

Angle Recoding 2. Wu

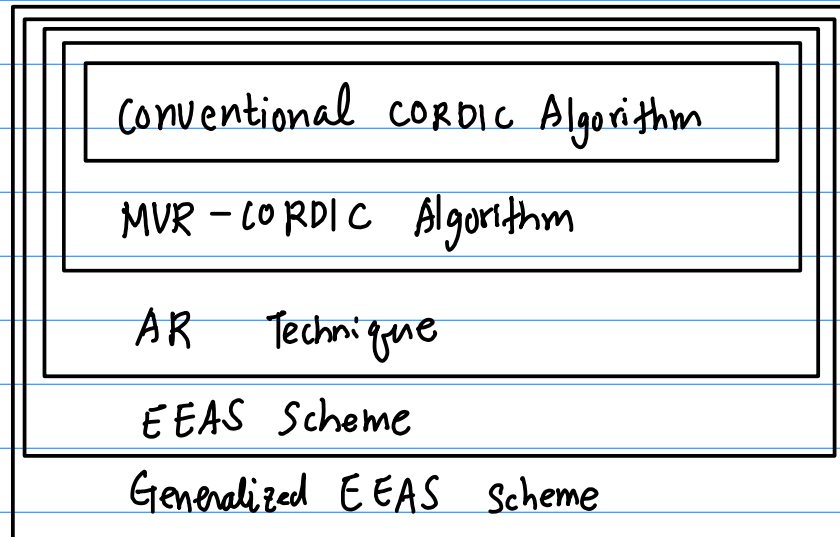
0. Overview

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Vector Rotational CORDIC



Vector Rotation Alg	Selection of Rotation Sequence	Elementary Angle Set	Micro Rotation	Angle Quantization	
				θ_i	N_A
Conventional CORDIC	$\mu = \{-1, +1\}$	EAS S	complete	$\mu(i) a(i)$	W Fixed
Angle Recoding	$\mu = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	N' Variable
MVR-CORDIC	$\alpha = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	R_m Fixed
EEAS	$\alpha_1, \alpha_2 = \{-1, 0, +1\}$	EEAS S_2	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} + \alpha_1(i) \cdot 2^{-s_1(i)})$	R_m Fixed
Generalized EEAS	$\alpha_1, \alpha_2, \dots, \alpha_{d-1} = \{-1, 0, +1\}$	EEAS S_d $d \geq 3$	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} + \alpha_{d-1}(i) \cdot 2^{-s_{d-1}(i)})$	R_m Fixed

Elementary Angle Set EAS

$$S = \{ a(i) = \tan^{-1}(2^{-i}) : 0 \leq i \leq N-1 \}$$

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, s^* \in \{0, 1, \dots, N-1\} \}$$

$$S_2 = \{ \tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \alpha_0^* \in \{-1, 0, +1\}, s_0^* \in \{0, 1, \dots, N-1\}, \alpha_1^* \in \{-1, 0, +1\}, s_1^* \in \{0, 1, \dots, N-1\} \}$$

$$S_d = \{ \tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} + \dots + \alpha_{d-1}^* \cdot 2^{-s_{d-1}^*}) : \alpha_0^* \in \{-1, 0, +1\}, s_0^* \in \{0, 1, \dots, N-1\}, \alpha_1^* \in \{-1, 0, +1\}, s_1^* \in \{0, 1, \dots, N-1\}, \vdots, \alpha_{d-1}^* \in \{-1, 0, +1\}, s_{d-1}^* \in \{0, 1, \dots, N-1\} \}$$

for the AQ perspective

$$S = \{ a(i) = \tan^{-1}(2^{-i}) : 0 \leq i \leq N-1 \}$$

$$S = \{ \tan^{-1}(\mu(i) \cdot 2^{-i}) : 0 \leq i \leq N-1, \mu(i) \in \{-1, +1\} \}$$

