Laurent Series and z-Transform Examples case 0.B

20171123

$$X(z) = \frac{-1}{(z-1)(z-2)} \xleftarrow{z^{-1}} f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$X(z) = \frac{-1}{(z-0.5)(z-2)} = \frac{2}{3}\left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{2}{3}\left(\frac{1}{z^{-1}-0.5} - \frac{1}{z^{-1}-2}\right)$$

$$= \frac{2}{3}\left(\frac{2}{2z^{-1}-1} - \frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \frac{2}{3}\left(\frac{2z}{2-z} - \frac{0.5z}{0.5-z}\right)$$

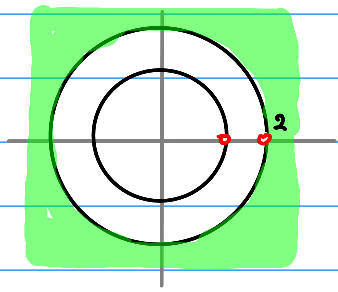
$$= \frac{2}{3}\left(\frac{-2z}{z-2} + \frac{0.5z}{z-0.5}\right)$$

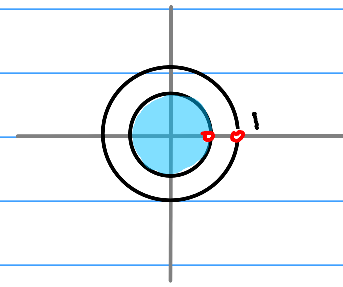
$$= \frac{2z}{3}\left(\frac{-2}{z-2} + \frac{0.5}{z-0.5}\right)$$

$$= \frac{2z}{3}\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

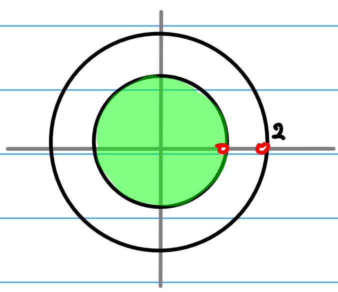
$$= \frac{-z^2}{(z-2)(z-0.5)}$$

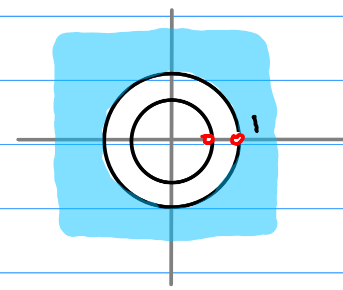
1.B $X(z) = \frac{-1}{(z-1)(z-2)} \xleftarrow{z^{-1}} f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$



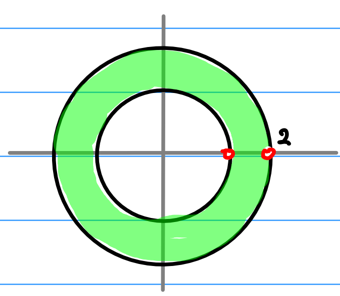


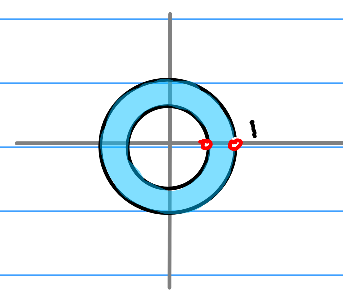
$$\sum_{n=1}^{\infty}\left[1-2^{-n+1}\right]z^{-n} = \sum_{n=0}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1}-1\right]z^{n}$$





$$\sum_{n=-1}^{-\infty}\left[-1+2^{n-1}\right]z^{-n} = \sum_{n=-1}^{-\infty}\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)z^{n}$$

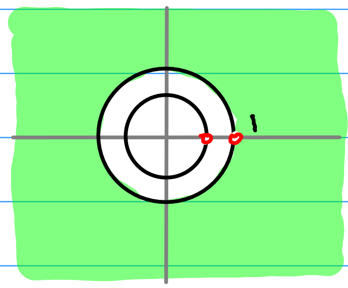


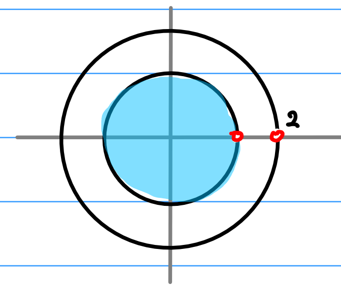


$$\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{-n} = \sum_{n=-1}^{-\infty} z^{n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\cdot z^{n}$$

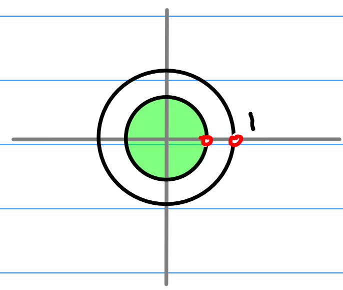
2. B

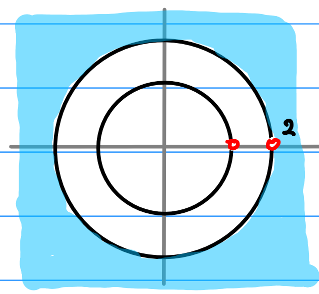
$$X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} \xleftarrow{z^{-1}} f(z) = \frac{-1}{(z-1)(z-2)}$$



