

Angle Recoding CORDIC

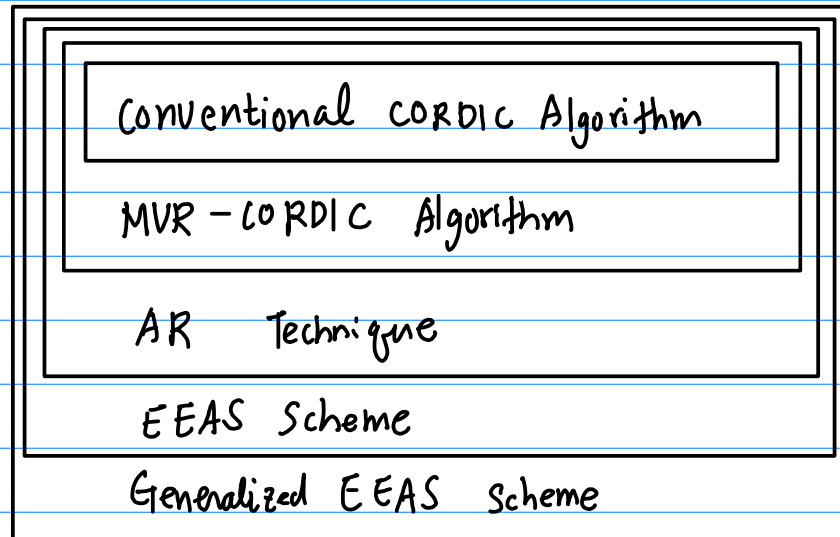
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20180804

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Vector Rotational CORDIC



Vector Rotation Alg	Selection of Rotation Sequence	Elementary Angle Set	Micro Rotation	Angle Quantization	
				θ_i	N_A
Conventional CORDIC	$\mu = \{-1, +1\}$	EAS S	complete	$\mu(i) a(i)$	W Fixed
Angle Recoding	$\mu = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	N' Variable
MVR-CORDIC	$\alpha = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	R_m Fixed
EEAS	$\alpha_1, \alpha_2 = \{-1, 0, +1\}$	EEAS S_2	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} + \alpha_1(i) \cdot 2^{-s_1(i)})$	R_m Fixed
Generalized EEAS	$\alpha_1, \alpha_2, \dots, \alpha_{d-1} = \{-1, 0, +1\}$	EEAS S_d $d \geq 3$	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} + \alpha_{d-1}(i) \cdot 2^{-s_{d-1}(i)})$	R_m Fixed

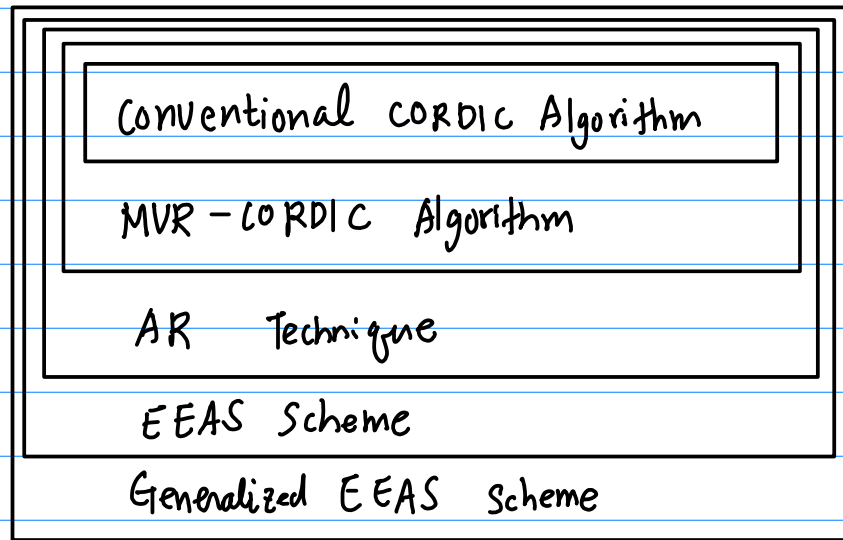
Family of Vector Rotational CORDIC

AQ process — } CORDIC
(Angle Quantization) } AR (Angle Recoding)
MVR - CORDIC (Modified Vector Rotation)
EEAS (Extended Elementary Angle Set)
Generalized EEAS

AQ process with various EAS and
and suitable combinations of subangles

to decompose the target rotational angle
into several easy-to-implement subangles

minimizing the angle quantization error ξ_m
to obtain the best precision performance



