

Laurent Series and z-Transform Examples case 0.B

20171123

Copyright (c) 2016 - 2017 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

$$X(z) = \frac{-1}{(z-1)(z-2)} \xleftarrow{z^{-1}} f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

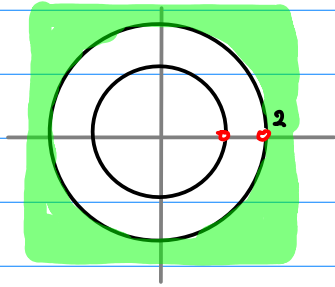
$$X(z) = \frac{-1}{(z-0.5)(z-2)} = \frac{2}{3} \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$$\begin{aligned} \frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} &= \frac{2}{3} \left(\frac{1}{z^{-1}-0.5} - \frac{1}{z^{-1}-2} \right) \\ &= \frac{2}{3} \left(\frac{2}{2z^{-1}-1} - \frac{0.5}{0.5z^{-1}-1} \right) \\ &= \frac{2}{3} \left(\frac{2z}{2-z} - \frac{0.5z}{0.5-z} \right) \\ &= \frac{2}{3} \left(\frac{-2z}{z-2} + \frac{0.5z}{z-0.5} \right) \\ &= \frac{2z}{3} \left(\frac{-2}{z-2} + \frac{0.5}{z-0.5} \right) \\ &= \frac{2z}{3} \left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)} \right) \\ &= \frac{-z^2}{(z-2)(z-0.5)} \end{aligned}$$

1.B

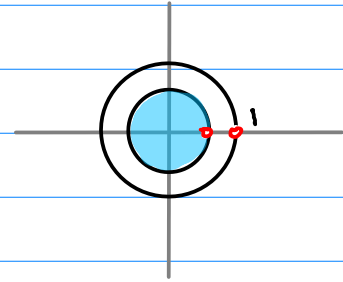
$$X(z) = \frac{-1}{(z-1)(z-2)} \quad \leftarrow z^{-1}$$

$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

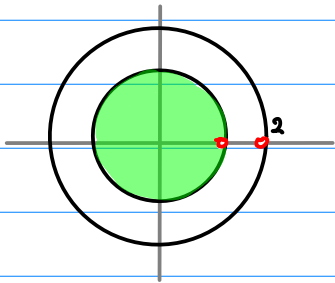


$$\sum_{n=-\infty}^{\infty} [1 - 2^{-n+1}] z^{-n}$$

≡

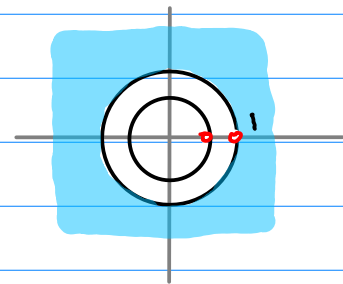


$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$

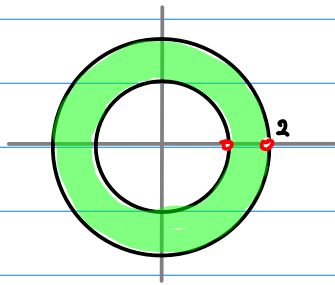


$$\sum_{n=-1}^{\infty} [-1 + 2^{n-1}] z^{-n}$$

≡

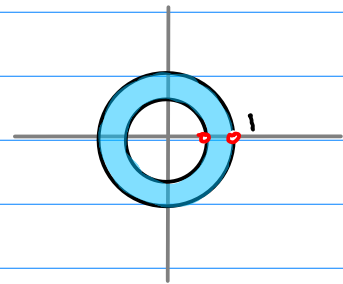


$$\sum_{n=-1}^{\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n$$



$$\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} \cdot z^{-n}$$

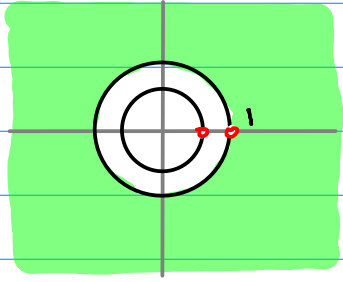
≡



$$\sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} \cdot z^n$$

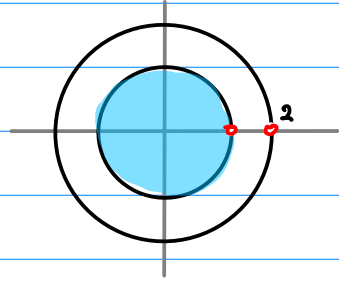
2. B

$$X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \xleftarrow{z^{-1}} f(z) = \frac{-1}{(z-1)(z-2)}$$