still $0 \notin S$ $S \neq S_1$

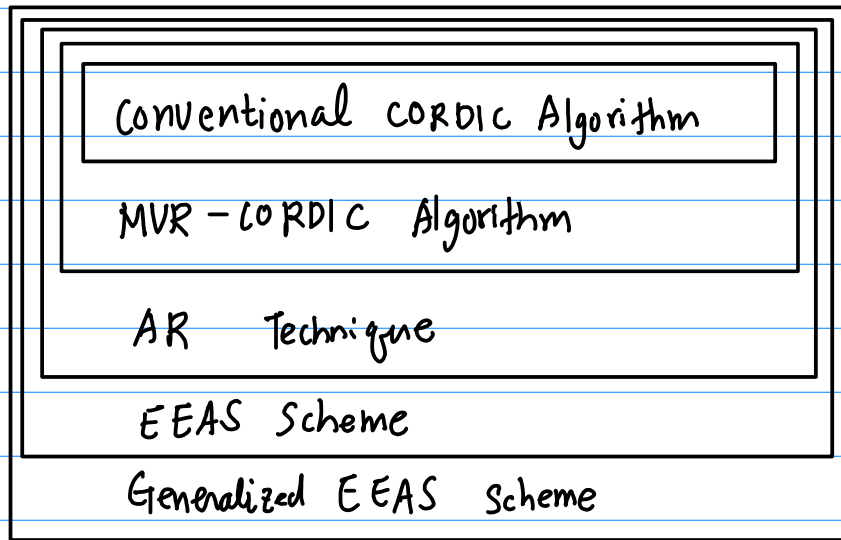
Family of Vector Rotational CORDIC

AQ process — } CORDIC
(Angle Quantization) } AR (Angle Recoding)
} MVR - CORDIC (Modified Vector Rotation)
} EEAS (Extended Elementary Angle Set)
} Generalized EEAS

AQ process with various EAS and
and suitable combinations of subangles

to decompose the target rotational angle
into several easy-to-implement subangles

minimizing the angle quantization error ξ_m
to obtain the best precision performance



EEAS covers $\left\{ \begin{array}{l} \text{MVR-CORDIC} \\ \text{AR} \end{array} \right.$

a subset of EEAS S_2 EAS S_1

MVR-CORDIC a subset of AR

one constraint on the iteration number

Angle Quantization

Quantization process on the rotational angle θ

decompose the original rotational angle θ
into several θ_i 's

$$\theta = \theta_0 + \theta_1 + \theta_2 + \dots + \theta_{N_A-1} + \xi_m$$

sum up those subangles to approximate
the original angle as close as possible

Minimize the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

N_A : the number of sub-angles

design issues in the AQ process

① Need to determine the sub-angles θ_i
each θ_i needs to be easy-to-implement

② how to select and combine these sub-angles ξ_m
such that the angle quantization error ξ_m
can be minimized

Angle Quantization

Quantization process on the rotational angle θ

decompose θ into several subangles θ_i 's

the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

N_A the number of subangles
 $\theta_0, \theta_1, \dots, \theta_{N_A-1}$

$$\theta = \theta_0 + \theta_1 + \dots + \theta_{N_A-1} + \xi_m$$

data : W -bit word length

the iteration number : N $N \leq W$

the restricted iteration number : R_m $R_m \ll W$

Vector Rotation CORDIC Family

① Conventional CORDIC

② AR

③ MVR

④ EEAS

CSD (Canonical Signed Digit) Quantization

digital filter designs

coefficients are recoded

in terms of SPT (Signed Power of Two) terms

multiplication can be easily realized
with shift-and-add operations

$$h_2 = (-0.156249)_{10} \Rightarrow (0.0\bar{1}011)_2$$

$W=8$, 3 non-zero digits

- ① CSD quantization decomposes coefficients into several SPT terms (sub-coefficients)
- ② the multiplication of a coefficient can be reformed through the combination of the non-zero SPT sub-coefficients

quantize the rotation angle θ

decompose the rotation angle θ
into several sub-angles θ_i 's

the rotational operation of each θ_i
should be easily realized

If each θ_i can be realized
using only shift-and-add operations

the rotation of θ can be performed
through successive applications of
sub-angle rotations
in a cost-effective way

Approximation target	Coefficient h_i	Rotation angle Θ
Basic Element	Non-zero digit 2^{-i}	Sub-angle $\alpha(i) = \tan^{-1}(2^{-i})$
Basic Operation	shift-and-add operation	2 shift-and-add operations
Approximation Equation	$h_i \approx \sum_{j=0}^{N_D-1} g_j \cdot 2^{-d_j}$	$\Theta \approx \sum_{j=0}^{N_A-1} \alpha(j) \cdot a(s(j))$
	$g_j \in \{-1, 0, +1\}$ $d_j \in \{0, 1, \dots, w-1\}$ $N_D =$ the number of non-zero digits	$N_A =$ the number of sub-angles

try to approach the target rotation angle θ
step by step

decisions are made in each step
by choosing the best combination of $\alpha(i)$ $a(s(i))$

