random variable when both t and s are fixed at a time and an outcome, respectively

$$X_i = X(t_i, s) = X(t_i)$$
 random variable

X(t,s) = X(t) random process

An alphabet

the **alphabet** of X(t)

the set of its possible values

- the values of time t for which a random process is defined
- the **alphabet** of the random variable X = X(t) at time t

Classification of Random Processes (1) Types of time and alphabet

- the values of time t for which a random process is defined
 - continuous time
 - discrete time
- the alphabet of the random variable X = X(t) at time t
 - continuous alphabet
 - discrete alphabet

Classification of Random Processes (2) types of the random variable X(t) and the time t

- a continuous alphabet continuous time random process

 X(t) has continuous values and t has continuous values

 a discrete alphabet continuous time random process

 X(t) has discrete values and t has continuous values

 a continuous alphabet discrete time random process

 X(t) has continuous values and t has discrete values

 a continuous alphabet discrete time random process

 X(t) has continuous values and t has discrete values

 a discrete alphabet discrete time random process

 X(t) has continuous values and t has discrete values
 - X(t) has discrete values and t has discrete values

Deterministic and Non-deterministic Random Processes

- A process is non-deterministic if future values of any sample function <u>cannot</u> be <u>predicted</u> exactly from observed past values
- A process is **deterministic** if future values of any sample function can be predicted from observed past values

Deterministic Random Process Example (1)

$$X(t) = A\cos(\omega_0 t + \Theta)$$

A, Θ , or ω_0 (or all) can be random variables.

a <u>sample function</u> corresponds to the above equation with particular values of these random variables.

 $x_i(t) = A_i \cos(\omega_{0,i} t + \Theta_i)$

Deterministic Random Process Example (2)

$$x_i(t) = A_i \cos(\omega_{0,i} t + \Theta_i)$$

the knowledge of the <u>sample function</u> prior to any time instance fully allows the prediction of the <u>sample function</u>'s future values because all the necessary information is known

$$\mathbf{x}_{i}(t)$$
 $t \leq 0$ \implies $\mathbf{x}_{i}(t)$ $t > 0$

Functions and variables of a random process $X(t, \theta)$ (1)

$X(t, \theta)$	a family of functions, an ensemble
$X(t, \theta_k)$	a single time function selected by the outcome $ heta_k$
$X(t_1, \theta)$	a random variable at the time $t = t_1$
$X(t_1, \theta_k)$	a number at the time $t = t_1$, of the outcome $ heta_k$

https://www.cis.rit.edu/class/simg713/notes/chap7-random-process.pdf

Functions and variables of a random process $X(t, \theta)$ (2)

- X(t, θ) is a family of functions. Imagine a giant strip chart recording in which each pen is identified with a different θ. This family of functions is traditionally called an ensemble.
- A single function X(t, θ_k) is selected by the outcome θ_k. This is just a time function that we could call X_k(t). Different outcomes give us different time functions.
- If t is fixed, say $t = t_1$, then $X(t_1, \theta)$ is a random variable. Its value depends on the outcome θ .
- If both t_1 and θ_k are given then $X(t_1, \theta_k)$ is just a number. https://www.cis.rit.edu/class/simg713/notes/chap7-random-process.pdf

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