Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Example Problem (1)

N Gaussian random variables

Definition

two statisticaly independent Gaussian random variables Y_1, Y_2 each with zero mean and unit variance, can be generated by te transformation

$$Y_1 = T_1(X_1, X_2) = \sqrt{-2\ln(X_1)}\cos(2\pi X_2)$$

$$Y_2 = T_2(X_1, X_2) = \sqrt{-2\ln(X_1)}\cos(2\pi X_2)$$

the joint density of Y_1 and Y_2

$$f_{Y_1 Y_2}(y1, y2) = \frac{e^{-Y_1^2/2}}{\sqrt{2\pi}} \frac{e^{-Y_2^2/2}}{\sqrt{2\pi}}$$

Example Problem (2)

N Gaussian random variables

Definition

$$[C_W] = \begin{bmatrix} \sigma_{W_1}^2 & \rho_W \sigma_{W_1} \sigma_{W_2} \\ \rho_W \sigma_{W_1} \sigma_{W_2} & \sigma_{W_2}^2 \end{bmatrix} = [T][T]^t$$

$$[T] = \begin{bmatrix} T_{11} & 0 \\ T_{21} & T_{22} \end{bmatrix}$$

the joint density of Y_1 and Y_2

$$T_{11} = \sigma_{W_1}, \qquad T_{21} = \rho_W \sigma_{W_1} \sigma_{W_2}, \qquad T_{22} = \sigma_{W_2} \sqrt{1 - \rho_W^2}$$

Example Problem (3)

N Gaussian random variables

Definition

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} T_{11} & 0 \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

the joint density of Y_1 and Y_2

$$W_1 = T_{11} Y_1 = \sigma_{W_1} Y_1$$

$$W_2 = T_{21} Y_1 + T_{22} Y_2 = \rho_W \sigma_{W_1} \sigma_{W_2} Y_1 + \sigma_{W_2} \sqrt{1 - \rho_W^2}\, Y_2$$

Example Problem (4)

N Gaussian random variables

Definition

$$W_1 = \sigma_{W_1} Y_1$$

$$W_2 = \rho_W \sigma_{W_1} \sigma_{W_2} Y_1 + \sigma_{W_2} \sqrt{1 - \rho_W^2}\, Y_2$$

$$W_1 = \overline{W_1} + \sigma_{W_1} Y_1$$

$$W_2 = \overline{W_2} + \rho_W \sigma_{W_1} \sigma_{W_2} Y_1 + \sigma_{W_2} \sqrt{1 - \rho_W^2}\, Y_2$$