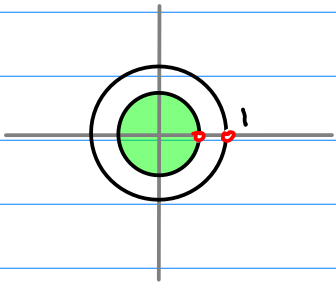


$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^n$$

==

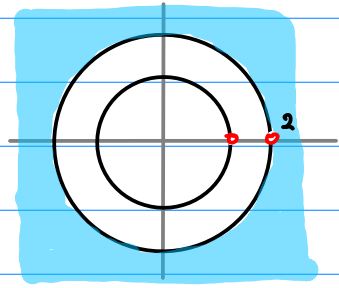


$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^n$$

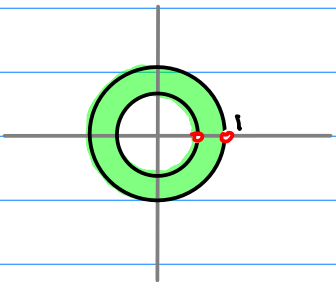


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

==

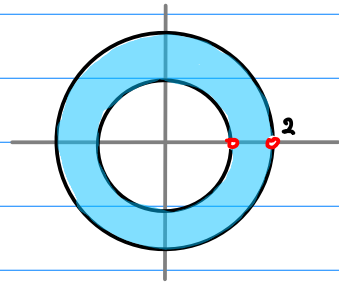


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$



$$\sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

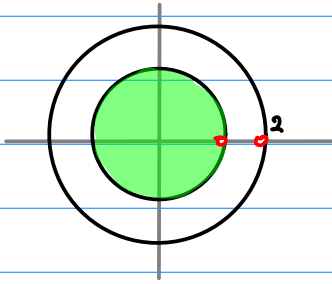
==



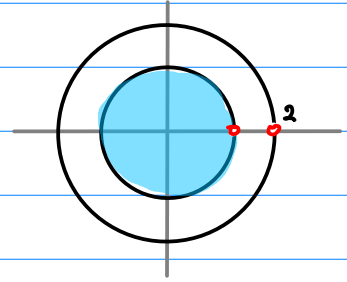
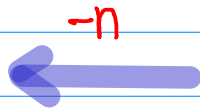
$$\sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

3.B

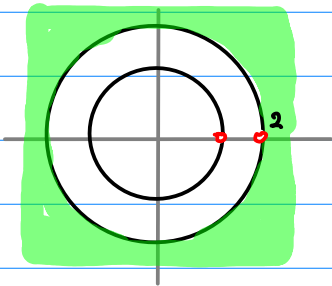
$$X(z) = \frac{-1}{(z-1)(z-2)} = f(z) = \frac{-1}{(z-1)(z-2)}$$



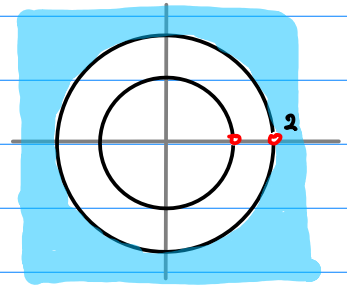
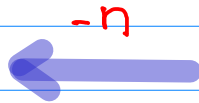
$$\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$



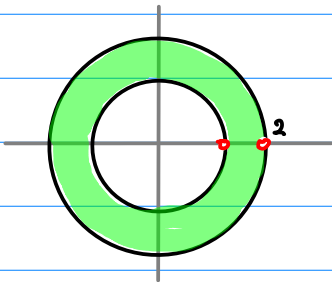
$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^{-n}$$



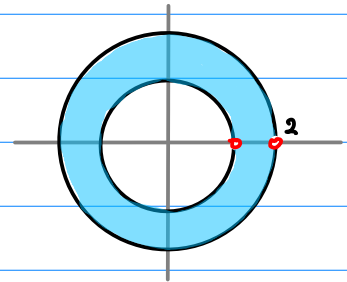
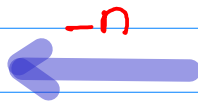
$$\sum_{n=-1}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$



$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^{-n}$$



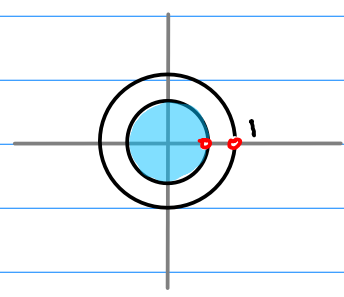
$$-\sum_{n=0}^{\infty} z^{-n} - \sum_{n=-1}^{\infty} 2^{n-1} z^{-n}$$



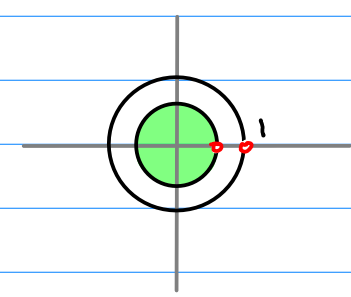
$$+\sum_{n=-1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{-n}$$

4.B

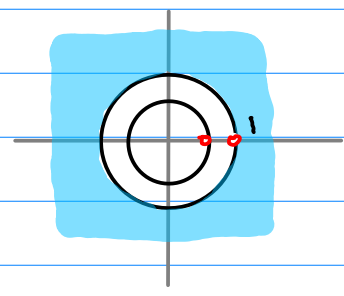
$$X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



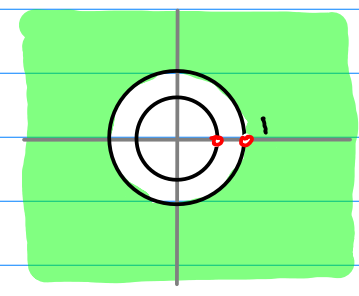
$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n \quad \leftarrow^{-n}$$



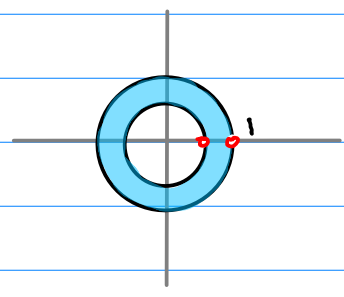
$$\sum_{n=-\infty}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$



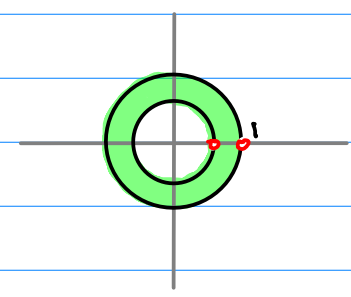
$$\sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^n \quad \leftarrow^{-n}$$



$$\sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$



$$+\sum_{n=-\infty}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n \quad \leftarrow^{-n}$$

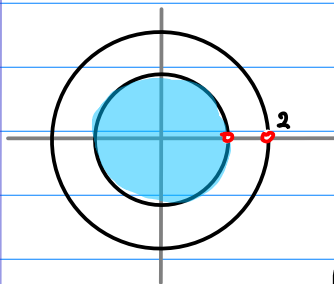


$$+\sum_{n=-\infty}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

1. B

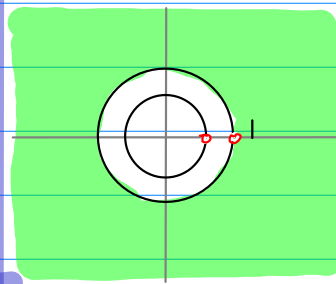
$$X(z) = \frac{-1}{(z-1)(z-2)} \quad \xleftarrow{z^{-1}} \quad f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

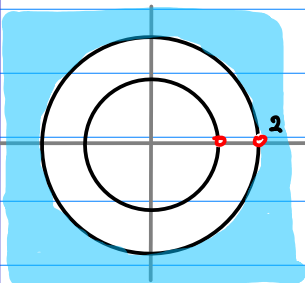
$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & (n \leq 0) \end{cases}$$

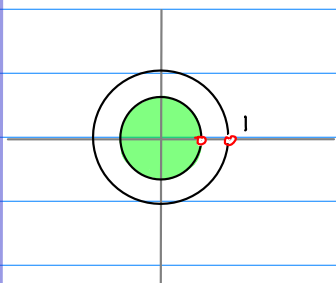
$$X(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

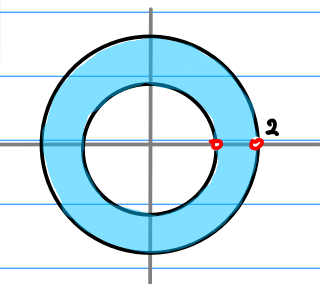
$$f(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1} \right) z^n$$



$$x_n = \begin{cases} \left(1 - \left(\frac{1}{2}\right)^{n+1} \right) & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

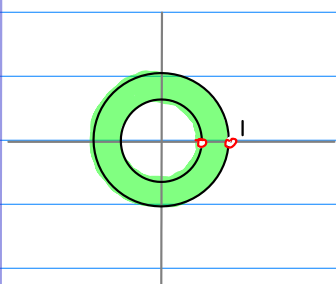
$$X(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1} \right) z^n$$

III



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$



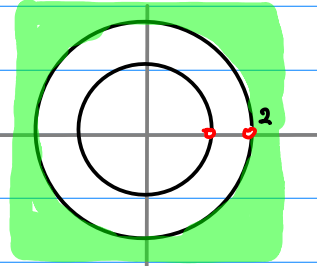
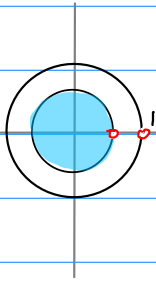
$$x_n = \begin{cases} 1 & (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$

2. B

$$X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \quad \leftarrow z^{-1} \quad f(z) = \frac{-1}{(z-1)(z-2)}$$

I



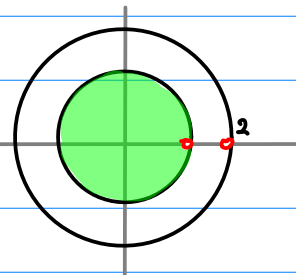
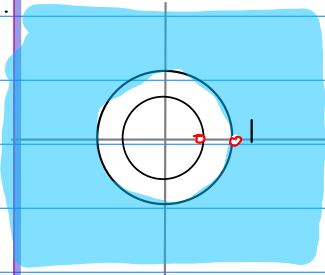
$$a_n = \begin{cases} [1 - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n \geq 0) \\ [1 - 2^{n-1}] & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n$$

$$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n$$

II



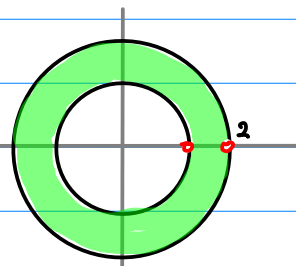
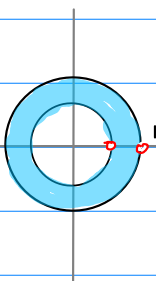
$$a_n = \begin{cases} 0 & (n > 0) \\ [2^{n-1} - 1] & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} [2^{n-1} - 1] & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

$$X(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

III



$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

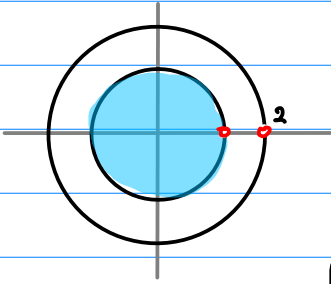
$$f(z) = \sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

3.B

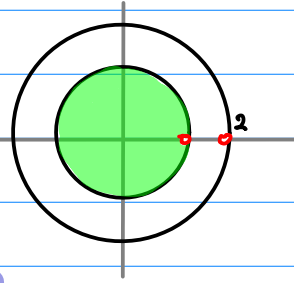
$$X(z) = \frac{-1}{(z-1)(z-2)} = f(z) = \frac{-1}{(z-1)(z-2)}$$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

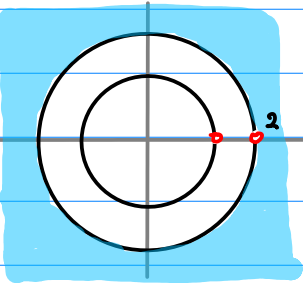
$$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

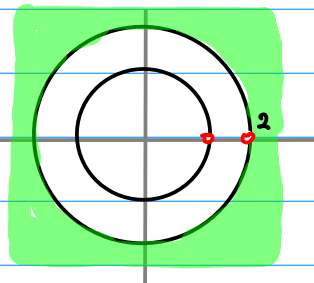
$$X(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

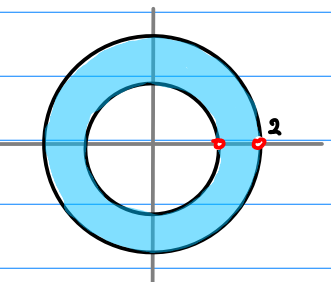
$$f(z) = \sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

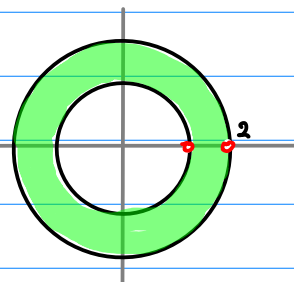
$$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$



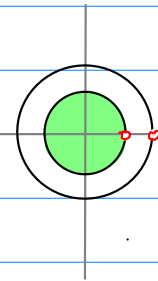
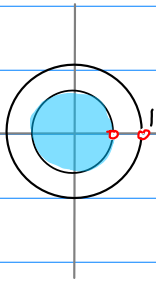
$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.B

$$X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

I



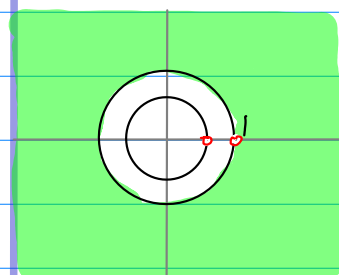
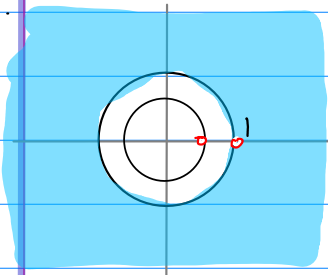
$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$

$$X(z) = \sum_{n=-\infty}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$

II



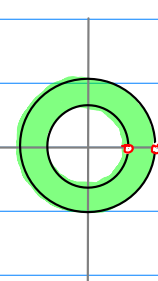
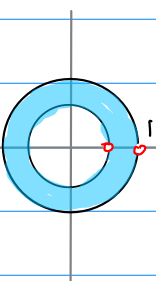
$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^n$$

$$X(z) = \sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$

III



$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

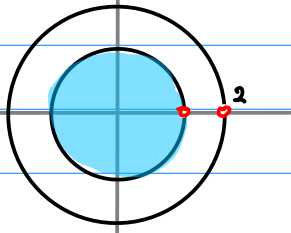
$$x_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = + \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$

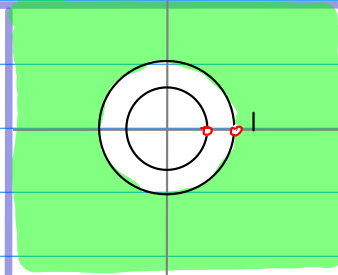
$$X(z) = + \sum_{n=1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$



I

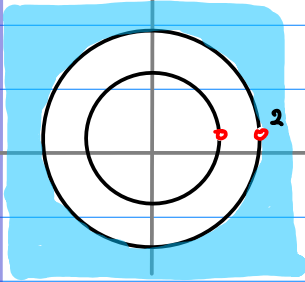


$$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$$

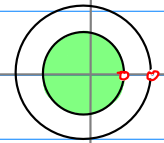


$$\begin{matrix} 0 & (n > 0) \\ (\frac{1}{2})^{n+1} - 1 & (n \leq 0) \end{matrix}$$

II

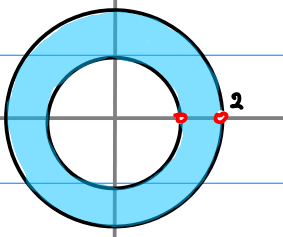


$$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}$$

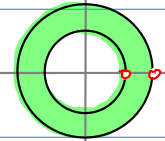


$$\begin{matrix} 1 - (\frac{1}{2})^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$$

III

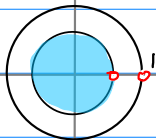


$$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$$

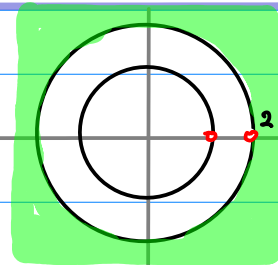


$$\begin{matrix} 1 & (n > 0) \\ (\frac{1}{2})^{n+1} & (n \leq 0) \end{matrix}$$

I

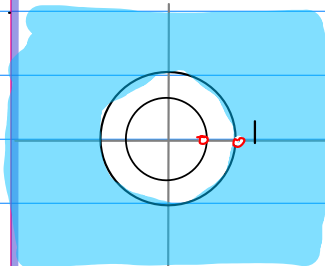


$$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$$

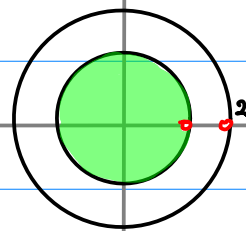


$$\begin{matrix} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{matrix}$$

II

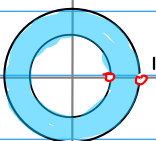


$$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$$

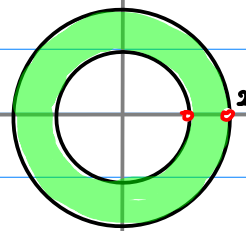


$$\begin{matrix} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$$

III

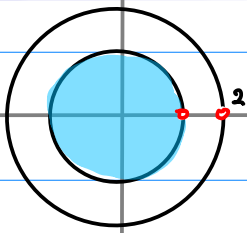


$$\begin{matrix} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{matrix}$$

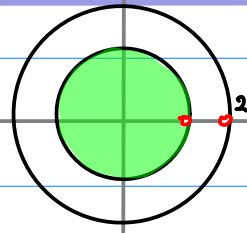


$$\begin{matrix} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$$

I

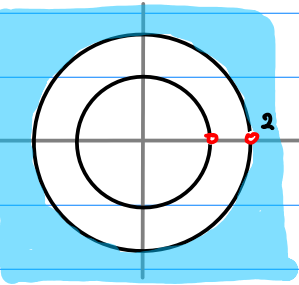


$$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$$

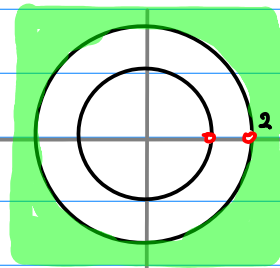


$$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$$

II

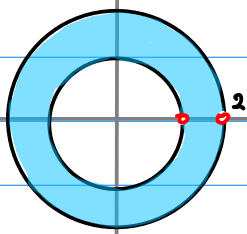


$$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}$$

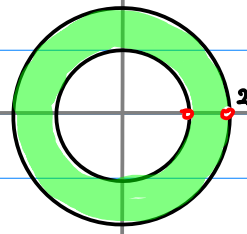


$$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$$

III

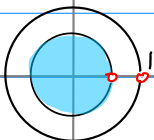


$$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$$

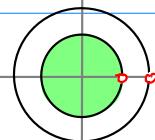


$$\begin{matrix} 1 & (n > 0) \\ 2^n & (n \leq 0) \end{matrix}$$

I

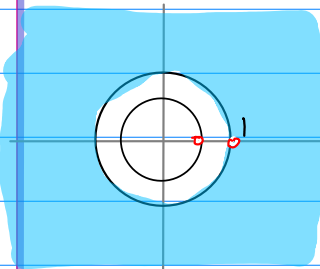


$$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$$

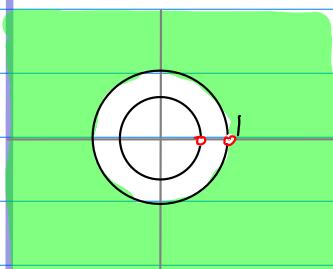


$$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}$$

II

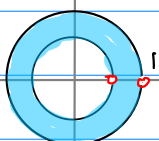


$$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$$

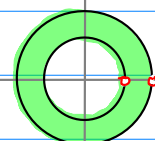


$$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$$

III

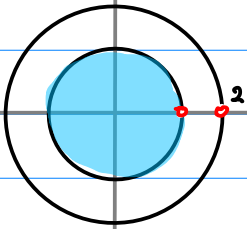


$$\begin{matrix} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{matrix}$$



$$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$$

I

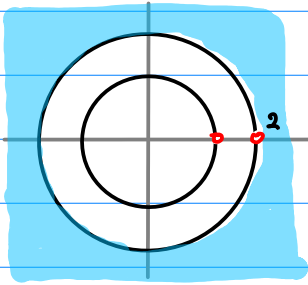


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

II

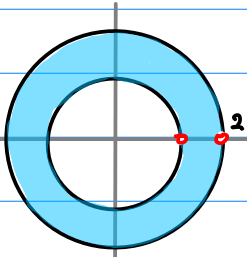


$$\begin{array}{ll} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

III

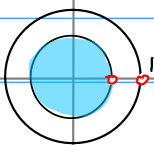


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

I

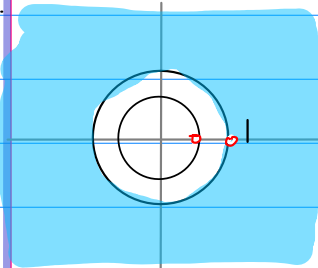


$$\begin{array}{ll} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

II

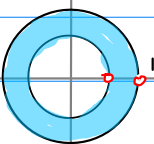


$$\begin{array}{ll} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

III



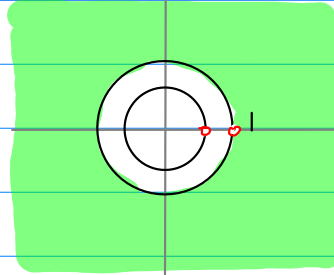
$$\begin{array}{ll} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

I

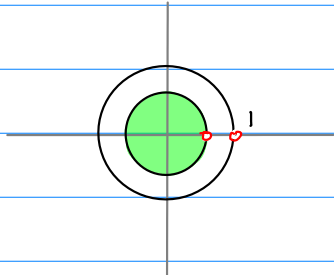
$$\begin{aligned} &0 \\ &(p_1)^{n+1} - (p_2)^{n+1} \\ &p_1 = 1/2 \quad p_2 = 1 \end{aligned}$$



$$\begin{aligned} &0 \quad (n > 0) \\ &(\frac{1}{2})^{n+1} - 1 \quad (n \leq 0) \end{aligned}$$

II

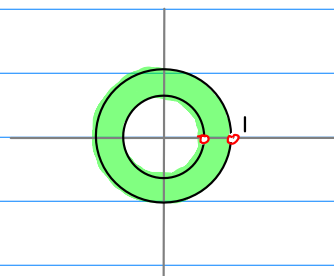
$$\begin{aligned} &(p_2)^{n+1} - (p_1)^{n+1} \\ &0 \\ &p_1 = 1/2 \quad p_2 = 1 \end{aligned}$$



$$\begin{aligned} &1 - (\frac{1}{2})^{n+1} \quad (n > 0) \\ &0 \quad (n \leq 0) \end{aligned}$$

III

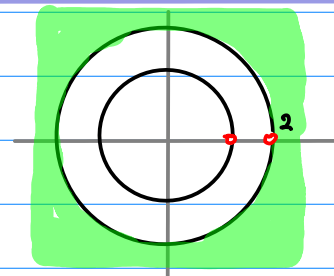
$$\begin{aligned} &(p_2)^{n+1} \\ &(p_1)^{n+1} \\ &p_1 = 1/2 \quad p_2 = 1 \end{aligned}$$



$$\begin{aligned} &1 \quad (n > 0) \\ &(\frac{1}{2})^{n+1} \quad (n \leq 0) \end{aligned}$$

I

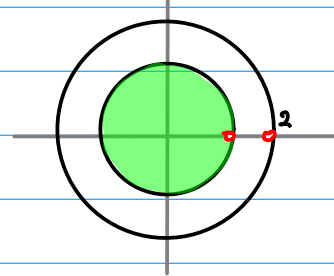
$$\begin{aligned} &0 \\ &(p_1)^{n+1} - (p_2)^{n+1} \\ &p_1 = 1 \quad p_2 = 2 \end{aligned}$$



$$\begin{aligned} &0 \quad (n \geq 0) \\ &1 - 2^{n+1} \quad (n < 0) \end{aligned}$$

II

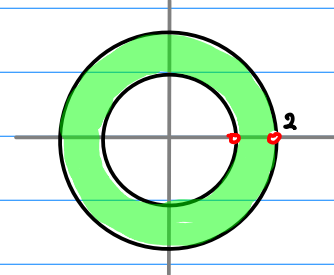
$$\begin{aligned} &(p_2)^{n+1} - (p_1)^{n+1} \\ &0 \\ &p_1 = 1 \quad p_2 = 2 \end{aligned}$$



$$\begin{aligned} &2^{n+1} - 1 \quad (n \geq 0) \\ &0 \quad (n < 0) \end{aligned}$$

III

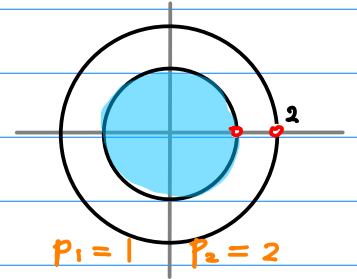
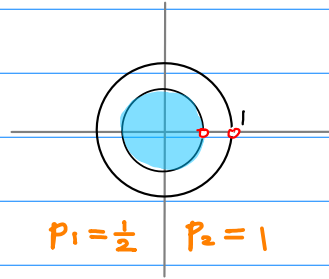
$$\begin{aligned} &(p_2)^{n+1} \\ &(p_1)^{n+1} \\ &p_1 = 1 \quad p_2 = 2 \end{aligned}$$



$$\begin{aligned} &2^{n+1} \quad (n \geq 0) \\ &1 \quad (n < 0) \end{aligned}$$

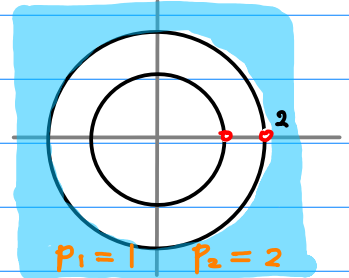
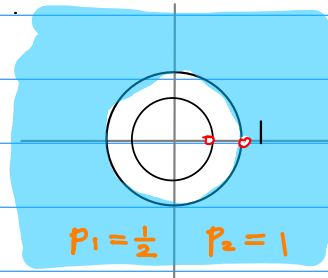
$$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$$

$$0$$



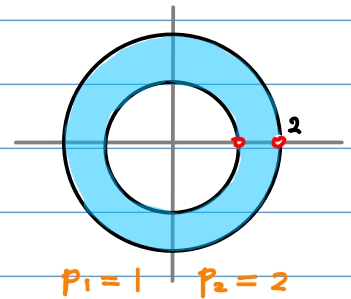
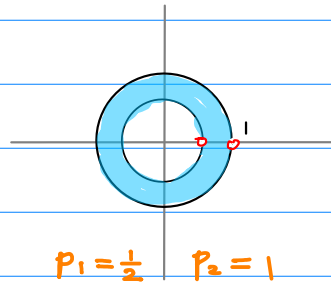
$$0$$

$$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$$



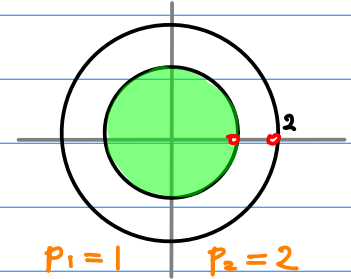
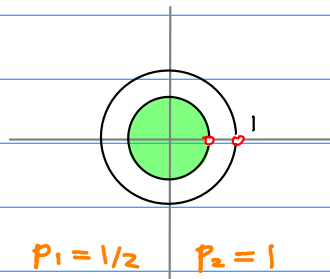
$$\left(\frac{1}{p_2}\right)^{n+1}$$

$$\left(\frac{1}{p_1}\right)^{n+1}$$



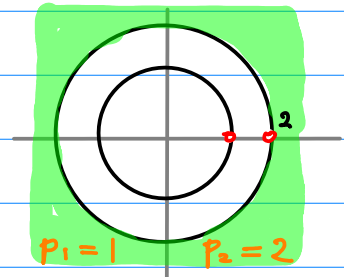
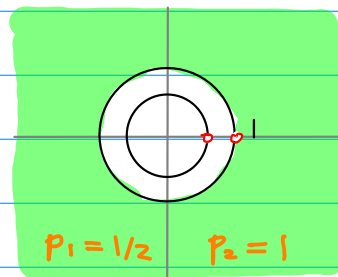
$$(p_2)^{n+1} - (p_1)^{n+1}$$

$$0$$



$$0$$

$$(p_1)^{n+1} - (p_2)^{n+1}$$



$$(p_2)^{n+1}$$

$$(p_1)^{n+1}$$