EEAS covers { MVR-CORDIC
AR

a subset of EEAS S_2 EAS S_1

MVR-CORDIC a subset of AR

one constraint on the iteration number

Angle Quantization

Quantization process on the rotational angle θ

decompose the original rotational angle θ
into several θ_i 's

sum up these subangles to approximate
the original angle as close as possible

Minimize the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

N_A : the number of sub-angles

$$\theta = \theta_0 + \theta_1 + \theta_2 + \dots + \theta_{N_A-1} + \xi_m$$

design issues in the AQ process

- ① Need to determine the sub-angles θ_i
each θ_i needs to be easy-to-implement
- ② how to select and combine these sub-angles ξ_m
such that the angle quantization error ξ_m
can be minimized

Angle Quantization

Quantization process on the rotational angle θ

decompose θ into several subangles θ_i 's

the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

N_A the number of subangles
 $\theta_0, \theta_1, \dots, \theta_{N_A-1}$

$$\theta = \theta_0 + \theta_1 + \dots + \theta_{N_A-1} + \xi_m$$

data : W -bit word length

the iteration number : N $N \leq W$

the restricted iteration number : R_m $R_m \ll W$

Vector Rotation CORDIC Family

① Conventional CORDIC

② AR

③ MVR

④ EEAS

② Conventional CORDIC

elementary angle $a(i) = \tan^{-1}(2^{-i})$

the number of elementary angles N

the rotation sequence $\mu(i) = \{-1, +1\}$
 $+1, -1, -1, +1, +1, \dots$

the i -th rotation angle $a(i)$

the w -bit word length

the iteration number $N \leq w$

the angle quantization error

$$\xi_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^{M-1} \mu(i) a(i)$$

① AR [Hu]

skip certain micro rotations

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$$\mu(i) = 0 \rightarrow \text{skip}$$

desire to minimize

$$\sum_{i=0}^N |\mu(i)|$$

so that the total number of CORDIC iterations can be minimized

Angle Recoding ← Multiplier Recoding

angle recoding method for efficient implementation of the CORDIC algorithm
Hu & Naganathan, ISCAS 89

Greedy algorithm

the angle quantization error

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^M \mu(i) a(i)$$

$$\theta(0) = \theta, \{ \mu(i) = 0, 0 \leq i \leq N-1 \}, k=0$$

repeat until $|\theta(k)| < a(N-1)$ Do

choose $i_k, 0 \leq i_k \leq N-1$

$$| |\theta(k)| - a(i_k) | = \text{Min}_{0 \leq i \leq N-1} | |\theta(k)| - a(i) |$$

$$\theta(k+1) = \theta(k) - \mu(i_k) a(i_k)$$

$$\mu(i_k) = \text{Sign}(\theta(k))$$

Angle Quantization

Quantization process on the rotational angle θ

decompose θ into several subangles θ_i 's

the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

N_A the number of subangles
 $\theta_0, \theta_1, \dots, \theta_{N_A-1}$

$$\theta = \theta_0 + \theta_1 + \dots + \theta_{N_A-1} + \xi_m$$

data : W -bit word length

the iteration number : N $N \leq W$

the restricted iteration number : R_m $R_m \ll W$

CSD (Canonical Signed Digit) Quantization

digital filter designs

coefficients are recoded

in terms of SPT (Signed Power of Two) terms

multiplication can be easily realized
with shift-and-add operations

$$h_2 = (-0.156249)_{10} \Rightarrow (0.0\bar{1}011)_2$$

$W=8$, 3 non-zero digits

- ① CSD quantization decomposes coefficients into several SPT terms (sub-coefficients)
- ② the multiplication of a coefficient can be reformed through the combination of the non-zero SPT sub-coefficients

quantize the rotation angle θ

decompose the rotation angle θ
into several sub-angles θ_i 's

the rotational operation of each θ_i
should be easily realized

If each θ_i can be realized
using only shift-and-add operations

the rotation of θ can be performed
through successive applications of
sub-angle rotations
in a cost-effective way

Approximation target	Coefficient h_i	Rotation angle Θ
Basic Element	Non-zero digit 2^{-i}	Sub-angle $\alpha(i) = \tan^{-1}(2^{-i})$
Basic Operation	shift-and-add operation	2 shift-and-add operations
Approximation Equation	$h_i \approx \sum_{j=0}^{N_D-1} g_j \cdot 2^{-d_j}$	$\Theta \approx \sum_{j=0}^{N_A-1} \alpha(j) \cdot a(s(j))$
	$g_j \in \{-1, 0, +1\}$ $d_j \in \{0, 1, \dots, w-1\}$ $N_D =$ the number of non-zero digits	$N_A =$ the number of sub-angles

try to approach the target rotation angle θ
step by step

decisions are made in each step
by choosing the best combination of $\alpha(i)$ $a(s(i))$

