

# FFT Octave Codes (1A)

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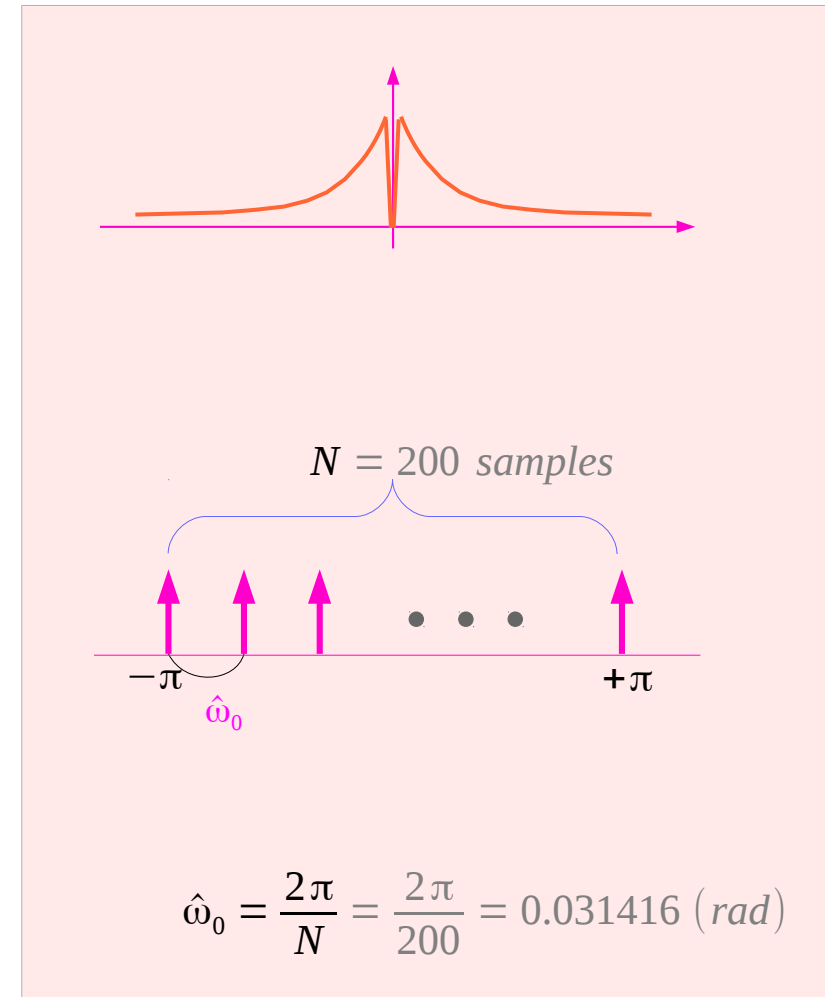
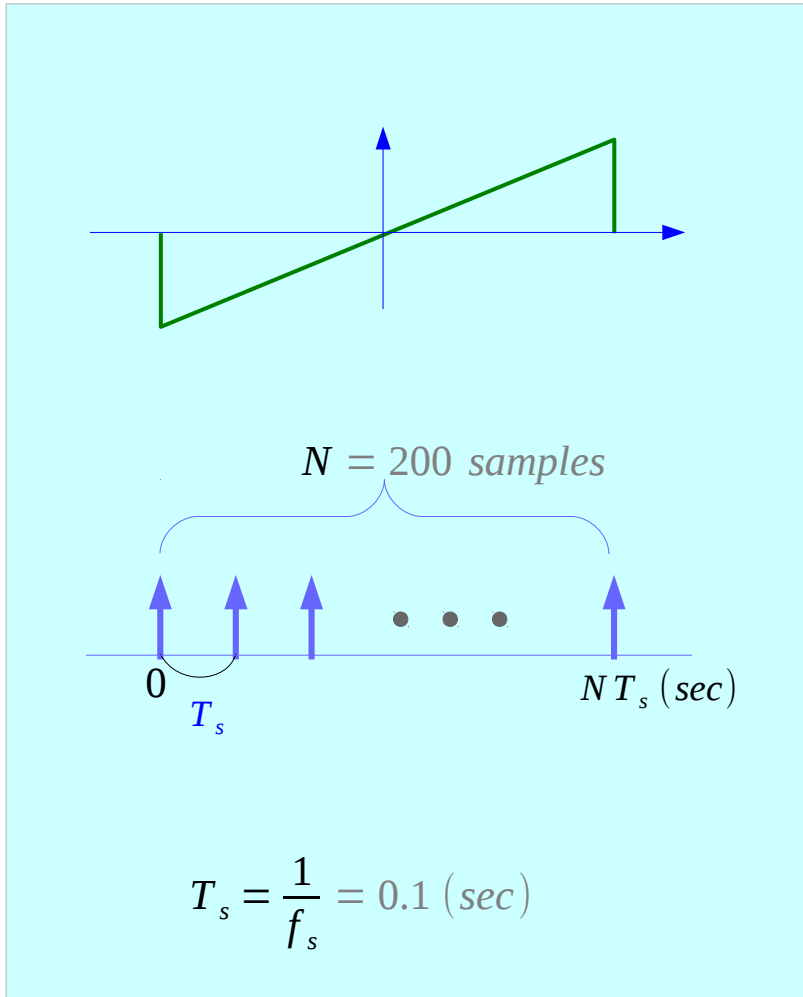
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Based on

B Ninness, Spectral Analysis using the FFT

# Time Scales and Frequency Scales



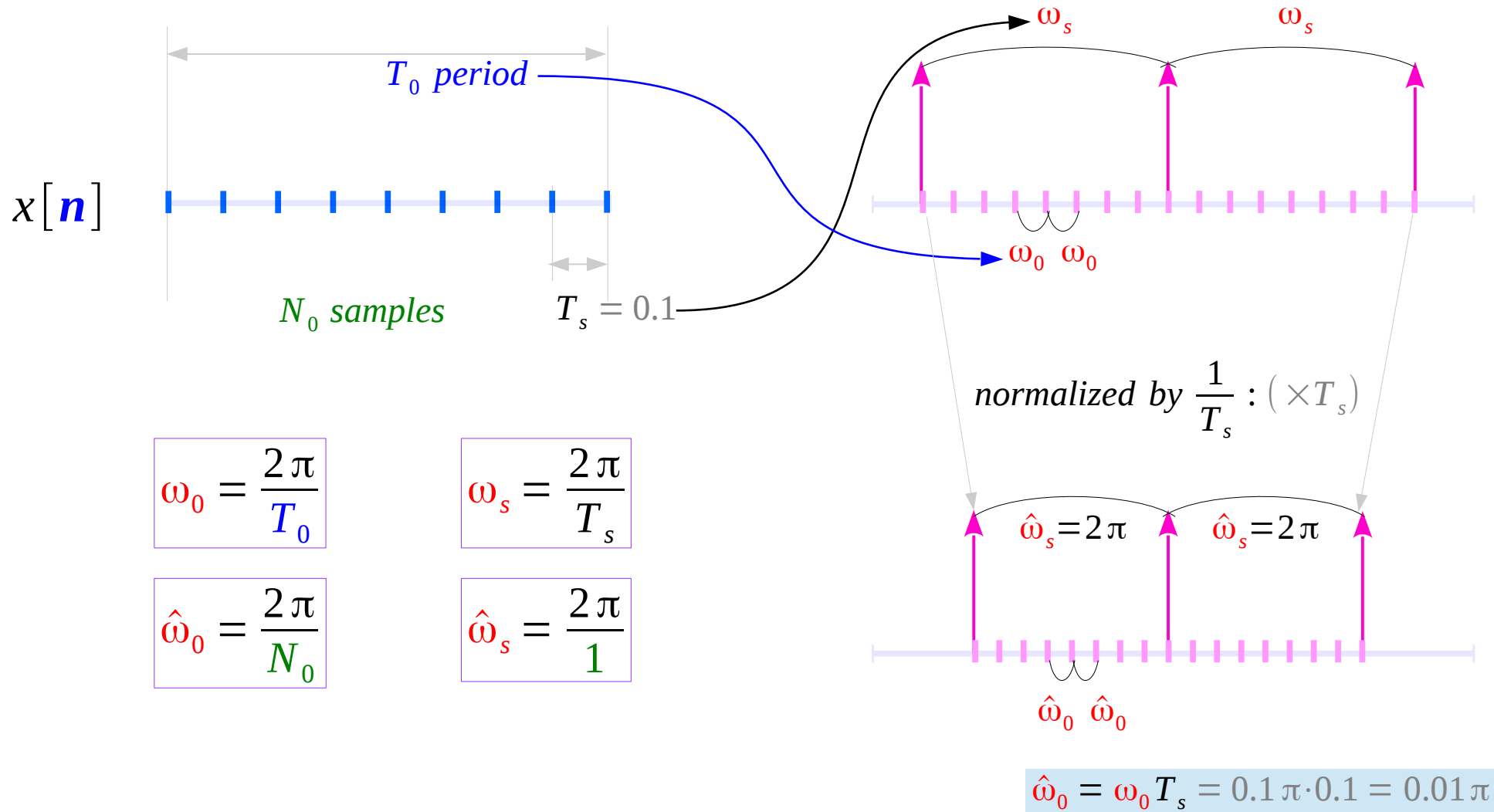
B Ninness, Spectral Analysis using the FFT

# Replication Frequency $\omega_s$ and Resolution $\omega_0$

$$T_0 = T_s N_0 = 200 \cdot 0.1$$

$$\omega_0 = \frac{2\pi}{200 \cdot 0.1} = 0.1\pi$$

$$\omega_s = \frac{2\pi}{0.1} = 20\pi$$



# Comparison of variable names

```
function s1( N )
% N = 200;
```

```
fs = 10;
t = (0:1:N-1)/fs;
x = cos(pi*t);
```

```
ws = 2*pi/N;
wnorm = -pi:w0:+pi;
wnorm = wnorm(1:N);
w = wnorm * fs;
X = fftshift(fft(x));
```

```
subplot(2,1,1);
plot(t, x);
subplot(2,1,2);
plot(w, abs(X));
```

Variables  
used in  
octave

Variables  
used in  
formulas

fs

$$f_s = \frac{1}{T_s}$$

ws

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

wnorm

$$k \hat{\omega}_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

w

$2\pi fs$

$$\hat{\omega}_0 = 2\pi \left( \frac{f_0}{f_s} \right) = 2\pi \left( \frac{T_s}{NT_s} \right) = \frac{2\pi}{N}$$

$$\omega_s = 2\pi f_s = \frac{2\pi}{T_s}$$

B Ninness, Spectral Analysis using the FFT

# FFT of a cosine wave

$$\begin{matrix} x & \longleftrightarrow & X \\ t & & \omega \end{matrix}$$

$$\cos(\pi t)$$

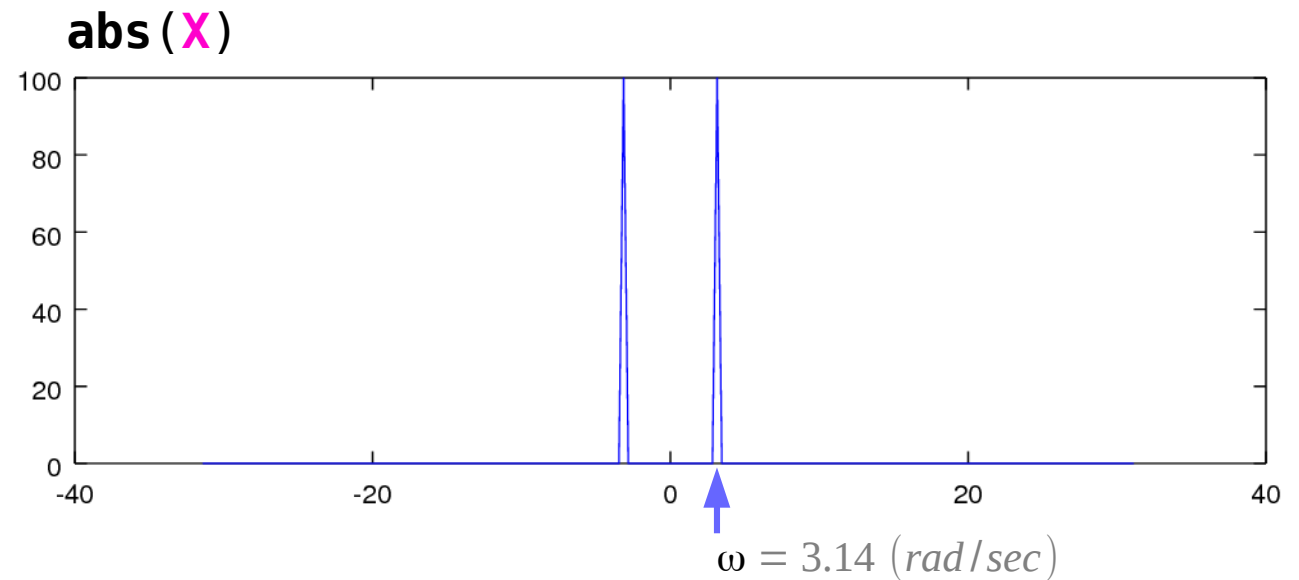
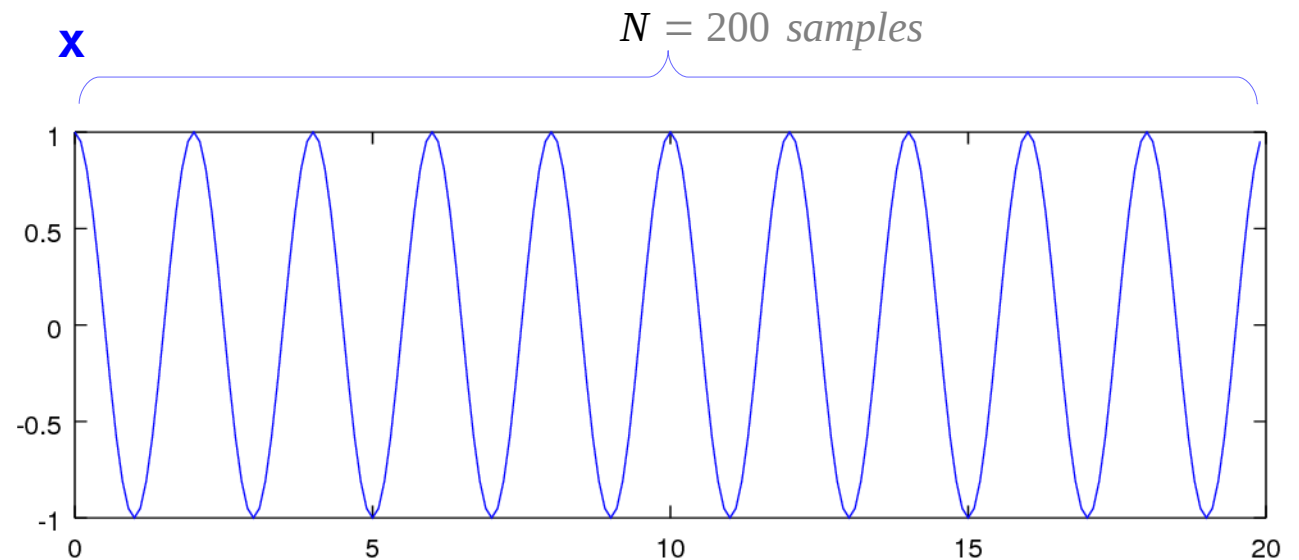
$$\cos(\pi n T_s)$$