So as to minimize $|\xi_m|$

$\alpha(i)$, $a(i)$ are determined such that
the error function is minimized

$$J(i) = |\theta(i) - \alpha(i)a(s(i))|$$

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

terminated if no further improvement can be found

$$J(i) \geq J(i-1)$$

or $\alpha(R_m-1)$ and $s(R_m-1)$
are determined at the end

Rotation Angle $\theta = \frac{13\pi}{64}$

Conventional CORDIC

$$\bar{\mu} = [1 \ -1 \ 1 \ 1 \ -1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1 \ 1 \ -1 \ 1 \ 1]$$

Angle Reordering - Greedy

$$\bar{\mu} = [1 \ 0 \ 0 \ -1 \ 0 \ 0 \ -1 \ -1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1]$$

MVR-CORDIC - Greedy

$$\bar{\alpha} = [1 \ -1 \ -1 \ -1]$$

$$\bar{s} = [0 \ 3 \ 6 \ 7]$$

MVR-CORDIC - Semi Greedy ($D=2$)

$$\bar{\alpha} = [1 \ -1 \ -1 \ 1]$$

$$\bar{s} = [0 \ 3 \ 5 \ 7]$$

MVR-CORDIC - TBS

$$\bar{\alpha} = [1 \ 1 \ -1 \ -1]$$

$$\bar{s} = [1 \ 2 \ 4 \ 7]$$

EEAS - Greedy

$$\bar{\alpha}_0 = [1 \ 1]$$

$$\bar{\alpha}_1 = [-1 \ -1]$$

$$\bar{s}_0 = [0 \ 2]$$

$$\bar{s}_1 = [8 \ 10]$$

EEAS - TBS

$R_m=2$

$$\bar{\alpha}_0 = [1 \ 1]$$

$$\bar{\alpha}_1 = [1 \ 1]$$

$$\bar{s}_0 = [0 \ 6]$$

$$\bar{s}_1 = [3 \ 5]$$

EEAS - TBS

$R_m=3$

$$\bar{\alpha}_0 = [1 \ -1 \ -1]$$

$$\bar{\alpha}_1 = [1 \ -1 \ 1]$$

$$\bar{s}_0 = [0 \ 3 \ 7]$$

$$\bar{s}_1 = [15 \ 6 \ 12]$$

```
>> mu = [1, -1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, -1, 1, 1]
>> length(mu)
ans = 16
>> s = [ 0: 15]
>> atan(1)
ans = 0.78540
>> pi/4
ans = 0.78540
>> sum(atan(2.^(-s)) .* mu)
ans = 0.63815
>> 13 * pi / 32
ans = 1.2763
```

```
>> mu = [1, 0, 0, -1, 0, 0, -1, -1, 0, 0, 0, 1, 0, 0, 0, 1]
>> length(mu)
ans = 16
>> s = [ 0: 15]
>> atan(1)
ans = 0.78540
>> pi/4
ans = 0.78540
>> sum(atan(2.^(-s)) .* mu)
ans = 0.63813
>> 13 * pi / 32
ans = 1.2763
```



```
>> s = [0, 3, 6, 7]
>> alpha = [1, -1, -1, -1]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63761
>>
```

```
>> s = [0, 3, 5, 7]
>> alpha = [1, -1, -1, 1]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63762
```

```
>> alpha = [1, 1, -1, -1]
>> s = [1, 2, 4, 7]
>> sum(atan(2.^(-s) .* alpha))
ans = 0.63840
```

```
alpha0=[1 -1]
alpha1=[-1 -1]
s0=[0 2]
s1=[8 10]
sum(atan(2.^(-s0)) .* alpha0 + atan(2.^(-s1)) .* alpha1)
ans = 0.53554          xxx
```

```
alpha0=[1 1]
alpha1=[-1 -1]
s0=[0 6]
s1=[3 5]
sum(atan(2.^(-s0)) .* alpha0 + atan(2.^(-s1)) .* alpha1)
ans = 0.64543          xxx
```

```
alpha0=[1 -1 -1]
alpha1=[1 -1 1]
s0=[0 3 7]
s1=[15 6 12]
sum(atan(2.^(-s0)) .* alpha0 + atan(2.^(-s1)) .* alpha1)
ans = 0.63788
```