So as to minimize $|\xi_m|$

$\alpha(i)$, $a(i)$ are determined such that
the error function is minimized

$$J(i) = |\theta(i) - \alpha(i)a(s(i))|$$

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

terminated if no further improvement can be found

$$J(i) \geq J(i-1)$$

or $\alpha(R_m-1)$ and $s(R_m-1)$
are determined at the end

$$\xi_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^M \mu(i) \alpha(i) \quad \mu(i) = \{-1, 0, +1\}$$

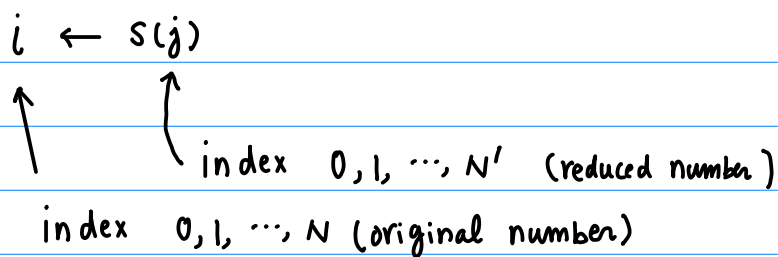
$$= \theta - \sum_{j=0}^{N'} \tilde{\theta}(j)$$

$$N' \equiv \sum_{i=0}^{N-1} |\mu(i)| \quad \{+1, 0, +1\}$$

the effective iteration number N'

$S(j)$ the rotational sequence

determines the micro-rotation angle in the j -th iteration



$$\begin{array}{ccc} \mu(S(j)) & \leftarrow & \alpha(j) \\ \downarrow & & \uparrow \\ & & \{-1, +1\} \end{array}$$

$$\mu(i) = \begin{cases} \mu(S(j)) & i = S(j) \\ 0 & i \neq S(j) \text{ --- reduced index} \end{cases}$$

er

$$\begin{aligned}
 i &= 0, \overset{\text{see}}{\boxed{1, 2}}, 3, \dots, N-1 \\
 s(j) &= 0, \boxed{1, 2}, 3, \dots, N-1 && \text{rotational sequence} \\
 \alpha(j) &= -1, \boxed{0, 0}, +1, \dots, -1 && \text{directional sequence} \\
 j &= 0, -, -, 1, \dots, N'-1 && \text{effective iteration number} \\
 N' &= N-2
 \end{aligned}$$

the j -th micro-rotation of $a(s(j))$

elementary angle

$$a(i) = \tan^{-1}(2^{-i})$$

$$a(s(j)) = \tan^{-1}(2^{-s(j)})$$

$$\alpha(j) a(s(j)) = \alpha(j) \tan^{-1}(2^{-s(j)}) \quad \alpha(j) \in \{-1, +1\}$$

$$\Leftrightarrow \mu(i) a(i) \quad \mu(i) \in \{-1, 0, +1\}$$

$$\sum_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^{N-1} \mu(i) a(i) \quad \mu(i) \in \{-1, 0, +1\}$$

$$= \theta - \left[\sum_{j=0}^{N'} \tilde{\theta}(j) \right]$$

$$= \theta - \left[\sum_{j=0}^{N'} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right] \quad \alpha(j) \in \{-1, +1\}$$

$$\tilde{\theta}(j) = \alpha(j) \tan^{-1}(2^{-s(j)})$$

$$= \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$

$$S_i = \{ \tan^{-1}(\alpha \cdot 2^{-s}) \mid \alpha \in \{-1, 0, +1\}, s \in \{0, 1, 2, \dots, N-1\} \}$$

② MVR (Modified Vector Rotational)

two modifications

① **repetition** of elementary angles

each micro-rotation of elementary angle
can be performed repeatedly

- more possible combinations
- smaller ξ_m

② **confinement** of total micro-rotation number

confine the iteration number
in the micro-rotation phase
to R_m ($R_m \ll W$)