$$\cos(0.1\pi n)$$

$$\hat{\omega} = 0.1\pi$$

$$\omega = \hat{\omega} \cdot f_s = 0.1\pi \cdot 10 = 3.14$$

$$\hat{\omega} = \omega \cdot T_s = \pi \cdot 0.1 = 0.314$$



# FFT of a cosine wave – non-integer multiple period

$$\cos(5\pi/3 \cdot t)$$

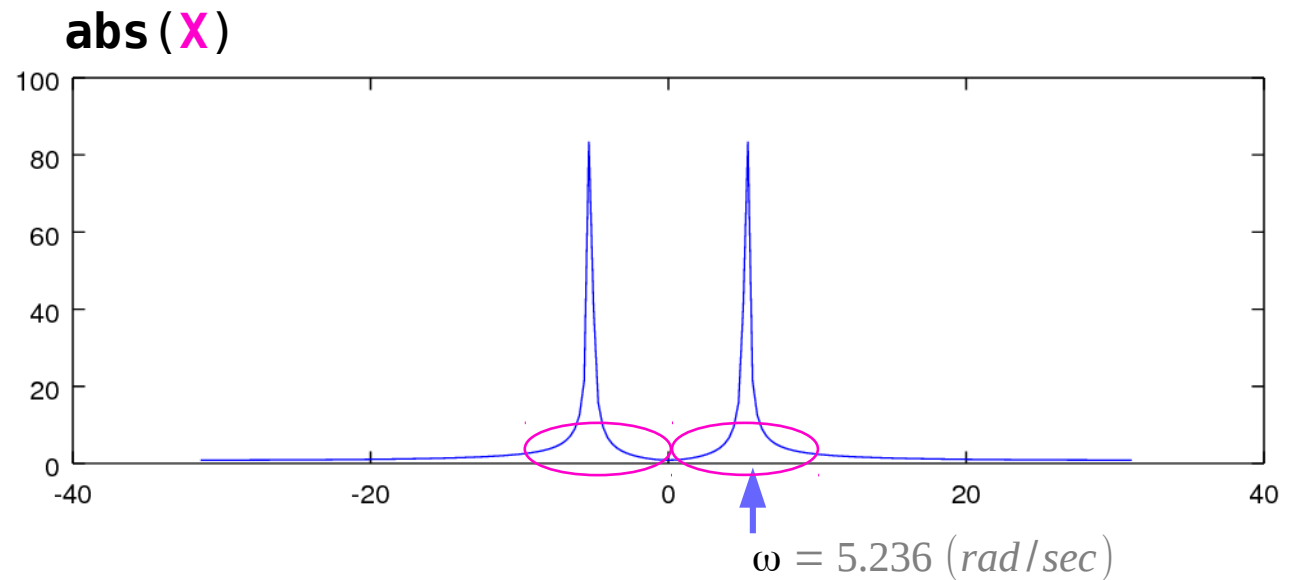
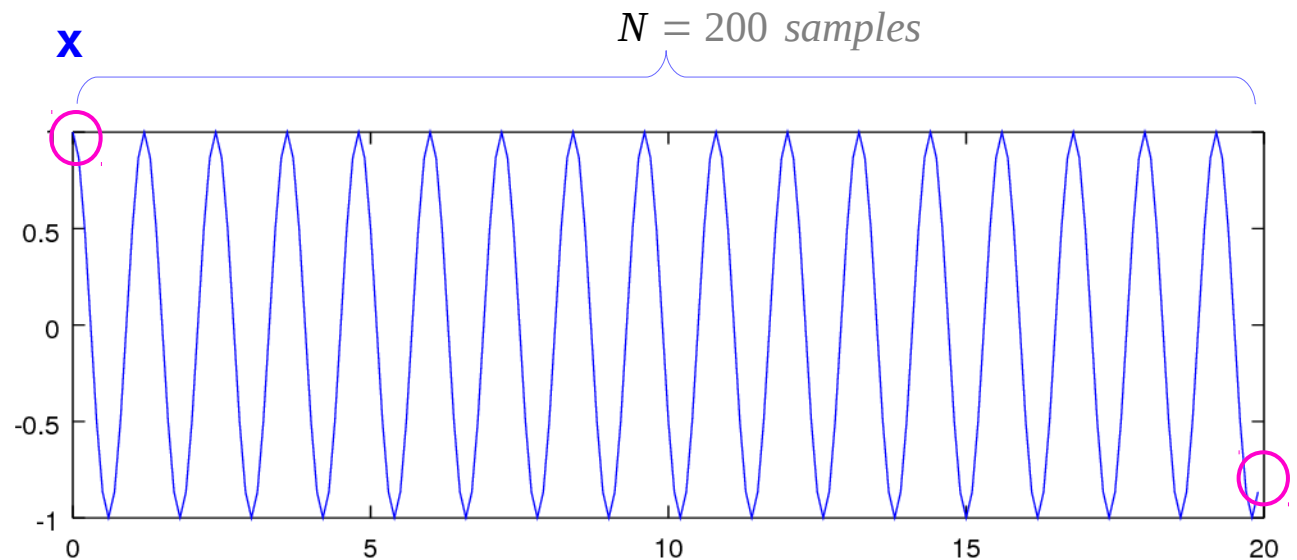
$$\cos(5\pi/3 \cdot nT_s)$$

$$\cos(0.1667\pi n)$$

$$\hat{\omega} = 0.1667\pi = 0.5236$$

$$\omega = \hat{\omega} \cdot f_s = 0.1667\pi \cdot 10 = 5.236$$

$$\hat{\omega} = \omega \cdot T_s = 1.667\pi \cdot 0.1 = 0.5236$$





$$\omega = k \omega_0$$

$$x(t) = \cos(\pi \cdot t) = \frac{1}{2}(e^{+j\pi t} + e^{-j\pi t})$$

$$N = 200, \quad T_s = 0.1$$

$$\omega = \frac{2\pi}{T} = \pi$$
$$T = \frac{2\pi}{\omega} = 2$$

$$\omega_0 = \frac{2\pi}{T_0} = 0.1\pi$$
$$T_0 = N T_s = 200 \cdot 0.1 = 20$$

$$k = 10$$

$$C_{10} = \frac{1}{2} \quad (\omega = 10 \omega_0 = \pi)$$

# Fourier Series and Spectrum

$$x(t) = \cos(\pi \cdot t) = \frac{1}{2}(e^{+j\pi t} + e^{-j\pi t})$$

$$\omega = \frac{2\pi}{T} = \pi \quad \omega_0 = \frac{2\pi}{T_0} = 0.1\pi$$
$$T = \frac{2\pi}{\omega} = 2 \quad T_0 = N T_s = 200 \cdot 0.1 = 20$$

$$T_0 = N T_s$$

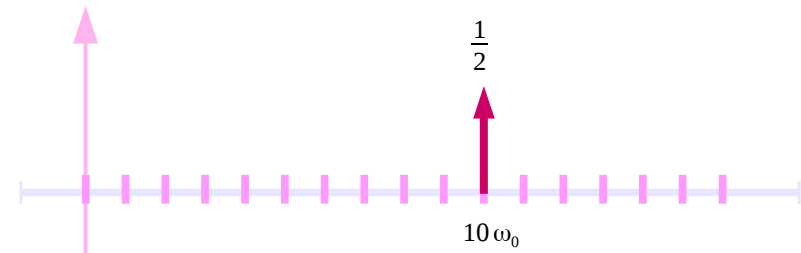
$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{N T_s} = \left(\frac{2\pi}{N}\right) \left(\frac{1}{T_s}\right) = \hat{\omega}_0 f_s$$

$$C_k = \frac{1}{2T_0} \int_0^{T_0} (e^{+j\omega t} + e^{-j\omega t}) e^{-jk\omega_0 t} dt$$
$$= \frac{1}{2T_0} \int_0^{T_0} (e^{+j(\omega - k\omega_0)t} + e^{-j(\omega + k\omega_0)t}) dt$$

$$C_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$C_{10} = \begin{cases} 0.5 & (\omega = \pm 10 \cdot \omega_0) \rightarrow k = 10 \\ 0 & (\omega \neq \pm 10 \cdot \omega_0) \end{cases}$$

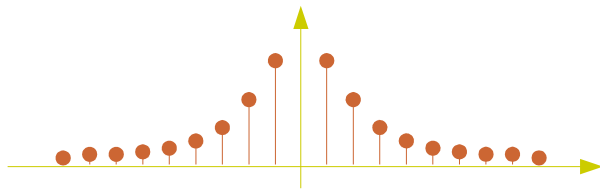
Only when  $\omega$  is an integer multiples of  $\omega_0$



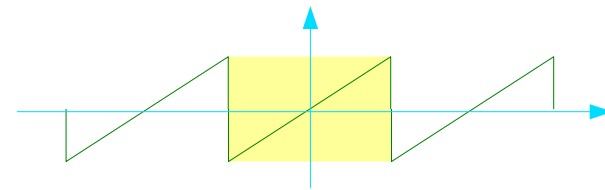
# Continuous Time – CTFS Computation

## Continuous Time Fourier Series

$$C_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k e^{+jk\omega_0 t}$$

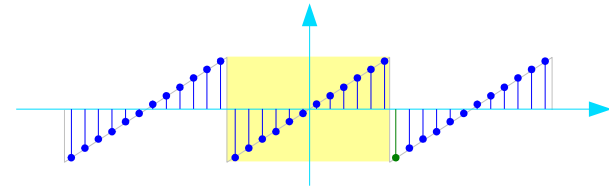
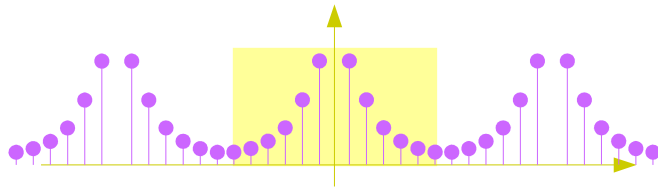


