

Full Adder

	x	y	z	f1 C	f2 S	
0	0	0	0		0	0
1	0	0	1		1	1
2	0	1	0		1	1
3	0	1	1	1	0	2
4	1	0	0		1	1
5	1	0	1	1	0	2
6	1	1	0	1	0	2
7	1	1	1	1	1	3

z = ci (carry in)
C = co (carry out)

$$C(x,y,z) = \text{Sum}(3,5,6,7)$$

$$S(x,y,z) = \text{Sum}(1,2,4,7)$$

	yz=00	yz=01	yz=11	yz=10	
	00	01	11	10	
x=0			1		x=0
x=1	1	1	1	1	x=1

$$C = xy + xz + yz$$

	yz=00	yz=01	yz=11	yz=10	
	00	01	11	10	
0		1		1	
1	1		1		

$$S = xy'z' + xyz + x'y'z + x'yz'$$

$$S = z'(xy' + x'y) + z(xy + x'y')$$

$$S = z'(x \oplus y) + z(x \oplus y)'$$

$$S = (z \oplus (x \oplus y))'$$

	yz=00	yz=01	yz=11	yz=10	
	00	01	11	10	
x=0			1		yz
x=1	1	1	1	1	x=0
					x=1

$$z = x'y + xy'$$

$$z' = (x'y + xy')'$$

$$(x'y)' (xy)'$$

$$(x+y)' (x'+y')$$

$$x(x'+y) + y'(x'+y)$$

$$xx' + xy + x'y' + y'y$$

$$xy + x'y'$$

	yz=00	yz=01	yz=11	yz=10	
	00	01	11	10	
x=0			1		xz
x=1	1	1	1	1	x=0
					x=1

	yz=00	yz=01	yz=11	yz=10	
	00	01	11	10	
x=0			1		xy
x=1	1	1	1	1	x=0
					x=1

$$C = xy + xz + yz$$

$$C = z(xy' + x'y) + xy$$

$$(x \downarrow y) \downarrow z$$

$$x \downarrow (y \downarrow z)$$

$$((x + y)' + z)'$$

$$(x + (y + z))'$$

$$(x + y) z'$$

$$(x'(y + z))$$

$$xz' + yz'$$

$$x'y + x'z$$

$$x \downarrow y \downarrow z$$

$$(x + y + z)'$$

$$x \uparrow y \uparrow z$$

$$(xyz)'$$

$$F = ((ABC)'(DE)')'$$

$$(ABC)'' + (DE)''$$

$$ABC + DE$$

$$F = x \oplus y$$

	x	y	z	$x \oplus y$	$(x \oplus y) \oplus z$	$y \oplus z$	$x \oplus (y \oplus z)$	$x \oplus y \oplus z$
0	0	0	0	0	0			0
1	1	0	1	0	1			1
2	0	1	0	1	1			1
3	0	1	1	1	0			0
4	1	0	0	1	1			1
5	1	0	1	1	0			0
6	1	1	0	0	0			0
7	1	1	1	0	1			1

$$F(x,y,z) = \text{Sum}(1,2,3,4,5,7)$$

		yz=00	yz=01	yz=11	yz=10
		00	01	11	10
x=0	0		1	1	1
x=1	1	1	1	1	

$F = z + x'y + xy'$

		yz=00	yz=01	yz=11	yz=10
		00	01	11	10
x=0	0		1	1	1
x=1	1	1	1	1	

z

		yz=00	yz=01	yz=11	yz=10
		00	01	11	10
x=0	0		1	1	1
x=1	1	1	1	1	

$x'y$

		yz=00	yz=01	yz=11	yz=10
		00	01	11	10
x=0	0		1	1	1
x=1	1	1	1	1	

xy'

	x	y	z	xy	(xy)z	(yz)	x(yz)	xyz
0	0	0	0	0	0			
1	0	0	1	0	0			
2	0	1	0	0	0			
3	0	1	1	0	0			
4	1	0	0	0	0			
5	1	0	1	0	0			
6	1	1	0	1	0			
7	1	1	1	1	1			

	x	y	z	xy	(xy)z	(yz)	x(yz)	xyz
0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
2	0	1	0	0	0	0	0	0
3	0	1	1	0	0	1	0	0
4	1	0	0	0	0	0	0	0
5	1	0	1	0	0	0	0	0
6	1	1	0	1	0	0	0	0
7	1	1	1	1	1	1	1	1

$(xy) + z'$

	x	y	z	$x \uparrow y$	$(x \uparrow y) \uparrow z$	$(y \uparrow z)$	$x \uparrow (y \uparrow z)$	$x \uparrow y \uparrow z$
0	0	0	0	1	1			
1	0	0	1	1	0			
2	0	1	0	1	1			
3	0	1	1	1	0			
4	1	0	0	1	1			
5	1	0	1	1	0			
6	1	1	0	0	1			
7	1	1	1	0	1			

$x' + (yz)$

	x	y	z	$x \uparrow y$	$(x \uparrow y) \uparrow z$	$(y \uparrow z)$	$x \uparrow (y \uparrow z)$	$x \uparrow y \uparrow z$
0	0	0	0	1	1	1	1	
1	0	0	1	1	0	1	1	
2	0	1	0	1	1	1	1	
3	0	1	1	1	0	0	1	
4	1	0	0	1	1	1	0	
5	1	0	1	1	0	1	0	
6	1	1	0	0	1	1	0	
7	1	1	1	0	1	0	1	

	x	y	z	$x \uparrow y$	$(x \uparrow y) \uparrow z$	$(y \uparrow z)$	$x \uparrow (y \uparrow z)$	$x \uparrow y \uparrow z$
0	0	0	0	1	1	1	1	1
1	0	0	1	1	0	1	1	1
2	0	1	0	1	1	1	1	1
3	0	1	1	1	0	0	1	1
4	1	0	0	1	1	1	0	1
5	1	0	1	1	0	1	0	1
6	1	1	0	0	1	1	0	1
7	1	1	1	0	1	0	1	0

$$\underline{(x \uparrow y) \uparrow z} \quad ((xy)'z)' \quad (xy) + z'$$

$$\underline{x \uparrow (y \uparrow z)} \quad (x(yz)')' \quad x' + (yz)$$