The role of R_m is quite similar
to the **number of non-zero digit**
 N_D in CSD recoding scheme

the angle quantization error

$$\sum_{m, \text{MVR}} \triangleq \theta - \sum_{i=0}^{P_m-1} \alpha(i) a(s(i))$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\}$$

the micro-rotation angle

in the i -th iteration

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\}$$

the direction of the i -th

micro-rotation of $a(s(i))$

$$\alpha(i) a(s(i)) = \tilde{\theta}(j)$$

$$\xi_{m, MVR} \cong \theta - \sum_{j=0}^{R_m-1} \alpha(j) a(s(j))$$

the rotational sequence $s(j)$

$$j = 0, 1, 2, \dots, R_m-1$$



$$s(j) \in \{0, 1, \dots, W-1\}$$

determines the micro-rotation angle $a(s(j))$
in the j -th iteration

the directional sequence $\alpha(j)$

$$\alpha(j) \in \{-1, 0, +1\}$$

controls the direction of the j -th
micro-rotation of $a(s(j))$

$$\alpha(j) a(s(j)) = \tilde{\theta}(j)$$

$i = 0, 1, 2, 3, \dots, W-1$	
$s(j) = 0, 1, 2, 3, \dots, W-1$	rotational sequence
$\alpha(j) = -1, 0, 0, +1, \dots, -1$	directional sequence
$j = 0, \dots, R_m-1$	effective iteration number
$R_m \ll W$	

sub-angle $(\alpha(j) a(s(j))) \sim \tilde{\theta}(j)$

$$\begin{aligned}\xi_{m,AR} &= \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right] \\ &= \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j) \right], \quad \tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})\end{aligned}$$

$$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)| \quad \text{the effective iteration number}$$

EAS formed by MVR-CORDIC
is the same as AR
also performs AQ

The major difference

1) the total number of sub-angles N_A

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of R_m

$$N_A = R_m$$

2) the sub-angle θ_j corresponds to $\alpha(j) a(s(j))$

$$\theta_j = \alpha(j) a(s(j)) = \tilde{\theta}_j$$

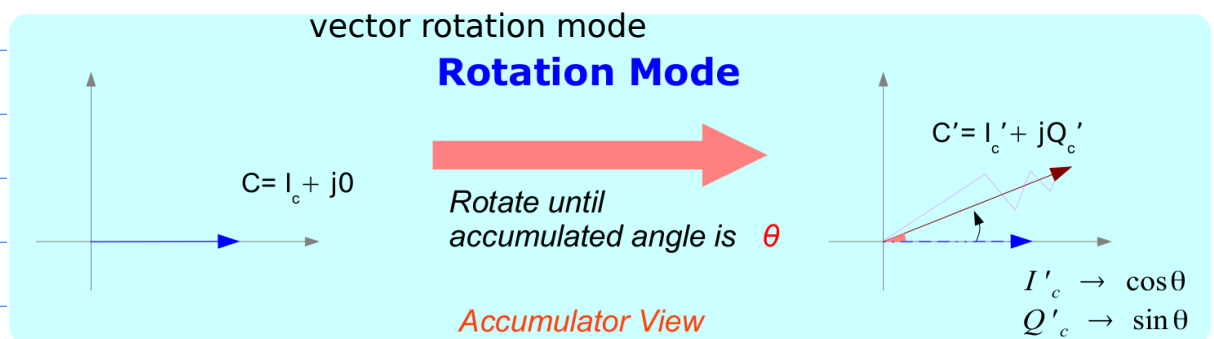
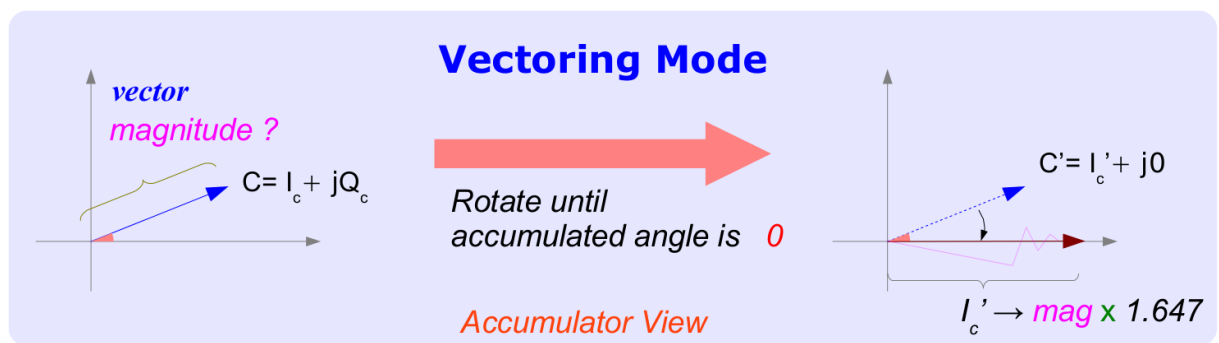
MVR (Modified Vector Rotation)

1) Repeat of Elementary Angles θ_i, θ_i

2) fixed total micro-rotation Number R_m

* Vector Rotation Mode

* and the rotation angles are known in advance



Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

Three Searching Algorithm

- ① the selective prerotation
- ② the selective scaling
- ③ iteration-tradeoff scheme

Optimization Problem

EAS point of view

Given θ , find the combination of R_m elementary angles from EAS S_i , such that the angle quantization error $|\xi_{m, \text{NR}}|$ is minimized.