$$\omega_0 = \frac{2\pi}{T}$$



$$C_k \approx \frac{1}{N} \text{DFT}\{x(nT_s)\}$$

$$x(nT_s) \approx N \text{IDFT}\{C_k\}$$



$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$



$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi}{N}kn}$$

# $X[k]$ and $C_k$

$$C_k \approx \frac{1}{N} \text{DFT}\{x(nT_s)\}$$

$$C_k \approx \frac{1}{N} X[k]$$

Scale down

$$N C_k \approx \text{DFT}\{x(nT_s)\}$$

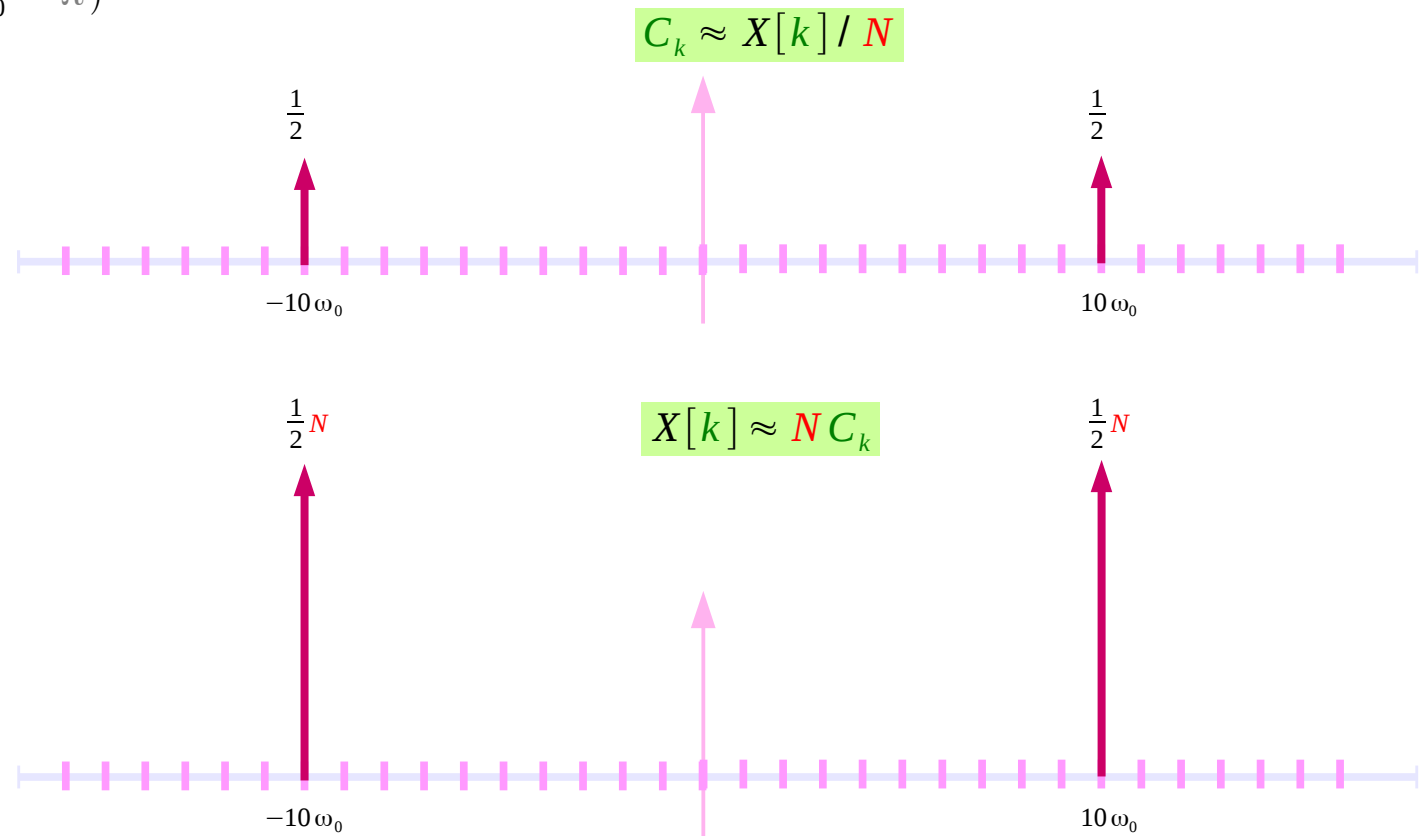
$$X[k] \approx N C_k$$

Scale up

# $X[10]$ and $C_{10}$

$$x(t) = \cos(\pi \cdot t) = \frac{1}{2}(e^{+j\pi t} + e^{-j\pi t})$$

$$C_{10} = \frac{1}{2} \quad (\omega = 10\omega_0 = \pi)$$



# Magnitude of a cosine spectrum

$$\cos(\pi \cdot t) = \frac{1}{2}(e^{+j\pi t} + e^{-j\pi t})$$

$$\cos(5\pi/3 \cdot nT_s) = \cos(\omega nT_s)$$

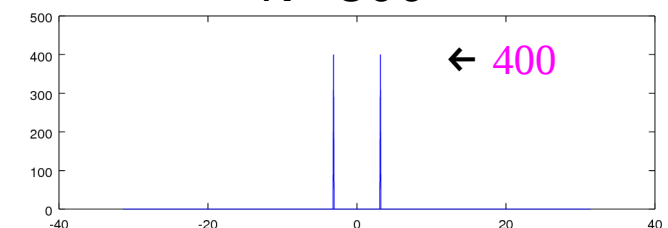
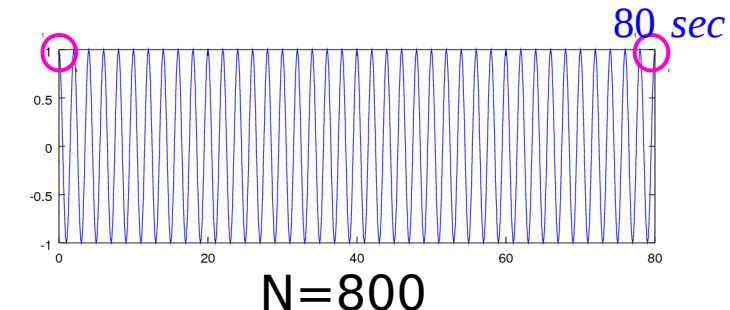
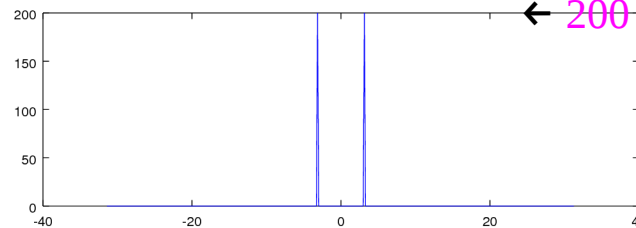
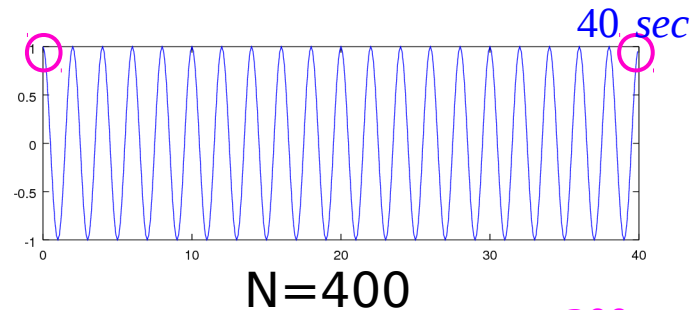
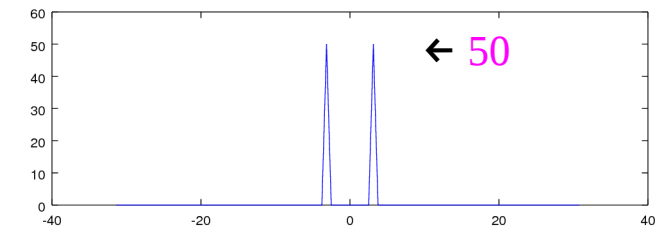
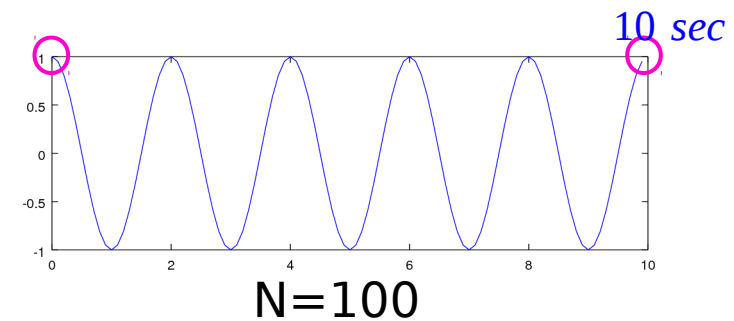
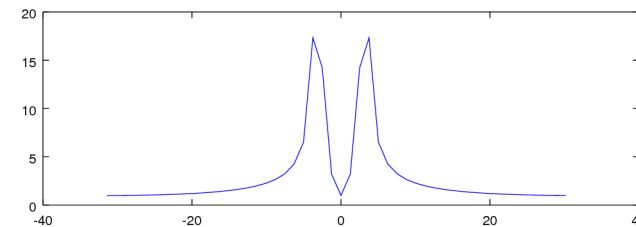
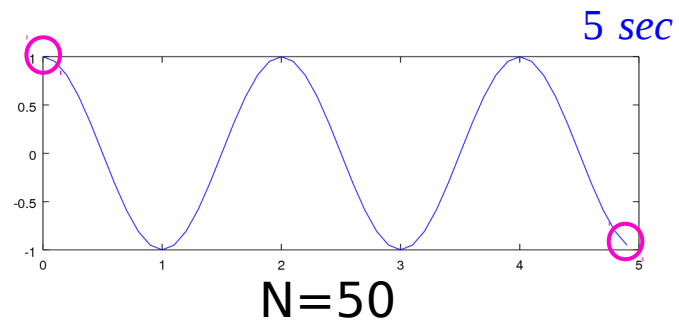
$$\cos(0.1667\pi n) = \cos(\hat{\omega} n)$$

$$\hat{\omega} = 0.1667\pi = 0.5236$$

$$\omega = \hat{\omega} \cdot f_s = 0.1667\pi \cdot 10 = 5.236$$

$$\hat{\omega} = \omega \cdot T_s = 1.667\pi \cdot 0.1 = 0.5236$$

$$N C_k \approx X[k]$$



# Generating a cosine signal with added noise

```
pkg load communications
```

```
N = 400;
```

```
fs = 10;
```

```
t = (0:1:N-1)/fs;
```

```
x = 0.8*cos(2*pi*0.8*t);
```

```
xn = awgn(x, -2);
```

```
X = fft(x);
```

```
Xn = fft(xn);
```

$$\begin{cases} \omega = 4.9770 \\ \text{amp} = 160 \end{cases} \quad \longrightarrow \quad \begin{cases} f = 0.8 \\ A = 160 \end{cases}$$

awgn : additive white Gaussian noise

$$T_s = 0.1 \text{ (sec)} \quad f_s = \frac{1}{T_s} = 10 \text{ (Hz)}$$

$$T_0 = NT_s = 400 \cdot 0.1 = 40 \text{ (sec)}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{NT_s} = \frac{2\pi}{40} = 0.15708 \text{ (rad/sec)}$$

$$\hat{\omega}_0 = \omega_0 T_s = \frac{2\pi}{N} = \frac{2\pi}{400} = 0.015708$$

$$\omega = 2\pi f = 4.9770 \quad \rightarrow \quad f = 0.79211 \approx 0.8$$

$$C_k = \frac{1}{N} X[k]$$

$$\frac{A}{2} = \frac{1}{400} \text{amp} = \frac{1}{400} |X[k]| = \frac{1}{400} 160$$

$$\rightarrow A = \frac{160}{400} * 2 = 0.7953 \approx 0.8$$

B. P. Ninniss, Spectral Analysis using the FFT

# FFT of a noisy cosine

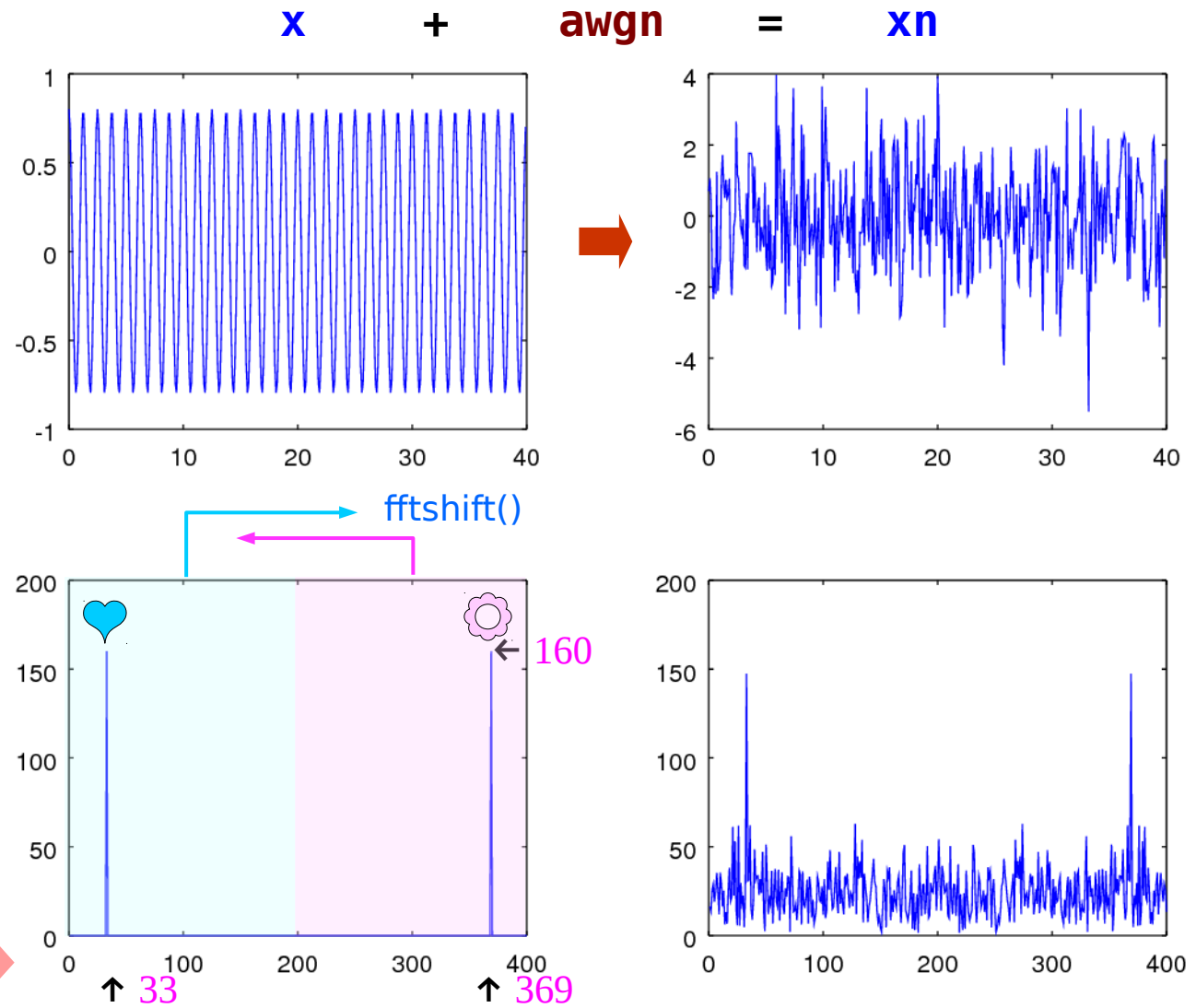
```
subplot(2, 2, 1);  
plot(t, x);
```

```
subplot(2, 2, 2);  
plot(t, xn);
```

```
subplot(2, 2, 3);  
plot(abs(X) );
```

```
subplot(2, 2, 4);  
plot(abs(Xn) );
```