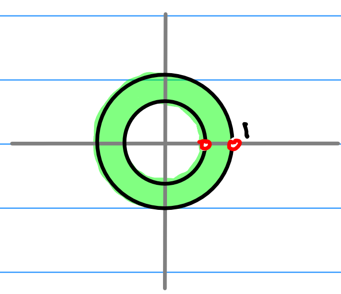


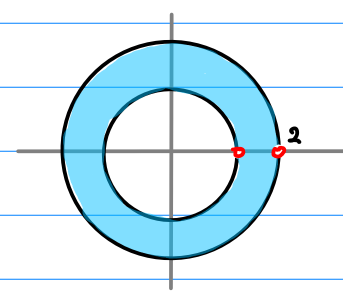
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^n = \sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^n$$





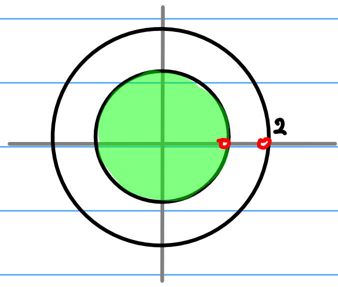
$$\sum_{n=0}^{-\infty}\left[2^{n-1}-1\right]z^n = \sum_{n=0}^{-\infty}\left[2^{n-1}-1\right]z^n$$

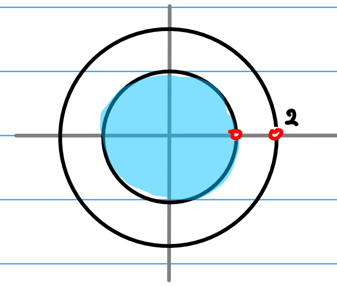




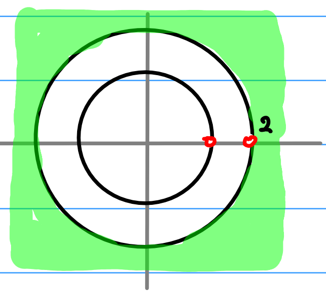
$$\sum_{n=1}^{\infty}1\cdot z^n + \sum_{n=0}^{-\infty}2^{n-1}\cdot z^n = \sum_{n=1}^{\infty}1\cdot z^n + \sum_{n=0}^{-\infty}2^{n-1}\cdot z^n$$

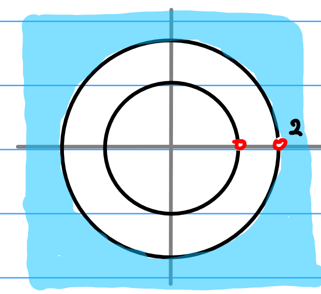
3.B $X(z) = \dfrac{-1}{(z-1)(z-2)}$ $=$ $f(z) = \dfrac{-1}{(z-1)(z-2)}$



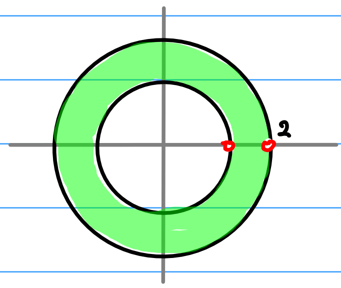


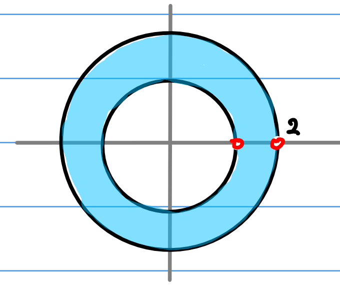
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^{-n} \xleftarrow{\;-n\;} \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^{-n} \xleftarrow{\;-n\;} \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$

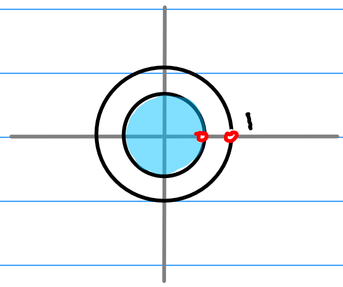


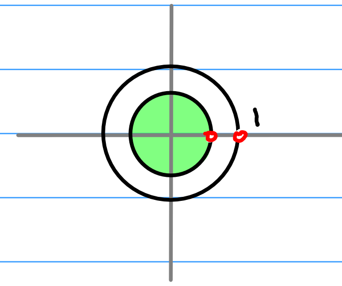


$$- \sum_{n=0}^{\infty} z^{-n} - \sum_{n=-1}^{\infty} 2^{-n-1} z^{-n} \xleftarrow{\;-n\;} + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{-n}$$

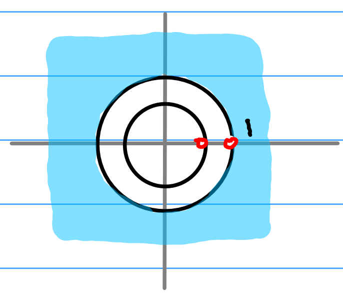
4.B

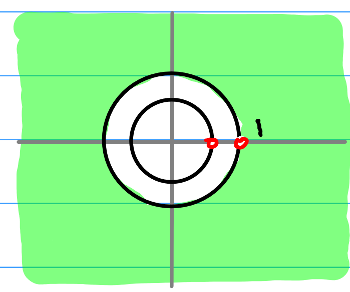
$$X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