Semi-greedy algorithm
trade offs between computational complexities
and performance

key issue in the MVR-CORDIC

is to find the best sequences of

$s(i)$ and $\alpha(i)$ to minimize $|\xi_m|$

subject to the constraint that

the total iteration number is confined to R_m

1) Greedy Algorithm

2) Exhaustive Algorithm

3) Semigreedy Algorithm

1) Greedy Algorithm

try to approach the target rotation angle, θ , step by step in each step, decisions are made on $\alpha(i)$ and $s(i)$ by choosing the best combination of $\alpha(i)$ and $s(i)$ so as to minimize $|\xi_m|$

$\alpha(i)$ and $s(i)$ are determined such that the error function $J(i) = |\theta(i) - \alpha(i) a(s(i))|$ is minimized

$\theta(i)$: the residue angle in the i -th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

the searching is terminated

if no further improvements can be found

$$J(i) \geq J(i-1)$$

$\alpha(R_m-1)$ and $s(R_m-1)$ are determined

at the end of the searching

the greedy algorithm terminates

Only when the residue angle error cannot be further reduced.

Initialization:

given θ , w , R_m

$$\theta^{(i)} = \theta - \sum_{m=0}^{i-1} \alpha^{(m)} a(s^{(m)})$$

Select $\alpha^{(i)} \in \{-1, 0, +1\}$
 $s^{(i)} \in \{0, 1, 2, \dots, w-1\}$
to minimize $J^{(i)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$