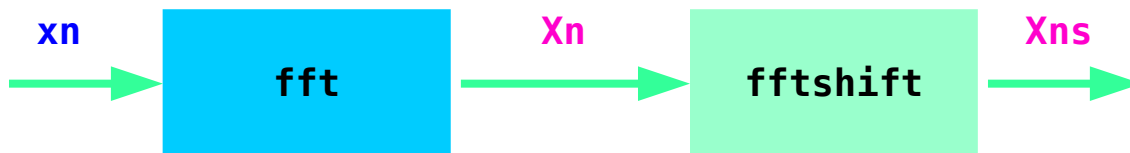
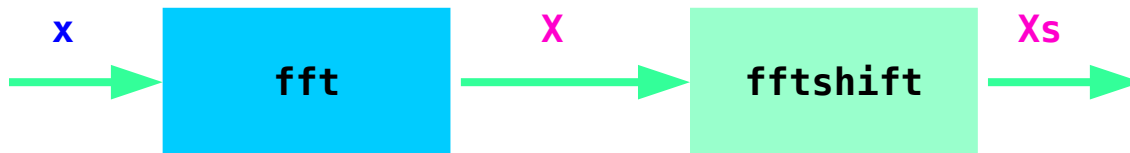
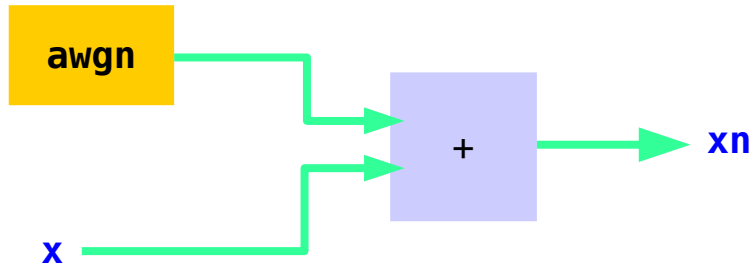
pause



B Ninness, Spectral Analysis using the FFT



# X, Xn, and Xns



# Computing frequency domain scales

```
ws = 2*pi/N;
wnorm = -pi:ws:+pi;
wnorm = wnorm(1:N);
w = wnorm * fs;
```

$$\hat{\omega} = \omega \cdot T_s$$

$$\hat{\omega} \cdot f_s = \omega$$



$$\hat{\omega}_0 = \frac{2\pi}{N} = \frac{2\pi}{NT_s} T_s$$

$$10\hat{\omega}_0$$



$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{N} \cdot \frac{1}{T_s}$$



$$10\omega_0 \quad 10 \frac{2\pi}{NT_s}$$

$$\text{ws} \quad : \quad \hat{\omega}_0 = \frac{2\pi}{N}$$

$$\text{wnorm} \quad : \quad [ -\pi, \dots, k\hat{\omega}_0, \dots, +\pi ]$$

$$\text{w} \quad : \quad [ -\pi f_s, \dots, k\omega_0, \dots, +\pi f_s ]$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{NT_s}$$

$$\hat{\omega}_0 = 2\pi \left( \frac{T_s}{T_0} \right) = \frac{2\pi}{N}$$

$$\omega_0 = \hat{\omega}_0 \cdot f_s$$

$$\hat{\omega}_0 = \omega_0 \cdot T_s$$

$$T_s = 0.1 \text{ (sec)} \quad f_s = \frac{1}{T_s} = 10 \text{ (Hz)}$$

$$T_0 = NT_s = 400 \cdot 0.1 = 40 \text{ (sec)}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{NT_s} = \frac{2\pi}{40} = 0.15708 \text{ (rad/sec)}$$

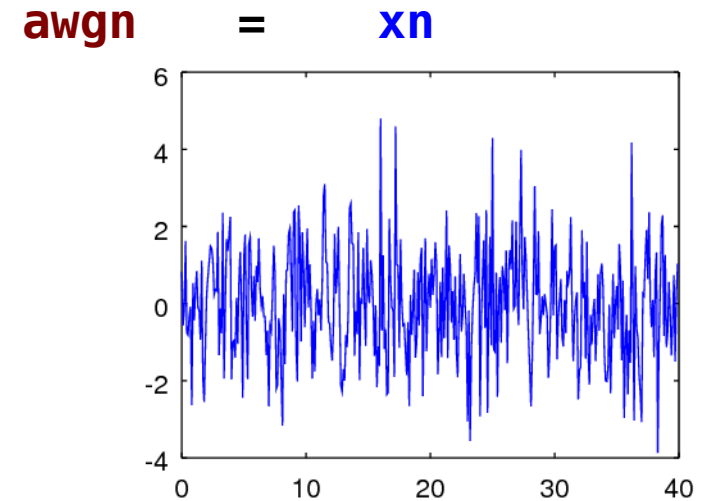
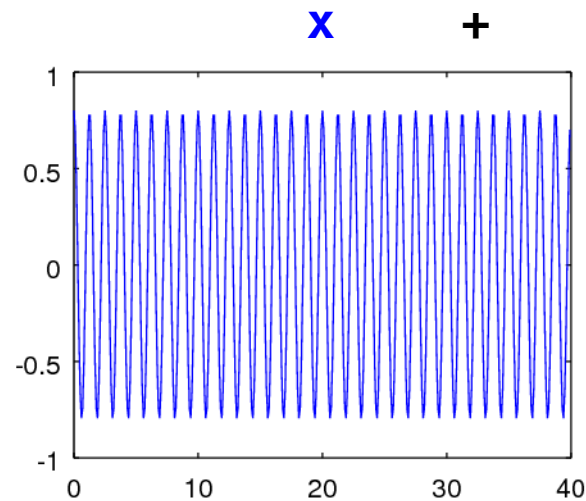
$$\hat{\omega}_0 = \omega_0 T_s = \frac{2\pi}{N} = \frac{2\pi}{400} = 0.015708$$

B Ninness, Spectral Analysis using the FFT

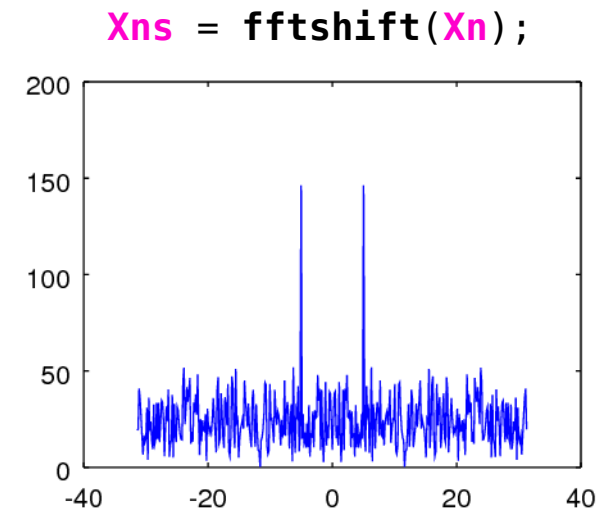
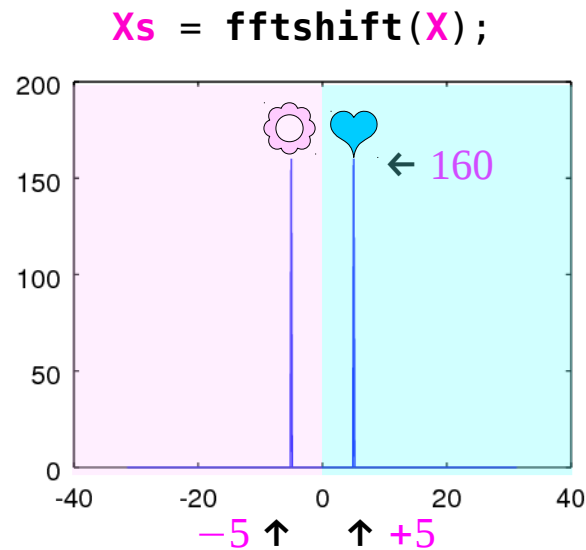
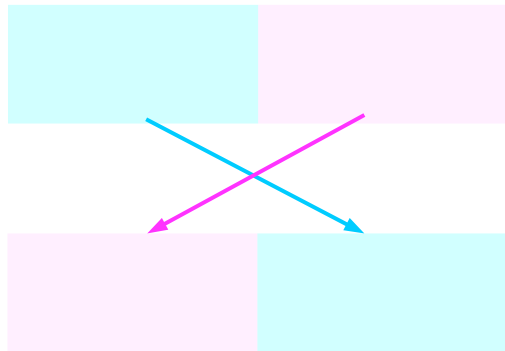
# fftshift

```
Xs = fftshift(X);  
Xns = fftshift(Xn);
```

```
subplot(2,2,3);  
plot(w, abs(Xs));  
subplot(2,2,4);  
plot(w, abs(Xns));
```



fftshift()



B Ninness, Spectral Analysis using the FFT

# Self inverting function : fftshift()

```
Xs ← fftshift(X);
```

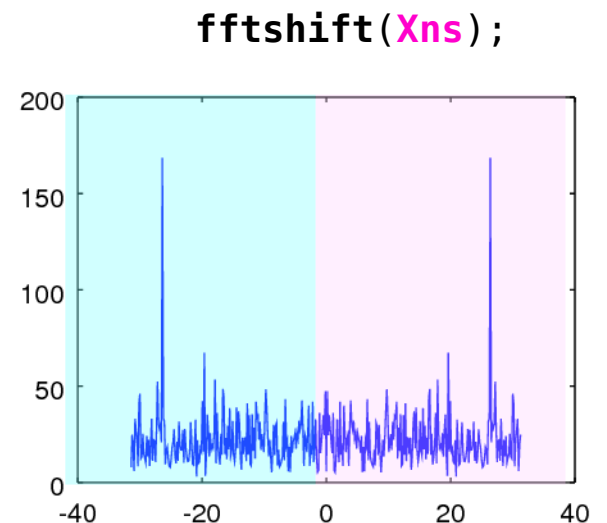
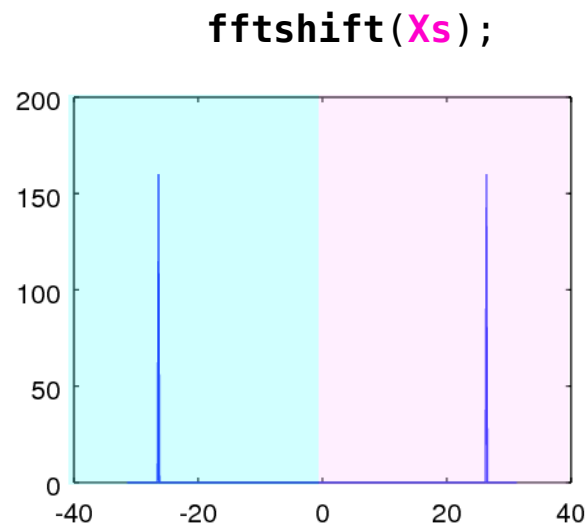
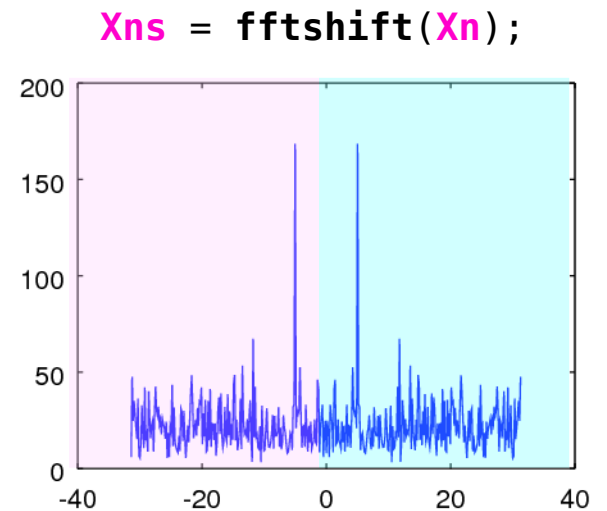
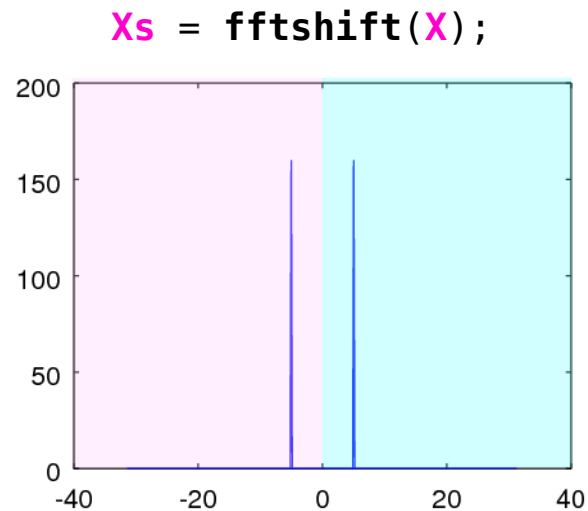
```
Xns ← fftshift(Xn);
```

```
X ← fftshift(Xs);
```

```
Xn ← fftshift(Xns);
```

```
X ← fftshift(fftshift(X));
```

```
Xn ← fftshift(fftshift(Xn));
```