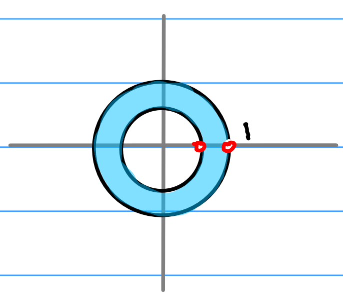


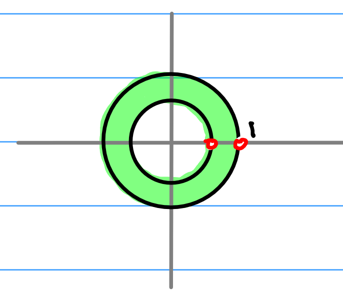
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right] z^{n} \xleftarrow{-n} \sum_{n=-1}^{\infty}\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$





$$\sum_{n=-1}^{-\infty}\left[2^{n-1}-1\right] z^{n} \xleftarrow{-n} \sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1}-1\right] z^{-n}$$





$$+\sum_{n=1}^{\infty} z^{n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{n} \xleftarrow{-n} +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

1.B $X(z) = \dfrac{-1}{(z-1)(z-2)} \xleftarrow{z^{-1}} f(z) = \dfrac{-0.5z^2}{(z-1)(z-0.5)}$

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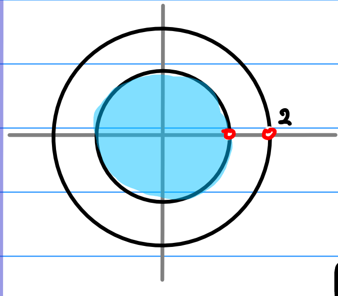

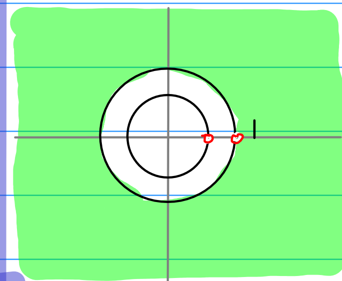

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^n$$

$$X(z) = \sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^n$$

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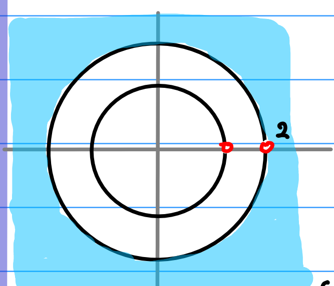

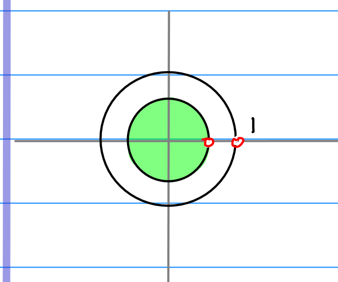

$$a_n = \begin{cases} 0 & (n \geq 0) \\ \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n < 0) \end{cases}$$

$$x_n = \begin{cases} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n$$

$$X(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\tfrac{1}{2}\right)^{n+1}\right) z^n$$

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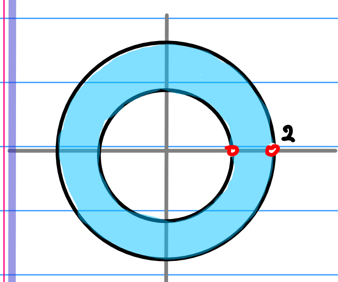

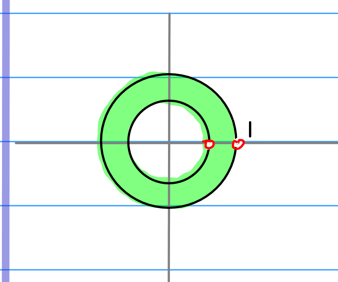

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} \cdot z^n$$

$$X(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} \cdot z^n$$

2.B

$$X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \xleftarrow{z^{-1}} f(z) = \frac{-1}{(z-1)(z-2)}$$

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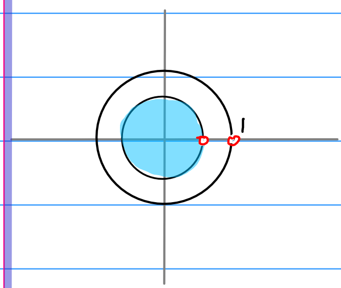

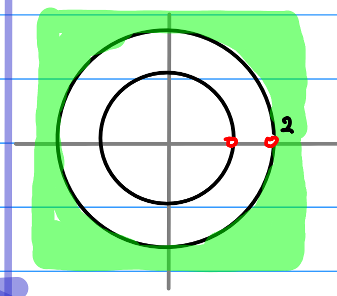

$$a_n = \begin{cases} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n\ge 0) \\ [1-2^{n-1}] & (n<0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^n \qquad X(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^n$$

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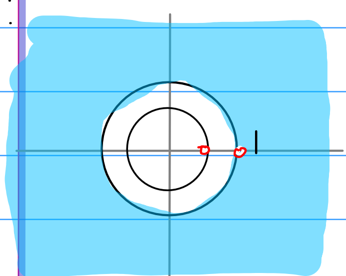

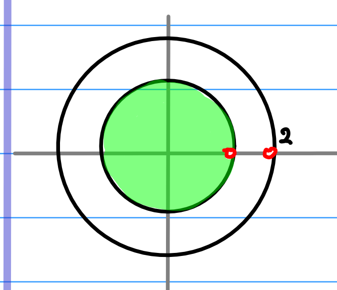

$$a_n = \begin{cases} 0 & (n>0) \\ [2^{n-1}-1] & (n\le 0) \end{cases} \qquad x_n = \begin{cases} [2^{n-1}-1] & (n\ge 0) \\ 0 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^n \qquad X(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^n$$

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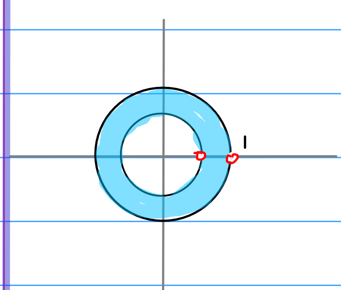

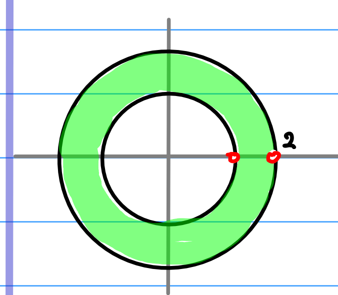

$$a_n = \begin{cases} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{cases} \qquad x_n = \begin{cases} 2^{n-1} & (n\ge 0) \\ 1 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n \qquad X(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n$$

3.B $X(z) = \frac{-1}{(z-1)(z-2)}$ $=$ $f(z) = \frac{-1}{(z-1)(z-2)}$

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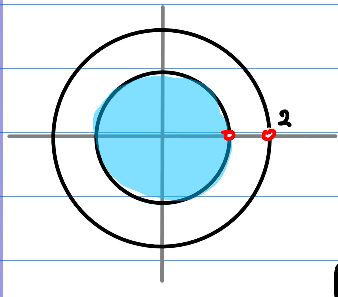

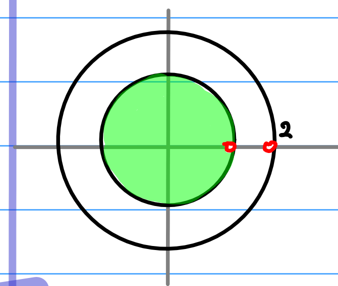

$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

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$$f(z) = \sum_{n=0}^{\infty} \left[(\tfrac{1}{2})^{n+1} - 1 \right] z^n$$

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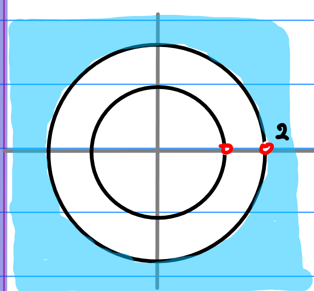

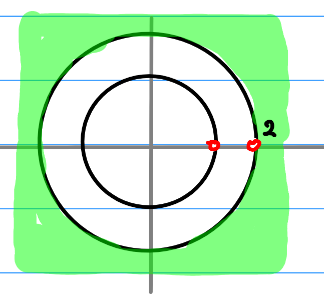

$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left[1 - (\tfrac{1}{2})^{n+1} \right] z^n$$

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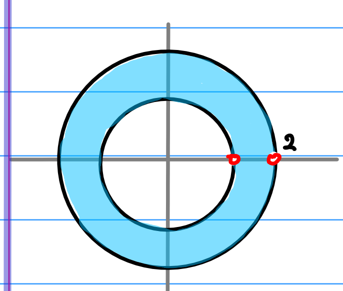

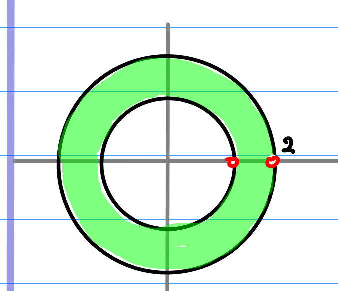

$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n+1} z^n$$

$$X(z) = + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4. B

$$X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \quad = \quad f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

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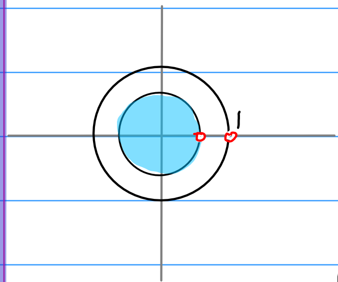

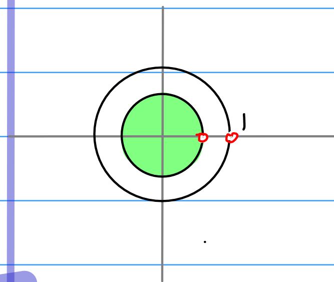

$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \le 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n \ge 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n \qquad X(z) = \sum_{n=-1}^{\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$