N
 $J^{(i)} < J^{(i-1)}$

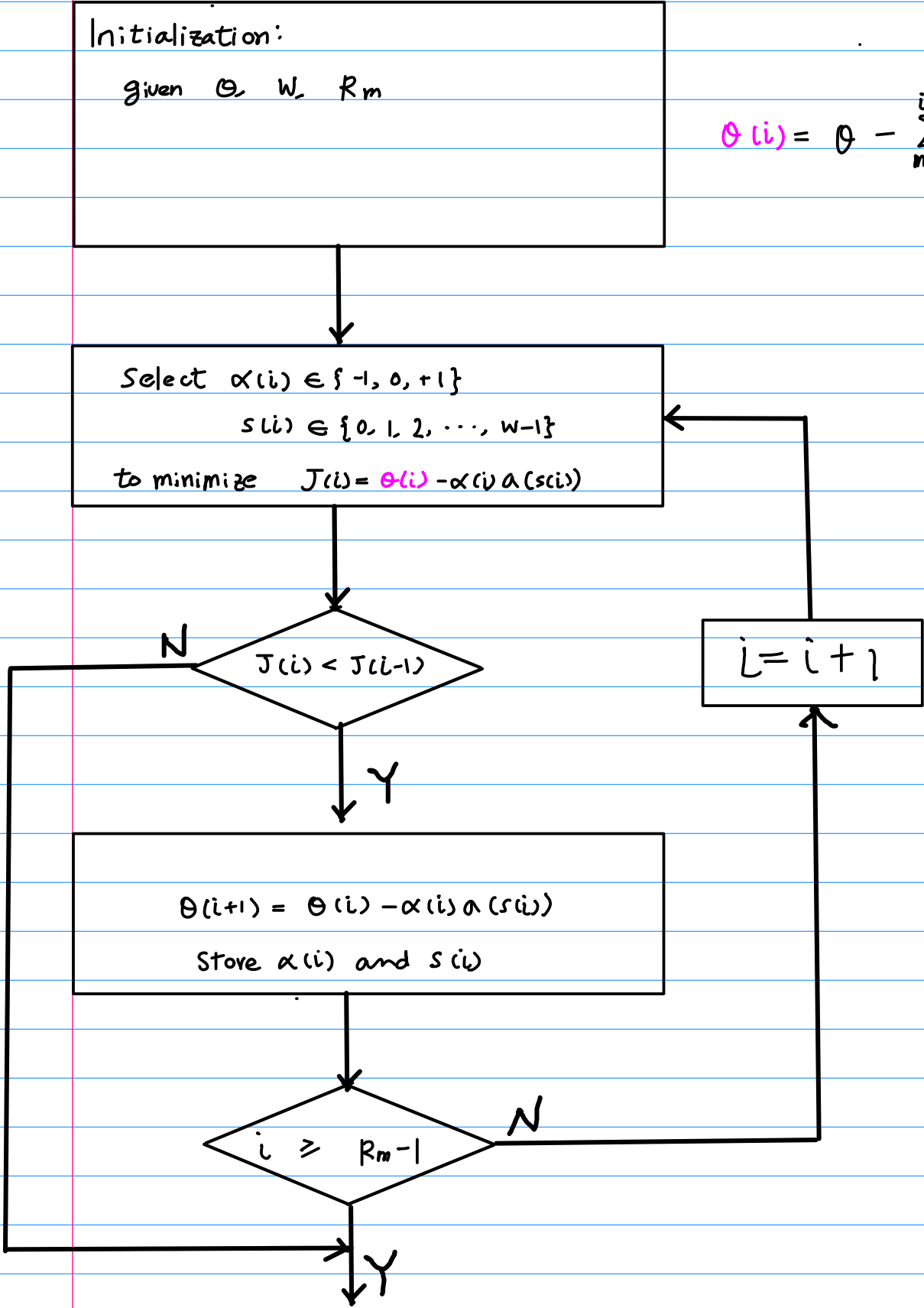
Y

$\theta^{(i+1)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$
Store $\alpha^{(i)}$ and $s^{(i)}$

$i \geq R_m - 1$
N

Y

$i = i + 1$



2) Exhaustive Algorithm

search for the entire solution space

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

in a single step

decisions for $\alpha(i)$ and $s(i)$, $0 \leq i \leq R_m-1$
by minimizing the error function

$$J = \left| 0 - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i)) \right|$$

global optimal solution

Initialization:

given Θ, W, R_m

let $\Theta(0) = \Theta,$

$i = 0$

$J(-1) = \infty$

Select $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \dots, W-1\}$

for $0 \leq i \leq R_m - 1$

to minimize $J(i) = \Theta - \sum_{l=0}^{R_m-1} \alpha(l) a(s(i))$

$$(3 \cdot W) \cdot (3 \cdot W) \dots (3 \cdot W) \\ = 3^{R_m} \cdot W^{R_m}$$

Store $\alpha(i)$ and $s(i)$

for $0 \leq i \leq R_m - 1$

in the i -th block

decision of $\alpha(k)$ and $s(k)$ for $iD \leq k \leq (i+1)D-1$

$$\text{minimizes } J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k)) \right|$$

$$\text{where } \theta(i) = \theta - \sum_{m=0}^{i-1} \left[\sum_{k=mD}^{(m+1)D-1} \alpha(k) a(s(k)) \right]$$

the residue angle in the i -th step

$$s = \left\lceil \frac{R_m}{D} \right\rceil$$

$$\theta(i) = \theta - \left[\sum_{k=0D}^{1D-1} \alpha(k) a(s(k)) + \sum_{k=1D}^{2D-1} \alpha(k) a(s(k)) + \dots + \sum_{k=(i-1)D}^{iD-1} \alpha(k) a(s(k)) \right]$$

Initialization:

given θ, W, R_m

let $\theta(0) = \theta,$

$i = 0$

$J(-1) = \infty$

Select $\alpha(k) \in \{-1, 0, +1\}$

$s(k) \in \{0, 1, 2, \dots, W-1\}$

for $iD \leq k \leq (i+1)D - 1$

to minimize $J(i) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

N
 $J(i) < J(i-1)$

$\theta(i+1) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

Store $\alpha(k), s(k)$

$i \geq \lceil \frac{R_m}{D} \rceil - 1$
N

Y

$i = i + 1$

③ Extended EAS-based CORDIC

$$\mathcal{S}_2 = \left\{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \right. \\ \left. \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\}, s_0^*, s_1^* \in \{0, 1, \dots, W-1\} \right\}$$

$$\theta_i = \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)})$$

the angle quantization error

$$|\xi_{m, \text{EAS}}| \triangleq \left| \theta - \sum_{j=0}^{R_m-1} \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}) \right|$$

Generalized EEAS Scheme

$$S_d = \left\{ \tan^{-1} \left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} + \dots + \alpha_{d-1}^* \cdot 2^{-s_{d-1}^*} \right) : \right. \\ \left. \begin{array}{l} \alpha_0^*, \alpha_1^*, \dots, \alpha_{d-1}^* \in \{-1, 0, +1\}, \\ s_0^*, s_1^*, \dots, s_{d-1}^* \in \{0, 1, \dots, W-1\} \end{array} \right\}$$