# Octave ginput()

```
% close all figure windows  
% must use this toolkit to use ginput()
```

```
close all;  
graphics_toolkit(fltk);
```

```
plot(w, abs(fftshift(Xn)));
```

```
[freq, amp, buttons]= ginput()
```

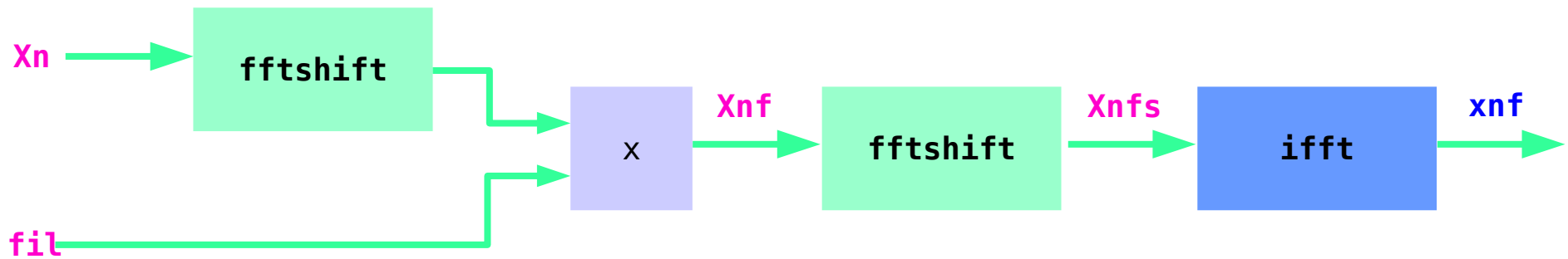
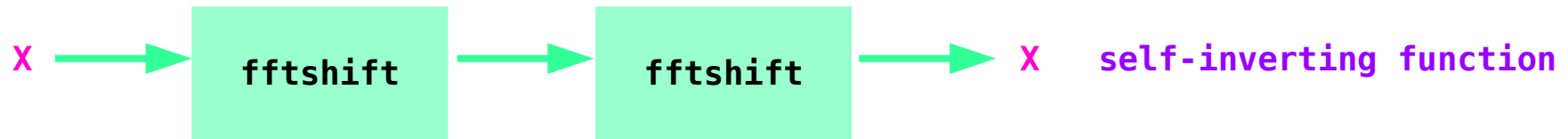
```
>> available_graphics_toolkits  
ans =  
{  
  [1,1] = fltk  
  [1,2] = gnuplot  
  [1,3] = qt  
}
```

```
freq =  
-5.3088  
5.0138  
4.8295  
4.8295  
4.8295  
4.6452  
4.6452
```

```
amp =  
142.448  
141.864  
42.536  
42.536  
42.536  
41.952  
41.952
```

```
buttons =  
1  
1  
1  
1  
1  
3  
3
```

# fftshift() before ifft()



# Filtering

```
% fil = [ zeros(1,160), ones(1,15), zeros(1,50),  
         ones(1,15), zeros(1,160)];  
fil = [ zeros(1,166), ones(1,5), zeros(1,59),  
       ones(1,5), zeros(1,165)];
```



```
Xnf= fil .* Xns;  
xnf= ifft(fftshift(Xnf));
```

```
subplot(2, 2, 3);  
plot(w, abs(fftshift(Xn)), 'b', w, 160*fil, '-.r');
```

black ↗ ↖ red

```
subplot(2, 2, 4);  
plot(t, real(xnf));
```

B Ninness, Spectral Analysis using the FFT

# Filtering results

$X_{ns} .* fil$

$X_{nsf}$

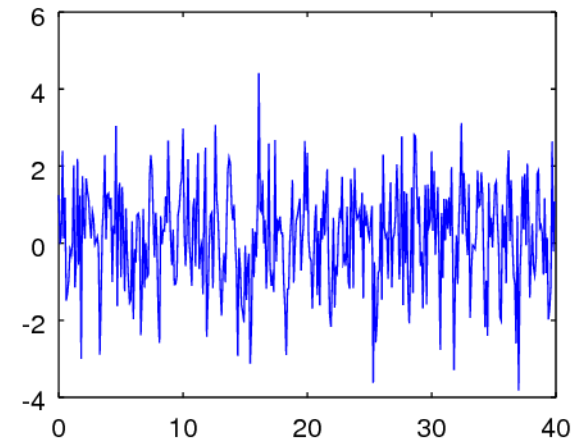
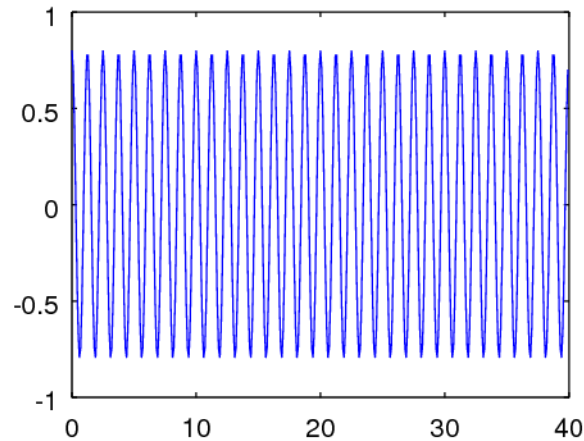
fftshift

$X_{nf}$

ifft

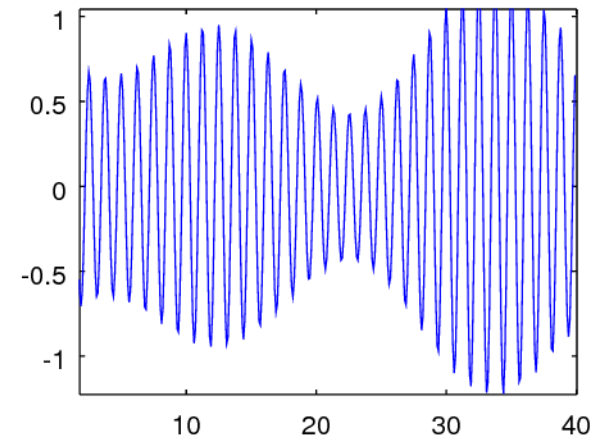
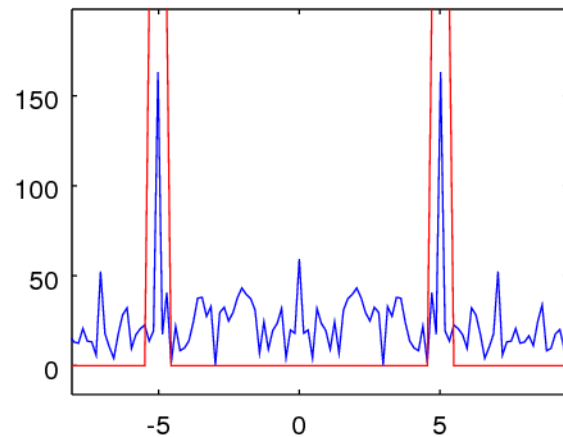
$x_{nf}$

$x + awgn = x_n$



$X_{ns} .* fil$

$x_{nf}$



B. P. Ninniss, Spectral Analysis using the FFT



# Window Function

```
win = hamming(N);
```

Hamming Window function **returns** a column vector

`win(:)` → Column access of the column vector

`xn(:)` → Column access of the row vector

`(:)` does not transpose the original input matrix

To get the correct element-wise multiplication

```
xw = win(:) .* xn(:);
```

`xn` : a row vector

`xw` : a column vector

# Colon Syntax (1)

As a special case, when a colon is used as a single index, the output is a **column vector** containing **all the elements** of the vector or matrix. For example:

```
a(:)      # result is a column vector  
a(:)'  
# result is a row vector
```

The above two code idioms are often used in place of reshape when a simple vector, rather than an arbitrarily sized array, is needed.

Given the matrix

```
a = [1, 2; 3, 4]
```

all of the following expressions are equivalent and select the first row of the matrix.

```
a(1, [1, 2])  # row 1, columns 1 and 2  
a(1, 1:2)    # row 1, columns in range 1-2  
a(1, :)      # row 1, all columns
```

<https://www.gnu.org/software/octave/doc/interpreter/Index-Expressions.html>

# Colon Syntax (2)

```
>> A = [1, 2; 3, 4]
A =
```

```
 1  2
 3  4
```

```
>> A(:)
ans =
```

```
 1
 3
 2
 4
```

```
>> >> B = [1, 2, 3; 4, 5, 6; 7, 8, 9]
B =
```

```
 1  2  3
 4  5  6
 7  8  9
```

```
>> B(:)
ans =
```

```
 1
 4
 7
 2
 5
 8
 3
 6
 9
```

```
>> B(1, :)
ans =
```

```
 1  2  3
```

```
>> B(2, :)
ans =
```

```
 4  5  6
```

```
>> B(:, 1)
ans =
```

```
 1
 4
 7
```

```
>> B(:, 2)
ans =
```

```
 2
 5
 8
```

# Column-wise Access (1)

```
>> a = [1 2 3 ]  
a =
```

```
    1    2    3
```

```
>> b = [2 4 6]  
b =
```

```
    2    4    6
```

```
>> a .* b  
ans =
```

```
    2    8   18
```

```
>> a * b  
error: operator *:  
nonconformant  
arguments (op1 is  
1x3, op2 is 1x3)
```

```
>> a(:)  
ans =
```

```
    1  
    2  
    3
```

```
>> b(:)  
ans =
```

```
    2  
    4  
    6
```

```
>> a(:) .* b(:)  
ans =
```

```
    2  
    8  
   18
```

```
>> a(:) * b(:)  
error: operator *:  
nonconformant  
arguments (op1 is  
3x1, op2 is 3x1)
```

```
>> a(:)  
ans =
```

```
    1  
    2  
    3
```

```
>> b  
b =
```

```
    2    4    6
```

```
>> a(:) .* b  
ans =
```

```
    2    4    6  
    4    8   12  
    6   12   18
```

```
>> a(:) * b  
ans =
```

```
    2    4    6  
    4    8   12  
    6   12   18
```

```
>>
```

```
>> a  
a =
```

```
    1    2    3
```

```
>> b(:)  
ans =
```

```
    2  
    4  
    6
```

```
>> a .* b(:)  
ans =
```

```
    2    4    6  
    4    8   12  
    6   12   18
```

```
>> a * b(:)  
ans = 28
```

```
>>
```

# Column-wise Access (2)

```
>> a = [1; 2; 3]
a =
```

```
1
2
3
```

```
>> b = [2; 4; 6]
b =
```

```
2
4
6
```

```
>> a .* b
ans =
```

```
2
8
18
```

```
>> a * b
error: operator *:
nonconformant
arguments (op1 is
3x1, op2 is 3x1)
```

```
>> a(:)
ans =
```

```
1
2
3
```

```
>> b(:)
ans =
```

```
2
4
6
```

```
>> a(:) .* b(:)
ans =
```

```
2
8
18
```

```
>> a(:) * b(:)
error: operator *:
nonconformant
arguments (op1 is
3x1, op2 is 3x1)
```

```
>> a(:)
ans =
```

```
1
2
3
```

```
>> b
b =
```

```
2
4
6
```

```
>> a(:) .* b
ans =
```

```
2
8
18
```

```
>> a(:) * b
error: operator *:
nonconformant
arguments (op1 is
3x1, op2 is 3x1)
```

```
>> a
a =
```

```
1
2
3
```

```
>> b(:)
ans =
```

```
2
4
6
```

```
>> a .* b(:)
ans =
```

```
2
8
18
```

```
>> a * b(:)
error: operator *:
nonconformant
arguments (op1 is
3x1, op2 is 3x1)
```

B Ninness, Spectral Analysis using the FFT

# Window Function

```
win = hamming(N);  
xw = win(:) .* xn(:);  
  
Xw = fft(xw);  
Xwf = fil(:) .* fftshift(Xw);  
Xwf = ifft(fftshift(Xwf));
```

