Temporal Characteristics of Random Processes

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Random Variable Definition

A random variable

a real **function** over a **sample space** $S = \{s_1, s_2, s_3, ..., s_n\}$

$$s \rightarrow X(s)$$

$$x = X(s)$$

a **function** of the possible **outcomes** s of an **experiment**

Random Variable Definition

A random variable

$$s \rightarrow X(s)$$

$$x = X(s)$$

$$s \rightarrow x$$

- a **random variable** : a capital letter X
- a particular value : a lowercase letter x
- a **sample space** $S = \{s_1, s_2, s_3, ..., s_n\}$
- an element of S : s

Random Variable Example

Example

$$X(s_1) = x_1 \qquad s_1 \longrightarrow x_1$$
$$X(s_2) = x_2 \qquad s_2 \longrightarrow x_2$$
$$\cdots \qquad \cdots$$
$$X(s_n) = x_n \qquad s_n \longrightarrow x_n$$

$S = \{s_1, s_2, s_3, ..., s_n\}$ a sample space

$X = \{x_1, x_2, x_3, ..., x_n\}$ a random variable

Random Process

A random process

a function of both **outcome** s and **time** t

$$X(t, s)$$

assigning a time function to every **outcome** s_i

$$s_i \rightarrow x(t, s_i)$$

the family of such time functions is called a random process

$$x(t, s_i) = X(t, s_i)$$

$$x(t, s) = X(t, s)$$

Ensemble of time functions

Time functions

A random process $X(t, s)$ represents
a family or ensemble of time functions

$X(t, s)$ represents

- a single time function $x(t, s)$
- when t is a variable and s is fixed at an outcome

$x(t, s)$ represents

- a sample function,
- an ensemble member,
- a realization of the process

Short-form notation for time functions

The short-form notation $x(t)$

to represent a specific waveform of a random process $X(t)$ for a given **outcome** s_i

$$x(t) = x(t, s)$$

$$X(t) = X(t, s)$$

Random Process Example

Example

$$X(t, s_1) = x_1(t) \qquad s_1 \longrightarrow x_1(t)$$
$$X(t, s_2) = x_2(t) \qquad s_2 \longrightarrow x_2(t)$$
$$\ldots \qquad \ldots$$
$$X(t, s_n) = x_n(t) \qquad s_n \longrightarrow x_n(t)$$

$$S = \{s_1, s_2, s_3, \ldots, s_n\}$$ a sample space

$$X(t) = \{x_1(t), x_2(t), x_3(t), \ldots, x_n(t)\}$$ a random process

Random variables with time

a **random process** $X(t, s)$ represents a single time function
when t is a variable and s is fixed at an outcome

a random process $X(t, s)$ represents a single **random variable**
when both t and s are fixed at a time and an outcome, respectively

$$X_i = X(t_i, s) = X(t_i) \qquad \textit{random variable}$$

$$X(t, s) = X(t) \qquad \textit{random process}$$

An alphabet

the alphabet of $X(t)$

the set of its possible values

- the values of **time** t for which a random process is defined
- the **alphabet** of the random variable $X = X(t)$ at time t

Classification of Random Processes

(1) Types of time and alphabet

- the values of **time** t for which a random process is defined
  - continuous time
  - discrete time
- the **alphabet** of the random variable $X = X(t)$ at time t
  - continuous alphabet
  - discrete alphabet

Classification of Random Processes

(2) types of the random variable $X(t)$ and the time t

- a continuous **alphabet** continuous **time** random process
  - $X(t)$ has continuous values and t has continuous values
- a discrete **alphabet** continuous **time** random process
  - $X(t)$ has discrete values and t has continuous values
- a continuous **alphabet** discrete **time** random process
  - $X(t)$ has continuous values and t has discrete values
- a discrete **alphabet** discrete **time** random process
  - $X(t)$ has discrete values and t has discrete values

Deterministic and Non-deterministic Random Processes

- A process is **non-deterministic**
  if future values of any sample function
  cannot be predicted exactly from observed past values
- A process is **deterministic**
  if future values of any sample function
  can be predicted from observed past values

Deterministic Random Process Example (1)

$$X(t) = A\cos(\omega_0 t + \Theta)$$

A, Θ, or ω_0(or all) can be random variables.

a sample function corresponds to the above equation with particular values of these random variables.

$$x_i(t) = A_i\cos(\omega_{0,i} t + \Theta_i)$$

Deterministic Random Process Example (2)

$$x_i(t) = A_i \cos(\omega_{0,i} t + \Theta_i)$$

the knowledge of the sample function
prior to any time instance fully allows
the prediction of the sample function's future values
because all the necessary information is known

$$x_i(t) \quad t \leq 0 \qquad \Longrightarrow \qquad x_i(t) \quad t > 0$$

Functions and variables of a random process $X(t, \theta)$ (1)

$X(t, \theta)$	a family of functions, an ensemble
$X(t, \theta_k)$	a single time function selected by the outcome θ_k
$X(t_1, \theta)$	a random variable at the time $t = t_1$
$X(t_1, \theta_k)$	a number at the time $t = t_1$, of the outcome θ_k

https://www.cis.rit.edu/class/simg713/notes/chap7-random-process.pdf

Functions and variables of a random process $X(t, \theta)$ (2)

- $X(t, \theta)$ is a family of functions. Imagine a giant strip chart recording in which each pen is identified with a different θ. This family of functions is traditionally called an ensemble.
- A single function $X(t, \theta_k)$ is selected by the outcome θ_k. This is just a time function that we could call $X_k(t)$. Different outcomes give us different time functions.
- If t is fixed, say $t = t_1$, then $X(t_1, \theta)$ is a random variable. Its value depends on the outcome θ.
- If both t_1 and θ_k are given then $X(t_1, \theta_k)$ is just a number.

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