Extended EAS (EEAS) - Wu

more flexible way of decomposing the rotation angle

better

the number of iterations
the error performance

$$S_{EAS} = \{ (\sigma \cdot \tan^{-1}(2^{-r})) : \sigma \in \{+1, 0, -1\}, r \in \{1, 2, \dots, n-1\} \}$$

$$S_{EEAS} = \{ (\sigma_1 \cdot \tan^{-1}(2^{-r_1}) + \sigma_2 \cdot \tan^{-1}(2^{-r_2})) : \\ \sigma_1, \sigma_2 \in \{+1, 0, -1\}, r_1, r_2 \in \{1, 2, \dots, n-1\} \}$$

The pseudo-rotation
for i -th micro rotations

$$\begin{aligned}x_{i+1} &= x_i - [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] y_i \\y_{i+1} &= y_i + [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] x_i\end{aligned}$$

The pseudo-rotated vector $[x_{R_m}, y_{R_m}]$
after R_m (the required number of micro-rotations)

Needs to be scaled by a factor $K = \prod K_i$

$$K_i = \left[1 + \left(\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)} \right)^2 \right]^{-\frac{1}{2}}$$

$$\begin{aligned}\tilde{x}_{i+1} &= \tilde{x}_i - [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{y}_i \\ \tilde{y}_{i+1} &= \tilde{y}_i + [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{x}_i\end{aligned}$$

$$\tilde{x}_0 = x_{R_m}$$

$$\tilde{y}_0 = y_{R_m}$$

$$k_1, k_2 \in \{-1, 0, 1\}$$

$$s_1, s_2 \in \{1, 2, \dots, n-1\}$$

- [21] C.-S. Wu, A.-Y. Wu, and C.-H. Lin, "A high-performance/low-latency vector rotational CORDIC architecture based on extended elementary angle set and trellis-based searching schemes," *IEEE Trans. Circuits Syst. II: Anal. Digital Signal Process.*, vol. 50, no. 9, pp. 589–601, Sep. 2003.

A Unified View for Vector Rotational CORDIC Algorithms and Architectures Based on Angle Quantization Approach

An-Yeu Wu and Cheng-Shing Wu

AR : to approximate θ
with the combination
of selected angle elements
from a pre-defined EAS
(Elementary Angle Set)

EAS : all possible values of $\theta(j)$

$$\text{EAS } \hat{\theta}_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, \\ s^* \in \{0, 1, \dots, N-1\} \}$$

EAS $\hat{\theta}_1$ consists of $\tan^{-1}(\text{Single signed power of two})$
 $\tan^{-1}(\text{Single SPT})$
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

SPT-based digital filter design

to increase the coefficient resolution

→ employ more SPT terms to represent filter coefficients

[12] H. Samuelli, "An improved search algorithm for the design of multiplierless FIR filters with power-of-two coefficients," *IEEE Trans. Circuits Syst.*, vol. 36, pp. 1044–1047, July 1989.

[13] Y. C. Lim, R. Yang, D. Li, and J. Song, "Signed power-of-two term allocation scheme for the design of digital filters," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 577–584, May 1999.

EAS S_1 consists of $\tan^{-1}(\text{Single signed power of two})$
 $\tan^{-1}(\text{Single SPT})$
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

EAS S_2 consists of $\tan^{-1}(\text{two signed power of two})$
 $\tan^{-1}(\text{two SPT})$
 $\tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*})$

Two Signed - Power - of - Two terms

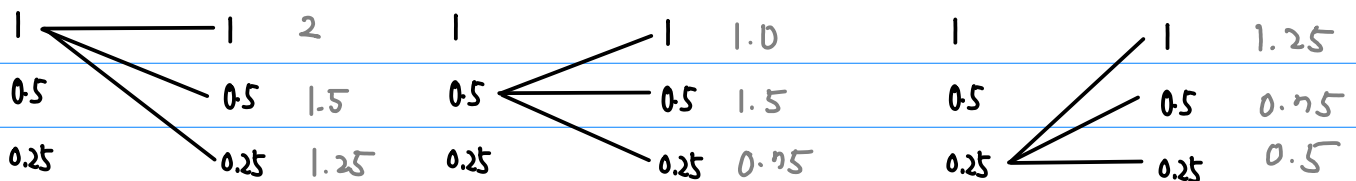
$$S_2 = \left\{ \tan^{-1} \left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} \right) : \right. \\ \left. \begin{array}{l} \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, w-1\} \end{array} \right\}$$