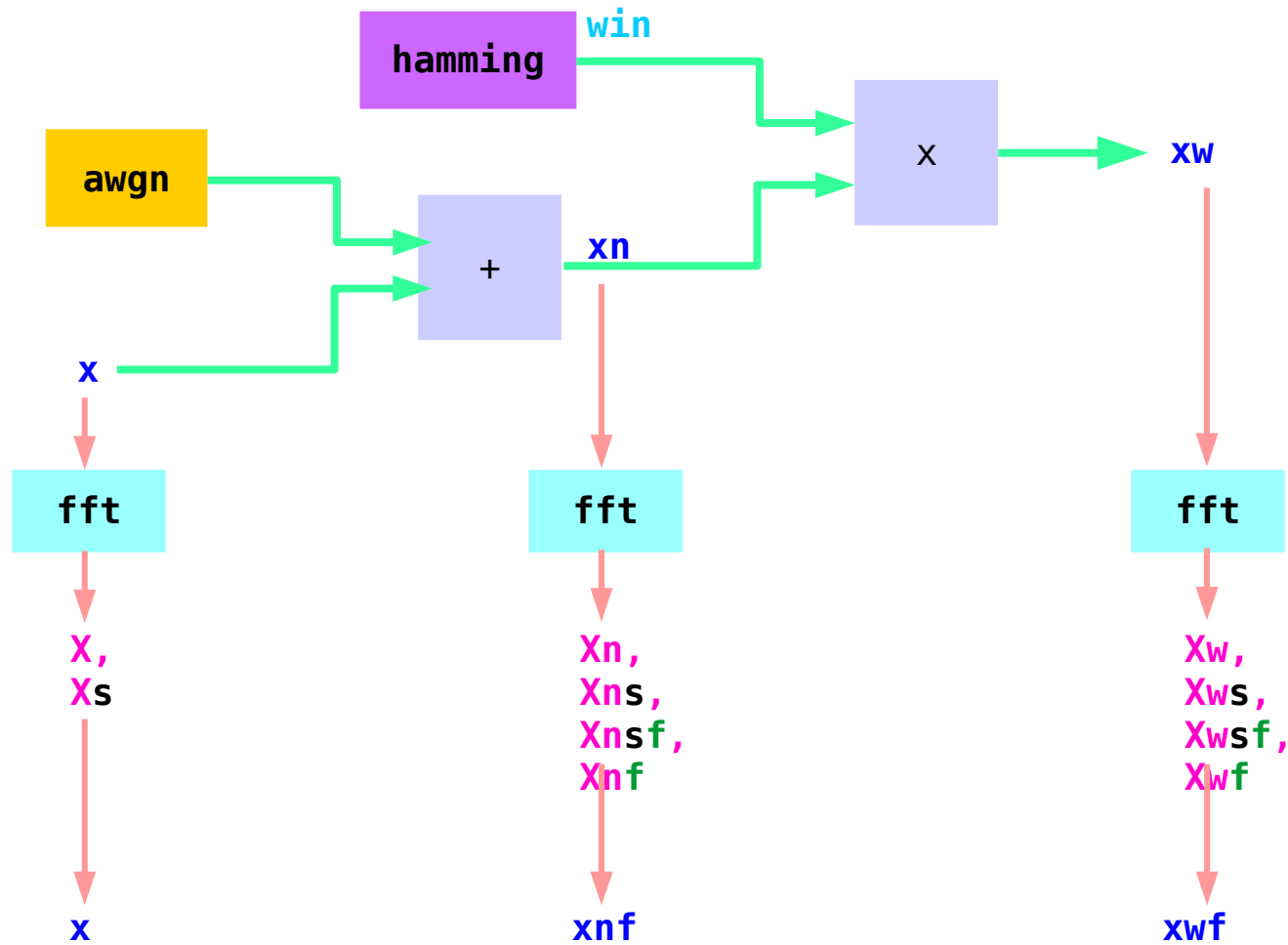
```
win : a column vector  
xn  : a row vector  
xw  : a column vector  
Xw  : a column vector  
fil : a row vector  
Xwf : a column vector
```

```
win_col = hamming(N);  
win = win_col';  
xw = win .* xn;  
  
Xw = fft(xw);  
Xws = fftshift(Xw);  
Xwf = fil .* fftshift(Xw);  
xwf = ifft(fftshift(Xwf));
```

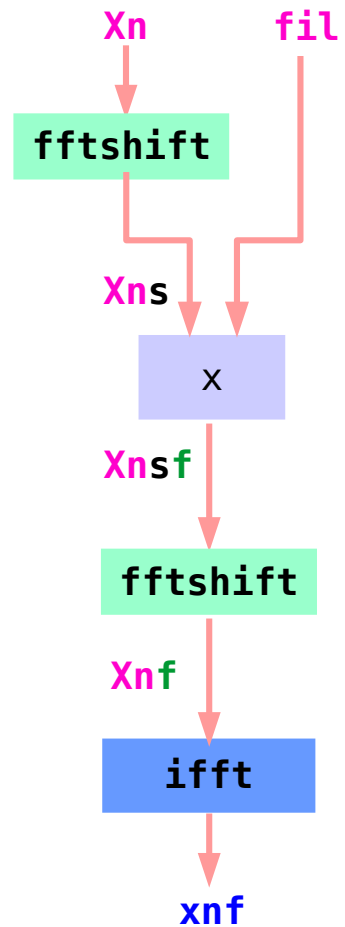
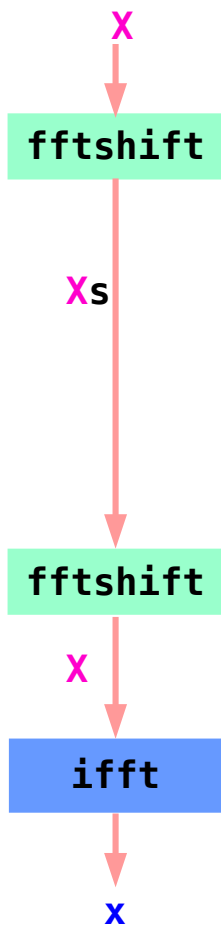
```
win : a row vector  
xn  : a row vector  
xw  : a row vector  
Xw  : a row vector  
fil : a row vector  
Xwf : a row vector
```

B Ninness, Spectral Analysis using the FFT

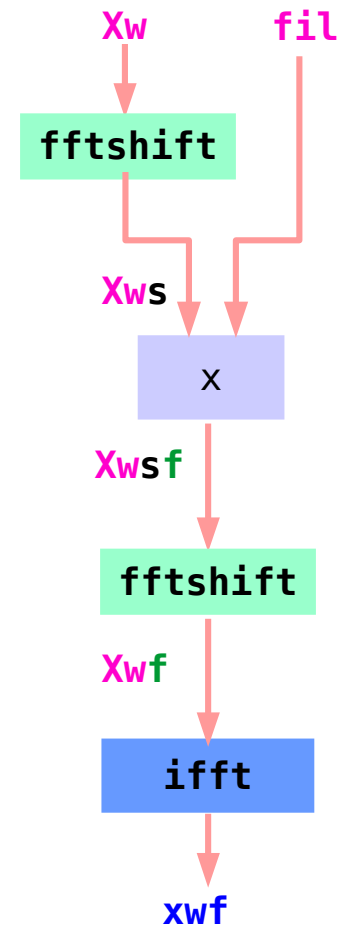
# Tap Signals - time domain



# Tap Signals - frequency domain



$n$  : noise



$w$  : window



# Window Result Plots

```
subplot(2, 2, 1);  
plot(t, win);
```

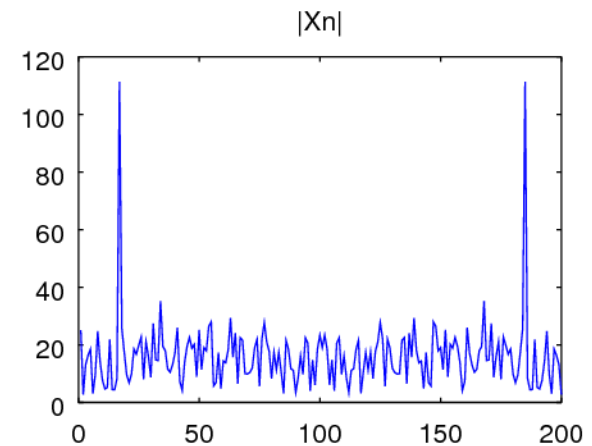
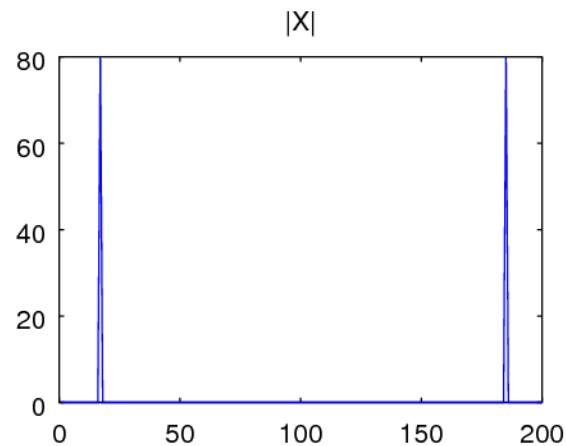
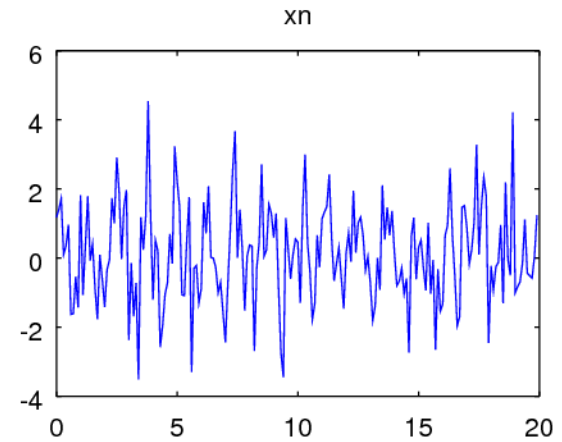
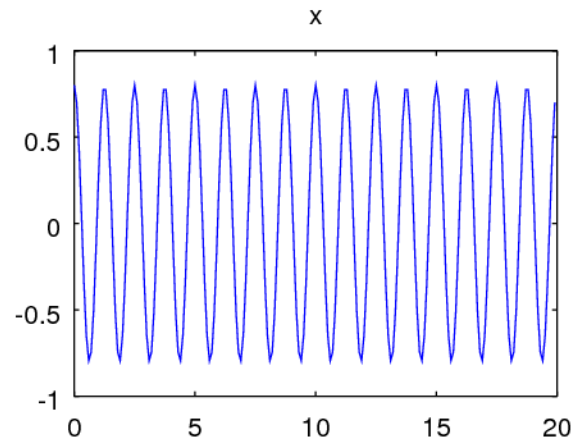
```
subplot(2, 2, 2);  
plot(t, xw);
```

```
subplot(2, 2, 3);  
plot(w, abs(fftshift(Xw)), 'b', w, 200*fil, '-r');
```

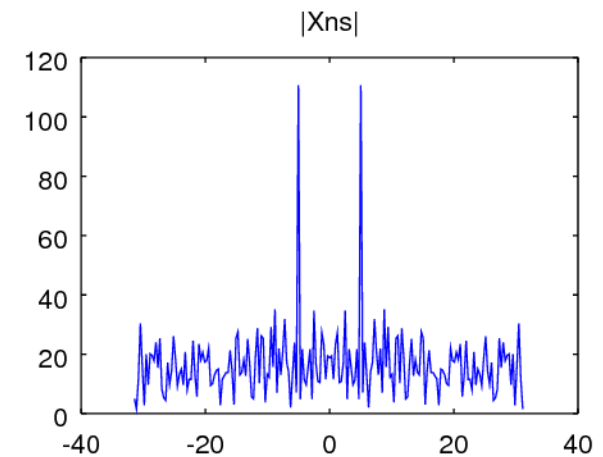
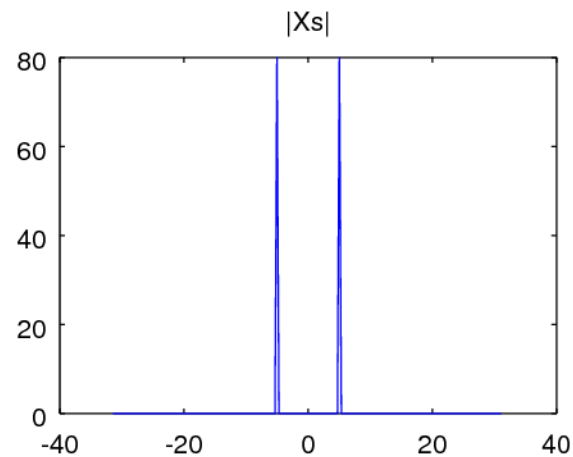
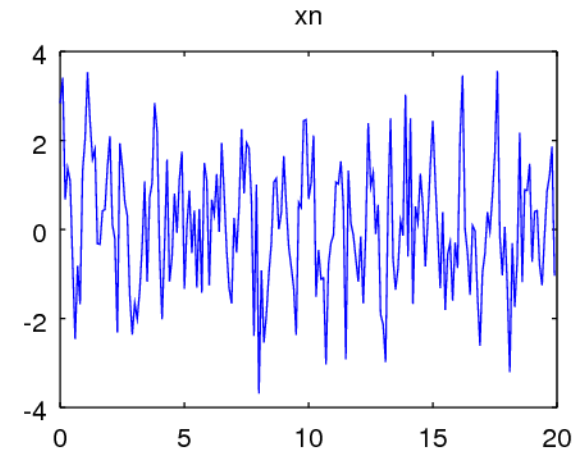
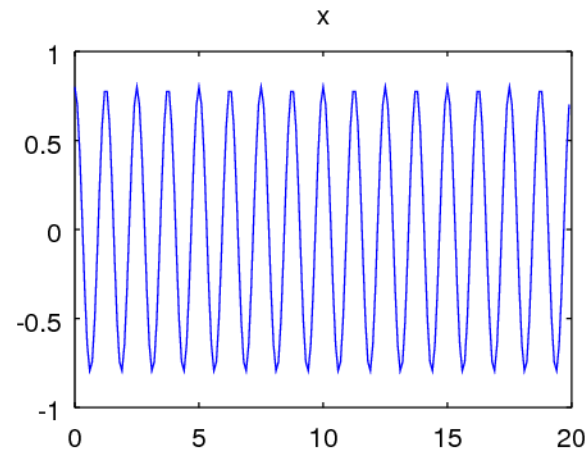
```
subplot(2, 2, 4);  
plot(t, real(xnf) ./ win(:) );
```

```
% plot(w, abs(fftshift(Xnf)), 'b', w, 200*fil, '-r');
```