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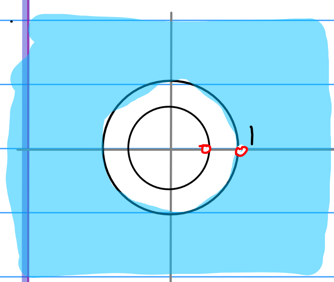

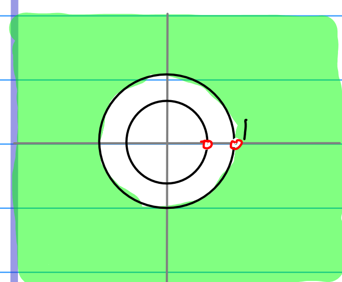

$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \le 0) \end{cases} \qquad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \ge 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left[2^{n-1} - 1\right] z^n \qquad X(z) = \sum_{n=1}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$

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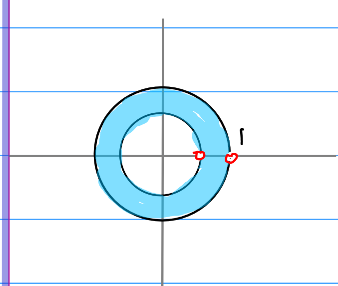

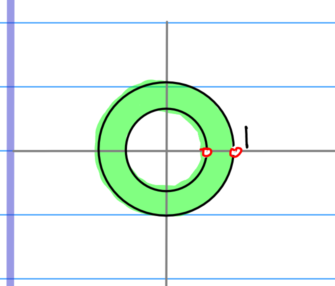

$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \le 0) \end{cases} \qquad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \ge 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n \qquad X(z) = +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

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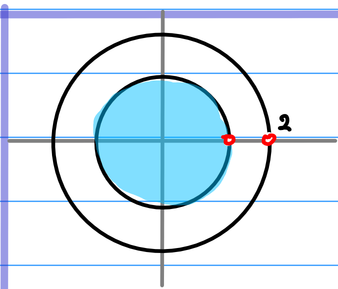
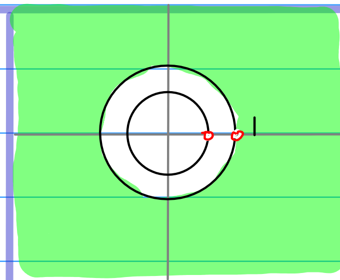


$$\boxed{\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}}$$



$$\boxed{\begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}}$$

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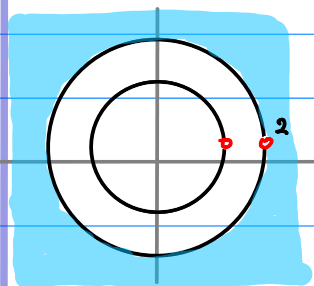
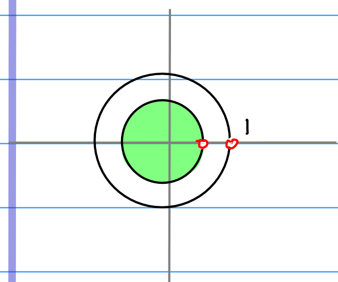


$$\boxed{\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}}$$



$$\boxed{\begin{cases} 1 - \left(\frac{1}{2}\right)^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}}$$

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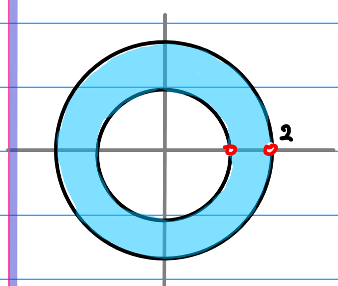
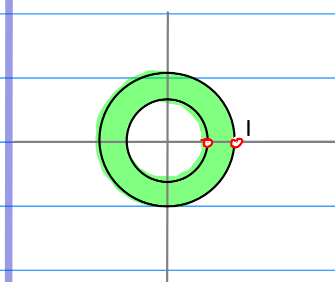
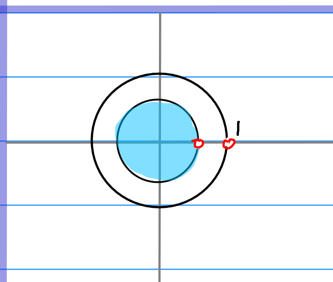


$$\boxed{\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}}$$

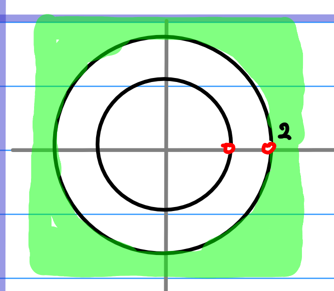


$$\boxed{\begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{cases}}$$

Ⅰ



$$\boxed{\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}}$$



$$\boxed{\begin{cases} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{cases}}$$

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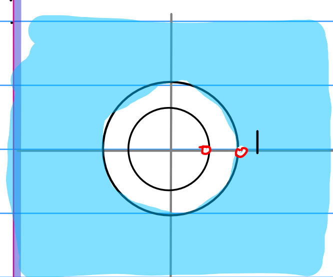
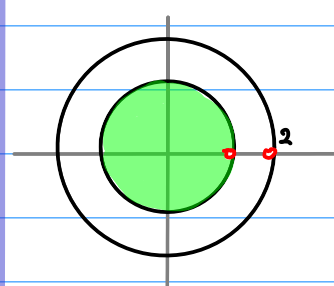
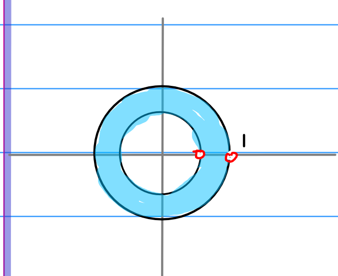


$$\boxed{\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}}$$

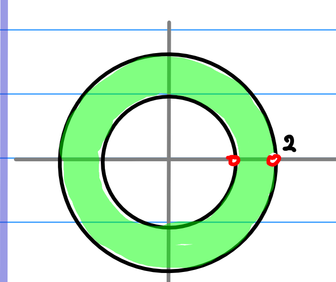


$$\boxed{\begin{cases} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}}$$

Ⅲ



$$\boxed{\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}}$$



$$\boxed{\begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}}$$

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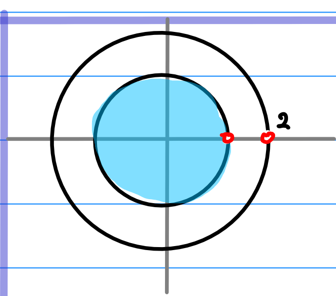
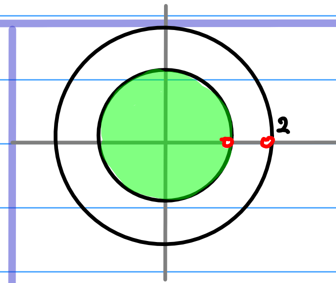


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$



$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

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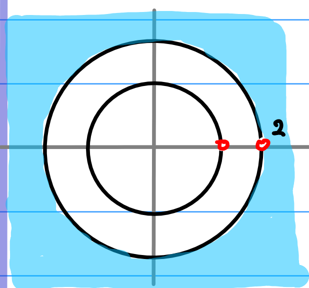
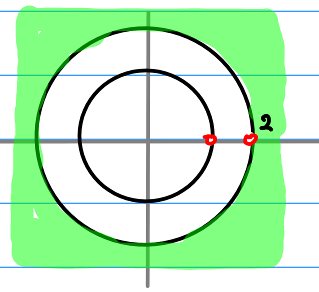
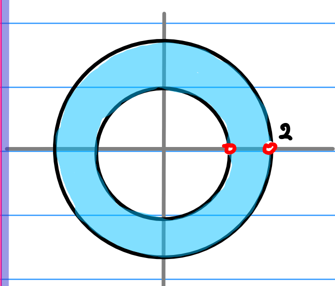


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

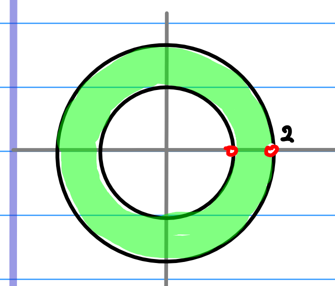


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⅲ

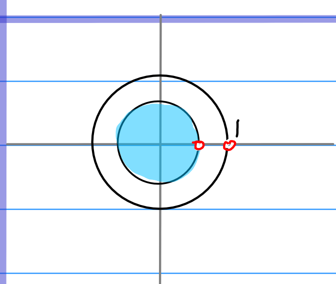


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

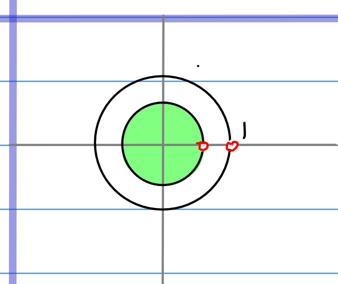


$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

Ⅰ

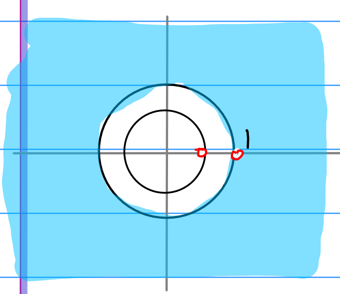


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

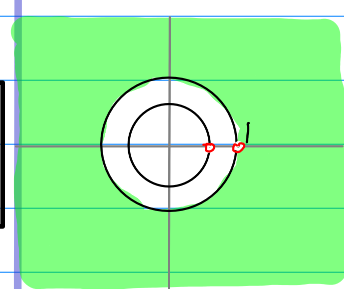


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

Ⅱ

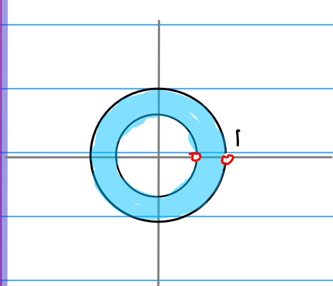


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

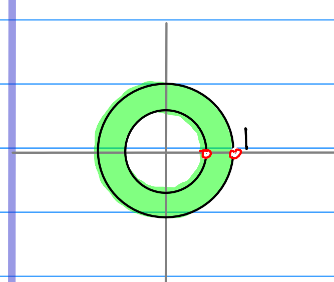


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⅲ

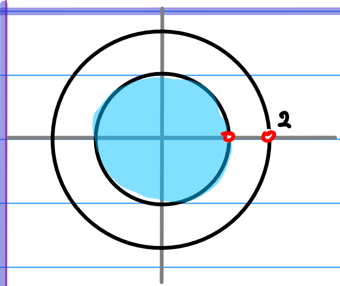


$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$



$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

Ⅰ

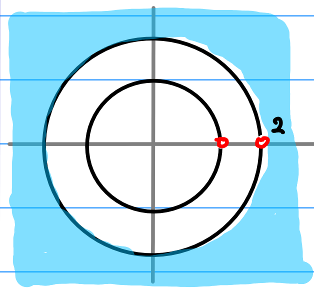


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

$p_1 = 1$ $p_2 = 2$

Ⅱ

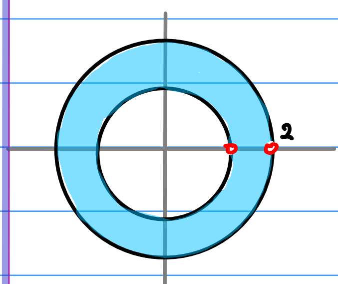


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

$p_1 = 1$ $p_2 = 2$

Ⅲ

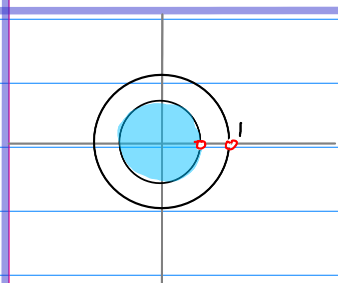


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

$p_1 = 1$ $p_2 = 2$

Ⅰ

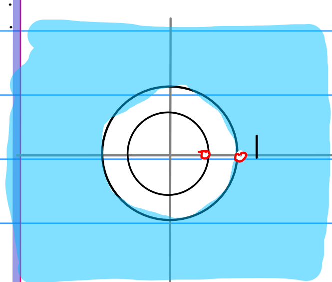


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

$p_1 = \frac{1}{2}$ $p_2 = 1$

Ⅱ

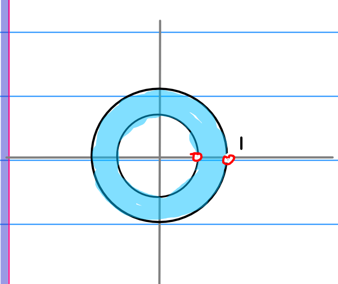


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

$p_1 = \frac{1}{2}$ $p_2 = 1$

Ⅲ



$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

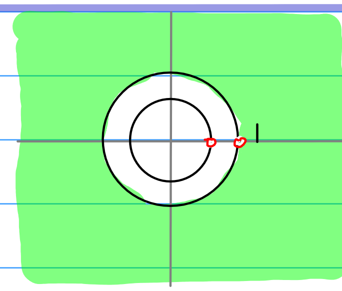
$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

$p_1 = \frac{1}{2}$ $p_2 = 1$

Ⅰ

$$\boxed{\begin{array}{l} 0 \\ (p_1)^{n+1} - (p_2)^{n+1} \end{array}}$$

$p_1 = 1/2 \qquad p_2 = 1$

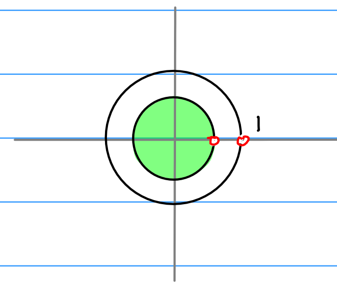


$$\boxed{\begin{array}{ll} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{array}}$$

Ⅱ

$$\boxed{\begin{array}{l} (p_2)^{n+1} - (p_1)^{n+1} \\ 0 \end{array}}$$

$p_1 = 1/2 \qquad p_2 = 1$

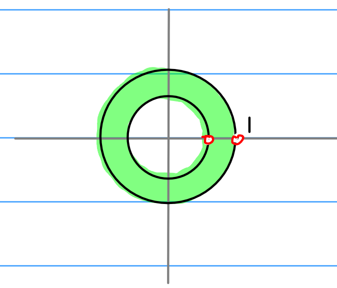


$$\boxed{\begin{array}{ll} 1 - \left(\frac{1}{2}\right)^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{array}}$$

Ⅲ

$$\boxed{\begin{array}{l} (p_2)^{n+1} \\ (p_1)^{n+1} \end{array}}$$

$p_1 = 1/2 \qquad p_2 = 1$

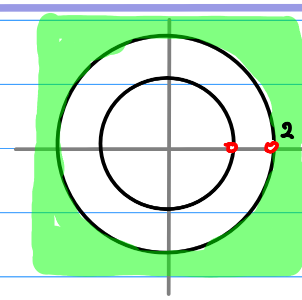


$$\boxed{\begin{array}{ll} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{array}}$$

Ⅰ

$$\boxed{\begin{array}{l} 0 \\ (p_1)^{n+1} - (p_2)^{n+1} \end{array}}$$

$p_1 = 1 \qquad p_2 = 2$

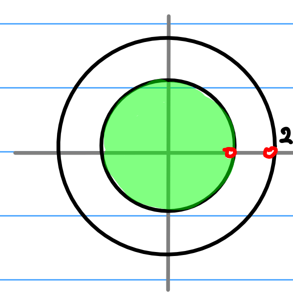


$$\boxed{\begin{array}{ll} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{array}}$$