S_1

1	$1 = 2^{-0}$	$\tan^{-1}(2^{-0})$
0.5	$\frac{1}{2} = 2^{-1}$	$\tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\tan^{-1}(2^{-2})$

S_2

2	$1+1 = 2^{-0} + 2^{-0}$	$\pm \tan^{-1}(2^{-0} + 2^{-0})$
1.5	$1+\frac{1}{2} = 2^{-0} + 2^{-1}$	$\pm \tan^{-1}(2^{-0} + 2^{-1})$
1.25	$1+\frac{1}{4} = 2^{-0} + 2^{-2}$	$\pm \tan^{-1}(2^{-0} + 2^{-2})$
1.0	$1 = 2^{-0}$	$\pm \tan^{-1}(2^{-0})$
0.75	$\frac{1}{2}+\frac{1}{4} = 2^{-1} + 2^{-2}$	$\pm \tan^{-1}(2^{-1} + 2^{-2})$
0.5	$\frac{1}{2} = 2^{-1}$	$\pm \tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\pm \tan^{-1}(2^{-2})$



$$2^{-0}, 2^{-1}, 2^{-2}$$

$$\{0, 1, 2\} = \{0, 1, w-1\}$$

$w=3$

$$s_0^*, s_1^* \in \{0, 1, 2\}$$

$$2^{s_0^*}, 2^{s_1^*} \in \{2^{-0}, 2^{-1}, 2^{-2}\}$$

as the wordsize W increases,
the size of the set S_2 increases exponentially

$$\theta_i = \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)})$$

R_m : the number of the subangle N_A

$$S_2 = \{ \theta_i \mid i = 0, 1, \dots, R_m \}$$

$$S_2 = \{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \\ \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, W-1\} \}$$

the optimization problem of the EEAS-based CORDIC algorithm

given θ and R_m

find $\alpha_0(j)$, $\alpha_1(j)$, $s_0(j)$, and $s_1(j)$

the combination of elementary angles
from EEAS S_2

Minimize the angle quantization error

$$| \xi_m, EEAS | \triangleq \theta - \sum_{j=0}^{R_m-1} \tan^{-1} (\alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)})$$

given $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

$$\begin{bmatrix} x(j+1) \\ y(j+1) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} \\ \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} & 1 \end{bmatrix} \begin{bmatrix} x(j) \\ y(j) \end{bmatrix}$$

$$\begin{bmatrix} x_f \\ y_f \end{bmatrix} = P \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix} = \frac{1}{\prod_{j=0}^{R_m-1} \sqrt{1 + [\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}]^2}} \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix}$$

Micro rotation procedure
the scaling operation

↳ additions

increased hardware
reduced iteration steps

Rotation Angle $\theta = \frac{13\pi}{64}$

Conventional CORDIC

$$\bar{\mu} = [1 \ -1 \ 1 \ 1 \ -1 \ 1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1 \ 1 \ -1 \ 1 \ 1]$$

Angle Reordering - Greedy

$$\bar{\mu} = [1 \ 0 \ 0 \ -1 \ 0 \ 0 \ -1 \ -1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1]$$

MVR-CORDIC - Greedy

$$\bar{\alpha} = [1 \ -1 \ -1 \ -1]$$

$$\bar{s} = [0 \ 3 \ 6 \ 7]$$

MVR-CORDIC - Semi Greedy ($D=2$)

$$\bar{\alpha} = [1 \ -1 \ -1 \ 1]$$

$$\bar{s} = [0 \ 3 \ 5 \ 7]$$

MVR-CORDIC - TBS

$$\bar{\alpha} = [1 \ 1 \ -1 \ -1]$$

$$\bar{s} = [1 \ 2 \ 4 \ 7]$$

EEAS - Greedy

$$\bar{\alpha}_0 = [1 \ 1]$$

$$\bar{\alpha}_1 = [-1 \ -1]$$

$$\bar{s}_0 = [0 \ 2]$$

$$\bar{s}_1 = [8 \ 10]$$

EEAS - TBS

$R_m=2$

$$\bar{\alpha}_0 = [1 \ 1]$$

$$\bar{\alpha}_1 = [1 \ 1]$$

$$\bar{s}_0 = [0 \ 6]$$

$$\bar{s}_1 = [3 \ 5]$$

EEAS - TBS

$R_m=3$

$$\bar{\alpha}_0 = [1 \ -1 \ 1]$$

$$\bar{\alpha}_1 = [1 \ 1 \ -1]$$

$$\bar{s}_0 = [0 \ 3 \ 7]$$

$$\bar{s}_1 = [15 \ 6 \ 2]$$