# Window Result Plots (1)

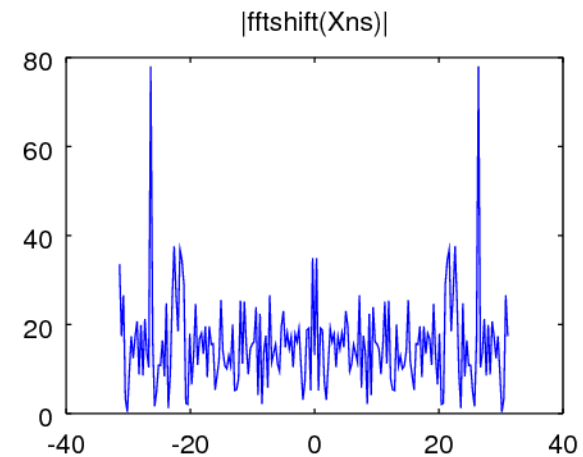
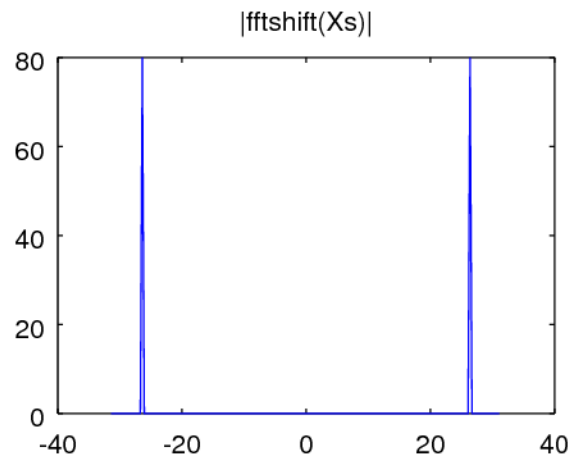
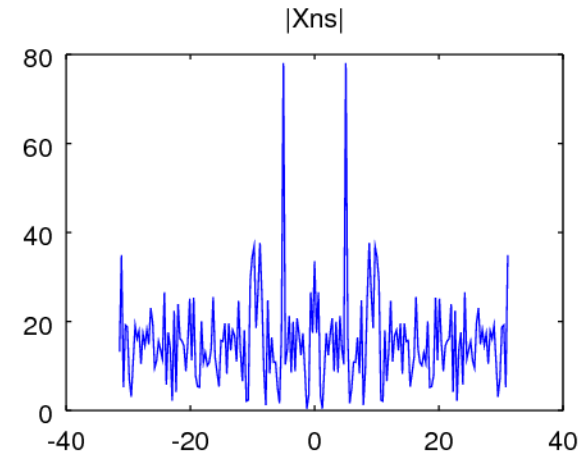
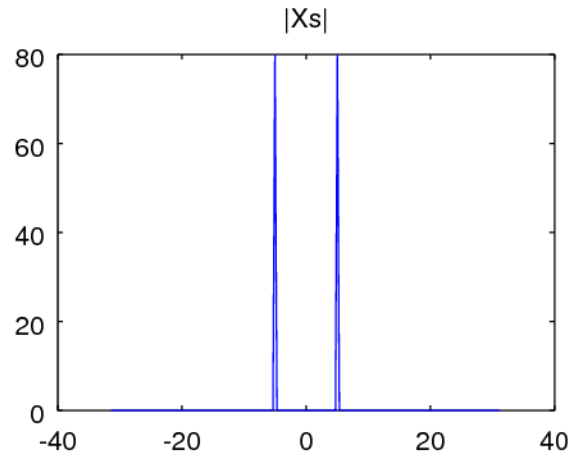


# Window Result Plots (2)



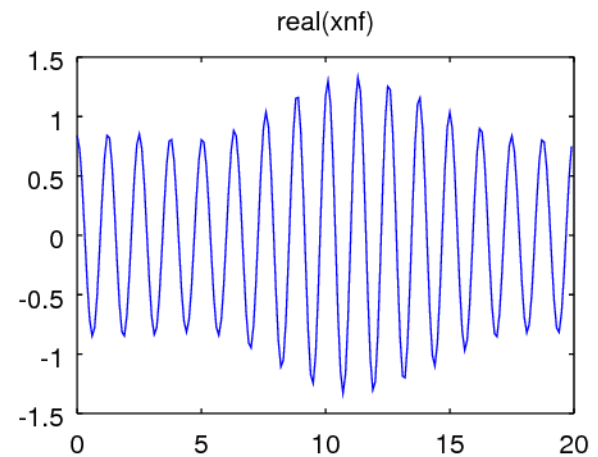
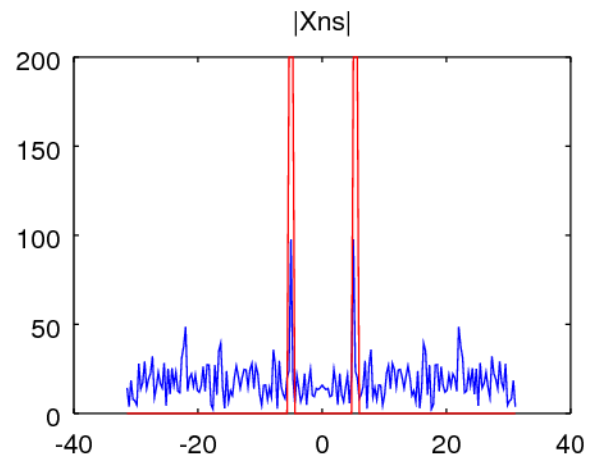
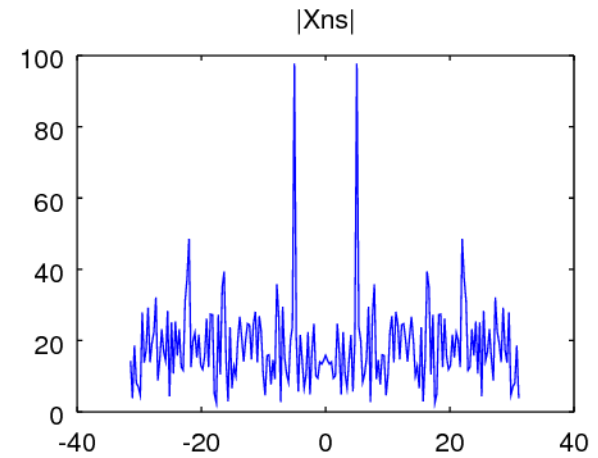
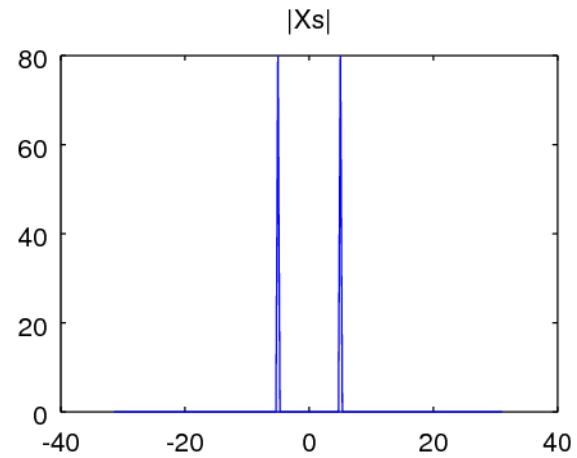
B Ninness, Spectral Analysis using the FFT

# Window Result Plots (3)



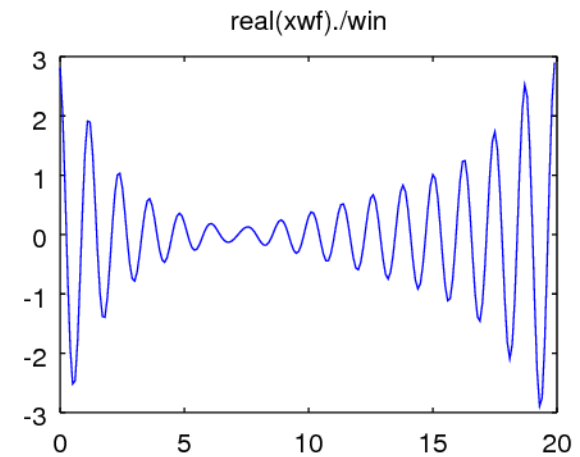
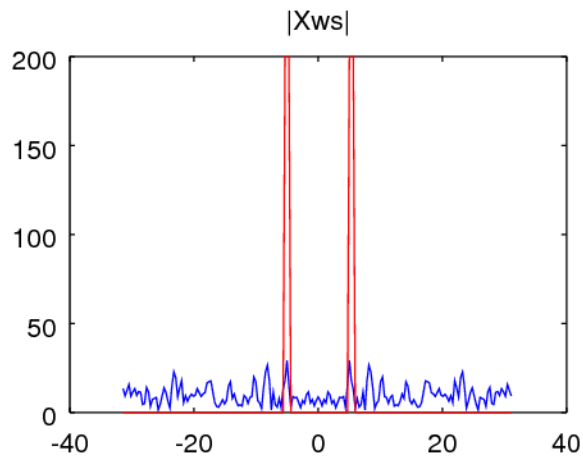
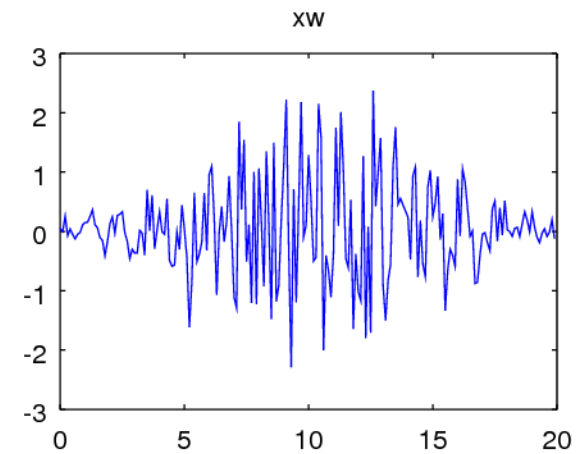
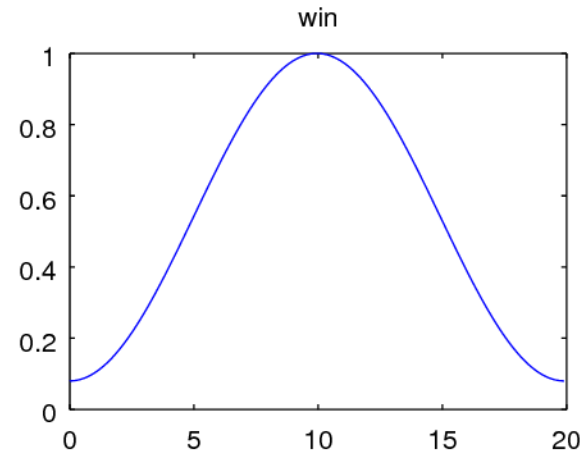
B. P. Ninness, Spectral Analysis using the FFT

# Window Result Plots (4)

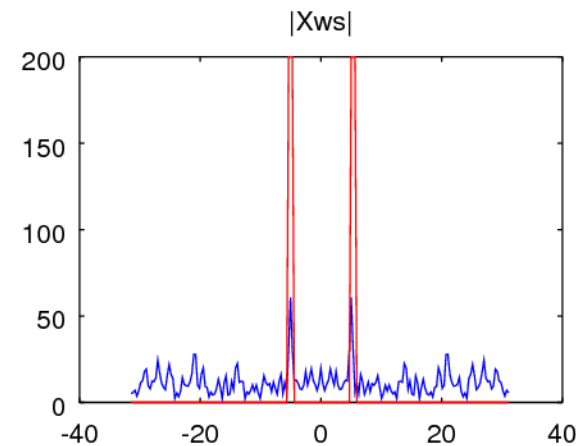
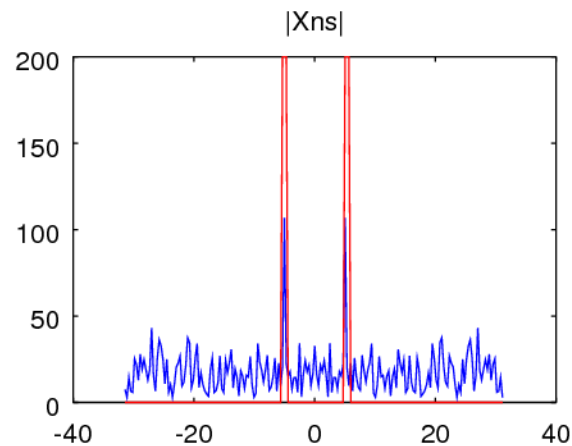
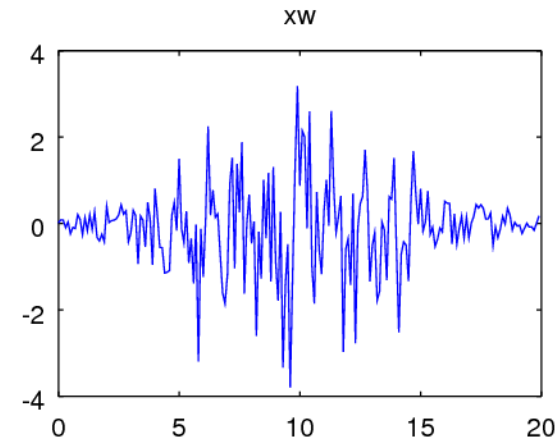
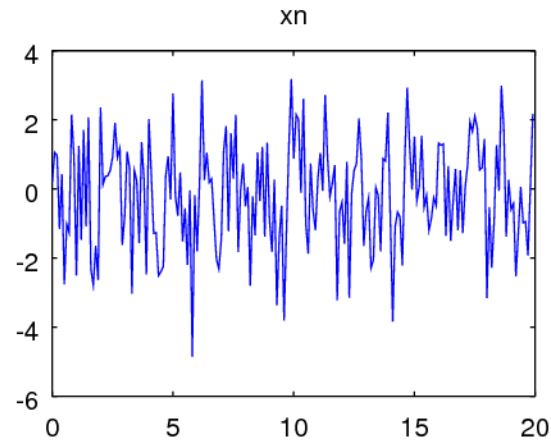