Ⅱ

$$\boxed{\begin{array}{l} (p_2)^{n+1} - (p_1)^{n+1} \\ 0 \end{array}}$$

$p_1 = 1 \qquad p_2 = 2$

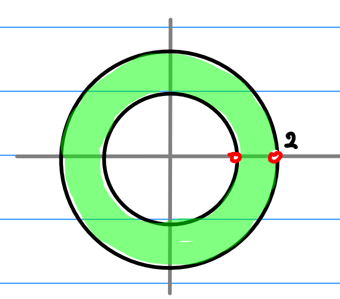


$$\boxed{\begin{array}{ll} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{array}}$$

Ⅲ

$$\boxed{\begin{array}{l} (p_2)^{n+1} \\ (p_1)^{n+1} \end{array}}$$

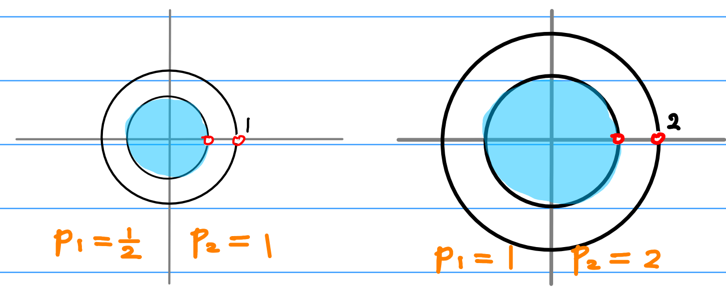
$p_1 = 1 \qquad p_2 = 2$



$$\boxed{\begin{array}{ll} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{array}}$$

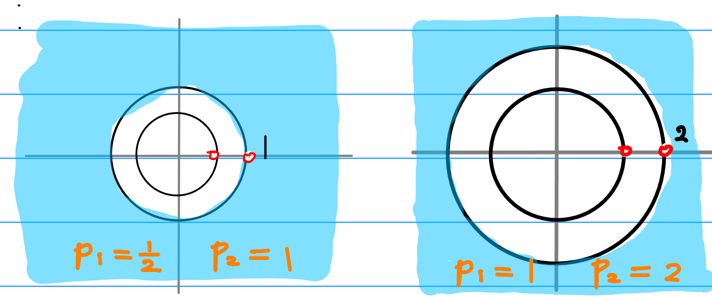
$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$

0



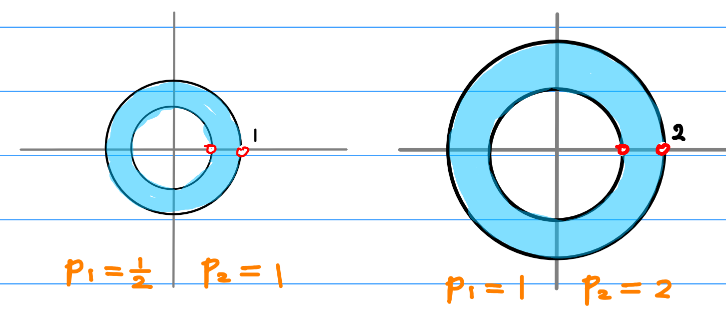
0

$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$



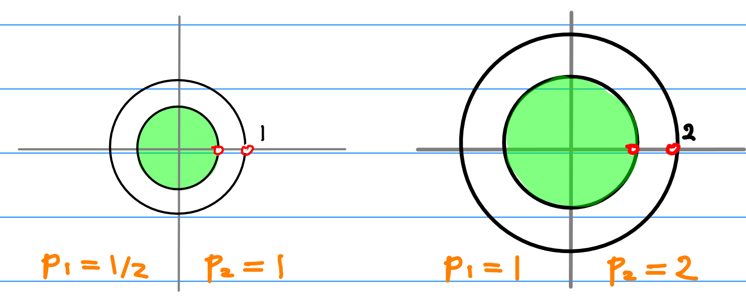
$\left(\frac{1}{p_2}\right)^{n+1}$

$\left(\frac{1}{p_1}\right)^{n+1}$



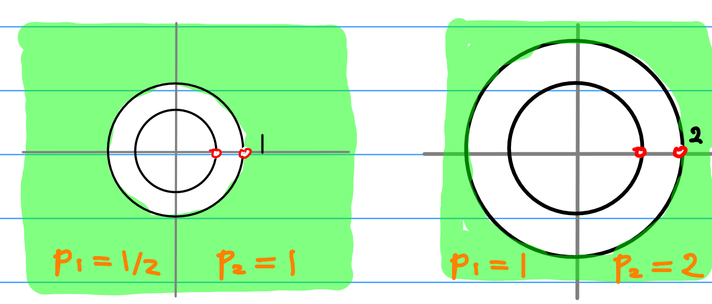
$(p_2)^{n+1} - (p_1)^{n+1}$

0



0

$(p_1)^{n+1} - (p_2)^{n+1}$



$(p_2)^{n+1}$

$(p_1)^{n+1}$