

Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

1 Central Limit Theorem

Central Limit Theorem

Definition

the central limit theorem says that the probability distribution function of the sum of large number of random variables approaches a Gaussian distribution.

This theorem is known to apply some cases of statistically independent random variables.

Central Limit Theorem

Unequal Distribution Case

Definition

the sum Y of N independent random variables X_1, X_2, \dots, X_N

Let $Y = X_1 + X_2 + \dots + X_N$, then

$$\bar{Y}_N = \bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_N$$

$$\sigma_{Y_N}^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 + \dots + \sigma_{X_N}^2$$

the probability distribution of Y asymptotically approaches

to Gaussian distribution function as $N \rightarrow \infty$

Sufficient Conditions

Unequal Distribution Case

Definition

$$\sigma_{X_i}^2 > B_1 > 0 \quad i = 1, 2, \dots, N$$

$$E[|X_i - \bar{X}_i|^3] < B_2 \quad i = 1, 2, \dots, N$$

where B_1 and B_2 are positive numbers

these conditions guarantee that no one random variable in the sum dominates

Distribution vs density functions

Unequal Distribution Case

the central limit theorem guarantees

- only that the distribution of the sum of random variables become Gaussian
- the density of the sum of random variables is not always Gaussian
- the sum of continuous random variables :
 - under certain conditions on individual random variables the density of the sum is always Gaussian
- the sum of discrete random variables :
 - the density function may contain impulses and thus is not Gaussian.