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# Window Result Plots (5)

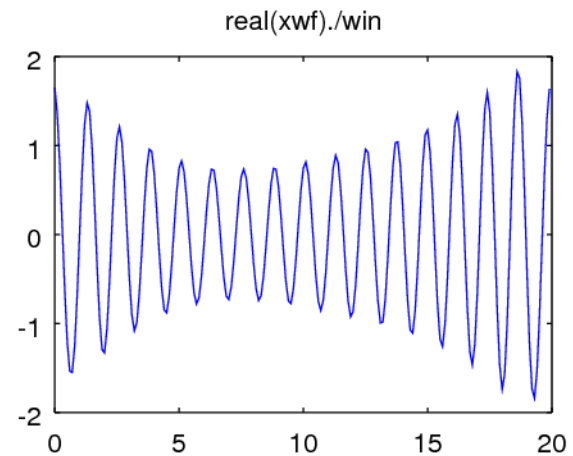
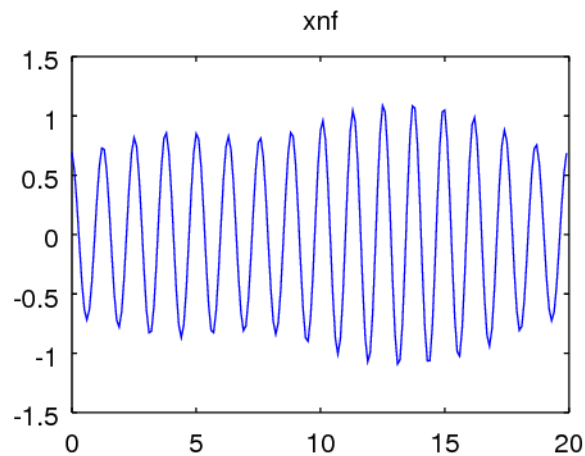
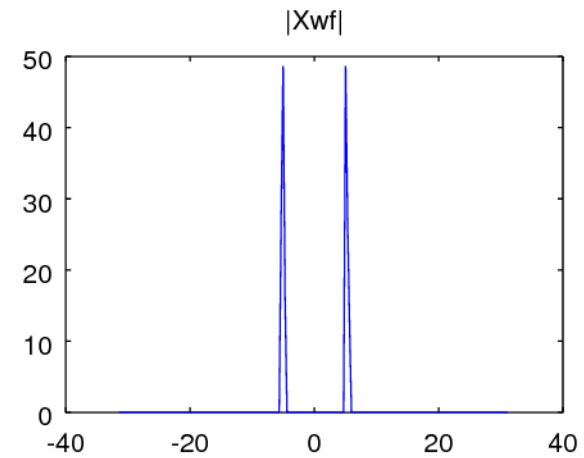
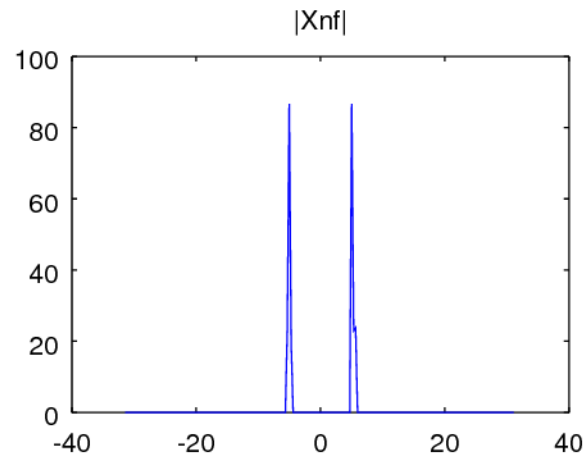


# Window Result Plots (6)



B. P. Ninnings, Spectral Analysis using the FFT

# Window Result Plots (7)



B Ninness, Spectral Analysis using the FFT



# Normalized $\omega_s$ and $\omega_0$

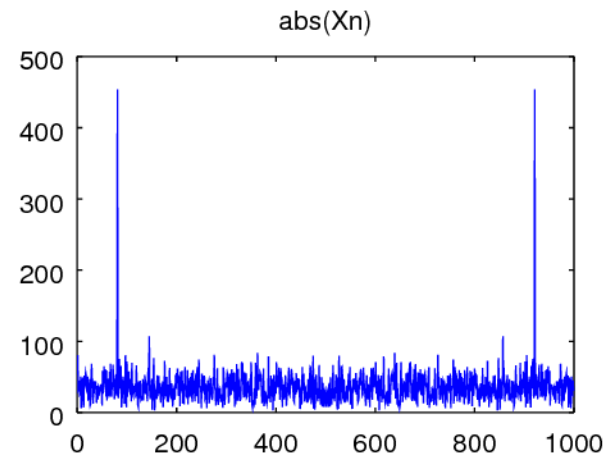
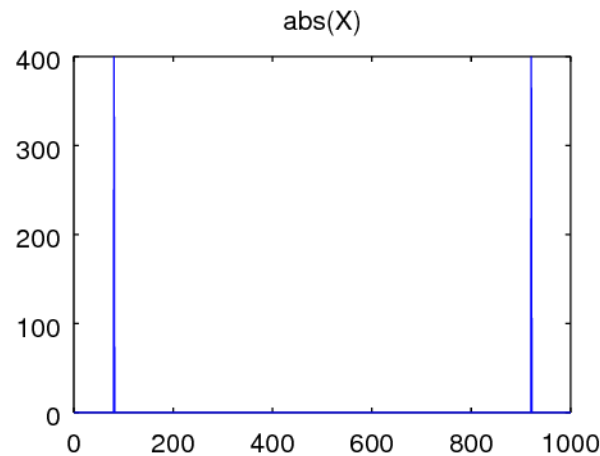
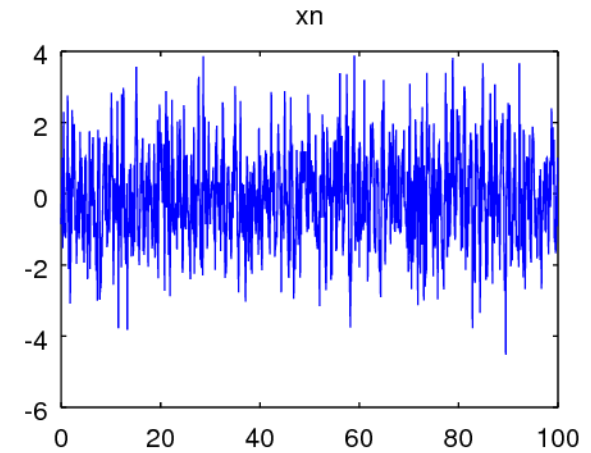
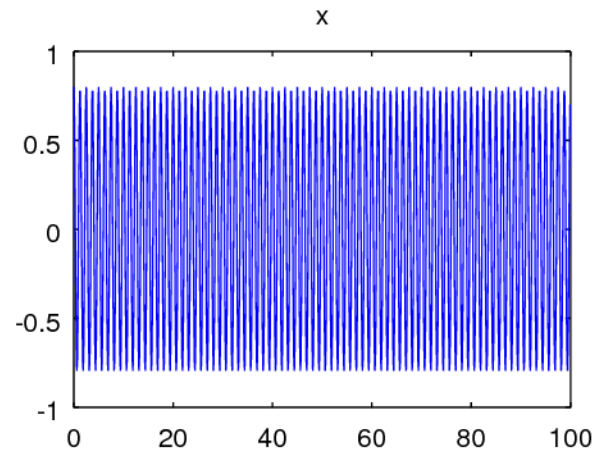
```
N = length(x);  
ws = 2*pi/N;  
wnorm = -pi:ws:pi;  
wnorm = wnorm(1:length(x));  
w = wnorm*fs;  
  
X= fft(x);  
  
plot(w,abs(fftshift(X)));  
axis([-30,30,0,160]);
```

# Normalized $\omega_s$ and $\omega_0$

```
win = hamming(N);  
xw = win(:) .* x(:);  
Xw = fft(xw);  
plot(w, abs(fftshift(Xw)));  
axis([-10, 10, 0, 80]);
```

# Normalized $\omega_s$ and $\omega_0$

N=1000



B Ninness, Spectral Analysis using the FFT

# FFT Example Code (1)

```
pkg load communications

N = 2000;

fs = 10;
t = (0:1:N-1)/fs;

x = 0.8*cos(2*pi*0.8*t);
xn = awgn(x, -2);

X = fft(x);
Xn = fft(xn);

display('[1] x, xn, |X|, |Xn| : freq index');
figure(1, 'name', 'Fig.1 x, xn, |X|, |Xn| : freq index');
subplot(2, 2, 1); plot(t, x); title("x");
subplot(2, 2, 2); plot(t, xn); title("xn");
subplot(2, 2, 3); plot(abs(X)); title("|X|");
subplot(2, 2, 4); plot(abs(Xn)); title("|Xn|");

pause

ws = 2*pi/N;
wnorm = -pi:ws:+pi;
wnorm = wnorm(1:N);
w = wnorm * fs;

Xs = fftshift(X);
Xns = fftshift(Xn);
```

B Ninness, Spectral Analysis using the FFT

# FFT Example Code (2)

```
display('[2] |Xs|, |Xn| : freq scales');
figure(1,'name','[2] |Xs|, |Xns| : freq scales');
subplot(2,2,3); plot(w, abs(Xs)); title("|Xs|");
subplot(2,2,4); plot(w, abs(Xns)); title("|Xns|");

pause

display('[3] |Xs|, |Xns|, |Xss|, |Xnss| : self-inverting');
figure(1,'name','[3] |Xs|, |Xns|, |Xss|, |Xnss| : self-inverting');
subplot(2,2,1); plot(w, abs(Xs)); title("|Xs|");
subplot(2,2,2); plot(w, abs(Xns)); title("|Xns|");
subplot(2,2,3); plot(w, abs(fftshift(Xs))); title("|fftshift(Xs)|");
subplot(2,2,4); plot(w, abs(fftshift(Xns))); title("|fftshift(Xns)|");

pause

% close all figure windows
% must use this toolkit to use ginput()
% close all;
% graphics_toolkit fltk
%
% plot(w, abs(fftshift(Xn)));
%
% [freq,amp, buttons]= ginput()

b1 = round( 5 / 400 * N);
a1 = round(166 / 400 * N);
a2 = round( 59 / 400 * N);
a3 = N -a1 -a2 -b1 -b1;
```

B Ninness, Spectral Analysis using the FFT

# FFT Example Code (3)

```
display(a1);
display(a2);
display(a3);
display(b1);
fil = [ zeros(1,a1), ones(1,b1), zeros(1,a2), ones(1,b1), zeros(1,a3)];
% fil = [ zeros(1,166), ones(1,5), zeros(1,59), ones(1,5), zeros(1,165)];
% fil = [ zeros(1,160), ones(1,15), zeros(1,50), ones(1,15), zeros(1,160)];

Xnf= fil .* fftshift(Xn);
xnf= ifft(fftshift(Xnf));

display('[4] |Xns|+fil, xnf');
figure(1, 'name', '[4] |Xns|+fil, xnf');
subplot(2, 2, 3); plot(w, abs(Xns),'b', w, 200*fil,'-r'); title("|Xns|");
subplot(2, 2, 4); plot(t, real(xnf)); title("real(xnf)");

pause

win_col = hamming(N);
win = win_col';
disp( length(win) );

xw = win .* xn;

Xw = fft(xw);
Xws= fftshift(Xw);
Xwf= fil .* fftshift(Xw);
xwf= ifft(fftshift(Xwf));
```

B Ninness, Spectral Analysis using the FFT

# FFT Example Code (4)

```
display('[5] win, xw, |Xws|+fil, xwf/win');
figure(1,'name','[5] win, xw, |Xws|+fil, xwf/win');
subplot(2, 2, 1); plot(t, win); title("win");
subplot(2, 2, 2); plot(t, xw); title("xw");
subplot(2, 2, 3); plot(w, abs(Xws),'b', w, 200*fil,'-r'); title("|Xws|");
subplot(2, 2, 4); plot(t, real(xwf) ./ win ); title("real(xwf)./win");
```

pause

```
display('[6] xn, xw, |Xns|+fil, |Xws|+fil');
figure(1,'name','[6] xn, xw, |Xns|+fil, |Xws|+fil');
subplot(2, 2, 1); plot(t, xn); title("xn");
subplot(2, 2, 2); plot(t, xw); title("xw");
subplot(2, 2, 3); plot(w, abs(Xns),'b', w, 200*fil,'-r'); title("|Xns|");
subplot(2, 2, 4); plot(w, abs(Xws),'b', w, 200*fil,'-r'); title("|Xws|");
```

pause

```
display('[7] |Xnf|, |Xwf|, xnf, xwf');
figure(1,'name','[7], |Xnf|, |Xwf|, xnf, xwf');
subplot(2, 2, 1); plot(w, abs(Xnf)); title("|Xnf|");
subplot(2, 2, 2); plot(w, abs(Xwf)); title("|Xwf|");
subplot(2, 2, 3); plot(t, xnf); title("xn timer");
subplot(2, 2, 4); plot(t, real(xwf) ./ win); title("real(xwf)./win");
```

pause

---

## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings
  
- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann